Determination of the quantum part of the truly nonperturbative Yang-Mills vacuum energy density in covariant gauge QCD

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(Received 1 February 2000; published 11 September 2000)

Using the effective potential approach for composite operators, we formulate a general method of calculation of the truly nonperturbative Yang-Mills vacuum energy density in the covariant gauge QCD ground state quantum models. It is defined as an integration of the truly nonperturbative part of the full gluon propagator over the deep infrared region (soft momentum region). A nontrivial minimization procedure makes it possible to determine the value of the soft cutoff in terms of the corresponding nonperturbative scale parameter, which is inevitably present in any nonperturbative model for the full gluon propagator. We have shown for specific models of the full gluon propagator explicitly that the use of the infrared-enhanced and finite gluon propagators leads to the vacuum energy density which is finite, always negative, and has no imaginary part (stable vacuum), while the infrared vanishing propagators lead to an unstable vacuum and therefore they are physically unacceptable.

PACS number(s): 11.15.Tk, 12.38.Lg

I. INTRODUCTION

The nonperturbative QCD vacuum is a very complicated medium and its dynamical and topological complexity [1–3] means that its structure can be organized at various levels (classical, quantum). It can contain many different components and ingredients which contribute to the truly nonperturbative vacuum energy density (VED), one of the main characteristics of the QCD ground state. Many models of the QCD vacuum involve some extra classical color field configurations such as randomly oriented domains of constant color magnetic fields, background gauge fields, averaged over spin and color, stochastic colored background fields, etc. (see Refs. [1,4,5] and references therein). The most elaborated classical models are random and interacting instanton liquid models (RILM and IILM, respectively) of the QCD vacuum [6]. These models are based on the existence of the topologically nontrivial instanton-type fluctuations of gluon fields, which are nonperturbative, weak coupling solutions to the classical equations of motion in Euclidean space (see Ref. [6] and references therein).

Here we are going to discuss the quantum part of the VED which is determined by the effective potential approach for composite operators [7–9]. It allows us to investigate the nonperturbative QCD vacuum, in particular, the Yang-Mills (YM) one, by substituting some physically well-justified ansatz for the full gluon propagator since the exact solutions are not known. In the absence of external sources the effective potential is nothing but the VED which is given in the form of the loop expansion where the number of the vacuum loops (consisting in general of the confining quarks and nonperturbative gluons properly regularized with the help of ghosts) is equal to the power of the Plank constant $\hbar$.

The full dynamical information of any quantum gauge field theory such as QCD is contained in the corresponding quantum equations of motion, the so-called Schwinger-Dyson (SD) equations for lower (propagators) and higher (vertices and kernels) Green's functions. It is a *highly nonlinear*, strongly coupled system of four-dimensional integral equations for the above-mentioned quantities. The kernels of these integral equations are determined by the infinite series of the corresponding skeleton diagrams [10–12]. It is a general feature of *nonlinear* systems that the number of exact solutions (if any) *cannot be fixed a priori*. Thus formally it may have several exact solutions. These equations should be also complemented by the corresponding Slavnov-Taylor (ST) identities [10–12] which in general relate the above-mentioned lower and higher Green's functions to each other. These identities are consequences of the exact gauge invariance and therefore *are exact constraints on any solution to QCD* [10]. Precisely this system of equations can serve as an adequate and effective tool for the nonperturbative approach to QCD [13,14].

Among the above-mentioned Green's functions, the two-point Green's function describing the full gluon propagator

$$iD_{\mu\nu}(q)=\{T_{\mu\nu}(q)d(-q^2,\xi)+\xi L_{\mu\nu}(q)\}\frac{1}{q^2} \qquad (1.1)$$

has a central place [10–15]. Here ξ is a gauge fixing parameter ($\xi=0$, Landau gauge) and $T_{\mu\nu}(q)=g_{\mu\nu}-q_\mu q_\nu/q^2 =g_{\mu\nu}-L_{\mu\nu}(q)$. Evidently, its free perturbative (tree level) counterpart is obtained by simply setting the full gluon form factor $d(-q^2,\xi)=1$ in Eq. (1.1). In particular, the solutions of the above-mentioned SD equation for the full gluon propagator, Eq. (1.1), are supposed to reflect the complexity of the quantum structure of the QCD ground state. As emphasized above, it is a *highly nonlinear* system of four-dimensional integrals containing many different, unknown in general, propagators, vertices, and kernels [10–12]. Because of truncation schemes, this system becomes the equation for the full gluon propagator only, but it remains *nonlinear*, nevertheless. Different truncations could lead to qualitatively different solutions, and the number of these solutions may be

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increased only. Moreover, to clearly distinguish between the exact or approximate solutions (if any), we do not know even the complete set of boundary conditions to attempt to uniquely fix solution of the truncated equation. We certainly know the boundary condition in the ultraviolet (UV) limit because of asymptotic freedom and certainly we do not know the corresponding boundary condition in the infrared (IR) precisely because of confinement (at this stage it is not even clear whether the two boundary conditions [in the UV and in the IR (if it can be established)] will be sufficient to completely fix the theory or not). Because of the above-discussed highly complicated mathematical structure of the SD equation for the full gluon propagator, there is no hope for exact solution(s). However, in any case the solutions of this equation can be distinguished by their behavior in the deep IR limit (the UV limit is uniquely determined by asymptotic freedom), describing thus many different types of quantum excitations and fluctuations of gluon field configurations in the QCD vacuum. Evidently, not all of them reflect the real structure of the QCD vacuum.

The deep IR asymptotics of the full gluon propagator can be generally classified into three different types: (1) the IR enhanced (IRE) or IR singular (IRS), (2) the IR finite (IRF), and (3) the IR vanishing (IRV) ones (for references see the corresponding sections below). Let us emphasize that any deviation in the behavior of the full gluon propagator in the deep IR domain from the free perturbative one automatically assumes its dependence on a scale parameter (at least one) in general different from the QCD asymptotic scale parameter Λ_{QCD}. It can be considered as responsible for the nonperturbative dynamics (in the IR region) in the QCD vacuum models under consideration. If QCD itself is a confining theory, then such a characteristic scale is very likely to exist. In what follows, let us denote it as, say, Λ_{NP}. This is very similar to asymptotic freedom which requires the above-mentioned asymptotic scale parameter associated with nontrivial perturbative dynamics in the UV region (scale violation). However, for calculation of the truly nonperturbative VED we do not exactly need the deep IR asymptotics of the full gluon propagator, but rather its truly nonperturbative part, which vanishes when the above-mentioned nonperturbative scale parameter goes formally to zero, i.e., when only the perturbative phase survives. So we define the truly nonperturbative part of the full gluon form factor in Eq. (1.1) as follows:

$$d^{NP}(-q^2,\Lambda_{NP}) = d(-q^2,\Lambda_{NP}) - d(-q^2,\Lambda_{NP}=0), \tag{1.2}$$

which, on the one hand, uniquely determines the truly nonperturbative part of the full gluon propagator. On the other hand, the definition (1.2) explains the difference between the truly nonperturbative part $d^{NP}(-q^2)$ and the full gluon propagator $d(-q^2)$ which is nonperturbative itself. Let us note in advance that in realistic models for the full gluon propagator, the limit $\Lambda_{NP}\to 0$ is usually equivalent to the limit $-q^2\to\infty$. In some cases, the model gluon propagator does not depend explicitly on the nonperturbative scale parameter (the dependence is hidden); then, its behavior at infinity should be subtracted. In realistic models of the full gluon propagator its truly nonperturbative part usually coincides with its deep IR asymptotics, emphasizing thus the strong intrinsic influence of the IR properties of the theory on its nonperturbative dynamics.

It is well known, however, that the VED in general is badly divergent in quantum field theory, in particular QCD [16]. Thus the main problem is how to extract the truly nonperturbative VED which is relevant for the QCD vacuum quantum model under consideration. It should be finite, negative, and have no imaginary part (stable vacuum). Why is it so important to calculate it from first principles, i.e., on the basis of some realistic ansatz for the full gluon propagator only? As was emphasized above, this quantity is important in its own right as being nothing else but the bag constant (the so-called bag pressure) apart from the sign, by definition [16]. Through the trace anomaly relation [17] it helps in the correct estimation of such an important phenomenological nonperturbative parameter as the gluon condensate introduced in the QCD sum rules approach to resonance physics [18]. Furthermore, the YM VED assists in the resolution of the $U(1)$ problem [19] via the Witten-Veneziano (WV) formula for the mass of the η' meson [20]. The problem is that the topological susceptibility [19–23] needed for this purpose is determined by a two-point correlation function from which the perturbative contribution is already subtracted, by definition [20,23–25]. The same is valid for the above-mentioned bag constant which is a much more general quantity than the string tension because it is relevant for light quarks as well. Thus to calculate correctly the truly nonperturbative VED means to understand correctly the structure of the QCD vacuum in different models.

We have already formulated a general method of calculation of the truly nonperturbative YM VED in the axial gauge QCD in Ref. [26], where the Abelian Higgs model [27] of the dual QCD [28] ground state was investigated. Moreover, we have calculated the truly nonperturbative VED (using a particular method) in the covariant gauge QCD quantum vacuum model as well [29,30]. The main purpose of this paper (Sec. II) is to formulate precisely a general method of calculation of the truly nonperturbative quantum part of the YM VED in the covariant gauge QCD. In Secs. III, IV, and V this is illustrated by considering different covariant gauge QCD quantum models of its ground state by choosing three different types of the deep IR asymptotics of the full gluon propagator, IRE, IRF, and IRV, respectively. The conclusions are presented in Sec. VI.

II. TRULY NONPERTURBATIVE VACUUM ENERGY DENSITY

In this section we formulate a general method of numerical calculation of the quantum part of the truly nonperturbative YM VED in the covariant gauge QCD. Let us start from the gluon part of the VED which to leading order[1] (log-loop

[1]Next-to-leading and higher contributions (two and more vacuum loops) are numerically suppressed by one order of magnitude in powers of $\hbar$ at least and are left for consideration elsewhere.

level $\sim\hbar$) is given by the effective potential for composite operators [7] as follows:

$$V(D)=\frac{i}{2}\int \frac{d^nq}{(2\pi)^n}\mathrm{Tr}\{\ln(D_0^{-1}D)-(D_0^{-1}D)+1\}, \tag{2.1}$$

where $D(q)$ is the full gluon propagator (1.1) and $D_0(q)$ is its free perturbative (tree level) counterpart. Here and below the traces over space-time and color group indices are understood. The effective potential is normalized as $V(D_0)=0$; i.e., the free perturbative vacuum is normalized to zero. In order to evaluate the effective potential (2.1) we use the well-known expression

$$\mathrm{Tr}\ln(D_0^{-1}D)=8\times\ln\det(D_0^{-1}D)=8\times 4\ln\left[\frac{3}{4}d(-q^2)+\frac{1}{4}\right]. \tag{2.2}$$

It becomes zero (in accordance with the above-mentioned normalization condition) when the full gluon form factor is replaced by its free perturbative counterpart. This composition does not depend explicitly on a gauge choice. Going over to four-dimensional ($n=4$) Euclidean space in Eq. (2.1), on account of Eq. (2.2), and evaluating some numerical factors, one obtains [$\epsilon_g=V(D)$]

$$\epsilon_g=-\frac{1}{\pi^2}\int dq^2q^2\left[\ln[1+3d(q^2)]-\frac{3}{4}d(q^2)+a\right], \tag{2.3}$$

where constant $a=(3/4)-2\ln 2=-0.6363$ and integration from zero to infinity is assumed. Substituting the definition (1.2) into Eq. (2.3) and doing some trivial rearrangements, one obtains

$$\epsilon_g=-\frac{1}{\pi^2}\int dq^2q^2\left[\ln[1+3d^{NP}(q^2,\Lambda_{NP})]-\frac{3}{4}d^{NP}(q^2,\Lambda_{NP})\right]-\frac{1}{\pi^2}I_{PT}, \tag{2.4}$$

where we introduce the following notation:

$$I_{PT}=\int dq^2q^2\left[\ln\left(1+\frac{3d(q^2,\Lambda_{NP}=0)}{1+3d^{NP}(q^2,\Lambda_{NP})}\right)-\frac{3}{4}d(q^2,\Lambda_{NP}=0)+a\right]. \tag{2.5}$$

It contains the contribution which is mainly determined by the perturbative part of the full gluon propagator, $d(q^2,\Lambda_{NP}=0)$. The constant a also should be included since it comes from the normalization of the free perturbative vacuum to zero. If we separate the deep IR region from the perturbative one [which consists of the intermediate (IM) and UV regions since the IM region remains *terra incognita* in QCD], by introducing the so-called soft cutoff explicitly then we get

$$\epsilon_g=-\frac{1}{\pi^2}\int_0^{q_0^2}dq^2q^2\left[\ln[1+3d^{NP}(q^2,\Lambda_{NP})]-\frac{3}{4}d^{NP}(q^2,\Lambda_{NP})\right]-\frac{1}{\pi^2}(\tilde{I}_{PT}+I_{PT}), \tag{2.6}$$

where evidently

$$\tilde{I}_{PT}=\int_{q_0^2}^{\infty}dq^2q^2\left[\ln[1+3d^{NP}(q^2,\Lambda_{NP})]-\frac{3}{4}d^{NP}(q^2,\Lambda_{NP})\right]. \tag{2.7}$$

Thus the first integral represents contribution to the YM VED which is determined by the truly nonperturbative piece of the full gluon propagator integrated over the deep IR region. In other words, just this term is the truly nonperturbative contribution to the YM VED. This means that the two remaining terms in Eq. (2.6) should be subtracted by introducing corresponding counterterms into the effective potential. Thus in general the integral (2.5) determining the contribution from the perturbative part of the full gluon propagator and the integral (2.7) determining the contribution from the perturbative region (IM plus UV) are of no importance for our present consideration. The above-mentioned necessary subtractions can be done in a more sophisticated way by means of ghost degrees of freedom (see below).

The effective potential at the log-loop level for the ghost degrees of freedom is

$$V(G)=-i\int \frac{d^np}{(2\pi)^n}\mathrm{Tr}\{\ln(G_0^{-1}G)-(G_0^{-1}G)+1\}, \tag{2.8}$$

where $G(p)$ is the full ghost propagator and $G_0(p)$ is its free perturbative (tree level) counterpart. The effective potential $V(G)$ is normalized as $V(G_0)=0$. Evaluating formally the ghost term $\epsilon_{gh}=V(G)$ in Eq. (2.8), we obtain $\epsilon_{gh}=\pi^{-2}I_{gh}$. The integral I_{gh} depends on the ghost propagator, which remains arbitrary (unknown) within our approach. In principle, we have to sum up all contributions to obtain the total VED (the confining quark part of the vacuum energy density is not considered here). However, upon substitution of definition (1.2) into the integral over the whole momentum range from zero to infinity, Eq. (2.3), some terms appear there which may have unphysical singularities below the scale Λ_{QCD} [integral (2.5)]. Thus the initial VED (2.3) is a formal one; it suffers from unphysical singularities briefly mentioned above and it is badly divergent as well. In order to get a physically meaningful expression, one has to subtract two integrals (2.5) and (2.7) from Eq. (2.3). We have done this subtraction with the help of a ghost term by imposing the following condition: $\Delta=\tilde{I}_{PT}+I_{PT}-I_{gh}=0$. The nonperturbative gluon contribution to the VED is determined by subtracting unwanted terms by means of the ghost contribution, i.e., defining $\epsilon_g+\epsilon_{gh}=\epsilon_{YM}$ at $\Delta=0$. Thus the truly nonperturbative YM VED becomes

$$\epsilon_{YM}=\frac{1}{\pi^2}\int_0^{q_0^2}dq^2q^2\left[\frac{3}{4}d^{NP}(q^2,\Lambda_{NP}) -\ln[1+3d^{NP}(q^2,\Lambda_{NP})]\right]. \tag{2.9}$$

In many cases this subtraction is sufficient to obtain the expression for the truly nonperturbative YM VED. However, in some other cases the truly nonperturbative part of the full gluon propagator which enters Eq. (2.9) continues to suffer from unphysical singularities below the scale Λ_{QCD} (see the discussion at the end of Sec. V). As was noticed, some additional terms should be included in our subtraction scheme in this case, indicating that the chosen ansatz for the full gluon propagator itself was not realistic.

A few general remarks are in order. In QCD nothing should explicitly depend on ghosts. By contributing to closed loops only, the main purpose of their introduction is to cancel the unphysical degrees of freedom of gauge bosons (maintaining thus the unitarity of the *S* matrix), for example, to exclude the longitudinal components, the above-mentioned unphysical singularities below the QCD scale, etc. This is the main reason why they are to be considered together with gluons always. In nonperturbative QCD in general and in our approach in particular the ghost propagator (or equivalently the ghost self-energy) still remains unknown (in this sense arbitrary) since the exact ghost-gluon vertex (which enters the corresponding SD equation) is not exactly known (in Refs. [31,32] some very specific truncation scheme is used in order to derive a particular expression for this vertex). We know, however, that the ghost propagator contribution to the VED, regular or singular, should be combined with the gluon contribution in order to cancel exactly the above-mentioned unphysical singularities of the gauge bosons which are inevitably present in any ansatz for the full gluon propagator. In other words, if one knows the ghost propagator exactly, then the above-mentioned cancellation should proceed automatically (as usual in perturbative calculus if, of course, all calculations are correct). But if it is not known exactly (as usual in nonperturbative calculus), then one has to impose the condition of cancellation as was done in our case, $\Delta=0$. Obviously, the above-mentioned condition of cancellation was imposed in the most general form. Instead of the introduction of some counterterms into the initial effective potential to cancel the most dangerous UV divergences presented in the integral (2.5), we have used the ghost term for this purpose as well. Thus our subtraction scheme is in agreement with the general physical interpretation of ghosts to cancel all unphysical degrees of freedom of the gauge bosons [10,33].

The expression (2.9) is our definition of the truly nonperturbative YM VED as integrated out of the truly nonperturbative part of the full gluon propagator over the deep IR region (soft momentum region, $0\leqslant q^2\leqslant q_0^2$). The soft cutoff q_0^2 (as a function of the nonperturbative scale) can be determined by the corresponding minimization procedure (see below).

A. Λ_{YM} as a fixed scale

From this point it is convenient to factorize scale dependence of the truly nonperturbative YM VED (2.9). As was already emphasized above, $d^{NP}(q^2)$ always contains at least one scale parameter (Λ_{NP}) responsible for the nonperturbative dynamics in the model under consideration. It is considered as a free one within our general method, i.e., ''running'' (when it formally goes to zero, then the perturbative phase only survives in the model). Its numerical value (if any) will be used at the final stage only to evaluate numerically the corresponding truly nonperturbative YM VED (if any). We can introduce dimensionless variables and parameters by using a completely extra scale (which is always fixed in comparison with Λ_{NP}), for example, the flavorless QCD asymptotic scale parameter Λ_{YM}, as follows:

$$z=\frac{q^2}{\Lambda_{YM}^2},\quad z_0=\frac{q_0^2}{\Lambda_{YM}^2},\quad b=\frac{\Lambda_{NP}^2}{\Lambda_{YM}^2}. \tag{2.10}$$

Here z_0 is a corresponding dimensionless soft cutoff while the parameter b has a very clear physical meaning. It measures the ratio between nonperturbative dynamics, symbolized by Λ_{NP}^2, and nontrivial perturbative dynamics (violation of scale, asymptotic freedom), symbolized by Λ_{YM}^2. When it is zero only the perturbative phase remains in the model. In this case, the gluon form factor obviously becomes a function of z and b, i.e., $d^{NP}(q^2)=d^{NP}(z,b)$, and the truly nonperturbative VED (2.9) is $[\epsilon_{YM}\equiv\epsilon_{YM}(z_0,b)]$

$$\Omega_g(z_0,b)=\frac{1}{\Lambda_{YM}^4}\epsilon_{YM}(z_0,b), \tag{2.11}$$

where the gluon effective potential at a fixed scale, Λ_{YM}, [26,29,34] is introduced:

$$\Omega_g\equiv\Omega_g(z_0,b) =\frac{1}{\pi^2}\int_0^{z_0}dzz\left[\frac{3}{4}d^{NP}(z,b)-\ln[1+3d^{NP}(z,b)]\right]. \tag{2.12}$$

This expression precisely allows us to investigate the dynamical structure of the YM vacuum. It is free of scale dependence since it has been already factorized in Eq. (2.11). It depends only on z_0 and b and a minimization procedure with respect to b, $\partial\Omega_g(z_0,b)/\partial b=0$ [usually after integrated out in Eq. (2.12)], can provide a self-consistent relation between z_0 and b; that is, we get q_0 as a function of Λ_{NP}. Let us note in advance that the final numerical results will depend on Λ_{NP} only as it should be for the nonperturbative part of the YM VED (see Secs. III and IV below). Obviously, minimization with respect to z_0 leads to a trivial zero. In principle, through the relation $\Lambda_{YM}^4=q_0^4z_0^{-2}$, it is possible to fix the soft cutoff q_0 itself, but this is not the case indeed since then z_0 cannot be varied.

B. Soft cutoff as a fixed scale

On the other hand, the scale dependence can be factorized as follows:

$$z=\frac{q^2}{\Lambda_{NP}^2}, \quad z_0=\frac{q_0^2}{\Lambda_{NP}^2}; \tag{2.13}$$

i.e., $b=1$. For simplicity (but not losing generality) we use the same notation for the dimensionless set of variables and parameters as in Eq. (2.10). In this case, the gluon form factor obviously becomes a function of z only, $d^{NP}(q^2)=d^{NP}(z)$, and the truly nonperturbative YM VED (2.9) becomes

$$\epsilon_{YM}(z_0)=\frac{1}{\pi^2}q_0^4 z_0^{-2}\int_0^{z_0}dzz\left[\frac{3}{4}d^{NP}(z)-\ln[1+3d^{NP}(z)]\right]. \tag{2.14}$$

Evidently, to fix the scale is possible in two different ways. In principle, we can fix Λ_{NP} itself, i.e., introducing

$$\tilde{\Omega}_g(z_0)=\frac{1}{\Lambda_{NP}^4}\epsilon_{YM}(z_0) =\frac{1}{\pi^2}\int_0^{z_0}dzz\left[\frac{3}{4}d^{NP}(z)-\ln[1+3d^{NP}(z)]\right]. \tag{2.15}$$

However, the minimization procedure again leads to a trivial zero, which shows that this scale cannot be fixed.

In contrast with the previous case, let us fix the soft cutoff itself, i.e., setting [26,29,30]

$$\bar{\Omega}_g(z_0)=\frac{1}{q_0^4}\epsilon_{YM}(z_0) =\frac{1}{\pi^2}z_0^{-2}\int_0^{z_0}dzz\left[\frac{3}{4}d^{NP}(z)-\ln[1+3d^{NP}(z)]\right]. \tag{2.16}$$

In this case the perturbative phase is recovered in the $z_0\to\infty$ $(\Lambda_{NP}\to 0)$ limit. Now the minimization procedure with respect to z_0 is nontrivial. Indeed, $\partial\bar{\Omega}_g(z_0)/\partial z_0=0$ yields the ''stationary'' condition

$$\int_0^{z_0}dzz\left[\frac{3}{4}d^{NP}(z)-\ln[1+3d^{NP}(z)]\right] =\frac{1}{2}z_0^2\left[\frac{3}{4}d^{NP}(z_0)-\ln[1+3d^{NP}(z_0)]\right], \tag{2.17}$$

the solutions of which (if any) allow one to find q_0 as a function of Λ_{NP}. On account of this ''stationary'' condition, the effective potential (2.16) itself becomes simpler for numerical calculations, namely,

$$\bar{\Omega}_g(z_0^{st})=\frac{1}{2\pi^2}\left[\frac{3}{4}d^{NP}(z_0^{st})-\ln[1+3d^{NP}(z_0^{st})]\right], \tag{2.18}$$

where z_0^{st} is a solution (if any) of the ''stationary'' condition (2.17) and corresponds to the minima (if any) of the effective potential (2.16). In the next sections, we illustrate how this method works by considering some quantum models of the covariant gauge QCD ground state explicitly.

III. IRE GLUON PROPAGATOR: ZME QUANTUM MODEL

Today there are no doubts left that the dynamical mechanisms of important nonperturbative quantum phenomena such as quark confinement and dynamical (or equivalently spontaneous) chiral symmetry breaking (DCSB) are closely related to the complicated topologically nontrivial structure of the QCD vacuum [1–4,10]. On the other hand, it also becomes clear that the nonperturbative IR dynamical singularities, closely related to the nontrivial vacuum structure, play an important role in the large distance behavior of QCD [35,36]. For this reason, any correct nonperturbative model of quark confinement and DCSB necessarily turns out to be a model of the true QCD vacuum and the other way around.

Our model of the true QCD ground state is based on the existence and importance of such a kind of nonperturbative, quantum excitations of the gluon field configurations (due to self-interaction of massless gluons only, i.e., without explicitly involving some extra degrees of freedom) which can be effectively correctly described by the q^{-4} behavior of the full gluon propagator in the deep IR domain (at small q^2) [29,30]. These excitations are topologically nontrivial also since they lead to the nontrivial YM VED (see below). Thus our main definition (1.2) becomes

$$d^{NP}(-q^2,\Lambda_{NP})=d(-q^2,\Lambda_{NP})-d(-q^2,\Lambda_{NP}=0) =\frac{\Lambda_{NP}^2}{(-q^2)}. \tag{3.1}$$

In the above-mentioned papers [29,30] the nonperturbative scale was denoted as $\bar{\mu}$, i.e., $\bar{\mu}\equiv\Lambda_{NP}$. In this way we obtain the generally accepted form of the deep IR singular asymptotics for the full gluon propagator (for some references see below),

$$D_{\mu\nu}(q)\sim(q^2)^{-2}, \quad q^2\to 0, \tag{3.2}$$

which may be referred equivalently to as the strong coupling regime [10]. It describes the zero-momentum-mode enhancement (ZMME) dynamical effect in QCD at large distances. We prefer to use simply ZME (zero-mode enhancement) since we work always in momentum space. This is our primary dynamical assumption in this section. The main problem due to this strong singularity is its correct treatment by the dimensional regularization method [37] within the distribution theory [38], which was one of the highlights of our previous publications [29,30] (see also Ref. [39]). There exist many arguments in favor of this behavior.

(a) Such singular behavior of the full gluon propagator in the IR domain leads to the area law for static quarks (indicative of confinement) within the Wilson loop approach [40].

(b) The cluster property of the Wightman functions in QCD fails and this allows such singular behavior like Eq. (3.2) for the full gluon propagator in the deep IR domain [41].

(c) After the pioneering papers of Mandelstam in the covariant (Landau) gauge [42] and Baker, Ball, and Zachariasen in the axial gauge [43], the consistency of the singular asymptotics (3.2) with direct solution of the SD equation for the full gluon propagator in the IR domain was repeatedly confirmed (see, for example, Refs. [13,14,44,45] and references therein).

(d) Moreover, let us underline that without this component in the decomposition of the full gluon propagator in continuum theory it is impossible to ''see'' the linearly rising potential between heavy quarks by lattice QCD simulations [46] not involving some extra (besides gluons and quarks) degrees of freedom. This should be considered as strong lattice evidence (though not direct) of the existence and importance of q^{-4}-type excitations of gluon field configurations in the QCD vacuum. There exists also direct lattice evidence that the zero modes are enhanced in the full gluon propagator indeed [47].

(e) Within the distribution theory [38] the structure of the nonperturbative IR singularities in four-dimensional Euclidean QCD is the same as in two-dimensional QCD, which confines quarks at least in the large N_c limit [48]. In this connection, let us note that the q^{-4} IR singularity is the simplest nonperturbative power singularity in four-dimensional QCD as well as the q^{-2} IR singularity being the simplest nonperturbative power singularity in two-dimensional QCD. The QCD vacuum is a much more complicated medium than its two-dimensional model; nevertheless, the above-mentioned analogy is promising even in the case of the nonperturbative dynamics of light quarks.

(f) Some classical models of the QCD vacuum also invoke the q^{-4} behavior of the gluon fields in the IR domain. For example, it appears in the QCD vacuum as a condensation of the color-magnetic monopoles (the QCD vacuum is a chromomagnetic superconductor) proposed by Nambu, Mandelstam, and 't Hooft and developed by Nair and Rosenzweig (see Ref. [49] and references therein. For recent developments in this model see Di Giacomo [1]) as well as in the classical mechanism of the confining medium [50] and in effective theory for the QCD vacuum proposed in Ref. [51].

(g) It is also required to derive the heavy quark potential within the recently proposed exact renormalization group flow equations approach [52].

(h) It has been shown in our papers that the singular behavior (3.2) is related directly to light quark confinement and DCSB [29,30]. Moreover, very good agreement has been obtained with the phenomenological values of the topological susceptibility, the mass of the η' meson, and the gluon condensate [21,22].

Thus we consider our main ansatz (3.1),(3.2) as physically well motivated. Let us emphasize that $d^{NP}(-q^2,\xi) = \Lambda_{NP}^2/(-q^2)$ is the truly nonperturbative part of the full gluon propagator since it vanishes in the perturbative limit ($\Lambda_{NP}^2 \to 0$, when the perturbative phase survives only) and simultaneously it correctly reproduces the deep IR asymptotics of the full gluon propagator; i.e., $d^{NP}(-q^2)$ coincides with $d^{IR}(-q^2)$.

A. Set of dimensionless variables of *A* type

The truly nonperturbative YM VED is given now by Eq. (2.9) with $d^{NP}(q^2) = \Lambda_{NP}^2/q^2$ in Euclidean space. Let us first introduce the *A*-type set of dimensionless variables, Eqs. (2.10). Then $d^{NP}(q^2)$ becomes $d^{NP}(z,b) = b/z$. Performing almost trivial integration in the effective potential at a fixed scale (2.12), one obtains

$$\Omega_g(z_0,b) = \frac{1}{2\pi^2}\left[9b^2 \ln\left(1+\frac{z_0}{3b}\right) - \frac{3z_0}{2}b - z_0^2 \ln\left(1+\frac{3b}{z_0}\right)\right]. \tag{3.3}$$

It is easy to show that as a function of b, the effective potential (3.3) linearly approaches zero from below and it diverges also linearly at infinity while as a function of z_0 itself it approaches zero from above and also diverges as $\sim -z_0$ at infinity. Thus as a function of b it has a local minimum (relating b to z_0) at which the truly nonperturbative YM VED will be always finite and negative. The minimization procedure with respect to b, $\partial\overline{\Omega}_g(z_0;b)/\partial b = 0$, yields the following ''stationary'' condition: $\nu = 4\ln[1+(\nu/3)]$, where $\nu = z_0/b$. Its solution is $\nu^{min} = 2.2$. Using this ''stationary'' condition, the effective potential (3.3) can be written as

$$\Omega_g(\nu^{min},b) = \frac{b^2\nu^{min}}{2\pi^2}\left[\frac{3}{4} - \nu^{min}\ln\left(1+\frac{3}{\nu^{min}}\right)\right] = -0.1273b^2, \tag{3.4}$$

so the truly nonperturbative YM VED (2.11) becomes

$$\epsilon_{YM} = -0.1273\Lambda_{NP}^4, \tag{3.5}$$

where the relation $\Lambda_{NP}^4 = b^2\Lambda_{YM}^4$ has been already used. Determined in this way, it is always finite (since the characteristic scale of our model, Λ_{NP}, is finite, evidently it cannot be arbitrarily large), automatically negative (as it should be for the truly nonperturbative energy), and it has no imaginary part (stable vacuum). Obviously the characteristic scale of our model, Λ_{NP}, cannot be determined within the YM theory alone. Its numerical value should be taken from the good physical observable in full QCD by implementing the physically well-motivated scale setting scheme. Precisely this has been done in our papers [29,30] where the nonperturbative VED was numerically evaluated from first principles. Moreover, in recent publications [21,22] it is shown that our numerical results are of the necessary order of magnitude in order to nicely saturate the large mass of the η' meson in the chiral limit as well as the phenomenological value of the

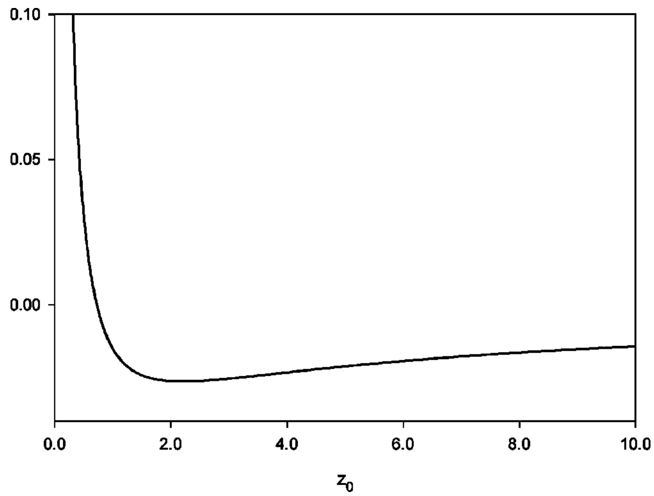

FIG. 1. Effective potential (3.7) as a function of z_0.

topological susceptibility. Thus the existence of the nontrivial VED in the ZME quantum model, which agrees well with QCD topology, is one more serious argument in its favor. It is worthwhile to present the numerical value for the soft cutoff in terms of Λ_{NP}, namely, $q_0 = 1.48324\Lambda_{NP}$. This follows from the solution of the ''stationary'' condition, of course.

B. Set of dimensionless variables of *B* type

It is instructive to calculate the truly nonperturbative YM VED by choosing the *B*-type set of dimensionless variables Eqs. (2.13). Then $d^{NP}(q^2) = \Lambda_{NP}^2/q^2$ becomes $d^{NP}(z) = 1/z$. Performing almost trivial integration in the effective potential at a fixed scale (2.16) in this case, one obtains

$$\overline{\Omega}_g(z_0) = \frac{1}{2\pi^2} z_0^{-2}\left[9 \ln\left(1+\frac{z_0}{3}\right) - \frac{3}{2}z_0 - z_0^2 \ln\left(1+\frac{3}{z_0}\right)\right]. \tag{3.6}$$

It is easy to show now that as a function of z_0, the effective potential (3.7) diverges as $\sim z_0^{-1}$ at small z_0 and converges as $\sim -z_0^{-1}$ at infinity (perturbative limit); see Fig. 1. Thus as a function of z_0 it has a local minimum at $z_0 = 4 \ln[1 + (z_0/3)]$, the so-called ''stationary'' condition in this case. Its solution again is $z_0^{min} = 2.2$. At the ''stationary'' state the effective potential (3.6) can be written

$$\overline{\Omega}_g(z_0^{min}) = \frac{1}{2\pi^2}\left[\frac{3}{4}(z_0^{min})^{-1} - \ln\left(1+\frac{3}{z_0^{min}}\right)\right] = -0.0263, \tag{3.7}$$

so the truly nonperturbative YM VED (2.16) becomes

$$\epsilon_{YM} = -0.0263 q_0^4 = -0.1273\Lambda_{NP}^4, \tag{3.8}$$

where the relation $q_0^4 = (z_0^{min})^2\Lambda_{NP}^4$ has been already used. Thus we have explicitly demonstrated that the truly nonperturbative YM VED does not indeed depend on how one introduces dimensionless variables into the effective potential, i.e., $\epsilon_{YM} = \Lambda_{NP}^4 \Omega_g(\nu^{min}, b) = q_0^4 \overline{\Omega}_g(z_0^{min}) = -0.1273\Lambda_{NP}^4$. In some cases, the *B*-type calculation is preferable. For example, to calculate the confining quark contribution into the total VED is much easier using precisely this set of the dimensionless variables (see our papers [29,30] and the next section as well).

IV. IRF GLUON PROPAGATOR

Let us consider now a possible IRF behavior of the full gluon propagator (in the Landau gauge) in the deep IR domain, which was suggested by recent lattice calculations in Ref. [53]. The main definition (1.2) in this case becomes

$$d^{NP}(-q^2, M) = d(-q^2, M) - d(-q^2, M=0) = \frac{ZAM^{2\alpha}(-q^2)}{(-q^2+M^2)^{1+\alpha}}. \tag{4.1}$$

Here M is the mass scale parameter responsible for the nonperturbative dynamics in this model, i.e., $M = \Lambda_{NP}$ in our notation. When the parameter M formally goes to zero, the perturbative phase only remains in this model. Again as in the previous case, the truly nonperturbative part vanishes in the perturbative limit $(M \to 0)$ and it reproduces the IR asymptotics of the full gluon propagator correctly as well. The best estimates for the parameters M and A are $M = (1020 \pm 100 \pm 25)$ MeV and $A = (9.8 + 0.1 - 0.9)$. As was emphasized above, the numerical value of the parameter M will be used only at the final stage in order to estimate numerically the truly nonperturbative YM VED in this model. The exponent in general is $\alpha = 2 + \delta$, where $\delta > 0$ and small, while $Z \approx 1.2$ is the renormalization constant.

In this case, it is convenient to choose the *B*-type set of variables and parameters, Eqs. (2.13). Then $d^{NP}(q^2)$ in Euclidean space becomes

$$d^{NP}(z) = \frac{a_1 z}{(1+z)^{1+\alpha}}, \tag{4.2}$$

where the parameter $a_1 = ZA = 11.76$ is fixed. Substituting this into the effective potential (2.16), one obtains

$$\overline{\Omega}_g(z_0; a_1) = \frac{1}{q_0^4}\epsilon_{YM} = -\frac{1}{\pi^2} z_0^{-2}\{I_1(z_0; a_1) - I_2(z_0; a_1)\}, \tag{4.3}$$

where the integrals are given as follows:

$$I_1(z_0; a_1) = \int_0^{z_0} dz z \ln\left(1 + \frac{3a_1 z}{(1+z)^{1+\alpha}}\right),$$

$$I_2(z_0; a_1) = \frac{3a_1}{4}\int_0^{z_0} dz z \frac{z}{(1+z)^{1+\alpha}}. \tag{4.4}$$

The asymptotic behavior of the effective potential (4.3) depends on the asymptotic properties of the integral $I_1(z_0; a_1)$

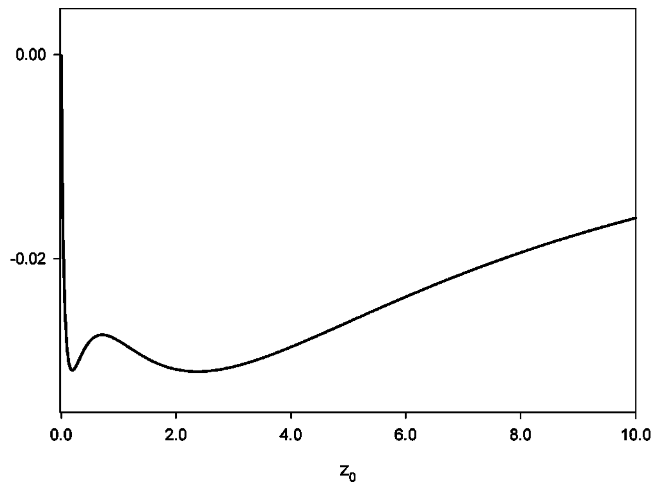

FIG. 2. Effective potential (4.3) as a function of z_0.

since the integral $I_2(z_0;a_1)$ in Eq. (4.4) can be taken explicitly: namely (in what follows in this section, $\alpha=2$),

$$I_2(z_0;a_1)=\frac{3a_1}{4}\Biggl(\ln(1+z_0)+2[(1+z_0)^{-1}-1]-\frac{1}{2}[(1+z_0)^{-2}-1]\Biggr). \tag{4.5}$$

From these expressions it is almost obvious that the asymptotics of the effective potential (4.3) at $z_0\to 0,\infty$ to leading order can be easily evaluated analytically. Thus the effective potential (4.3) as a function of the soft cutoff z_0 has two local minima; see Fig. 2. The corresponding ''stationary'' condition can be evaluated as follows:

$$[I_1(z_0;a_1)-I_2(z_0;a_1)]=\frac{1}{2}z_0^2\Biggl\{\ln\Biggl(1+\frac{3a_1z_0}{(1+z_0)^3}\Biggr)-\frac{3a_1z_0}{4(1+z_0)^3}\Biggr\}. \tag{4.6}$$

Using this ''stationary'' condition, the effective potential (4.3) at the ''stationary'' state becomes

$$\bar{\Omega}_g(z_0^{st};a_1)=-\frac{1}{2\pi^2}\Biggl\{\ln\Biggl(1+\frac{3a_1z_0^{st}}{(1+z_0^{st})^3}\Biggr)-\frac{3a_1z_0^{st}}{4(1+z_0^{st})^3}\Biggr\}, \tag{4.7}$$

where z_0^{st} is a solution(s) to the ''stationary'' condition (4.6). The two solutions of the ''stationary'' condition (4.6) corresponding to the two local minima are $z_0^{st}=0.19$ and $z_0^{st}=2.37$ with almost equal numerical values for the corresponding effective potentials at the ''stationary'' states, namely, $\bar{\Omega}_g(0.19;a_1)=-0.0309$ and $\bar{\Omega}_g(2.37;a_1)=-0.0310$, respectively. However, the numerical values of the nonperturbative YM VED (4.3) are drastically different,

$$\epsilon_{YM}(0.19)=-0.0309q_0^4(0.19)=-0.00123M^4 \tag{4.8}$$

and

$$\epsilon_{YM}(2.37)=-0.0310q_0^4(2.37)=-0.174M^4, \tag{4.9}$$

where the relation $q_0^4=(z_0^{min})^2M^4$ and the corresponding values of $z_0^{min}(\equiv z_0^{st})$ were applied. How to distinguish between the two solutions for the truly nonperturbative YM VED (4.8) and (4.9)? This question is discussed in the following.

Discussion

In the first case, on account of the numerical value of the nonperturbative scale $M\approx 1$ GeV, Eq. (4.8) numerically becomes

$$\epsilon_{YM}(0.19)=-0.00123\ \text{GeV}^4. \tag{4.10}$$

It is the same order of magnitude as the VED due to instantons [22]. Thus summing up this and instantons with ZME values, one obtains a fair agreement with chiral QCD topology [20]. Also the soft cutoff in this case is $q_0\approx 0.463M\approx 463$ MeV. This is quite reasonable value for the deep IR region (in continuum theory) where the smooth-type behavior of the full gluon propagator effectively takes place.

In the second case, on account of the numerical value of the nonperturbative scale $M\approx 1$ GeV, Eq. (4.9) numerically becomes

$$\epsilon_{YM}(2.37)=-0.174\ \text{GeV}^4. \tag{4.11}$$

In Refs. [21,22] an analytical formalism has been developed which allows one to calculate the topological susceptibility as a function of the truly nonperturbative YM VED. The corresponding expression is

$$\chi_t=-\left(\frac{4\xi}{3}\right)^2\epsilon_{YM}, \tag{4.12}$$

where the parameter ξ has two different values, namely, $\xi_{NSVZ}=2/11$ and $\xi_{HZ}=4/33$ (see Ref. [22]). Evaluating Eq. (4.12) numerically, on account of Eq. (4.11), one obtains $\chi_t^{NSVZ}=(550.8\ \text{MeV})^4$ and $\chi_t^{HZ}=(259.6\ \text{MeV})^4$, while its phenomenological value is $\chi_t^{phen}=(180.36\ \text{MeV})^4$. Thus, Eq. (4.11) substantially overestimates the phenomenological value of the topological susceptibility (in both modes) and consequently the mass of the η' meson in the chiral limit, indeed. The soft cutoff in this case is $q_0\approx 1.54M\approx 1.54$ GeV. It is also hard to imagine that the deep IR region (in continuum theory) can be effectively extended up to ≈ 1.54 GeV especially for the smooth-type behavior of the full gluon propagator there. The continuum limit of the scale parameter M is not known, so its realistic numerical value still remains to be well established, and so does the selection from solutions, Eqs. (4.8) and (4.9). Let us note that in accordance with the general scheme of our method we distinguish the nonperturbative scale of this model from the perturbative one but for simplicity we retain the same notation. Evidently, one will obtain the same numerical results for the truly nonperturbative YM VED by choosing the set of variables of A type.

V. IRV GLUON PROPAGATOR

The IRV full gluon propagator is represented by the so-called Zwanziger-Stingle (ZS) formula [54,55]

$$d(-q^2)=\frac{(-q^2)^2}{(-q^2)^2+\mu^4}, \tag{5.1}$$

in the whole range, where μ^4 is again the mass scale parameter responsible for the nonperturbative dynamics in this model, i.e., $\mu \equiv \Lambda_{NP}$, in our notation. When it is zero, then the ZS gluon propagator (5.1) becomes a free perturbative one, indeed. Though the full gluon propagator (5.1) is nonperturbative itself, however, its truly nonperturbative part is determined by the subtraction (1.2), i.e.,

$$d^{NP}(q^2)=d(q^2,\mu^4)-d(q^2,\mu^4=0)=-\frac{\mu^4}{(-q^2)^2+\mu^4}. \tag{5.2}$$

Since this expression is rather simple, it will be instructive to perform calculations in both schemes, *A*, Eqs. (2.10), and *B*, Eqs. (2.13). So let us start from the *A* scheme.

A. Fixing Λ_{YM}

Within the *A*-type set of variables, Eqs. (2.10), $d^{NP}(q^2)$ from Eq. (5.2) becomes $d^{NP}(z,b)=-(b^2/b^2+z^2)$ (Euclidean space). After integration over four-dimensional Euclidean space in Eq. (2.12), one obtains

$$\Omega_g(z_0,b)=\frac{1}{8\pi^2}\{-8b^2\ln 2b^2+8b^2\ln(2b^2-z_0^2)+(b^2+4z_0^2)\ln(b^2+z_0^2)-4z_0^2\ln(z_0^2-2b^2)-b^2\ln b^2\}. \tag{5.3}$$

From this expression it follows obviously that the effective potential (5.3) at any finite relation between the soft cutoff z_0 and parameter b will always contain the imaginary part, which is a direct manifestation of the vacuum instability [56] in this model. Its asymptotics at $b\to 0,\infty$ to leading order can be easily evaluated analytically. Omitting all intermediate calculations, one finally obtains, $\Omega_g(z_0,b)\sim_{b\to 0}-(9/8\pi^2)b^2\ln b^2$ and $\Omega_g(z_0,b)\sim_{b\to\infty}-(1/8\pi^2)[3+4\ln(-2)]z_0^2$, confirming the vacuum instability. Let us also consider the corresponding formal ''stationary'' condition $\partial\Omega_g(z_0,b)/\partial b=0$, which yields

$$3t_0^2+(1+t_0^2)\ln(1+t_0^2)+8(1+t_0^2)\ln\left(1-\frac{t_0^2}{2}\right)=0, \tag{5.4}$$

where $t_0^2=(z_0^2/b^2)$. It has only a trivial solution $t_0=z_0=0$.

Thus the vacuum of this model is unstable, indeed, so it has no relation to quark confinement and DCSB. Our conclusion is in full agreement with conclusion given in Ref. [57]. The particular type of expressions for the dressed-quark-gluon vertex free from ghost contributions were used in their investigation. Our result, however, is a general one since it does not require the particular choice of the dressed-quark-gluon vertex.

B. Fixing the soft cutoff

Within the *B*-type set of variables, Eqs. (2.13), $d^{NP}(q^2)$ from Eq. (5.2) becomes $d^{NP}(z)=-(1/1+z^2)$ (Euclidean space). After almost trivial integration over four-dimensional Euclidean space in Eq. (2.16), one obtains

$$\bar{\Omega}_g(z_0)=\frac{1}{8\pi^2}z_0^{-2}\{-8\ln 2+8\ln(2-z_0^2)+(1+4z_0^2)\ln(1+z_0^2)-4z_0^2\ln(z_0^2-2)\}. \tag{5.5}$$

From this expression it obviously follows that the effective potential at any finite value of the soft cutoff z_0 will always contain the imaginary part, which is a direct manifestation of the vacuum instability [56] as was indicated above. Its asymptotics at $z_0\to 0,\infty$ to leading order can be easily evaluated analytically. Omitting all intermediate calculations, one finally obtains $\bar{\Omega}_g(z_0)\sim_{z_0\to 0}-(1/8\pi^2)[3+4\ln(-2)]$ and $\bar{\Omega}_g(z_0)\sim_{z_0\to\infty}(9/8\pi^2)z_0^{-2}\ln z_0^2$, so the vacuum of this model is unstable, indeed. In order to confirm this, let us consider the corresponding formal ''stationary'' condition which is

$$3z_0^2+(1+z_0^2)\ln(1+z_0^2)+8(1+z_0^2)\ln\left(1-\frac{z_0^2}{2}\right)=0. \tag{5.6}$$

It has only a trivial solution $z_0=0$.

In Ref. [57] a modification of the ZS propagator (5.1) was proposed which took into consideration the renormalization group improvements to leading order for the running coupling constant in the UV region, namely,

$$d(-q^2)=\frac{(-q^2)^2}{(-q^2)^2+\mu^4}\,\frac{\text{const}}{\ln\left(\tau+\frac{q^2}{\Lambda_{QCD}^2}\right)}. \tag{5.7}$$

Here ''const'' obviously depends on the first coefficient of the β function and an unphysical parameter τ is introduced in order to regulate the unphysical singularity—Landau pole—at $q^2=\Lambda_{QCD}^2$ (Euclidean space). The truly nonperturbative part now is

$$d^{NP}(q^2)=d(q^2,\mu^4)-d(q^2,\mu^4=0)=-\frac{\mu^4}{(-q^2)^2+\mu^4}\,\frac{\text{const}}{\ln\left(\tau+\frac{q^2}{\Lambda_{QCD}^2}\right)}. \tag{5.8}$$

However, it is possible to show that the YM VED continues to contain imaginary part in this case as well. It is worth noting that in the derivation of the corresponding expression for the YM VED (2.9) all terms depending in general on some unphysical parameters (in this case τ) should be additionally subtracted by means of ghosts [as was mentioned above in Sec. II just after Eq. (2.9)]. Concluding, let us note that neither Eq. (5.2) nor (5.8) coincides with the deep IR asymptotics of the corresponding full gluon propagators (5.1) and (5.7).

VI. CONCLUSIONS

In summary, we have formulated a general method as to how to numerically calculate the quantum part of the truly nonperturbative YM VED (the bag constant, apart from the sign, by definition) in the covariant gauge QCD quantum models of its ground state using the effective potential approach for composite operators. It is defined as integrated out of the truly nonperturbative part of the full gluon propagator over the deep IR region (soft momentum region), Eq. (2.9). The nontrivial minimization procedure makes it possible to determine the value of the soft cutoff as a function of the corresponding nonperturbative scale parameter which is inevitably present in any nonperturbative full gluon propagator model. If the chosen ansatz for the full gluon propagator is a realistic one, then our general method gives the truly nonperturbative YM VED which is always finite, automatically negative, and has no imaginary part (stable vacuum) (Secs. III and IV). Its numerical value does not, of course, depend on how one introduces the scale dependence by choosing different scale parameters as was described above in Secs. II A and II B; i.e., both sets of variables lead to the same numerical value of the truly nonperturbative YM VED.

From a comparison of Eqs. (2.3) and (2.9), a prescription can be derived as to how one can obtain the relevant expression for the truly nonperturbative YM VED. For this purpose the full gluon propagator in Eq. (2.3) should be replaced by its truly nonperturbative part in accordance with Eq. (1.2). The constant a should be omitted (it has already been explained why) and the soft cutoff q_0^2 on the upper limit should be introduced. Now it looks like the UV cutoff. Nevertheless, let us underline once more that it separates the deep IR region from the perturbative one, which includes the IM region as well. It has a clear physical meaning as determining the range where the deep IR asymptotics of the full gluon propagator is valid. By definition it cannot be arbitrary large as the UV cutoff is. As far as one chooses the ansatz for the full gluon propagator, the separation of ''nonperturbative versus perturbative'' is exact because of the definition (1.2). The separation of ''soft versus hard'' momenta is also exact because of the above-mentioned minimization procedure. Thus the proposed determination of the truly nonperturbative YM VED is uniquely defined. The nontrivial minimization procedure can be done only by two ways: first, to minimize the effective potential at a fixed scale (2.11),(2.12) with respect to the physically meaningful parameter. When it is zero, the perturbative phase only survives in all models of the QCD ground state. Equivalently, we can minimize the auxiliary effective potential (2.16) as a function of the soft cutoff z_0 itself. When it goes to infinity, then again the perturbative phase survives only. On the other hand, both effective potentials (2.12) and (2.16) should go to zero in the perturbative limit since the perturbative contributions have been already subtracted from the very beginning (see Sec. II). As was emphasized above, both methods lead to the same numerical value for the truly nonperturbative YM VED.

We have shown explicitly that the IRE gluon propagator (3.2) as well as IRF (4.1) corresponds to the nontrivial VED which is always finite, negative, and has no imaginary part (stable vacuum). In this way they reflect some physical types of excitations of gluon field configurations in the QCD vacuum. At the same time, the IRV gluon propagators (5.1) and (5.2) lead to an unstable vacuum and therefore are physically impossible. However, these results are by no means general. For example, to come to the same conclusion for the IRV gluon propagator obtained and investigated in Refs. [31,32] it is necessary to proceed along the lines of our method. Thus the proposed method is a precisely general one and each particular model for the full gluon propagator should be separately analyzed within its framework. However, it seems to us that the unstable vacuum is a fundamental defect of all vacuum models based on the IRV-type behavior of the full gluon propagator. It is worthwhile also noting that, in contrast to the IRE gluon propagator, the smooth behavior of the full gluon propagator in the IR domain is hard to relate to quark confinement and DCSB.

Thus our method can serve as a test of any different QCD vacuum models (quantum or classical) since it provides an exact criterion for the separation of ''stable versus unstable vacuum.'' Vacuum stability in classical models is important as well. For example, we have already shown [26] that the vacuum of the Abelian Higgs model without string contributions is unstable against quantum corrections.

There is no general method of calculation of the confining quark contribution to the total VED. In quantum theory it heavily depends on the particular solutions of the corresponding quark SD equation, on account of the chosen ansatz for the full gluon propagator. If it is correctly calculated, then it is of opposite sign to the nonperturbative gluon part and it is one order of magnitude less (see, for example, our papers [21,22,29,30]). Our method is not a solution for the fundamental badly divergent problem of VED in QCD. Moreover, it is even not necessary to deal with this problem. What is necessary, indeed, is to be able to extract the finite part of the truly nonperturbative VED in a self-consistent way. Just this is provided by our method which thus can be applied to any nontrivial QCD vacuum quantum and classical models.

In conclusion, let us make some remarks. In some cases together with the nonperturbative scale some other parameter(s) should be considered as ''running'' in accordance with the general scheme of our method. For example, such a situation will arise in the IRF model gluon propagator suggested by lattice calculations in Ref. [58] (see also Ref. [59]). In this case the general procedure of calculation of the truly nonperturbative YM VED (if any) remains, of course, unchanged. However, because of some technical details [for example, the corresponding ''stationary'' condition (2.17) will be more complicated], this case requires a separate consideration. Brief recent reviews on both continuum and lattice gluon propagators can be found in Refs. [15,53]. An attempt at a VED calculation by the introduction of a rather controversial gluon mass was made in a recent paper [60].

ACKNOWLEDGMENTS

One of the authors (V.G.) is grateful to M. Polikarpov, M. Chernodub, A. Ivanov, and especially to V.I. Zakharov for useful and critical discussions which finally led the authors to the formulation of the general method presented here. He also would like to thank H. Toki for many discussions on this subject during his stay at RCNP, Osaka University. This work was supported by OTKA Grant Nos. T016743 and T029440.

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