

Naturalness of the Coleman-Glashow mass relation in the $1/N_c$ expansion: An update

Elizabeth Jenkins*

Department of Physics, University of California at San Diego, La Jolla, California 92093

Richard F. Lebed†

Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606

(Received 5 May 2000; published 8 September 2000)

A new measurement of the Ξ^0 mass verifies the accuracy of the Coleman-Glashow relation at the level predicted by the $1/N_c$ expansion. Values for other baryon isospin mass splittings are updated, and continue to agree with the $1/N_c$ hierarchy.

PACS number(s): 11.15.Pg, 13.40.Dk, 14.20.-c

The recent measurement of the Ξ^0 mass, $1314.82 \pm 0.06 \pm 0.2$ MeV, by the NA48 Collaboration [1] represents a significant improvement over the 30-year-old value 1314.9 ± 0.6 MeV [2]. The Ξ^0 mass now is known to an uncertainty comparable to that of the other baryons of the lowest-lying spin-1/2 octet. This improvement makes it possible to test the precision of the famous Coleman-Glashow (CG) mass relation [3]

$$\Delta_{\text{CG}} = (p-n) - (\Sigma^+ - \Sigma^-) + (\Xi^0 - \Xi^-) = 0. \quad (1)$$

Using the old and new experimental values for Ξ^0 yields $\Delta_{\text{CG}} = 0.39 \pm 0.61$ and 0.29 ± 0.26 MeV, respectively: For the first time, Δ_{CG} has been measured to have a nonzero value, though only at the one-sigma level. It is of theoretical interest to understand the size of this breaking. In this Brief Report, we observe that the experimental value agrees with the theoretical accuracy of the CG relation as predicted in the $1/N_c$ expansion of QCD [4].

The mass spectrum of the baryon spin-1/2 octet and spin-3/2 decuplet was analyzed in Ref. [4] in a combined expansion in $1/N_c$ and flavor-symmetry breaking. It was found that all of the baryon mass splittings have a natural explanation in terms of powers of $1/N_c$, $SU(3)$ breaking ϵ , and isospin breaking ϵ' (from $m_d - m_u$) or ϵ'' (from electromagnetic effects). Our analysis differs from the standard flavor-symmetry breaking analysis in that it incorporates the enhanced symmetry of baryons present in the large- N_c limit. Large- N_c baryons respect an exact $SU(6)$ spin-flavor symmetry [5–7].¹ For arbitrary N_c , the ground state baryons fill the N_c -quark completely symmetric representation of the spin-flavor algebra, which for $N_c=3$ reduces to the usual **56**-plet of $SU(6)$. The spin-flavor symmetry is broken by corrections of subleading order in $1/N_c$, while flavor symmetry is broken in the usual manner. Our analysis in Ref. [4] showed that the CG mass combination is $O(\epsilon' \epsilon / N_c^2)$ relative to the average mass of the baryon **56** spin-flavor multiplet, which is of order $N_c \Lambda_{\text{QCD}}$. For $N_c=3$, this result implies

that the CG mass combination is predicted to be an order of magnitude smaller than expected from an $SU(3)$ flavor symmetry-breaking analysis alone.

In this work, we update the experimental values of mass combinations affected by the new mass measurement of the Ξ^0 . First, we briefly review the notation introduced in Ref. [4]: The isospin I combinations of baryon masses are denoted by a subscript I . Thus, the $I=0$ and $I=1$ mass combinations of the Ξ^0 and Ξ^- masses are denoted by

$$\begin{aligned} \Xi_0 &= \frac{1}{2}(\Xi^0 + \Xi^-), \\ \Xi_1 &= (\Xi^0 - \Xi^-), \end{aligned} \quad (2)$$

respectively. Using the new value of the Ξ^0 mass changes the experimental values of these mass combinations to

$$\begin{aligned} \Xi_0 &= 1318.07 \pm 0.12 \quad (\text{was } 1318.11 \pm 0.31) \text{ MeV}, \\ \Xi_1 &= -6.50 \pm 0.25 \quad (\text{was } -6.4 \pm 0.6) \text{ MeV}. \end{aligned} \quad (3)$$

The improvement in the experimental value of the $I=0$ mass combination Ξ_0 is small, and does not appreciably affect the numerical evaluation performed in Ref. [4] of $I=0$ mass combinations. Thus, we restrict our attention here to $I=1$ mass combinations. The remaining $I=1$ mass combinations are denoted by

$$\begin{aligned} N_1 &= (p-n), \quad \Sigma_1 = (\Sigma^+ - \Sigma^-), \\ \Delta_1 &= (3\Delta^{++} + \Delta^+ - \Delta^0 - 3\Delta^-), \\ \Sigma_1^* &= (\Sigma^{*+} - \Sigma^{*-}), \quad \Xi_1^* = (\Xi^{*0} - \Xi^{*-}), \end{aligned} \quad (4)$$

and the Λ - Σ^0 mixing parameter. In terms of these definitions, the CG mass combination is given by $\Delta_{\text{CG}} = N_1 - \Sigma_1 + \Xi_1$. There are large uncertainties in the Δ isospin mass splittings, so the $I=1$ mass combination Δ_1 does not figure in the mass combinations we consider.

As in Ref. [4], we define the *relative accuracy* R of a linear combination of masses written in the form $l-r$ (where the combinations l and r are uniquely defined to contain baryon masses with only positive coefficients) by

*Email address: ejenkins@ucsd.edu

†Email address: lebed@jlab.org

¹For a recent review, see Ref. [8] and references therein.

TABLE I. $I=1$ mass combinations before (old) and after (new) the recent measurement of the Ξ^0 mass. The relative accuracy R and its theoretical value R_T are defined in the text.

Mass combination	R (old)	R (new)	R_T	R_T (num)
$N_1 - \Sigma_1 + \Xi_1$	$(1.1 \pm 1.8) \times 10^{-4}$	$(8.4 \pm 7.5) \times 10^{-5}$	$\epsilon' \epsilon / N_c^2$	1.1×10^{-4}
$25(\Sigma_1 + \Xi_1) - 3(4\Sigma_1^* - 3\Xi_1^*)$	$(3.6 \pm 0.2) \times 10^{-3}$	$(3.7 \pm 0.1) \times 10^{-3}$	ϵ' / N_c	1.4×10^{-3}
$N_1 - \Xi_1$	$(2.3 \pm 0.3) \times 10^{-3}$	$(2.3 \pm 0.1) \times 10^{-3}$	ϵ' / N_c	1.4×10^{-3}
$5(2N_1 + \Sigma_1 - \Xi_1) - 3(4\Sigma_1^* - 3\Xi_1^*)$	$(0.5 \pm 1.8) \times 10^{-4}$	$(0.6 \pm 1.7) \times 10^{-4}$	ϵ' / N_c^3	1.5×10^{-4}

$R \equiv |l-r|/[(l+r)/2]$. The quantity R yields a scale-independent measure of the breaking of the relation compared to the average baryon mass. The theoretical expectation R_T for R of a particular mass combination is given by the combined flavor and $1/N_c$ suppressions of the mass combination, which are listed in Table II of Ref. [4], divided by N_c since the average baryon mass is $O(N_c^1)$. As an example, $R_T = \epsilon' \epsilon / N_c^2$ for the CG combination. If the $1/N_c$ expansion is natural, then R/R_T should be a number of order unity.

In Table I we present values of R and R_T for the four mass combinations depending upon Ξ_1 in Table II of Ref. [4]. We obtain numerical values for R_T by taking $\epsilon \approx 1/4$ and $\epsilon' \approx 1/3^5$ (and of course $N_c=3$); these are typical values one finds for flavor breaking in the meson mass spectrum, but one could also in principle fit to them using the observed baryon masses. One sees first that only the CG combination central value changes substantially from the improvement of the Ξ^0 mass measurement, although three of the four uncertainties drop significantly. Most importantly, one observes that the combined $1/N_c$ and flavor expansion continues to explain the size of the mass combinations in a natural way. It is also clear from Table I that the agreement of R and R_T would simply fail without the explicit $1/N_c$ factors.

The improvement in the measured value of Ξ_1 also permits a better estimate of the Λ - Σ^0 mixing parameter. Using Eq. (4.10) of Ref. [4], we find

$$\Lambda \Sigma^0 = \frac{1}{2\sqrt{3}}(\Xi_1 - N_1) = -1.50 \pm 0.07$$

(was -1.47 ± 0.17) MeV, (5)

up to a theoretical uncertainty of $O(\epsilon' \epsilon / N_c^2)$ times the average baryon mass, which yields a comparable theoretical error.

In summary, the new Ξ^0 mass measurement leads to a one-sigma determination of the magnitude of the Coleman-Glashow mass combination. The current experimental value of the CG mass combination is naturally explained in the $1/N_c$ expansion, which yields an additional suppression factor of $1/N_c^2$ beyond flavor symmetry-breaking factors; an $SU(3)$ flavor symmetry-breaking analysis alone fails to explain the observed accuracy of the CG mass combination. Further testing of the mass hierarchy predicted in the combined $1/N_c$ and flavor-symmetry breaking expansion is possible by improving the measurements of isospin mass splittings in the decuplet, particularly those of the Δ baryon.

E.J. was supported by the Department of Energy under Contract No. DOE-FG03-97ER40546 and by the National Young Investigator program through Grant No. PHY-9457911 from the National Science Foundation. R.F.L. was supported by the Department of Energy under Contract No. DE-AC05-84ER40150.

- [1] NA48 Collaboration, V. Fanti *et al.*, Eur. Phys. J. C **12**, 69 (2000).
 [2] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
 [3] S. Coleman and S.L. Glashow, Phys. Rev. Lett. **6**, 423 (1961).
 [4] E. Jenkins and R.F. Lebed, Phys. Rev. D **52**, 282 (1995).
 [5] R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993);

- 315**, 438 (1993).
 [6] E. Jenkins, Phys. Lett. B **315**, 431 (1993); **315**, 441 (1993); **315**, 447 (1993).
 [7] R. Dashen, E. Jenkins, and A.V. Manohar, Phys. Rev. D **49**, 4713 (1994); **51**, 3697 (1995).
 [8] E. Jenkins, Annu. Rev. Nucl. Part. Sci. **48**, 81 (1998).