Extending the frontiers: Reconciling accelerator and cosmic ray *p*-*p* cross sections

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We simultaneously fit a QCD-inspired parametrization of all accelerator data on forward proton-proton and antiproton-proton scattering amplitudes, *together* with cosmic ray data (using Glauber theory), to predict proton-air and proton-proton cross sections at energies near $\sqrt{s}\approx 30$ TeV. The *p*-air cosmic ray measurements provide a strong constraint on the inclusive particle production cross section, as well as greatly reducing the errors on the fit parameters—in turn, greatly reducing the errors in the high-energy proton-proton and proton-air cross section predictions.

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The energy range of cosmic ray experiments covers not only the energy of the CERN Large Hadron Collider (LHC), but extends beyond it. Cosmic ray experiments can measure the penetration in the atmosphere of these very high-energy protons—however, extracting proton-proton cross sections from cosmic ray observations is far from straightforward [1]. By a variety of experimental techniques, cosmic ray experiments map the atmospheric depth at which cosmic ray initiated showers develop. The measured quantity is the shower attenuation length (Λ_m), which is not only sensitive to the interaction length of the protons in the atmosphere ($\lambda_{p\text{-air}}$), with

$$\Lambda_m = k\lambda_{p\text{-air}} = k\frac{14.5 m_p}{\sigma_{p\text{-air}}^{\text{inel}}}, \tag{1}$$

but also depends critically on the proton inelasticity and the properties of the pion interactions, which determines the rate at which the energy of the primary proton is dissipated into electromagnetic shower energy observed in the experiment. The latter effect is taken into account in Eq. (1) by the parameter k; m_p is the proton mass and $\sigma_{p\text{-air}}^{\text{inel}}$ the inelastic proton-air cross section. The departure of k from unity depends on the inclusive particle production cross section in nucleon and meson interactions on the light nuclear target of the atmosphere and its energy dependence.

The extraction of the pp cross section from the cosmic ray data is a two stage process. First, one calculates the *p*-air total cross section from the inelastic cross section inferred in Eq. (1), where

$$\sigma_{p\text{-air}}^{\text{inel}} = \sigma_{p\text{-air}} - \sigma_{p\text{-air}}^{\text{el}} - \sigma_{p\text{-air}}^{q-\text{el}}. \tag{2}$$

Next, the Glauber method [2] transforms the value of $\sigma_{p\text{-air}}^{\text{inel}}$ into a proton-proton total cross section σ_{pp}; all the necessary steps are calculable in the theory, but depend sensitively on a knowledge of B, the slope of $d\sigma_{pp}^{\text{el}}/dt$, the pp differential elastic scattering cross section, where

$$B = \left[\frac{d}{dt}\left(\ln\frac{d\sigma_{pp}^{\text{el}}}{dt}\right)\right]_{t=0}. \tag{3}$$

In Eq. (2) the cross section for particle production is supplemented with $\sigma_{p\text{-air}}^{\text{el}}$ and $\sigma_{p\text{-air}}^{q-\text{el}}$, the elastic and quasielastic cross section, respectively, as calculated by the Glauber theory, to obtain the total cross section $\sigma_{p\text{-air}}$. We show in Fig. 1 plots of B as a function of σ_{pp}, for five curves of different values of $\sigma_{p\text{-air}}^{\text{inel}}$. This summarizes the reduction procedure from the measured quantity Λ_m [of Eq. (1)] to σ_{pp} [1]. Also plotted in Fig. 1 is a curve (dashed) of B vs σ_{pp} which will be discussed later. Two significant drawbacks of this extraction method are that one needs the following.

(1) A model of proton-air interactions to complete the loop between the measured attenuation length Λ_m and the cross section $\sigma_{p\text{-air}}^{\text{inel}}$, i.e., the value of k in Eq. (1).

(2) A simultaneous relation between B and σ_{pp} at very high energies—well above the region currently accessed by accelerators.

A proposal to minimize the impact of theory on these needs is the topic of this paper.

We have constructed a QCD-inspired parametrization of the forward proton-proton and proton-antiproton scattering amplitudes [3] which is analytic, unitary and fits all accelerator data [4] of σ_{tot}, B, and ρ, the ratio of the real-to-imaginary part of the forward scattering amplitude; see Fig. 2. In addition, the high energy cosmic ray data of Fly's Eye [5] and Akeno Giant Air Shower Array (AGASA) [6] experiments are also simultaneously used, i.e., k from Eq. (1) is also a fitted quantity—we refer to this fit as a global fit [7]. We emphasize that in the global fit, all four quantities σ_{tot}, B, ρ, and k, are *simultaneously* fitted. Because our parametrization is both unitary and analytic, its high-energy predictions are effectively model independent, if you require that the proton is asymptotically a black disk. Using vector meson dominance and the additive quark models, we find fur-

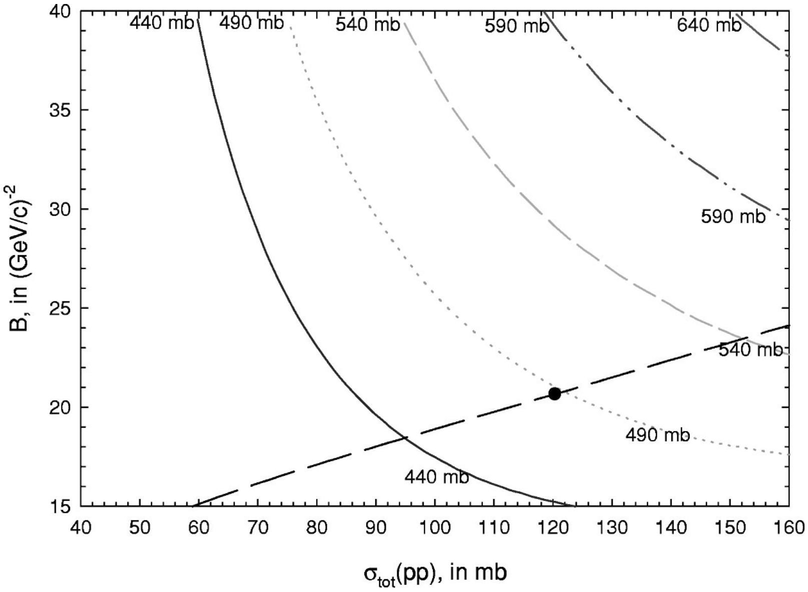

FIG. 1. The B dependence of the pp total cross section σ_{pp}. The five curves are lines of constant $\sigma_{p\text{-air}}^{\text{inel}}$, of 440, 490, 540, 590, and 640 mb—the central value is the published Fly's Eye value, and the others are $\pm 1\sigma$ and $\pm 2\sigma$. The dashed curve is a plot of our QCD-inspired fit of B against σ_{pp}. The dot is our fitted value for $\sqrt{s}=30$ TeV, the Fly's Eye energy.

ther support for our QCD fit—it accommodates a wealth of data on photon-proton and photon-photon interactions without the introduction of new parameters [8]. In particular, it also *simultaneously* fits σ_{pp} and B, forcing a relationship between the two. Specifically, the B vs σ_{pp} prediction of our fit completes the relation needed (using the Glauber model) between σ_{pp} and $\sigma_{p\text{-air}}^{\text{inel}}$. The percentage error in the prediction of σ_{pp} at $\sqrt{s}=30$ TeV is $\approx 1.2\%$, due to the statistical error in the fitting parameters (see Refs. [3,8]). A *major* difference between the present result, in which we simultaneously fit the cosmic ray and accelerator data, and our earlier result [7], in which only accelerator data are used, is a *significant* reduction (about a factor of 2.5) in the errors of σ_{pp} at $\sqrt{s}=30$ TeV.

In Fig. 3, we have plotted the values of σ_{pp} vs $\sigma_{p\text{-air}}^{\text{inel}}$ that are deduced from the intersections of our B-σ_{pp} curve with the $\sigma_{p\text{-air}}^{\text{inel}}$ curves of Fig. 1. Figure 3 allows the conversion of measured $\sigma_{p\text{-air}}^{\text{inel}}$ cross sections to σ_{pp} total cross sections. The percentage error in $\sigma_{p\text{-air}}^{\text{inel}}$ is $\approx 0.8\%$ near $\sigma_{p\text{-air}}^{\text{inel}}=450$ mb, due to the errors in σ_{pp} and B resulting from the errors in the fitting parameters. Again, the global fit gives an error of a factor of about 2.5 smaller than our earlier result [7], a *distinct* improvement.

When we confront our predictions of the p-air cross sections ($\sigma_{p\text{-air}}^{\text{inel}}$) as a function of energy with published cross section measurements of the Fly's Eye [5] (see Fig. 1) and AGASA [6] groups, we find that the predictions systematically are about one standard deviation below the *published* cosmic ray values. It is at this point important to recall Eq. (1) and remind ourselves that the measured experimental quantity is Λ_m and *not* $\sigma_{p\text{-air}}^{\text{inel}}$. We emphasize that the extraction of $\sigma_{p\text{-air}}^{\text{inel}}$ from the measurement of Λ_m requires *knowledge* of the parameter k. The measured depth $X_{\max}$ at which a shower reaches maximum development in the atmosphere, which is the basis of the cross section measurement in Ref.

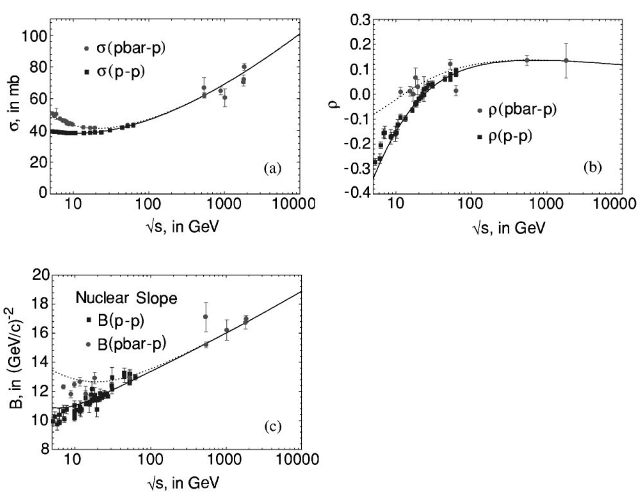

FIG. 2. The simultaneous QCD-inspired fit of total cross section σ_{pp}, ρ, and B vs $\sqrt{s}$, in GeV, for pp (squares) and $\overline{p}p$ (circles) accelerator data: (a) σ_{pp}, in mb, (b) ρ, (c) Nuclear slope B, in GeV^{-2}.

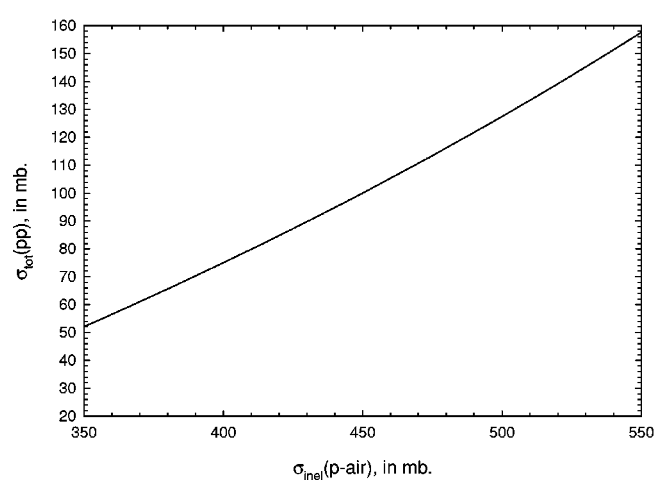

FIG. 3. A plot of the predicted total pp cross section σ_{pp}, in mb vs the measured p-air cross section $\sigma_{p\text{-air}}^{\text{inel}}$, in mb.

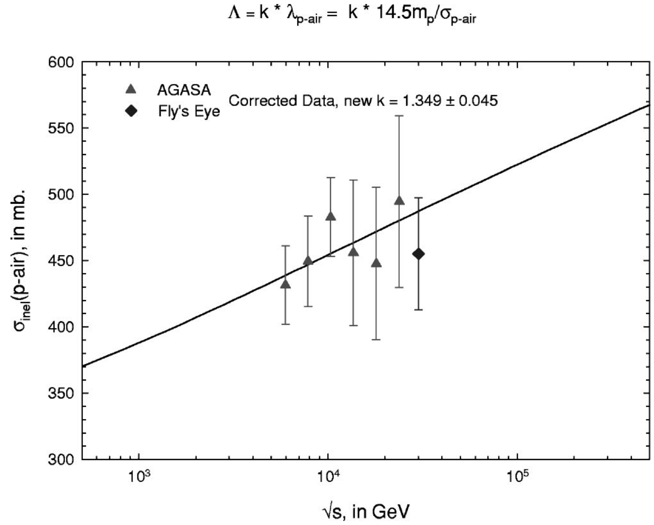

FIG. 4. The AGASA and Fly's Eye data for $\sigma_{p\text{-air}}^{\text{inel}}$, in mb, as a function of the energy, $\sqrt{s}$, in GeV, as found in our global fit, using the common value of $k=1.349$.

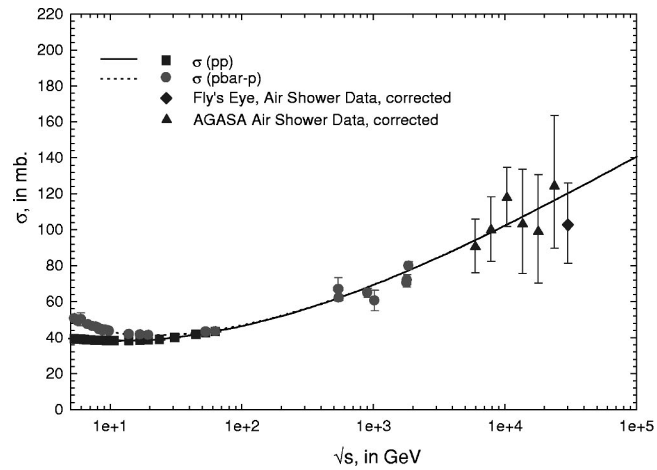

FIG. 5. A plot of the QCD-inspired fit of the total nucleon-nucleon cross section σ_{pp}, in mb vs $\sqrt{s}$, in Gev. The cosmic ray data that are shown have been converted from $\sigma_{p\text{-air}}^{\text{inel}}$ to σ_{pp} using the results of Fig. 3 and the common value of $k=1.349$, found from our global fit.

[5], is a combined measure of the depth of the first interaction, which is determined by the inelastic cross section, and of the subsequent shower development, which has to be corrected for. $X_{\max}$ increases logarithmically with energy with elongation rate ($\Delta X_{\max}$ per decade of lab energy) of 50–60 g/cm^2 in calculations with QCD-inspired hadronic interaction models. The position of $X_{\max}$ directly affects the rate of shower attenuation with atmospheric depth, which is the alternative procedure for extracting $\sigma_{p\text{-air}}^{\text{inel}}$. The rate of shower development and its fluctuations are the origin of the deviation of k from unity in Eq. (1). Its predicted values range from 1.5 for a model where the inclusive cross section exhibits Feynman scaling, to 1.1 for models with large scaling violations [1]. The comparison between prediction and experiment is further confused by the fact that the AGASA [6] and Fly's Eye [5] experiments used different values of k in the analysis of their data, i.e., AGASA used $k=1.5$ and Fly's Eye used $k=1.6$.

We therefore decided to let k be a free parameter and to make a global fit to the accelerator and cosmic ray data, as emphasized earlier. This neglects the possibility that k may show a weak energy dependence over the range measured. Recently, Pryke [9] has made Monte Carlo model simulations that indicate that k is compatible with being energy independent. Using an energy-independent k, we find that $k=1.349\pm0.045$, where the error in k is the statistical error of the global fit. By combining the results of Figs. 2(a) and 3, we can predict the variation of $\sigma_{p\text{-air}}^{\text{inel}}$ with energy, $\sqrt{s}$. In Fig. 4 we have *rescaled* the published high-energy data for $\sigma_{p\text{-air}}^{\text{inel}}$ (using the common value of $k=1.349$), and plotted the revised data against our prediction of $\sigma_{p\text{-air}}^{\text{inel}}$ vs $\sqrt{s}$.

The plot of σ_{pp} vs $\sqrt{s}$, including the rescaled cosmic ray data is shown in Fig. 5. Clearly, we have an excellent fit, with good agreement between AGASA and Fly's Eye. In order to extract the cross sections' energy dependence from the cosmic ray data, the experimenters of course assigned energy values to their cross sections. Since the cosmic ray spectra vary so rapidly with energy, we must allow for systematic errors in k due to possible energy misassignments. At the quoted experimental energy resolutions, $\Delta\log_{10}[E_{\text{lab}}(\text{ev})]=0.12$ for AGASA [6] and $\Delta\log_{10}[E_{\text{lab}}(\text{ev})]=0.4$ for Fly's Eye [5], where E_{lab} is the proton laboratory energy, we find from the curve in Fig. 4 that $\Delta k/k=0.0084$ for AGASA [6] and $\Delta k/k=0.0279$ for Fly's Eye [5]. We estimate conservatively that experimental energy resolution introduces a systematic error in k such that $\Delta k_{\text{systematic}}=\sqrt{(\Delta k_{\text{AGASA}}^2+\Delta k_{\text{flys eye}}^2)/2}=0.028$. Thus, we write our final result as $k=1.349\pm0.045\pm0.028$, where the first error is statistical and the last error is systematic.

Recently, Pryke [9] has published a comparative study of high statistics simulated air showers for proton primaries, using four combinations of the MOCCA [10] and CORSIKA [11] program frameworks, and SIBYLL [12] and QGSJET [13] high energy hadronic interaction models. He finds $k=1.30\pm0.04$ and $k=1.32\pm0.03$ for the CORSIKA-QGSJET and MOCCA INTERNAL models, respectively, which are in excellent agreement with our measured result, $k=1.349\pm0.045\pm0.028$.

Further, Pryke [9] obtains $k=1.15\pm0.03$ and $k=1.16\pm0.03$ for the CORSIKA-SIBYLL and MOCCA-SIBYLL models, respectively, whereas the SYBILL [1] group finds $k=1.2$, which is not very different from the Pryke value. However, the SYBILL-based models, with $k=1.15$–1.20, are significantly different from our measurement of $k=1.349\pm0.045\pm0.028$. At first glance, this appears somewhat strange, since our model for forward scattering amplitudes and SIBYLL share the same underlying physics. The increase of the total cross section with energy to a black disk of soft partons is the shadow of increased particle production which is modeled in SYBILL by the production of (mini)-jets in QCD. The difference between the k values of 1.15–1.20 and 1.349 results from the very rapid rise of the pp cross section in SIBYLL at the highest energies. This is not a natural consequence of the physics in the model—it is an artifact of a fixed transverse momentum cutoff. In most other codes (QG-SJET [13]), it is remedied by the use of an energy-dependent transverse momentum cutoff in the computation of the minijet production cross section.

In conclusion, the overall agreement between the accelerator and the cosmic ray pp cross sections with our QCD-inspired fit, as shown in Fig. 5, is striking. We find that the accelerator and cosmic ray pp cross sections are readily reconcilable using a value of $k=1.349\pm0.045\pm0.028$, which is both model independent and energy independent—this determination of k severely constrains any model of high-energy hadronic interactions. We predict high-energy σ_{pp} and $\sigma_{p\text{-air}}^{\text{inel}}$ cross sections that are accurate to ≈ 1.2 and 0.8 %, respectively, at $\sqrt{s}=30$ TeV.

At the LHC ($\sqrt{s}=14$ TeV), we predict $\sigma_{\text{tot}}=107.9\pm1.2$ mb for the total cross section, $B=19.59\pm0.11$ $(\text{GeV}/c)^{-2}$ for the nuclear slope and $\rho=0.117\pm0.001$, where the quoted errors are due to the statistical errors of the fitting parameters. In the near future, we look forward to the possibility of repeating this analysis with the higher statistics of the HIRES [14] cosmic ray experiment that is currently in progress and the Auger [15] Observatory.

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