

Dynamical gauge symmetry breaking in an $SU(3)_L \otimes U(1)_X$ extension of the standard model

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We study the $SU(3)_L \otimes U(1)_X$ extension of the standard model with a strong $U(1)$ coupling. We argue that current experiments limit this coupling to be relatively large. The model is dynamically broken to the standard $SU(2)_L \otimes U(1)$ model at the scale of a few TeV with all the extra gauge bosons and the exotic quarks acquiring masses much larger than the scale of electroweak symmetry breaking. Furthermore we find that the model naturally displays the top condensation mechanism for the dynamical breakdown of electroweak gauge symmetry. By adding an $SU(3)_L$ singlet quark χ , we find that the model can fit the experimental masses of the top quark and the W, Z bosons. Based on the dynamical symmetry breaking mechanism we predict the masses of the exotic quarks in this model to be of the order of a few TeV.

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I. INTRODUCTION

The $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ extension [1,2] of the standard model predicts interesting new physics at the TeV scale. In the minimal version of the model, one requires four Higgs multiplets in order to generate experimentally acceptable mass spectra. The scale of symmetry breaking, M_Y , is in the range of several hundred GeV to a few TeV and is fixed by the value of $\sin^2(\theta_W)$ which at this scale is given by $4 \sin^2(\theta_W) = 1/[1 + g^2/(4g_X^2)]$ [3], g and g_X are the $SU(3)_L$ and $U(1)_X$ coupling constants. The scale M_Y is fixed by evolving this coupling so that its value at the electroweak symmetry breaking scale agrees with the experimental result.

The model predicts five new gauge particles: one neutral Z' , which is dominantly the $U(1)_X$ gauge boson and four charged bileptons. It also predicts three new exotic quarks with charges $-\frac{4}{3}$, $-\frac{4}{3}$, and $\frac{5}{3}$, respectively for the three generations. The mass of the neutral gauge boson Z' is experimentally constrained to be above 1.7 TeV [3]. This also requires that the gauge coupling for the $U(1)_X$ interaction is relatively large. For example, if $M(Z') = 1.7$ TeV, then the $U(1)_X$ coupling g_X is roughly 1.5 and increases with further rise in the mass of Z' . This raises the possibility that this coupling may in fact be strong enough to dynamically break the gauge symmetry of this model without requiring the introduction of fundamental Higgs particles. In the present paper we investigate this possibility and determine its phenomenological consequences.

The fermion representations [2] consist of the lepton triplets

$$\begin{pmatrix} e \\ \nu_e \\ e^c \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \\ \mu^c \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \\ \tau^c \end{pmatrix}_L,$$

which transform as 3^* under $SU(3)_L$ with the $U(1)_X$ hypercharge equal to 0, $\bar{3}$ and the quark triplets

$$\begin{pmatrix} u \\ d \\ D \end{pmatrix}_L, \begin{pmatrix} c \\ s \\ S \end{pmatrix}_L, \begin{pmatrix} b \\ t \\ T \end{pmatrix}_L,$$

transforming as 3, 3, and 3^* , respectively. The $U(1)_X$ hypercharges of these three quark triplets are $\frac{2}{3}$, $\frac{2}{3}$, and $-\frac{4}{3}$, respectively. The right handed quarks are singlets under $SU(3)$ with $U(1)_X$ hypercharges $Y(u_R) = Y(c_R) = Y(t_R) = -\frac{4}{3}$, $Y(d_R) = Y(s_R) = Y(b_R) = \frac{2}{3}$, $Y(D_R) = Y(S_R) = \frac{8}{3}$, and $Y(T_R) = -\frac{10}{3}$. Note that the third quark generation is treated asymmetrically from the first two. The third component of the quark generations corresponds to new exotic quarks. The current experimental lower limits on the exotic quarks are 200 GeV [4]. The new charged gauge bosons Y^+ and Y^{++} in this model are constrained to have masses above 270 GeV [5]. Neutrino oscillations further constrain these masses to be above 300 GeV [6]. Muonium-antimuonium conversion data, however, give the most stringent limit of 800 GeV [7].

One of the unique and very interesting features of this model is that it requires at least three generations of fermions for anomaly cancellation, hence providing a justification for their existence. The model has been well studied in the literature and has been found to lead to phenomenological predictions which are consistent with current experimental data. It predicts interesting new physics at the next generation of colliders such as lepton number violation due to the existence of bilepton gauge bosons.

II. DYNAMICAL SYMMETRY BREAKING

The experimental lower bound of 1.5 on g_X leads to the Landau pole in the TeV scale. The one loop evolution of the $U(1)_X$ coupling α_X leads to the Landau pole at $Q = \mu \exp(24\pi^2/g_X^2[\Sigma Y^2])$. Here the summation over Y^2 of all the quarks is equal to $\frac{288}{9}$. For $g_X(M_{Z'}) = 1.53$ ($M_{Z'} = 1.7$ TeV) leads to Landau pole at 40 TeV, which indicates the strong nature of the interaction and that we may expect

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strong dynamical effects at TeV energies.

The pattern of dynamical symmetry breaking can be obtained by analyzing the Schwinger-Dyson equation for quarks. We will include only the $U(1)_X$ interaction for this purpose since this gives dominant contribution. Furthermore we confine ourselves to the rainbow approximation. For a quark with left hypercharge Y_L and right hypercharge Y_R the corresponding equation for the self energy can be written as

$$\begin{aligned} \Sigma(q) = & \int \frac{d^4k}{(2\pi)^4} \Gamma_\mu \frac{i}{\not{k} - \Sigma(k)} \Gamma_\nu \\ & \times \frac{[-g^{\mu\nu} + (k-q)^\mu(k-q)^\nu / (k-q)^2]}{(k-q)^2 - M_{Z'}^2}. \end{aligned} \quad (1)$$

The dressed propagator $S(q) = i/[\not{q}A(q^2) - \Sigma(q)]$ where q and $(k-q)$ are the four momentum of the fermion and the gauge particle. In the Schwinger-Dyson equation we have set the wave function renormalization $A(q^2) = 1$. Going beyond this approximation requires a much more extensive calculation [8], which is unnecessary for our purpose. If there exists a solution to this equation such that $\Sigma(q) \neq 0$, then chiral symmetry is dynamically broken. Assuming that such a solution is indeed obtained then the gauge boson Z' will become massive. Since we are interested in obtaining a non-perturbative solution we have included a Z' mass term in this equation. It remains to be verified later that the Z' mass generated by this mechanism is equal to the mass assumed while solving this equation. The form of Γ_μ used in Eq. (1) is given by

$$\begin{aligned} \Gamma_\mu &= g_V \gamma_\mu + g_A \gamma_\mu \gamma_5, \\ g_V &= \frac{g_X}{2} \frac{(Y_L + Y_R)}{2}, \\ g_A &= \frac{g_X}{2} \frac{(-Y_L + Y_R)}{2}. \end{aligned}$$

The final equation for the quark self-energy Σ is

$$\begin{aligned} \Sigma(q) = & -i \frac{1}{2} (g_X^2 Y_L Y_R) \\ & \times \int \frac{d^2k}{(2\pi)^4} \frac{\Sigma(k)}{[k^2 - \Sigma^2(k)][(k-q)^2 - M_{Z'}^2]} \end{aligned} \quad (2)$$

The relevant coupling factor is therefore $g_X^2 Y_L Y_R$. We find that the factor $Y_L Y_R$ is equal to $\frac{40}{9}$ for the third generation exotic quark T and $\frac{16}{9}$ for the other two exotic quarks as well for the top quark. For the remaining quarks this factor is considerably smaller or negative being $\frac{4}{9}$ for down and strange quarks and $-\frac{8}{9}$ for up, charm, and bottom quarks. Based on these hypercharges we find that only the three exotic quarks and the top quark can be above the critical value for dynamical symmetry breaking. This leads to a mass pat-

tern where the exotic T quark is very heavy and the other two exotic quarks have masses comparable to top with the rest of the quarks remaining massless. The reason why only four of these quarks acquire dynamical masses is because the effective coupling for the top quark has to be very close to the critical coupling necessary for dynamical symmetry breaking in order to maintain the large ratio Λ/m_t , where Λ is the scale of $SU(3)_L \otimes U(1)_X$ breaking which is expected to be of the order of a few TeV. In order to assure that at least the top and the exotic quarks, corresponding to the first two generations, acquire dynamical masses, we have to demand that the effective coupling is slightly above the critical value. A certain amount of fine tuning is required in order to maintain the small mass of the top quark. The fine tuning required is not excessive since the mass of the top is only one order of magnitude smaller than the scale of $SU(3)_L \otimes U(1)_X$ breaking. The effective couplings of the remaining quarks are therefore necessarily below the critical value required for dynamical symmetry breaking.

In order to get an estimate of the dynamically generated fermion and gauge boson masses we numerically solve the gap equation in the ladder approximation, imposing an ultraviolet cutoff Λ_c on this equation. If the gap equation accepts a $M_T \neq 0$ solution, where M_T is the T quark mass, then the gauge symmetry is dynamically broken and the Z' , Y^\pm , and $Y^{\pm\pm}$ gauge bosons become massive with masses equal to $g_X F_\Pi (Y_R - Y_L)/4$, $g F_\Pi/2$, and $g F_\Pi/2$, respectively, where F_Π is the pseudoscalar decay constant and we calculate it using the Pagels-Stokar approximation [9], which should be sufficient for our purpose. More detailed analysis [8] requires considerable more numerical effort and is expected to give results within 20% of this formula. A consistency requirement imposed on our solution is that the mass of the exchanged particle Z' has to be equal to the mass generated by the dynamical mechanism. The solutions to the gap equation are therefore iteratively improved by starting with a trial guess for the exchanged boson mass and then comparing it with the predicted mass obtained using the decay constant. Solutions to this equation for different cutoff and coupling g_X choice are shown in Table I.

Next we consider the top quark and the first two generation exotic quarks (D,S). The product of hypercharges in this case is $Y_R Y_L = \frac{16}{9}$ which is considerably smaller than the corresponding value for T . Since the mass of T quark is in the TeV regime and mass of the top quark $m_t = 175$ GeV, the effective coupling for the case of the top quark has to be very close to critical value and require a certain fine tuning. However since the ratio of the these two scales is only about 0.1, the fine tuning required is not severe. Since the effective coupling of the exotic quarks belonging to the first two generations is identical to that of the top quark, we predict that the mass of these exotic quarks should be of the same order as that of the top quark.

The degeneracy between the mass of top quarks and these two exotic quarks is broken after we include the effect of Z boson and photon. We include these contributions also in the rainbow approximation. The contribution of Z as well as the photon to the exotic quark mass is larger than the top. This is because the electric charge as well as the effective Z cou-

TABLE I. The dynamically generated gauge boson and heavy quark masses for different $U(1)_X$ gauge coupling and cutoff. The dynamically generated top quark and W boson mass is fixed to be 175 GeV and 80 GeV, respectively. The first two generation exotic quarks D, S are degenerate.

$M_{Z'}$ (TeV)	M_Y (TeV)	Λ_c (TeV)	m_T (TeV)	g_X	m_D, m_S (TeV)
2.13	0.26	42	5.7	2.576	0.78
3.07	0.37	90	8.2	2.540	1.0
4.08	0.50	162	10.9	2.524	1.3
4.47	0.55	196	11.9	2.521	1.4
5.00	0.61	247	13.4	2.514	1.6
7.09	0.87	504	19.0	2.507	2.2

pling factor of these quarks is larger in comparison to the top. These coupling, although relatively small compared to the strong $U(1)_X$ coupling, give a substantial contribution to the top, D and S quark masses. This is because the effective $U(1)_X$ coupling of these quarks is close to critical value and even the small effects including the Z boson and photon makes a significant contribution.

Once the $t\bar{t}$ condensate is formed we get a direct prediction for the mass of the electroweak gauge bosons W and Z , assuming that there is no fundamental Higgs interaction present in the model. The situation here is very similar to the one studied in Ref. [10] except that in our case we do not need to introduce any four fermi interaction by hand. Unfortunately the prediction turns out to be about half the experimental masses. For the range of parameters given in Table I, we find that the mass of W boson ranges from 30 GeV to 40 GeV. Alternatively if we fix the W boson mass to its experimental value the top quark turns out to be too heavy. An analogous situation was found in [10] where the authors had to choose very large cutoffs in order that with a top quark mass of around 175 GeV, the experimental value of electroweak gauge particles is obtained. The relevant scale in our model cannot however be much larger than a few TeV and hence our prediction for m_W and m_Z is necessarily much smaller. This might indicate the need for introducing some fundamental Higgs multiplets or further modifying the gauge structure of the model. The final result will also depend to some extent on the precise truncation scheme chosen for the integral equations. It may be better, for example, to also include the scale dependence of the $U(1)_X$ coupling since it varies rapidly in this region. However we have tested the sensitivity of our prediction by including this scale dependence as well as by using a truncation scheme of the coupled Schwinger-Dyson and Bethe-Salpeter equation proposed in Ref. [8] and find that it changes by less than 20%. The prediction of the decay constant and hence of the W boson mass is thus expected to be quite reliable.

The lower prediction for the W boson mass in this model is expected since it arises in all models which propose to break electroweak symmetry purely by top condensation [10–12]. A solution to this problem has recently been proposed [13,14] which lowers the mass of the top quark through a seesaw mechanism. This arises due to the mixing of the top quark with another particle which is a weak $SU(2)$ singlet. An analogous mechanism can be implemented in our model by the introduction of new quarks χ_L and χ_R

which transform as $(3, 1, -\frac{4}{3})$ under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. The model remains anomaly free after introduction of these fermions. Since χ do not couple to $SU(3)_L$, we can add the mass terms $m_{\chi\chi}\bar{\chi}_L\chi_R + m_{\chi t}\bar{\chi}_L t_R + \text{H.c.}$ to the Lagrangian without explicitly breaking the gauge symmetry. Following Refs. [13,14] the mass matrix of the top, χ quarks is given by

$$\begin{pmatrix} m & m \\ m_1 & m_2 \end{pmatrix},$$

where m is the mass of top quark in the absence of the quark χ and the value of $m_2, m_1 (m_2 > m_1 > m)$ is determined by gauge coupling and the explicit mass parameters $m_{\chi\chi}$ and $m_{\chi t}$. After diagonalization the top quark mass is given by $m_t = m(1 - m_1/m_2) < m$ and the ratio of m_t/F_Π can be reduced by adjusting the explicit masses $m_{\chi\chi}$ and $m_{\chi t}$ in order to obtain the experimental result for W and Z masses. The results shown in Table I include the contribution of the quark χ . The ratio m_1/m_2 ranges from 0.48 to 0.30 for the results shown in Table I. We therefore find that the present model can accommodate the observed mass spectrum. It may also be possible to extend the current model by introducing mirror fermions which appear to be very helpful in formulating a dynamical symmetry breaking model [15] and also leads to a seesaw mechanism for the top quark mass.

We point out that there may exist another resolution to the problem of heavy top quark mass. This arises due to the possibility of low scale gravity in a universe with extra dimensions [16,17]. As shown in Ref. [18], if the right handed top is assumed to propagate in the extra dimensions while the remaining standard model fermions are assumed to be confined to four dimensions, the top quark mass can be lowered while maintaining the experimental scale of electroweak symmetry breaking. The same mechanism can also be implemented in the present model.

Current experiments constrain the masses of exotic quarks to be higher than 200 GeV [4]. Our prediction for the mass of the lightest exotic quark is a few TeV which can only be probed in the next generation of colliders. There also exist stringent limits on the mass of the bilepton gauge bosons Y^{--} and Y^- . The most stringent bound is obtained from muonium to antimuonium conversion of about 800 GeV [7]. We note that since we do not require our model be perturbative, our upper limit on the mass of these particles is much

higher by the order of a few TeV [3] and the model is not currently ruled out. We also point out that this bound of 800 GeV is obtained with the assumption that the CKM matrix for the coupling of leptons to SU(3) gauge bosons Y^{--} and Y^- is essentially equal to identity. This assumption is quite reasonable since as shown in [19], current experimental data already demands that the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix be close to one. However there is one other possibility that is not ruled out currently by experiments. This is the case of maximal mixing in the first two generations with $V_{11} \approx V_{22} \approx 0$ and $V_{12} \approx 1$. Reference [19] claims to rule this out also at 95% confidence level by using the upper limit on the mass of M_Y to be 430 GeV. However, as long as we do not impose the constraint that the theory remains within the perturbative regime, the limit on the Y mass is much higher [3]. Hence we argue that this region of the parameter space is so far not thoroughly explored and some of the limits on this model claimed in the literature are not valid if the CKM matrix has this form. This includes, for example, the recent bound on Y mass obtained by using the muonium to antimuonium conversion [7]. In the case of maximal mixing between first two generations the model predicts zero conversion rate and hence, in this case, no limit can be imposed on m_Y based on this process.

III. CONCLUSIONS

In conclusion, we have shown that the $SU(3)_L \otimes U(1)_X$ extension of the standard model naturally displays the top condensation mechanism [10–12,20–22] for electroweak symmetry breaking. The model has several other interesting features such as the requirement of three generations for anomaly cancellation. In the present paper we have shown that current experiments require the U(1) coupling in this model to be strong, which leads to dynamical breakdown of $SU(3)_L \otimes U(1)_X$ to $SU(2)_L \otimes U(1)$ and finally to $U(1)_{EM}$ through top condensation. The model, unfortunately, also suffers from the problem of very heavy top quark mass that arises in all top condensation models. However this problem may be solved by introducing additional fermions [13,14] or by speculating the existence of extra dimensions [18]. We find that the dynamical symmetry breakdown mechanism predicts the masses of the first two generation exotic quarks to be of the order of a few TeV and should be observable at the CERN Large Hadron Collider LHC.

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