Extrapolating SU(3) breaking from D to B decays

Michael Gronau
Physics Department, Technion–Israel Institute of Technology, 32000 Haifa, Israel

Dan Pirjol
Department of Physics, University of California at San Diego, La Jolla, California 92093

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We consider two SU(3) breaking parameters $R_1(m_B)$ and $R_2(m_B)$ appearing in a relation between $B^+\to K\pi$ and $B^+\to\pi\pi$ amplitudes, which plays an important role in determining the weak phase γ. In the heavy quark limit, we identify an isospin-related quantity $R_2(m_D)$ measured in D decays, exhibiting large SU(3) breaking which is likely due to nonfactorizable effects. Applying heavy quark symmetry to semileptonic D and B decay form factors, we find that factorizable SU(3) breaking in $R_2(m_B)/R_1(m_B)$ may be significantly larger than estimated from certain model calculations of form factors.

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Flavor SU(3) symmetry of strong interactions plays an essential role in some of the methods proposed to determine Cabibbo-Kobayashi-Maskawa (CKM) weak phases from B meson hadronic decays [1]. First order SU(3) breaking may be parametrized in a completely general way in terms of several unknown parameters [2], some of which can be determined from experiments. In $b\to c$ decays, such as in $B\to\bar{D}\pi$, experimental evidence exists for factorization [3], and SU(3) breaking parameters are given by ratios of K and π decay constants and ratios of B/B_s to D/D_s form factors. In charmless decays, which are useful for weak phase determinations [4], experimental evidence for factorization of hadronic matrix elements is still lacking. It was argued recently [5] that nonfactorizable corrections due to hard gluon exchange are calculable and those which are due to soft exchanges are suppressed by $\Lambda_{\rm QCD}/m_b$. Actual calculations of these corrections, controlling the former in a model-independent manner and showing that the latter are indeed small, are both desirable and challenging. Furthermore, in order to treat SU(3) breaking within the factorization approximation, one still needs the values of certain ratios of unmeasured form factors, for which one oftens relies on theoretical models.

The purpose of this Brief Report is to learn about SU(3) breaking in B decays from the corresponding measured effects in D decays. SU(3) breaking does not necessarily decrease monotonically with the decaying heavy quark mass. We will address the two relevant questions, of factorizable and nonfactorizable SU(3) violating corrections to hadronic decays, and of SU(3) breaking in semileptonic form factors which are used in the factorization approximation.

Soft final state interactions which spoil factorization are expected to affect D and B decays differently. It was often argued [6], and it has recently been shown by an actual calculation [7], that D decay amplitudes involve large contributions from nearby light $q\bar{q}$ resonances which induce large SU(3) breaking effects. Such effects are not expected in B decays. To avoid resonance effects, and thus study D and B decays on common grounds, we will consider only decays to ‘‘exotic’’ final states involving $\pi\pi$ in $I=2$ and $K\pi$ in $I=3/2$.

We consider an SU(3) relation between the isospin $I=3/2$ amplitude in $B\to K\pi$ and the $I=2$ amplitude in $B\to\pi\pi$ [8]:

$$
\begin{aligned}
&A(B^+\to K^0\pi^+)+\sqrt{2}A(B^+\to K^+\pi^0)\\
&\qquad=\sqrt{2}\tan\theta_c(R_1-\delta_+e^{-i\gamma}R_2)A(B^+\to\pi^0\pi^+),\\
&\delta_+\equiv-[3/(2\lambda|V_{ub}/V_{cb}|)][(c_9+c_{10})/(c_1+c_2)]\\
&\qquad=0.66\pm0.15. \qquad (1)
\end{aligned}
$$

This generalizes a triangle relation proposed in [9] by including, in addition to the current-current (‘‘tree’’) contributions, also the effects of dominant electroweak penguin (EWP) amplitudes given by the second term on the right-hand side (RHS). Equation (1) and its charge conjugate were proposed as a way for determining the weak phase $\gamma\equiv{\rm Arg}V_{ub}^*$.

The complex coefficients $R_{1,2}$ in Eq. (1) parametrize SU(3) breaking effects. Knowledge of the precise values of R_1 and R_2/R_1, in the presence of SU(3) breaking, is crucial for an accurate determination of γ [8,10,11]. Using the factorization approximation, it is customary to apply the value $R_1\simeq f_K/f_\pi=1.22$ to the tree part. SU(3) breaking corrections to the EWP-to-tree ratio R_2/R_1 were estimated in the generalized factorization approximation, assuming a certain model-dependent value for the ratio of B to K and B to π form factors, and were found to amount to a few percent [8]. Our main concern will be the SU(3) breaking parameter R_2.

For completeness, and in order to define R_1 and R_2 in broken SU(3) and to prove Eq. (1), we start by quickly reviewing the SU(3) structure of the amplitudes entering Eq. (1). The tree and electroweak penguin four-quark operators describing charmless decays transform under flavor SU(3) as a sum of $\bar{\mathbf{3}}$, $\mathbf{6}$, and $\overline{\mathbf{15}}$ [12]:

$$
\mathcal{H}_T^{\Delta S=1}+\mathcal{H}_T^{\Delta S=0}+\mathcal{H}_{EWP}^{\Delta S=1}=\frac{G_F}{\sqrt{2}}\lambda_u^{(s)}\left[\frac{1}{2}(c_1-c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0}-\mathbf{6}_{I=1})+\frac{1}{2}(c_1+c_2)\left(-\overline{\mathbf{15}}_{I=1}-\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0}\right)\right]
$$

$$+\frac{G_F}{\sqrt{2}}\lambda_u^{(d)}\left[\frac{1}{2}(c_1-c_2)(\mathbf{6}_{I=1/2}-\overline{\mathbf{3}}^{(a)}_{I=1/2})+\frac{1}{2}(c_1+c_2)\left(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=3/2}-\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=1/2}+\frac{1}{\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=1/2}\right)\right]$$

$$-\frac{G_F}{\sqrt{2}}\frac{\lambda_t^{(s)}}{2}\left[\frac{c_9-c_{10}}{2}(3\cdot\mathbf{6}_{I=1}+\overline{\mathbf{3}}^{(a)}_{I=0})+\frac{c_9+c_{10}}{2}\left(-3\cdot\overline{\mathbf{15}}_{I=1}-\frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=0}\right)\right], \quad (2)$$

where $\lambda_q^{(q')}=V^*_{qb}V_{qq'}$.

The left side of Eq. (1) receives only contributions from the $I=1$ terms which transform as **6** and $\overline{\mathbf{15}}$, while the $I=3/2$ amplitude on the RHS transforms as pure $\overline{\mathbf{15}}$:

$$A(B^+\to K^0\pi^+)+\sqrt{2}A(B^+\to K^+\pi^0)$$
$$=\lambda_u^{(s)}[(C_{\overline{15}_{I=1}}+C_{6_{I=1}})-\delta_+e^{-i\gamma}(C_{\overline{15}_{I=1}}-C_{6_{I=1}})], \quad (3)$$

$$\sqrt{2}A(B^+\to\pi^+\pi^0)=\lambda_u^{(d)}C_{\overline{15}_{I=3/2}}, \quad (4)$$

where $C_{\overline{15}_{I=1}}(m_B)=\frac{G_F}{\sqrt{2}}\frac{1}{2}(c_1+c_2)(\langle K^0\pi^+|-\overline{\mathbf{15}}_{I=1}|B^+\rangle + \sqrt{2}\langle K^+\pi^0|-\overline{\mathbf{15}}_{I=1}|B^+\rangle),$

$$C_{6_{I=1}}(m_B)=\frac{G_F}{\sqrt{2}}\frac{1}{2}(c_1-c_2)(\langle K^0\pi^+|-\mathbf{6}_{I=1}|B^+\rangle + \sqrt{2}\langle K^+\pi^0|-\mathbf{6}_{I=1}|B^+\rangle),$$

$$C_{\overline{15}_{I=3/2}}(m_B)=\frac{G_F}{\sqrt{2}}(c_1+c_2)\sqrt{\frac{2}{3}}\langle\pi^+\pi^0|-\overline{\mathbf{15}}_{I=3/2}|B^+\rangle.$$

In $B\to K\pi$ we used $(c_9+c_{10})/(c_1+c_2)\approx(c_9-c_{10})/(c_1-c_2)$, which holds to better than 3% [13], and in $B\to\pi\pi$ we neglected very small EWP contributions [12].

Taking the ratio of Eqs. (3) and (4) reproduces the factor on the right-hand side of Eq. (1) with $R_1(m_B)=(C_{\overline{15}_{I=1}}+C_{6_{I=1}})/(C_{\overline{15}_{I=3/2}})$, $R_2(m_B)=(C_{\overline{15}_{I=1}}-C_{6_{I=1}})/(C_{\overline{15}_{I=3/2}})$. Both final states on the left side of Eqs. (3) and (4) belong to a **27** multiplet of SU(3), such that the matrix elements of $\overline{\mathbf{15}}_{I=1}$ and $\overline{\mathbf{15}}_{I=3/2}$ are related in the SU(3) limit, $C_{\overline{15}_{I=1}}=C_{\overline{15}_{I=3/2}}$. The matrix element of **6** in Eq. (3) vanishes in the same limit, such that $R_1=R_2=1$. In broken SU(3) $C_{\overline{15}_{I=1}}\neq C_{\overline{15}_{I=3/2}}$, $C_{6_{I=1}}\neq 0$, and hence $R_1\neq 1$, $R_2\neq 1$.

Whereas $R_1(m_B)$ and $R_2(m_B)$ are purely theoretical quantities, we prove now that another SU(3) breaking parameter,

$$R_2(m_D)=-\frac{V_{us}}{V_{ud}}\frac{A(D^-\to K^0\pi^-)}{\sqrt{2}A(D^-\to\pi^-\pi^0)}, \quad (5)$$

measured in D decays, is related to $R_2(m_B)$ by isospin in a fictitious heavy quark limit $m_c=m_b$.

The final states in the numerator and denominator of $R_2(m_D)$ have quantum numbers $|I=\frac{3}{2},I_3=-\frac{3}{2}\rangle$ and $|I=2,I_3=-1\rangle$, respectively, and belong to the same isospin multiplets as the states $|K^0\pi^+\rangle+\sqrt{2}|K^+\pi^0\rangle$ and $|\pi^+\pi^0\rangle$ in Eq. (1). The initial states D^- and B^+ are related to each other by isospin in the limit of identical heavy quarks. The weak Hamiltonian responsible for the relevant $\overline{D}$ decays is

$$\mathcal{H}_W=\frac{G_F}{\sqrt{2}}V^*_{ud}V_{cs}\left[\frac{1}{2}(c_1-c_2)\sqrt{2}\mathbf{6}_{I=1}-\frac{1}{2}(c_1+c_2)\sqrt{2}\;\overline{\mathbf{15}}_{I=1}\right]$$
$$+\frac{G_F}{\sqrt{2}}V^*_{us}V_{cs}\left[(c_1+c_2)\left(\frac{1}{\sqrt{3}}\overline{\mathbf{15}}_{I=3/2}-\sqrt{\frac{2}{3}}\overline{\mathbf{15}}_{I=1/2}\right)+(c_1-c_2)\mathbf{6}_{I=1/2}\right], \quad (6)$$

where we neglect a small CP-violating contribution proportional to $\frac{1}{2}(V^*_{us}V_{cs}+V^*_{ud}V_{cd})=\mathcal{O}(\lambda^5)$ in the Cabibbo-suppressed part and very small contributions of penguin operators [14].

The $\Delta S=1$ $(\Delta S=0)$ $I=1$, $I_3=-1$ $(I=\frac{3}{2},I_3=-\frac{1}{2})$ operators in (6) are the isospin partners of the $I=1,I_3=0$ $(I=\frac{3}{2},I_3=\frac{1}{2})$ operators in the B decay Hamiltonian (2). Therefore, in the limit of identical heavy quarks, isospin symmetry of strong interactions implies $A(D^-\to K^0\pi^-)=V^*_{ud}V_{cs}[C_{\overline{15}_{I=1}}(m_D)-C_{6_{I=1}}(m_D)]$, $\sqrt{2}A(D^-\to\pi^-\pi^0)=-V^*_{us}V_{cs}C_{\overline{15}_{I=3/2}}(m_D)$. The ratio of these amplitudes yields $R_2(m_D)$ as defined in Eq. (5).

The experimental value of $R_2(m_D)$ is [15] $|R_2(m_D)|=0.56\pm0.08$. This large SU(3) breaking is somewhat surprising since the relevant final states are exotic, $I=\frac{3}{2}$ and 2, and receive no resonant contributions [7]. The large deviation of the ratio $|R_2(m_D)|$ from 1 raises the concern of a similar large SU(3) breaking effect in the B case. In view of this possibility, let us review previous attempts and difficulties in explaining the numerical value of $R_2(m_D)$.

In the generalized factorization approach [16] one finds

$$R_2(m_D)=\frac{a_2^{(DK\pi)}}{a_1^{(D\pi\pi)}+a_2^{(D\pi\pi)}}\frac{f_K}{f_\pi}\frac{F_0^{D\pi}(m_K^2)}{F_0^{D\pi}(m_\pi^2)}+\frac{a_1^{(DK\pi)}}{a_1^{(D\pi\pi)}+a_2^{(D\pi\pi)}}\frac{m_D^2-m_K^2}{m_D^2-m_\pi^2}\frac{F_0^{DK}(m_\pi^2)}{F_0^{D\pi}(m_\pi^2)}. \quad (7)$$

The phenomenological parameters $a_{1,2}$, describing the external and internal W-emission amplitudes, respectively, are related to corresponding Wilson coefficients through $a_{1,2}$

$=c_{1,2}+\zeta c_{2,1}$. The parameter ζ is process and scale dependent and is determined from experiments. When fitting nonleptonic two-body $D\to K\pi$ decays, using $F_0^{DK}(m_\pi^2)=0.77$ [17] and $F_0^{D\pi}(m_\pi^2)=0.7$ [18], one obtains [16] $a_1^{(DK\pi)}=1.26$ and $a_2^{(DK\pi)}=-0.51$, corresponding to $\zeta(m_c)=0$. This fit neglects, however, resonance contributions in nonexotic channels which, when included, modify the extracted values of $a_{1,2}$ to become $a_1^{(DK\pi)}=1.06$, $a_2^{(DK\pi)}=-0.64$ [7].

An attempt was made [19] to explain the large SU(3) breaking in $R_2(m_D)$ by using Eq. (7). This attempt faced three kinds of problems. First, there is an uncertainty in the values of $a_i^{(DK\pi)}$ due to resonance contributions in fitted nonexotic D decays. Second, the values of $a_i^{(D\pi\pi)}$ may differ from those of $a_i^{(DK\pi)}$. A determination of $a_i^{(D\pi\pi)}$ from the corresponding Cabibbo suppressed decays (neglecting resonance contributions) gives very different results [20] for a_2 compared with the $D\to K\pi$ case, $a_1^{(D\pi\pi)}=1.05$, $a_2^{(D\pi\pi)}=-0.07$, when $F_0^{D\pi}(m_\pi^2)=0.7$ [18] is used. Finally, there is an uncertainty due to the present experimental error in the ratio of form factors $F_0^{DK}(0)/F_0^{D\pi}(0)$. The average value obtained from four experiments [21] is $F_0^{DK}(0)/F_0^{D\pi}(0)=1.00\pm0.08$.

We conclude that it is difficult to evaluate $R_2(m_D)$ and to explain its experimental value in a reliable manner within the generalized factorization approach. It is not entirely impossible that the failure to account for this large SU(3) breaking is due to resonant contributions in other D decay processes which modify the extracted values of a_i. Assuming, for instance, $a_2^{(DK\pi)}/a_1^{(DK\pi)}=-0.6$ [7], $a_i^{(D\pi\pi)}=a_i^{(DK\pi)}$, $F_0^{DK}(0)/F_0^{D\pi}(0)=1.1$, one finds using Eq. (7) the value $R_2(m_D)=0.64$, consistent with experiment. Still, a probable explanation for this failure is the presence of significant nonfactorizable nonresonant contributions.

In view of the situation of $R_2(m_D)$, one should be aware of the possible presence of nonfactorizable SU(3) breaking terms at the B mass. Keeping this in mind, we disregard such terms for the rest of the discussion and study $R_1(m_B)$ and $R_2(m_B)$ in the generalized factorization approximation

$$R_{1,2}(m_B)=\frac{a_{1,2}^{(BK\pi)}}{a_1^{(B\pi\pi)}+a_2^{(B\pi\pi)}}\frac{f_K}{f_\pi}\frac{F_0^{B\pi}(m_K^2)}{F_0^{B\pi}(m_\pi^2)}+\frac{a_{2,1}^{(BK\pi)}}{a_1^{(B\pi\pi)}+a_2^{(B\pi\pi)}}\frac{m_B^2-m_K^2}{m_B^2-m_\pi^2}\frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_\pi^2)}. \tag{8}$$

The parameters $a_i^{(BK\pi)}$ and $a_i^{(B\pi\pi)}$ cannot be determined direcly from experiments. The closest one can get empirically is to measure these parameters at a different scale in hadronic $b\to c$ decays. An analysis of $B\to D^{(*)}\pi(\rho)$ yields values [3,22] $a_1^{BD\pi}\simeq1$ and $a_2^{BD\pi}=0.2-0.3$. A recent perturbative QCD calculation of $B\to\pi\pi$ decays [5], including nonfactorizable contributions due to hard gluon exchange, suggests that the corresponding value of the effective a_2 for two light pions is smaller, around $|a_2^{(B\pi\pi)}|=0.1$, involving a sizable complex phase. This calculation does not include nonfactorizable terms due to soft exchanges, which are argued to be power suppressed in the heavy quark limit. In our estimate below of $R_{1,2}$ we will use the range $a_2=0.1$-0.3, assuming for simplicity $a_i^{(BK\pi)}=a_i^{(B\pi\pi)}$ and neglecting complex phases which have a small effect on our estimates. Note that under these assumptions the sum R_1+R_2 can be estimated more reliably than the difference, since it is independent of $(a_1-a_2)/(a_1+a_2)$.

The form factors $F_0^{B\pi(K)}$ at $q^2=0$ were computed in a variety of quark models [18,23], light front model [24], MIT bag model [25], QCD sum rules [26–28] and lattice QCD [29]. The results span a wide range of values for $F_0^{BK}(0)/F_0^{B\pi}(0)$, from 0.7 [23] to 1.3 [24]. The ratio of form factors $F_0^{B\pi}(m_K^2)/F_0^{B\pi}(m_\pi^2)$ is expected to differ from 1 by less than 1%; this difference will be neglected in the following discussion. Using the numerical values from [18,28] gives a typical value for the form factor ratio appearing in the second term of Eq. (7), $F_0^{BK}(m_\pi^2)/F_0^{B\pi}(m_\pi^2)=1.16$. It is hard to assign a theoretical uncertainty to this value, considering the large spread of model predictions, some of which [23] involve values smaller than one. This particular value implies $R_1=1.21$ (1.20) and $R_2=1.16$ (1.17), corresponding to $a_2=0.1$ (0.3). Thus, with this choice of the form factor ratio, SU(3) breaking in R_2/R_1 is at most about 4%.

In view of the wide range of model-dependent results for $F_0^{BK}(0)/F_0^{B\pi}(0)$, and in order to narrow this range, we propose an alternative calculation of this ratio, which is based on the measured ratio of corresponding form factors in D decays, $F_0^{DK}(0)/F_0^{D\pi}(0)=1.00\pm0.08$. Semileptonic B and D decay form factors, at points of equal $\pi(K)$ energy in the rest frame of the decaying meson, are related by a heavy quark symmetry scaling law [30]

$$F_0^{BP}(q_*^2)=\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25}\sqrt{\frac{m_D}{m_B}}F_0^{DP}(0),\quad P=\pi,K.$$

The momentum transfer for B form factors corresponding to $q^2=0$ in D decays is $q_*^2=18.0$ GeV^2, for K in the final state, and $q_*^2=17.6$ GeV^2 for π. Taking the double ratio of B and D form factors [31] cancels the leading $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(m_s/\Lambda_{\chi SB})$ corrections to the scaling laws of the individual form factors:

$$\frac{F_0^{BK}(q_*^2)/F_0^{B\pi}(q_*^2)}{F_0^{DK}(0)/F_0^{D\pi}(0)}=1+\mathcal{O}(m_s/m_c-m_s/m_b). \tag{9}$$

We use this relation to predict the ratio of B form factors in terms of the corresponding ratio for D decays. The extrapolation of the former from q_*^2 down to $q^2=0$ is made by assuming pole dominance by the 0^+ states $B_{0(s)}$ for which we take $m_{B_0}=5.7$–5.8 GeV, $m_{B_{s0}}=5.8$–5.9 GeV. This gives

$$\frac{F_0^{BK}(0)}{F_0^{B\pi}(0)}=(1.013\pm0.002)\frac{F_0^{BK}(q_*^2)}{F_0^{B\pi}(q_*^2)}\simeq1.01\pm0.11, \tag{10}$$

where we introduced an error of 7% associated with the $\mathcal{O}(m_s/m_c)$ term in Eq. (9) [31]. The rest of the uncertainty is due to the error in $F_0^{DK}(0)/F_0^{D\pi}(0)$. This uncertainty is expected to be reduced in future experiments of semileptonic D

decays. The relation between ratios of form factors in D and B decays can be tested by measuring $B \to \pi \ell \nu$ and $B \to K \ell^+ \ell^-$.

The value (10) is somewhat lower than the above-mentioned result, $F_0^{BK}(0)/F_0^{B\pi}(0) = 1.16$, taken from certain models. Inserting Eq. (10) into Eq. (8) we find the central values $R_1 = 1.20$ (1.17) and $R_2 = 1.02$ (1.05) for $a_2 = 0.1$ (0.3). This implies very small SU(3) breaking in R_2 and larger SU(3) breaking in R_2/R_1, at a level of 15% (10%). This is significantly higher than the 4% effect estimated from $F_0^{BK}(0)/F_0^{B\pi}(0) = 1.16$. An even larger SU(3) breaking in R_2/R_1 is obtained in the factorization approximation for values of the form factor ratio which are smaller than 1.

We conclude with an interesting observation. Our discussion of the large measured SU(3) breaking in hadronic D decays indicates the likely need for a significant nonfactorizable nonresonant contribution. Such effects may be smaller in B decays but ought to be considered with care. In spite of this warning, one may argue from rather simple grounds that in the generalized factorization approximation SU(3) breaking in $R_2(m_D)$ is expected to be much larger than in $R_2(m_B)$. Assuming universal values for a_i, separately for B and D decays, both $R_2(m_B)$ in Eq. (8) and $R_2(m_D)$ in Eq. (7) consist of two SU(3) breaking contributions weighed by $a_2/(a_1 + a_2)$ and $a_1/(a_1 + a_2)$. In B decays, where $a_2/a_1 \sim 0.1$–0.3, the dominant a_1 term involves SU(3) breaking given by $F_0^{BK}(0)/F_0^{B\pi}(0) - 1$ which is expected to be at a level of 10%. On the other hand, in D decays in which $a_2/a_1 \sim (-0.6)$–(-0.4) is large and negative, the 22% SU(3) breaking of f_K/f_π in the a_2 term may be effectively roughly doubled by the destructive interference of this term with the a_1 term.

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