Heavy quark mass expansion and intrinsic charm in light hadrons

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We review the technique of heavy quark mass expansion of various operators made of heavy quark fields using a semiclassical approximation. It corresponds to an operator product expansion in the form of a series in the inverse heavy quark mass. This technique applied recently to the axial vector current is used to estimate the charm content of the η, η' mesons and the intrinsic charm contribution to the proton spin. The derivation of heavy quark mass expansion for $\langle \bar{Q}\gamma_5 Q\rangle$ is given here in detail and the expansions of the scalar, vector and tensor current and of $\langle \bar{Q}\nabla_\mu\gamma_\nu Q\rangle$ (a contribution to the energy-momentum tensor) are presented as well. The obtained results are used to estimate the intrinsic charm contribution to various observables.

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I. INTRODUCTION

Nowadays it is established beyond any doubt that the naive picture of light hadrons as made of three constituent quarks (for baryons) or $q\bar{q}$ pairs of constituent quarks (for mesons) is not complete. The deep inelastic scattering experiments revealed the rich sea structure of the nucleon; these experiments showed in particular that a considerable portion of the nucleon spin is carried by the strange component of the nucleon sea. Furthermore there are experimental facts which seem to suggest that a nonvanishing nonperturbative component of intrinsic charm is present in light hadrons [1,2].

We address the problem of intrinsic charm content of light hadrons from the point of view of the heavy quark mass expansion. The $c\bar{c}$ pairs in light hadrons, due to a parametrically large mass of charm quarks, can appear in a light hadron as a virtual state whose lifetime is short, of order $1/m_c$. The nonperturbative (with typical momenta below heavy quark mass m_c) gluon and light quark fluctuations are slowly varying from the "point of view" of the virtual $c\bar{c}$ pair; hence, the heavy quark mass expansion is equivalent to the semiclassical expansion. This expansion allows one to rewrite operators made of heavy quarks in terms of light degrees of freedom (gluons and light quarks). For a detailed discussion of the heavy quark mass expansion see [3].

Let us note also that in the absence of a direct probe of gluons the open charm production is considered as the main source of information on nucleon's gluon distributions. In hard leptoproduction heavy quarks are produced in the leading order via the photon-gluon fusion (PGF). The leading graph for PGF can be related directly to gluon distributions *if one assumes* that there is no intrinsic charm content in the nucleon [no $c(x),\bar{c}(x)$ and no $\Delta c(x),\Delta\bar{c}(x)$ at normalization point $\mu=m_c$]. However now there is much evidence that, in principle, there might be considerable intrinsic charm component in the nucleon wave function even at a low normalization point. For reliable extraction of gluon distributions from open charm electroproduction experiments it is necessary to have quantitative estimates of the intrinsic charm content of the nucleon.

This paper will be organized as follows: In the first part we present the calculation of the expectation value of heavy quark currents in the background of gluon fields using a semiclassical approximation. This corresponds to an expansion in the inverse of the heavy quark mass

$$\langle Q^\dagger(x)\Gamma Q(x)\rangle=\sum_n \frac{1}{m^n}X_\Gamma^{(n)}, \tag{1}$$

where Γ denotes the Lorentz structure of the current and the $X_\Gamma^{(n)}$ are local expressions of the field strength depending on Γ. In Sec. II A we review the large m expansion of the fermion determinant appearing in our definition of the expectation value. In Sec. II B we then outline the expansion of color singlet currents in general before we present our results for the expansion of the axial current, the axial vector current using the axial anomaly equation, as well as for the expansion of the scalar, vector and tensor currents. Finally we show the result of the expansion of $\langle Q^\dagger(x)\nabla_\mu\gamma_\nu Q(x)\rangle$, appearing in the energy-momentum tensor of QCD.

In the second part we discuss the calculation of intrinsic heavy quark content of light hadrons as an application of the heavy quark mass expansion. In the case of charm content of η', η mesons and intrinsic charm contributions to the proton spin we reduce the calculations of these quantities to matrix elements which are already known either phenomenologically or were computed previously. In other cases, like intrinsic charm contribution to the nucleon tensor charge and to energy momentum tensor, the problem is reduced to matrix elements of gluon operators which can be estimated using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, and theory of instanton vacuum.

In the Appendix we present some details of our calculation of the fermion determinant and the expansion of the axial current, which has already been used in [4].

II. HEAVY QUARK EXPANSION OF CURRENTS IN THE BACKGROUND OF GLUON AND LIGHT QUARK FIELDS

The expectation value of a color-singlet quark current made of heavy quarks in the background of gluon and light quark fields can be written after integrating out heavy degrees of freedom as

$$\langle Q^\dagger(x)\Gamma Q(x)\rangle = \det D\ \mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\Gamma|x\rangle, \tag{2}$$

Here Γ denotes an arbitrary Lorentz structure. Note that all calculations will be performed in the Euclidean space-time, so the QCD Dirac operator reads

$$D = i\nabla\!\!\!/ + im, \tag{3}$$

where the covariant derivative is defined as

$$\nabla_\mu = \left(\partial_\mu - i\frac{\lambda^a}{2}A^a_\mu(x)\right) \tag{4}$$

and m is the heavy quark mass. For the conventions and the Euclidization used see Appendix A. Equation (2) can now be expanded in a power series of the inverse heavy quark mass $1/m$ under the assumption that the gradient of the background field strength is small compared to m. The expansion of determinant of the Dirac operator in Eq. (2) has been calculated by a large number of authors, see e.g., [5–7]. We briefly review the calculation of the determinant following [7] since we use the result of this calculation as a check of the expansion of a scalar current of heavy quarks in Sec. II B 3.

A. Expansion of the determinant

The expansion of the determinant for heavy quarks in Eq. (2) yields divergences of various types. Since most of these divergences are connected with the determinant of the free Dirac operator we normalize the determinant with that in zero external field. For the remaining infinity which can be related to the logarithmic renormalization of the coupling constant, we use the so-called ζ regularization. Using the identity

$$\det D = \det D^\dagger = (\det D^\dagger D)^{1/2} \tag{5}$$

the normalized and regularized determinant can be written as follows:

$$\left(\frac{\det D}{\det D_0}\right)_{\zeta-\mathrm{reg}} = \exp\Bigg[-\frac{1}{2}\lim_{s\to 0}\frac{d}{ds}\frac{M^{2s}}{\Gamma(s)}\int_0^\infty dt\, t^{s-1} \times \mathrm{Tr}[e^{-tD^\dagger D} - e^{-tD_0^\dagger D_0}]\Bigg], \tag{6}$$

where D_0 denotes the Dirac operator in the absence of external gluon fields and M is the regulator mass. The functional trace denoted by Tr in Eq. (6) can be calculated with respect to any complete set of states. For further calculations it is convenient to compute functional traces in the basis of plane waves, so that

$$\begin{aligned}\mathrm{Tr}[e^{-tD^\dagger D}] &= \mathrm{tr}_{c,\gamma}\int d^4x\int\frac{d^4k}{(2\pi)^4}e^{-ikx}[e^{-tD^\dagger D}]e^{ikx}\\ &= \mathrm{tr}_{c,\gamma}\int d^4x\int\frac{d^4k}{(2\pi)^4}\\ &\quad\times[e^{-tD^\dagger(\partial_\mu\to\partial_\mu+ik_\mu)D(\partial_\nu\to\partial_\nu+ik_\nu)}]\times 1.\end{aligned} \tag{7}$$

The unity in Eq. (7) points out that the operators here act on unity, so that $\partial_\mu\cdot 1=0$. The further calculations are straightforward: the expression in Eq. (7) can be expanded in powers of the covariant derivative, integrated with respect to k and the Lorentz indices summed. The square of the Dirac operator in Eq. (7) with all differentiation operators shifted, $\partial_\mu\to\partial_\mu+ik_\mu$, is given by

$$\begin{aligned}&D^\dagger(\partial_\mu\to\partial_\mu+ik_\mu)D(\partial_\nu\to\partial_\nu+ik_\nu)\\ &\quad = -\nabla^2+\frac{\sigma}{2}F-2ik\nabla+k^2+m^2,\end{aligned} \tag{8}$$

where we have used

$$F^a_{\mu\nu} = i[\nabla_\mu,\nabla_\nu]^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu \tag{9}$$

$$\Rightarrow -\nabla\!\!\!/\nabla\!\!\!/ = -\nabla^2+\frac{\sigma}{2}F, \tag{10}$$

with the notations $\sigma F=\sigma_{\mu\nu}(\lambda^a/2)F^a_{\mu\nu}$ and $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$ applied.

The details of the expansion of the exponential function in Eq. (7) can be found in Appendix B. Up to order $1/m^2$ in the heavy quark mass expansion we end up with the following result for the fermion determinant:

$$\left(\frac{\det D}{\det D_0}\right)_{\zeta-\mathrm{reg}} = \exp\Bigg[\int d^4x\Bigg(-\frac{1}{48\pi^2}\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta} - \frac{i}{720\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha} - \frac{11}{1440\pi^2}\frac{1}{m^2}\mathrm{tr}_c[\nabla_\gamma,[\nabla_\alpha,F_{\alpha\beta}]]F_{\gamma\beta} + \frac{1}{1440\pi^2}\frac{1}{m^2}\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}] - \frac{7}{5760\pi^2}\frac{1}{m^2}\partial^2\mathrm{tr}_c F_{\gamma\beta}F_{\gamma\beta}\Bigg) + \mathcal{O}\left(\frac{1}{m^4}\right)\Bigg] \tag{11}$$

$$= \exp\Bigg[\int d^4x\Bigg(-\frac{1}{48\pi^2}\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta} - \frac{i}{720\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha} + \frac{1}{120\pi^2}\frac{1}{m^2}\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}]\Bigg) + \mathcal{O}\left(\frac{1}{m^4}\right)\Bigg]. \tag{12}$$

Note that in the last step partial integration has been used with all total derivatives left out. The effective action $S_{\mathrm{eff},E} = -\ln\det D$ which can be yielded from Eq. (12), rotated back to Minkowski space, corresponds exactly to the result of [6,7]

$$S_{\mathrm{eff,M}} = -\frac{1}{48\pi^2}\int d^4x\Bigg[\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F^{\alpha\beta} - \frac{i}{15\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F^{\beta}{}_{\gamma}F^{\gamma\alpha}\Bigg] + \mathcal{O}\left(\frac{1}{m^4}\right), \tag{13}$$

where equation of motion terms, which vanish in pure Yang-Mills theory $[\nabla_\alpha,F_{\alpha\beta}]=0$ have been neglected.

B. Expansion of heavy quark currents

In order to expand $\mathrm{tr}_{c,\gamma}\langle x|(1/D)\Gamma|x\rangle$ in Eq. (2) in a series of the inverse heavy quark mass we can use Eq. (8) to rewrite it as

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\Gamma|x\rangle = \mathrm{tr}_{c,\gamma}\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{1}{D^\dagger D}D^\dagger\Gamma e^{ikx} = \mathrm{tr}_{c,\gamma}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\sum_{n=0}^{\infty}\left(\frac{\nabla^2-\frac{\sigma}{2}F+2ik\nabla}{k^2+m^2}\right)^n \times(i\not\nabla-\not k-im)\Gamma\cdot 1. \tag{14}$$

The expansion in Eq. (14) is again justified for small gradients of the gluonic fields compared to the heavy quark mass m. Depending on the Lorentz structure Γ some of the integrals might be divergent and need to be regularized. For this we choose the dimensional regularization, since the integrals in Eq. (14) can then be calculated using

$$\int\frac{d^dk}{(2\pi)^d}\frac{k_{\mu_1}k_{\mu_2}\dots k_{\mu_{2m}}}{(k^2+m^2)^n} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-m-d/2)}{\Gamma(n)2^m}\delta_{\mu_1\dots\mu_{2m}}\left(\frac{1}{m^2}\right)^{n-m-d/2}, \tag{15}$$

with $\delta_{\mu_1\dots\mu_{2n}}$ denoting all possible contractions:

$$\delta_{\mu_1\dots\mu_{2n}} = \exp\left[\frac{1}{2}\frac{\partial^2}{\partial\phi_\nu\partial\phi_\nu}\right]\phi_{\mu_1}\dots\phi_{\mu_{2n}}\big|_{\phi=0}. \tag{16}$$

The number of terms contributing to a given order of $1/m$ is reduced by the fact that terms containing an odd number of γ matrices or an odd number of k's vanish due to the trace over Lorentz indices and the integration with respect to k. The expansion of Eq. (14) then is straightforward. The result of the expansion must be gauge invariant because we expand the gauge invariant operator. In order to obtain explicitly the gauge invariant result for heavy quark mass expansion a number of helpful identities based on the Bianchi identity

$$[\nabla_\alpha,F_{\beta\gamma}]+[\nabla_\gamma,F_{\alpha\beta}]+[\nabla_\beta,F_{\gamma\alpha}]=0 \tag{17}$$

can be derived.

In the following we present the result of the expansion of the pseudoscalar density and the divergency of the axial-vector current, which are related to each other by the axial anomaly. Since recently confusing results for these cases were reported in the literature [8–10], the full details of our calculation are given in Appendices A–C. Further we present the result of the expansion of scalar, vector and tensor currents and of $\langle\overline{Q}\nabla_\mu\gamma_\nu Q\rangle$ appearing in the energy-momentum tensor of QCD.

1. The pseudoscalar density

For $\Gamma=\gamma_5$ the expansion Eq. (14) has the form

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\ \gamma_5|x\rangle=-im\ \mathrm{tr}_{c,\gamma}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\times\sum_{n=0}^{\infty}\left(\frac{\nabla^2-\frac{\sigma}{2}F+2ik\nabla}{k^2+m^2}\right)^n\gamma_5\times 1 \tag{18}$$

Following the steps outlined before the expansion of the pseudoscalar density Eq. (18) can be rewritten as (see Appendix C)

$$\begin{aligned}\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle=&\frac{i}{16\pi^2 m}\mathrm{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta}\\&+\frac{i}{192\pi^2 m^3}\partial^2\mathrm{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta}\\&+\frac{i}{48\pi^2 m^3}\partial_\alpha\mathrm{tr}_c[\nabla_\rho,F_{\rho\beta}]\tilde{F}_{\alpha\beta}+\mathcal{O}\left(\frac{1}{m^5}\right),\end{aligned} \tag{19}$$

where we have introduced the common notation $\tilde{F}_{\alpha\beta}=\frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$.

2. The divergence of the axial vector current

Instead of expanding the axial vector current $j_\mu^5(x)=Q^\dagger(x)\gamma_\mu\gamma_5 Q(x)$ separately, we can use the divergence of the axial vector current given by

$$\partial_\mu j_\mu^5=2mQ^\dagger\gamma_5 Q-\frac{i}{16\pi^2}F_{\mu\nu}^a\tilde{F}_{\mu\nu}^a, \tag{20}$$

where the first term contains the axial current and the second is the axial anomaly term which arises due to quantum effects. The expansion of the divergence of the axial vector current in terms of the inverse of the heavy quark mass is therefore reduced to the expansion of the axial current, which we have already performed before. Further the axial anomaly equation (20) has some general properties, which can be used to check our result for the axial current:

First the right-hand side (rhs) of Eq. (20) vanishes in the limit of infinite quark mass. This can be understood by the fact that the regulator mass cancels the physical mass in the infinite mass limit because of the different sign in the definition of the regulator. Therefore we expect the order $\mathcal{O}(1/m)$ term in the expectation value of the axial current multiplied by $2m$ to cancel exactly the anomaly term. Indeed Eq. (19) gives

$$2m\ \mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle^{\mathcal{O}(1/m)}=\frac{i}{8\pi^2}\mathrm{tr}_c F_{\mu\nu}\tilde{F}_{\mu\nu}. \tag{21}$$

Second if one thinks of the expectation value of the axial vector current as a local large m expansion in the background of gluon fields

$$\langle j_\mu^5(x)\rangle=\sum_n\frac{1}{m^n}X_{\mu 5}^{(n)}(x), \tag{22}$$

then due to Eq. (20) the expectation value of the axial current in the large m expansion is

$$2m\ \mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle=\sum_n\frac{1}{m^n}\partial_\mu X_{\mu 5}^{(n)}(x). \tag{23}$$

This means that terms appearing in the expansion of the axial current must be of the form of a total derivative. The order $\mathcal{O}(1/m^3)$ term in Eq. (19) exactly obeys this form

$$2m\ \mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\ \gamma_5|x\rangle^{\mathcal{O}(1/m^3)}=\frac{i}{96\pi^2 m^2}\partial_\mu R_\mu, \tag{24}$$

$$R_\mu=\partial_\mu\mathrm{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta}+4\ \mathrm{tr}_c[\nabla_\alpha,F_{\alpha\nu}]\tilde{F}_{\mu\nu}. \tag{25}$$

The term $f^{abc}F_{\mu\nu}^a\tilde{F}_{\nu\alpha}^b F_{\alpha\mu}^c$ appearing falsely in the expansion of the axial current in [8,9] cannot be represented as a total derivative of a local expression[1] and therefore violates the general argument given above.

The expectation value for the divergence of the axial vector current in the background of gluon fields finally reads up to order $\mathcal{O}(1/m^4)$

$$\langle\partial_\mu j_\mu^5(x)\rangle=\frac{i}{96\pi^2 m^2}\partial_\mu(\partial_\mu\mathrm{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta}+4\ \mathrm{tr}_c[\nabla_\alpha,F_{\alpha\nu}]\tilde{F}_{\mu\nu})+\mathcal{O}\left(\frac{1}{m^4}\right). \tag{26}$$

3. The scalar current

Following the steps outlined in the introduction to this section the expansion of a scalar current in series of the inverse heavy quark mass yields up to order $\mathcal{O}(1/m^3)$:

[1] A straightforward calculation for the instanton field shows that $\int d^4x f^{abc}F_{\mu\nu}^a\tilde{F}_{\nu\alpha}^b F_{\alpha\mu}^c\neq 0$. But for dimensional reasons this nonvanishing contribution can be excluded from being generated by a surface term if the instanton field is taken in the regular gauge. Therefore the integrand cannot be a total derivative.

$$
\begin{aligned}
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}|x\rangle &= \mathrm{tr}_{c,\gamma}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\sum_{n=0}^{\infty}\left(\frac{\nabla^2-\frac{\sigma}{2}F+2ik\nabla}{k^2+m^2}\right)^n (i\not{\nabla}-\not{k}-im)\cdot 1 \\
&= -\frac{i}{(4\pi)^{d/2}}\left(\frac{1}{m^2}\right)^{1-(d/2)} m\Gamma\left(1-\frac{d}{2}\right) d\,\mathrm{tr}_c 1 \\
&\quad -\frac{i}{24\pi^2 m}\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}+\frac{1}{360\pi^2 m^3}\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\gamma}F_{\beta\gamma}-\frac{7i}{2880\pi^2 m^3}\partial^2\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta} \\
&\quad -\frac{i}{720\pi^2 m^3}(11\,\mathrm{tr}_c[\nabla_\alpha,[\nabla_\beta,F_{\beta\gamma}]]F_{\alpha\gamma}-\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}])+\mathcal{O}\left(\frac{1}{m^5}\right).
\end{aligned}
\tag{27}
$$

The infinite constant term can be cancelled by substracting the expectation value of the scalar current for vanishing gluonic background fields. Our result Eq. (27) coincides with that obtained in [11] if we neglect the total derivative terms which were ignored in [11].

Actually the result Eq. (27) with the total derivative terms neglected (and hence that of [11]) can be easily obtained from the expansion of the determinant of the Dirac operator Eq. (11), since

$$
\begin{aligned}
\int d^4x\,\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}|x\rangle &= \frac{d}{dm}[-i\ln(\det D)] \\
&= \frac{d}{dm}(-i\,\mathrm{Tr}\ln D).
\end{aligned}
\tag{28}
$$

Our expansion of the scalar current Eq. (27) is in agreement with the result for the determinat in Eq. (11).

4. The vector current

The heavy quark expansion of the vector current up to order $1/m^3$ gives exactly zero

$$
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_\mu|x\rangle = 0+\mathcal{O}\left(\frac{1}{m^4}\right). \tag{29}
$$

This result can be easily anticipated from the fact that the vector current is C parity odd. This implies that the first operator contributing to heavy quark mass expansion should contain at least three gluon fields, additionally the vector current conservation requires that this operator has the following structure: ∇G^3. From counting of dimensions we see that such an operator can contribute only at $1/m^4$ order.

5. The tensor current

For the color singlet tensor current we find that the first nonvanishing order of the expansion is $\mathcal{O}(1/m^3)$, yielding

$$
\begin{aligned}
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\,\sigma_{\mu\nu}|x\rangle &= \frac{i}{24\pi^2}\frac{1}{m^3} \\
&\quad \times\mathrm{tr}_c[F_{\alpha\beta}F_{\alpha\beta}F_{\mu\nu}+F_{\alpha\nu}F_{\beta\mu}F_{\alpha\beta} \\
&\quad -F_{\alpha\mu}F_{\beta\nu}F_{\alpha\beta}]+\mathcal{O}\left(\frac{1}{m^5}\right).
\end{aligned}
\tag{30}
$$

We note that the rhs of the above equation vanishes in the case of the SU(2) gauge group. Actually one can show that the left hand side (lhs) of Eq. (30) is identically zero in the case of the SU(2) gauge group. Therefore the fact that the rhs of Eq. (30) vanishes for the SU(2) gauge group is a powerful check of our calculations.

In order to prove that the lhs of Eq. (30) is zero in the case of the SU(2) gauge group we use the following transformation:

$$
G=C\tau^2,
$$

where C is charge conjugation matrix in Dirac spinor space and τ^2 is the color SU(2) matrix. Under this transformation we have:

$$
G\tau^a G^{-1}=-\tau^{aT},
$$

$$
G\sigma_{\mu\nu}G^{-1}=-\sigma^T_{\mu\nu},
$$

$$
GDG^{-1}=D^T,
$$

where ^{T} is the transposition operation. The lhs of Eq. (30) should be zero since

$$
\begin{aligned}
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\sigma_{\mu\nu}|x\rangle &= \mathrm{tr}_{c,\gamma}\langle x|G\frac{1}{D}\sigma_{\mu\nu}G^{-1}|x\rangle \\
&= \mathrm{tr}_{c,\gamma}\langle x|\left(\frac{1}{D}\right)^T(-\sigma^T_{\mu\nu})|x\rangle \\
&= -\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\sigma_{\mu\nu}|x\rangle.
\end{aligned}
\tag{31}
$$

Nullification of the heavy quark mass expansion of the tensor current for the SU(2) gauge group implies that the lhs of Eq. (30) is zero if it is computed in the field of a single instanton.

6. Expansion of $\langle \bar{Q}\nabla_\mu\gamma_\nu Q\rangle$

The energy-momentum tensor of QCD can be written in Minkowski space as

$$T^{\mu\nu}=-g^{\mu\nu}\mathcal{L}_{\mathrm{QCD}}-F^{\mu\alpha}F^{\nu}_{\alpha}+\frac{i}{2}\,\bar{\psi}\overleftrightarrow{\nabla}^{(\mu}\gamma^{\nu)}\psi, \qquad (32)$$

where $(\mu\nu)$ denotes the symmetrization of the indices. The large m expansion of the (not symmetrized) last term in Eq. (32) yields in Euclidean space

$$\begin{aligned}\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle=&\frac{-2i}{(4\pi)^{d/2}}\left(\frac{1}{m^2}\right)^{-(d/2)}\Gamma\left(-\frac{d}{2}\right)\delta_{\mu\nu}\mathrm{tr}_c 1\\
&+\frac{i}{(4\pi)^{d/2}}\left(\frac{1}{m^2}\right)^{2-(d/2)}\Gamma\left(2-\frac{d}{2}\right)\left(-\frac{1}{3}\delta_{\mu\nu}\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}+\frac{4}{3}\mathrm{tr}_c F_{\alpha\nu}F_{\alpha\mu}\right)\\
&+\frac{1}{720\pi^2}\frac{1}{m^2}\delta_{\mu\nu}\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha}-\frac{7i}{5760\pi^2}\frac{1}{m^2}\delta_{\mu\nu}\partial^2\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}\\
&-\frac{i}{1440\pi^2}\frac{1}{m^2}\delta_{\mu\nu}(11\,\mathrm{tr}_c[\nabla_\alpha,[\nabla_\beta,F_{\beta\gamma}]]F_{\alpha\gamma}-\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}])\\
&+\frac{1}{2880\pi^2}\frac{1}{m^2}(-4\,\mathrm{tr}_c F_{\alpha\nu}F_{\beta\mu}F_{\alpha\beta}-4\,\mathrm{tr}_c F_{\alpha\mu}F_{\beta\nu}F_{\alpha\beta}-30i\,\mathrm{tr}_c[\nabla_\alpha,F_{\mu\nu}][\nabla_\beta,F_{\alpha\beta}]\\
&+74i\,\mathrm{tr}_c[\nabla_\alpha,[\nabla_\beta,F_{\beta\nu}]]F_{\alpha\mu}+14i\,\mathrm{tr}_c[\nabla_\alpha,[\nabla_\beta,F_{\beta\mu}]]F_{\alpha\nu}\\
&-26i\,\mathrm{tr}_c[\nabla_\nu,[\nabla_\alpha,F_{\alpha\beta}]]F_{\beta\mu}-26i\,\mathrm{tr}_c[\nabla_\mu,[\nabla_\alpha,F_{\alpha\beta}]]F_{\beta\nu}\\
&+22\,i\,\partial^2\mathrm{tr}_c F_{\beta\nu}F_{\beta\mu}-3i\partial_\mu\partial_\nu\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}-26i\,\mathrm{tr}_c[\nabla_\mu,F_{\alpha\nu}][\nabla_\beta,F_{\alpha\beta}]-26i\,\mathrm{tr}_c[\nabla_\nu,F_{\alpha\mu}][\nabla_\beta,F_{\alpha\beta}]\\
&+40i\,\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\nu}][\nabla_\beta,F_{\beta\mu}]+4i\,\mathrm{tr}_c[\nabla_\nu,F_{\alpha\beta}][\nabla_\mu,F_{\alpha\beta}])+\mathcal{O}\left(\frac{1}{m^4}\right),\end{aligned} \qquad (33)$$

and

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}=\mathrm{tr}_{c,\gamma}\langle x|\nabla_\mu\frac{1}{D}\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}. \qquad (34)$$

The Lorentz trace of Eq. (33) can be compared with the expansion of the scalar current in Eq. (27) since

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\nu\gamma_\nu|x\rangle=-m\,\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}|x\rangle. \qquad (35)$$

For the trace of the rhs of Eq. (33) we find

$$\begin{aligned}\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\nu\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}\\
=\frac{i}{24\pi^2}\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}-\frac{1}{360\pi^2m^2}\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\gamma}F_{\beta\gamma}\\
+\frac{7i}{2880\pi^2m^2}\,\partial^2\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}\\
+\frac{i}{720\pi^2m^2}(11\,\mathrm{tr}_c[\nabla_\alpha,[\nabla_\beta,F_{\beta\gamma}]]F_{\alpha\gamma}\\
-\mathrm{tr}_c[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}]),\end{aligned} \qquad (36)$$

which exactly coincides with the rhs of Eq. (27) multiplied by $(-m)$. Equation (34) further agrees with the expansion of the vector current being zero up to $\mathcal{O}(1/m^3)$ since

$$\mathrm{tr}_{c,\gamma}\langle x|\nabla_\mu\frac{1}{D}\gamma_\nu|x\rangle-\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle=\partial_\mu\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_\nu|x\rangle. \qquad (37)$$

III. INTRINSIC HEAVY QUARKS IN LIGHT HADRONS

In light hadron processes heavy quarks may give contributions only through virtual effects which are suppressed by the mass of the heavy quarks. Especially for the charm quark, whose mass $m_c\approx 1.4$ GeV is not too large, virtual processes nevertheless may not give negligible contributions. In this section we discuss the applications of heavy quark mass expansion obtained in the previous sections.

In the following sections we use the ''perturbative'' normalization for the gluon field strength $G^a_{\mu\nu}=F^a_{\mu\nu}/g_s$ and rotate all expressions to Minkowski space (see Appendices A–C).

A. Intrinsic charm in η and η'

For the decay of the B meson into η' and K mesons in [8] a mechanism with virtual charm quarks was suggested. In this approach the Cabibbo favored process $b\to\bar{c}cs$ is followed by the conversion of the $\bar{c}c$ pair directly into η'. Its contribution to the decay amplitude is therefore direct depending on the ''intrinsic charm'' component of the η' meson which is usually characterized by the matrix element

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'(q)\rangle = i f^{(c)}_{\eta'} q_\mu . \tag{38}$$

Using the heavy mass expansion of the divergence of the axial vector current Eq. (26), the constant $f^{(c)}_{\eta'}$ can be expressed up to the order $1/m_c^2$ by

$$f^{(c)}_{\eta'} = -\frac{1}{12m_c^2}\langle 0|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}|\eta'\rangle. \tag{39}$$

We now can estimate the value of the constant $f^{(c)}_{\eta'}$

$$f^{(c)}_{\eta'}\approx -2 \text{ MeV}, \tag{40}$$

where we have used

$$\langle 0|\frac{\alpha_s}{4\pi}\, G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}|\eta'\rangle = 0.056 \ \ \text{GeV}^3, \tag{41}$$

obtained in [12].

In Eq. (39) we neglected the term proportional to $[\nabla_\alpha, G^\alpha_\nu]$ of Eq. (26) which vanishes in pure Yang–Mills theory and hence is suppressed by the strong coupling constant α_s at the heavy quark mass scale. In QCD the omitted term can be related to the matrix element

$$\frac{\alpha_s}{4\pi}\langle 0|g_s\sum_{q=u,d,s}\bar{q}\gamma_\nu\tilde{G}^\nu_\mu q|\eta'\rangle \tag{42}$$

using the equation of motion. Let us note that the expansion parameter in the heavy quark mass expansion is $g_s G/m_c$, because the nonperturbative gluon field strength $G\sim 1/g_s$ (cf. instanton field). Therefore $g_s(\mu)$ accompanied by gluon field strength is not counted as suppression. Additionally the matrix element Eq. (42) is $1/N_c$ suppressed relative to the main contribution Eq. (39)

A rough order of magnitude estimate for the contribution of the omitted term Eq. (42) to $f^{(c)}_{\eta'}$ using the results of [13] indicates a deviation at the level of 0.3 MeV to the value Eq. (40). A more careful analysis of the omitted matrix element Eq. (42) can be done by using the instanton methods developed in [14] which already have been applied by [13] to calculations of higher twist corrections to deep-inelastic scattering.

Our estimated value for $f^{(c)}_{\eta'}$ is consistent with the phenomenological analysis in [15] where the authors dervived the bound -65 MeV$\leqslant f^{(c)}_{\eta'}\leqslant 15$ MeV from the analysis of $\gamma\eta'$ transition form factors. From the analysis of (η,η',η_c) mixing in [16] the small value $f^{(c)}_{\eta'}=-(6.3\pm 0.6)$ MeV was derived. Taking into account off-shellness effects in the $\bar{c}c$ component of η' also, the value $|f^{(c)}_{\eta'}|\approx 2.4$ MeV was found in [17]. Further our value for $f^{(c)}_{\eta'}$ is in agreement with the phenomenological bound $|f^{(c)}_{\eta'}|<12$ MeV, obtained in [18], and corresponds to the result $f^{(c)}_{\eta'}\approx -2.3$ MeV presented in [19]. In [19] the divergence of the axial vector current was computed using the triangle graph for the axial anomaly with massive fermions, neglecting possible $1/m_c^2$ contributions such as

$$f^{abc}G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu} \tag{43}$$

from higher order diagrams. Indeed our calculation shows that such ''truly nonabelean'' operators do not contribute to the order $1/m_c^2$ and our result Eq. (39) therefore is exactly given by the first term of the expansion in $1/m_c^2$ of the triangle graph [20].

The small value Eq. (40) for $f^{(c)}_{\eta'}$ implies that the $b\to\bar{c}cs$ mechanism does not play a major role in the $B\to K\eta'$ decay mode.

Bigger values of $f^{(c)}_{\eta'}$ due to the operator Eq. (43) in the expansion of the axial current up to order $1/m_c^3$ have been given in [8], where $f^{(c)}_{\eta'}\approx(50-180)$ MeV and in [9] with $f^{(c)}_{\eta'}\approx-(12.3-18.4)$ MeV. These results have been used by a number of authors for the analysis of the charm content in noncharmed hadrons (see e.g., [21,10,22,23]), but since the operator Eq. (43) violates general properties of the axial anomaly and it also does not appear in explicit calculations (see Sec. II B 2), results relying on [8,9] should be reconsidered.

Analogously we can immediately estimate the constant $f^{(c)}_{\eta}$ characterizing the intrinsic charm contribution to the η meson. Using

$$\langle 0|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}|\eta\rangle = 0.020 \ \ \text{GeV}^3, \tag{44}$$

obtained in [12] we find

$$f^{(c)}_{\eta}\approx -0.7 \ \ \text{MeV}. \tag{45}$$

Since in the case of the η meson the contribution of the omitted term Eq. (42) can be of the same order as $f^{(c)}_{\eta}$ itself, the estimate Eq. (45) must be considered as poor.

B. Intrinsic charm contribution to the proton spin

Another application of our result for the heavy quark mass expansion of the divergence of axial vector current Eq. (26) has been given in [20]. In this paper the authors have shown that the intrinsic charm contribution to the first moment of the spin structure function $g_1(x,Q^2)$ of the nucleon is small contrary to the result of [21,22,10].

In [20] it was proven that the forward matrix element of the axial current in the leading order of heavy quark mass expansion can be computed as

$$\langle N(p,S)|\bar{c}\gamma_\mu\gamma_5 c(0)|N(p,S)\rangle = \frac{\alpha_s}{48\pi m_c^2}\langle N(p,S)|R_\mu(0)|N(p,S)\rangle. \quad (46)$$

Here the current $R_\mu(0)$ is given by Eq. (25). Note that the first term in R_μ does not contribute to the forward matrix element because of its gradient form, while the contribution of the second one is rewritten, by making use of the equation of motion, as matrix element of the operator

$$\langle N(p,S)|\bar{c}\gamma_\mu\gamma_5 c(0)|N(p,S)\rangle = \frac{\alpha_s}{12\pi m_c^2}\langle N(p,S)|g_s\sum_{f=u,d,s}\bar{\psi}_f\gamma_\nu\tilde{G}_\mu{}^\nu\psi_f|N(p,S)\rangle \equiv \frac{\alpha_s}{12\pi m_c^2}\,2\omega_N^3 S_\mu f_S^{(2)}. \quad (47)$$

The parameter $f_S^{(2)}$ was determined before in calculations of the power corrections to the first moment of the singlet part of g_1. QCD sum rule calculations gave $f_S^{(2)}=0.09$ [24], estimates using the renormalon approach led to $f_S^{(2)}=\pm 0.02$ [25] and calculations in the instanton model of the QCD vacuum give a result very close to that of the QCD sum rule [13].

Inserting these numbers for the charm axial constant of the nucleon we get finally the estimate

$$g_A^{(c)}=-\frac{\alpha_s}{12\pi}f_S^{(2)}\frac{m_N^2}{m_c^2}\approx -5\times 10^{-4} \quad (48)$$

with probably a 100% uncertainty. Note that this contribution is of nonperturbative origin (therefore we call it intrinsic), so that it is sensitive to large distances, as soon as the factorization scale is of order m_c.

C. Intrinsic charm contribution to the nucleon tensor charge

Using the results of Sec. II B 5 we can estimate the intrinsic charm contribution to the tensor charge of the nucleon. The tensor charge of the nucleon is defined as

$$\langle N(p,S)|\bar{c}\sigma_{\mu\nu}\gamma_5 c(0)|N(p,S)\rangle = 2i\,g_T^{(c)}(p_\mu S_\nu - p_\nu S_\mu). \quad (49)$$

Using the result of Sec. II B 5 and the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta},$$

we obtain for the charm contribution to the nucleon tensor charge the following result:

$$g_T^{(c)}=\frac{1}{96\pi^2}\frac{1}{m_c^3 m_N^2}\varepsilon^{\lambda\rho\mu\nu}p_\lambda S_\rho\langle N(p,S)|\mathrm{tr}_c g_s^3[G_{\alpha\beta}G^{\alpha\beta}G_{\mu\nu} + 2G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}]|N(p,S)\rangle + \mathcal{O}\left(\frac{1}{m_c^5}\right). \quad (50)$$

The matrix element on the rhs of the above equation can be roughly estimated in the instanton vacuum using the method of [14]. As discussed in Sec. II B 5 the gluonic operator on the rhs of Eq. (50) is identically zero on one instanton. Therefore it is the first nontrivial contribution one can get from an instanton–anti-instanton pair. If we compare the expression for the charm contribution to the nucleon charge with that for axial charge we see that the charm contribution to the tensor charge is suppressed by the additional power of m_N/m_c and one power of instanton packing fraction $(\pi^2\bar{\rho}^4/\bar{R}^4)$, however the tensor charge is enhanced by one power of $\alpha_s(m_c)$.[2] This allows us to make a rough estimate for the charm contribution of the tensor charge

$$g_T^{(c)}\sim\frac{m_N}{\alpha_s(m_c)m_c}\frac{(N_c-2)\pi^2\bar{\rho}^4}{N_c\bar{R}^4}g_A^{(c)}\sim 10^{-4}. \quad (51)$$

Factors of N_c are written in a way to reproduce the large N_c behavior of the matrix element and to account for the fact that the operator on the rhs of Eq. (50) is identically zero at $N_c=2$.

D. Intrinsic charm contribution to the nucleon momentum

The charm contribution to the nucleon momentum can be defined as

$$M_2^{(c)}(\mu^2)=\int_0^1 dx_B x_B[c(x_B)+\bar{c}(x_B)] = \frac{i}{2(P\cdot n)^2}\langle N(P)|\bar{c}\not{n}(n\cdot\nabla)c(0)|N(P)\rangle, \quad (52)$$

where $c(x_B)$ is the charm parton distribution normalized at the scale μ, which is assumed to be $\mu\approx m_c$. The light cone vector n is arbitrarily noncollinear to nucleon momentum P.

Now we can use the result of Eq. (33) in order to estimate the charm contribution to the nucleon momentum carried by intrinsic charm quarks:

[2] We stress once more that the expansion parameter in the heavy quark mass expansion is g_sG/m_c, because the non-perturbative gluon field strength $G\sim 1/g_s$ (cf. instanton field). Therefore $g_s(\mu)$ accompanied by gluon field strength is not counted as suppression.

$$M_2^{(c)}(\mu)=\frac{i}{2(P\cdot n)^2}\Bigg[\frac{i\alpha_s(\mu)}{4\pi}\frac{1}{\left(2-\frac{d}{2}\right)}\frac{4}{3}\times\langle N(P)|n^\mu n^\nu \mathrm{tr}_c G^\alpha{}_\nu G_{\alpha\mu}|N(P)\rangle+\frac{1}{120\pi^2}\frac{g_s^3(\mu)}{m_c^2}\times\langle N(P)|n^\mu n^\nu \mathrm{tr}_c G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}|N(P)\rangle\Bigg]+\mathcal{O}\left(\frac{1}{m_c^4}\right). \tag{53}$$

In derivation of this expression we neglected terms that are proportional to $[\nabla^\alpha, G_{\alpha\beta}]$ which are suppressed by one power of $g_s^2(\mu)$. The first term in Eq. (53) is divergent[3] and actually is related to the mixing of quark and gluon operators. We can rewrite Eq. (53) as follows:

$$M_2^{(c)}(\mu)=\frac{4}{3}\frac{\alpha_s(\mu)}{4\pi}\frac{1}{\left(2-\frac{d}{2}\right)}M_2^{(G)}(\mu)+\frac{i}{2(P\cdot n)^2}\frac{1}{120\pi^2}\frac{g_s^3(\mu)}{m_c^2}\times\langle N(P)|n^\mu n^\nu \mathrm{tr}_c G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}|N(P)\rangle+\mathcal{O}\left(\frac{1}{m_c^4}\right). \tag{54}$$

Here the first term, which is proportional to the momentum fraction carried by gluons $M_2^{(G)}(\mu)$, accounts for extrinsic charm. Note that the coefficient in front of this term is exactly the leading anomalous dimension $\gamma_{qG}=4/3$ which accounts for mixing quark and gluon twist-2 operators under QCD evolution. The intrinsic charm contribution is given by the second term, so that we have estimates of

$$M_2^{(c),\,\mathrm{intrinsic}}(\mu)=\frac{i}{2(P\cdot n)^2}\frac{1}{120\pi^2}\frac{g_s^3(\mu)}{m_c^2}\times\langle N(P)|n^\mu n^\nu \mathrm{tr}_c G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}|N(P)\rangle+\mathcal{O}\left(\frac{1}{m_c^4}\right). \tag{55}$$

We see that the momentum fraction carried by intrinsic charm in the nucleon is related to the value of the nucleon matrix element:

$$\langle N(P)|n^\mu n^\nu i g_s^3(\mu)\mathrm{tr}_c G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}|N(P)\rangle.$$

One can easily see that this matrix element in the theory of instanton vacuum [14] is zero in one-instanton approximation, the same as the matrix element

$$\langle N(P)|n^\mu n^\nu g_s^2(\mu)\mathrm{tr}_c G^\alpha{}_\nu G_{\alpha\mu}|N(P)\rangle.$$

Keeping in mind that for instanton field $G\sim 1/g_s$ we can write

$$\frac{\langle N(P)|n^\mu n^\nu i g_s^3(\mu)\mathrm{tr}_c G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}|N(P)\rangle}{\langle N(P)|n^\mu n^\nu g_s^2(\mu)\mathrm{tr}_c G^\alpha_\nu G_{\alpha\mu}|N(P)\rangle}=\Lambda^2, \tag{56}$$

where Λ is parameter of the dimension of mass whose value can be obtained using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, and theory of instanton vacuum. Generically we expect that this mass parameter is on the order of typical strong interaction scale $\Lambda\sim 1$ GeV. Now we can rewrite Eq. (55) in terms of this parameter and momentum fraction carried by gluons in the nucleon at scale $\mu\approx m_c$ as

$$M_2^{(c),\,\mathrm{intrinsic}}(\mu)=\frac{\alpha_s(\mu)}{30\pi}\frac{\Lambda^2}{m_c^2}M_2^G(\mu)+\mathcal{O}\left(\frac{1}{m_c^4}\right). \tag{57}$$

If we assume that $\Lambda^2=\mathrm{few\ GeV}^2$ than we get the estimate for the charm contribution to the nucleon momentum

$$M_2^{(c),\,\mathrm{intrinsic}}(\mu)=\mathrm{few}\times 10^{-3}. \tag{58}$$

We see that the heavy quark mass expansion of local currents allows us to reduce the problem of estimate of intrinsic charm content of the nucleon to the calculation of the ratio Eq. (56). The latter ratio can be computed using various methods of nonperturbative QCD; probably the most promising would be a calculation of this ratio in lattice QCD.

Recent analysis of [26,27] gives for $M_2^{(c),\,\mathrm{intrinsic}}$ values at the level of fraction of percent which are in agreement with our estimate Eq. (58).

Let us note that, since we performed the heavy quark mass expansion of the heavy quark part of the energy momentum tensor, without neglecting total derivatives, one can also compute its nonforward nucleon matrix element. From the nonforward matrix element of energy momentum one can obtain the total angular momentum carried by intrinsic heavy quarks in the nucleon using Ji's sum rules [28]. We shall report the corresponding estimates elsewhere.

IV. CONCLUSIONS

In this paper we have computed the heavy quark mass expansion of various local heavy quark currents. The details of the technique are illustrated on the example of heavy quark mass expansion of the pseudoscalar density $\bar{Q}\gamma_5 Q$. This operator plays an important role in problems related to intrinsic charm contribution to the proton spin and to intrinsic charm content of η,η' mesons.

We corrected the mistakes in [8,9] for heavy quark mass expansion of the operator $\bar{Q}\gamma_5 Q$. In these papers large in-

[3]We show only the most singular term.

TABLE I. Results for intrinsic charm contribution to various observables.

Quantity	Our estimate
$f_{\eta'}^{(c)}$	-2 MeV
$f_{\eta}^{(c)}$	-0.7 MeV
$g_A^{(c)}$	-5×10^{-4}
$g_T^{(c)}$	$\sim10^{-4}$
$M_2^{(c),\text{ intrinsic}}$	$\sim10^{-3}$

trinsic charm contribution to the proton spin and to intrinsic charm content of η,η' mesons was obtained due to contribution of the operator $f^{abc}G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}$ which appeared in heavy quark mass expansion of the operator $\bar{Q}\gamma_5 Q$ presented in [8,9]. We showed that the coefficient in front of this operator is identically zero (the result which actually follows from general properties of the axial anomaly [4]), so that the physical effects based on the presence of the above operator discussed in [8,9,21,10,22,23] are absent.

For the first time we presented the full results[4] for heavy quark mass expansion of the operators $\bar{Q}Q$ (to the order $1/m^3$), $\bar{Q}\gamma_5 Q$ (to the order $1/m^3$), $\partial^\mu\bar{Q}\gamma_\mu\gamma_5 Q$ (to the order $1/m^2$), $\bar{Q}\gamma_\mu Q$ (to the order $1/m^3$), $\bar{Q}\sigma_{\mu\nu}Q$ (to the order $1/m^3$), and $\bar{Q}\gamma_\mu\nabla_\nu Q$ (to the order $1/m^2$).

The results obtained for heavy quark mass expansion allowed us to estimate the intrinsic charm content of η',η mesons as well as the charm contribution to the proton spin, nucleon tensor charge and to the fraction of nucleon momentum carried by intrinsic charm. In the case of charm content of η',η mesons and intrinsic charm contributions to the proton spin we reduce the calculations of these quantities to matrix elements which are already known either phenomenologically or were computed previously. In other cases, like intrinsic charm contribution to the nucleon tensor charge and to energy momentum tensor, the problem is reduced to matrix elements of gluon operators which can be estimated using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, and theory of instanton vacuum.

We made here a rough order of magnitude estimate of matrix elements of gluon operators appearing in heavy quark mass expansion of tensor current and of energy momentum tensor using the instanton model of QCD vacuum. More quantitative estimates will be given elsewhere. The predictions for intrinsic charm contribution to various observables are summarized in Table I.

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[4]Not neglecting total derivatives and terms proportional to $[\nabla^\mu, G_{\mu\nu}]$.

APPENDIX A: EUCLIDIZATION

In this appendix we present our conventions regarding the Euclidization as well as details of the heavy quark mass expansion of the fermion determinant and the pseudoscalar density.

For the Euclidization we use the follwing conventions:

$$ix^0_M = x_{4,E}, \quad x^k_M = x_{k,E} \to d^4x_M = -i d^4 x_E,$$

$$\partial^0_M = i\partial_{4,E}, \quad \partial^k_M = -\partial_{k,E},$$

$$A^0_M = iA_{4,E}, \quad A^k_M = -A_{k,E}. \tag{A1}$$

The covariant derivative therefore reads in Minkowski and in Euclidean space-time:

$$\nabla^\mu_M = [\partial^\mu - iA^\mu(x)]_M, \tag{A2}$$

$$\nabla_{\mu,E} = [\partial_\mu - iA_\mu(x)]_E. \tag{A3}$$

The field strength, defined as

$$F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + f^{abc}A^b_\mu A^c_\nu = i[\nabla_\mu, \nabla_\nu] \tag{A4}$$

transforms as

$$F_{ij,M} = F_{ij,E}, \quad F_{0j,M} = iF_{4j,E}. \tag{A5}$$

For the Dirac matrices we choose the conventions

$$\gamma^0_M = \gamma_{4,E}, \quad \gamma^k_M = i\gamma_{k,E}, \tag{A6}$$

and γ_5 is defined within this paper as

$$\gamma_{5,M} = \gamma^5_M = -i(\gamma^0\gamma^1\gamma^2\gamma^3)_M = (\gamma_1\gamma_2\gamma_3\gamma_4)_E = \gamma_{5,E}. \tag{A7}$$

With

$$\varepsilon^{0123}_M = -\varepsilon_{0123,M} = +1 = \varepsilon_{1234,E}, \tag{A8}$$

it yields

$$\mathrm{tr}_\gamma[\gamma_5\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta]_E = 4\varepsilon_{\alpha\beta\gamma\delta,E}. \tag{A9}$$

The fermionic fields transform as

$$\psi_M = \psi_E, \quad \bar{\psi}_M = -i\psi^\dagger_E, \tag{A10}$$

so the Dirac operator in Euclidean space-time reads

$$D = i\nabla\!\!\!/ + im. \tag{A11}$$

In Sec. II A we have used the following transformation properties for the effective action and the appearing operators:

$$S_{\text{eff},M} = iS_{\text{eff},E}, \tag{A12}$$

$$(F_{\alpha\beta}F^{\alpha\beta})_M=(F_{\alpha\beta}F_{\alpha\beta})_E,$$

$$(F_{\alpha\beta}F^{\beta}{}_{\gamma}F^{\gamma\alpha})_M=-(F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha})_E. \tag{A13}$$

APPENDIX B: THE EXPANSION OF $\mathrm{Tr}[e^{-tD^\dagger D}]$

In Sec. II A we have shown that the normalized fermion determinant is given by

$$\left(\frac{\det D}{\det D_0}\right)_{\zeta-\mathrm{reg}}=\exp\left[-\frac{1}{2}\lim_{s\to 0}\frac{d}{ds}\frac{M^{2s}}{\Gamma(s)}\int_0^\infty dt\,t^{s-1}\times\mathrm{Tr}[e^{-tD^\dagger D}-e^{-tD_0^\dagger D_0}]\right], \tag{B1}$$

using the ζ regularization, where the functional trace can be written as

$$\mathrm{Tr}[e^{-tD^\dagger D}]=\mathrm{tr}_{c,\gamma}\int d^4x\int\frac{d^4k}{(2\pi)^4}e^{-t(k^2+m^2)}\sum_{n=0}^{\infty}(-1)^n\times\frac{t^n}{n!}\left(-\nabla^2+\frac{\sigma}{2}F-2ik\nabla\right)^n\cdot 1 \tag{B2}$$

in the Euclidean space. The occurring integrations with respect to k can be performed using

$$\int\frac{d^4k}{(2\pi)^4}k_{\mu_1}\ldots k_{\mu_{2n}}e^{-t(k^2+m^2)}=\frac{1}{4\pi^2}(2t)^{-(n+2)}e^{-tm^2}\delta_{\mu_1\ldots\mu_{2n}}, \tag{B3}$$

whereas integrals containing an odd number of k's vanish. Collecting then all terms which will contribute up to order $1/m^2$ after the t integration and summing over the Lorentz indices yields, for Eq. (B2),

$$\begin{aligned}\mathrm{Tr}[e^{-tD^\dagger D}]=\frac{1}{4\pi^2}\mathrm{tr}_c\int d^4x\,e^{-tm^2}\Bigg[&\frac{1}{t^2}+t^0\left(\frac{1}{6}\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta-\frac{1}{6}\nabla_\alpha\nabla^2\nabla_\alpha+\frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}\right)\\&+t\Bigg(\frac{1}{180}\nabla^2\nabla^2\nabla^2-\frac{1}{36}(\nabla_\alpha\nabla^2\nabla_\alpha\nabla^2+\nabla_\alpha\nabla^2\nabla^2\nabla_\alpha+\nabla^2\nabla_\alpha\nabla^2\nabla_\alpha)+\frac{1}{45}(\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta\nabla^2+\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla^2\nabla_\beta\\&+\nabla_\alpha\nabla_\beta\nabla^2\nabla_\alpha\nabla_\beta+\nabla_\alpha\nabla^2\nabla_\beta\nabla_\alpha\nabla_\beta+\nabla^2\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta+\nabla_\alpha\nabla_\beta\nabla^2\nabla_\beta\nabla_\alpha)-\frac{1}{90}(\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\gamma\nabla_\beta\nabla_\gamma\\&+\nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\alpha\nabla_\beta\nabla_\gamma+\nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\alpha\nabla_\gamma\nabla_\beta+\nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\beta\nabla_\alpha\nabla_\gamma+\nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\beta\nabla_\gamma\nabla_\alpha)+\frac{1}{6}(F_{\alpha\beta}F_{\alpha\beta}\nabla^2\\&+F_{\alpha\beta}\nabla^2F_{\alpha\beta}+\nabla^2F_{\alpha\beta}F_{\alpha\beta}-\nabla_\gamma F_{\alpha\beta}\nabla_\gamma F_{\alpha\beta}-\nabla_\gamma F_{\alpha\beta}F_{\alpha\beta}\nabla_\gamma-F_{\alpha\beta}\nabla_\gamma F_{\alpha\beta}\nabla_\gamma)-\frac{i}{6}F_{\alpha\beta}F_{\gamma\beta}F_{\gamma\alpha}\Bigg)+\mathcal{O}(\nabla^8)\Bigg]\end{aligned} \tag{B4}$$

$$\begin{aligned}=\frac{1}{4\pi^2}\mathrm{tr}_c\int d^4x\,e^{-tm^2}\Bigg[&\frac{1}{t^2}+\frac{1}{6}F_{\alpha\beta}F_{\alpha\beta}+t\Bigg(\frac{2}{15}iF_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha}-\frac{1}{180}[\nabla_\alpha,F_{\alpha\beta}][\nabla_\gamma,F_{\gamma\beta}]\\&+\frac{11}{360}[\nabla_\alpha,[\nabla_\alpha,F_{\beta\gamma}]]F_{\beta\gamma}+\frac{7}{720}[\nabla_\alpha,[\nabla_\alpha,F_{\beta\gamma}F_{\beta\gamma}]]\Bigg)+\mathcal{O}(\nabla^8)\Bigg].\end{aligned} \tag{B5}$$

In the last step we have rearranged all terms into explicit gauge invariants using the cyclic property of the color trace and some commutator algebra. From the Bianchi identity Eq. (17) we obtain the relation[5]

$$[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]]=-2i[F_{\rho\alpha},F_{\rho\beta}]+[\nabla_\alpha,[\nabla_\rho,F_{\rho\beta}]]-[\nabla_\beta,[\nabla_\rho,F_{\rho\alpha}]]. \tag{B6}$$

Its application to Eq. (B5) and reinsertion into Eq. (B1) then yields the result Eq. (11) for the heavy quark mass expansion of the fermion determinant.

[5]In the calculation of [9] the factor of 2 was missing in this identity, which led to the wrong result of the heavy quark mass expansion of the pseudoscalar density.

APPENDIX C: THE EXPANSION OF $\mathbf{Tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle$

The heavy quark mass expansion of the pseudoscalar density up to order $1/m^3$ reads in detail:

$$\mathrm{tr}_{c,\gamma}\left\langle x\left|\frac{1}{D}\gamma_5\right|x\right\rangle=-im\,\mathrm{tr}_{c,\gamma}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\sum_{n=0}^{\infty}\left(\frac{\nabla^2-\frac{\sigma}{2}F+2ik\nabla}{k^2+m^2}\right)^n\gamma_5\cdot 1 \tag{C1}$$

$$\begin{aligned}=&-im\,\mathrm{tr}_{c,\gamma}\int\frac{d^4k}{(2\pi)^4}\Bigg[\frac{1}{(k^2+m^2)^3}\frac{1}{4}\sigma F\sigma F\gamma_5\\&+\frac{1}{(k^2+m^2)^4}\left(\frac{1}{4}\nabla^2\sigma F\sigma F+\frac{1}{4}\sigma F\nabla^2\sigma F+\frac{1}{4}\sigma F\sigma F\nabla^2-\frac{1}{8}\sigma F\sigma F\sigma F\right)\gamma_5\\&-\frac{1}{(k^2+m^2)^5}(\sigma F\sigma Fk\nabla k\nabla+\sigma Fk\nabla\sigma Fk\nabla+\sigma Fk\nabla k\nabla\sigma F+k\nabla\sigma Fk\nabla\sigma F\\&+k\nabla k\nabla\sigma F\sigma F+k\nabla\sigma F\sigma Fk\nabla)\gamma_5\Bigg]+\mathcal{O}\left(\frac{1}{m^5}\right)\end{aligned} \tag{C2}$$

$$\begin{aligned}=&\frac{i}{32\pi^2m}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_cF_{\alpha\beta}F_{\gamma\delta}-\frac{1}{48\pi^2m^3}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_cF_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}\\&+\frac{i}{192\pi^2m^3}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[F_{\alpha\beta}F_{\gamma\delta}\nabla^2-F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho+F_{\alpha\beta}\nabla^2F_{\gamma\delta}\\&-\nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}+\nabla^2F_{\alpha\beta}F_{\gamma\delta}-\nabla_\rho F_{\alpha\beta}F_{\gamma\delta}\nabla_\rho]+\mathcal{O}\left(\frac{1}{m^5}\right).\end{aligned} \tag{C3}$$

Here we have used Eq. (15) for the integration over k and the following results for Dirac traces:

$$\begin{aligned}F_{\alpha\beta}F_{\gamma\delta}\mathrm{tr}_\gamma[\sigma_{\alpha\beta}\sigma_{\gamma\delta}\gamma_5]&=-F_{\alpha\beta}F_{\gamma\delta}\mathrm{tr}_\gamma[\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\gamma_5]\\&=-4\varepsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta},\end{aligned} \tag{C4}$$

$$\begin{aligned}F_{\alpha\beta}F_{\gamma\delta}F_{\epsilon\varphi}\mathrm{tr}_\gamma[\sigma_{\alpha\beta}\sigma_{\gamma\delta}\sigma_{\epsilon\varphi}\gamma_5]&=-iF_{\alpha\beta}F_{\gamma\delta}F_{\epsilon\varphi}\mathrm{tr}_\gamma[\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\gamma_\epsilon\gamma_\varphi\gamma_5]\\&=16i\varepsilon_{\alpha\beta\gamma\delta}F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}.\end{aligned} \tag{C5}$$

Using the identities

$$[\nabla_\rho,F_{\alpha\beta}][\nabla_\rho,F_{\gamma\delta}]=\nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}+F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho-\nabla_\rho F_{\alpha\beta}F_{\gamma\delta}\nabla_\rho-F_{\alpha\beta}\nabla^2F_{\gamma\delta}, \tag{C6}$$

$$[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]]F_{\gamma\delta}=\nabla^2F_{\alpha\beta}F_{\gamma\delta}+F_{\alpha\beta}\nabla^2F_{\gamma\delta}-2\nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}, \tag{C7}$$

$$F_{\alpha\beta}[\nabla_\rho,[\nabla_\rho,F_{\gamma\delta}]]=F_{\alpha\beta}\nabla^2F_{\gamma\delta}+F_{\alpha\beta}F_{\gamma\delta}\nabla^2-2F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho, \tag{C8}$$

Eq. (C3) can be written as

$$\begin{aligned}\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle=&\frac{i}{32\pi^2m}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_cF_{\alpha\beta}F_{\gamma\delta}+\frac{i}{192\pi^2m^3}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[2[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]]F_{\gamma\delta}\\&+[\nabla_\rho,F_{\alpha\beta}][\nabla_\rho,F_{\gamma\delta}]+4iF_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}]+\mathcal{O}\left(\frac{1}{m^5}\right).\end{aligned} \tag{C9}$$

Further we can use the relation (B6) and $\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[\nabla_\rho,F_{\rho\beta}][\nabla_\alpha,F_{\gamma\delta}]=0$ obtained from the Bianchi identity so that

$$\begin{aligned}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]]F_{\gamma\delta} &= -2\,i\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}-F_{\rho\beta}F_{\rho\alpha}F_{\gamma\delta}]\\ &\quad+\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[[\nabla_\alpha,[\nabla_\rho,F_{\rho\beta}]]F_{\gamma\delta}-[\nabla_\beta,[\nabla_\rho,F_{\rho\alpha}]]F_{\gamma\delta}]\\ &=\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[-4iF_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}+2[\nabla_\alpha,[\nabla_\rho,F_{\rho\beta}]]F_{\gamma\delta}]\\ &=\varepsilon_{\alpha\beta\gamma\delta}(-4i\,\mathrm{tr}_cF_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}+2\partial_\alpha\mathrm{tr}_c[\nabla_\rho,F_{\rho\beta}]F_{\gamma\delta}).\end{aligned} \tag{C10}$$

On the other hands it yields

$$\begin{aligned}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]]F_{\gamma\delta}+[\nabla_\rho,F_{\alpha\beta}][\nabla_\rho,F_{\gamma\delta}]] &= \varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_c[\nabla_\rho,[\nabla_\rho,F_{\alpha\beta}]F_{\gamma\delta}]=\varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\,\mathrm{tr}_c[\nabla_\rho,F_{\alpha\beta}]F_{\gamma\delta}\\ &=\varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\mathrm{tr}_c[[\nabla_\rho,F_{\alpha\beta}F_{\gamma\delta}]-F_{\alpha\beta}[\nabla_\rho,F_{\gamma\delta}]]\\ &=\tfrac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\mathrm{tr}_c[\nabla_\rho,F_{\alpha\beta}F_{\gamma\delta}]=\tfrac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\partial^2\mathrm{tr}_cF_{\alpha\beta}F_{\gamma\delta}.\end{aligned} \tag{C11}$$

So we finally end up with the result Eq. (19)

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle=\frac{i}{32\pi^2m}\varepsilon_{\alpha\beta\gamma\delta}\mathrm{tr}_cF_{\alpha\beta}F_{\gamma\delta}+\frac{i}{384\pi^2m^3}\varepsilon_{\alpha\beta\gamma\delta}\partial^2\mathrm{tr}_cF_{\alpha\beta}F_{\gamma\delta}+\frac{i}{96\pi^2m^3}\varepsilon_{\alpha\beta\gamma\delta}\partial_\alpha\mathrm{tr}_c[\nabla_\rho,F_{\rho\beta}]F_{\gamma\delta}+\mathcal{O}\left(\frac{1}{m^5}\right). \tag{C12}$$

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