Modified initial state for perturbative QCD

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A particular initial state for the construction of the perturbative expansion of QCD is investigated. It is formed as a coherent superposition of zero momentum gluon pairs and shows Lorentz as well as global SU(3) symmetries. It follows that the gluon and ghost propagators determined by it coincide with the ones used in an alternative of the usual perturbation theory proposed in a previous work. Therefore, the ability of such a procedure to produce a finite gluon condensation parameter already in the first orders of perturbation theory is naturally explained. It also follows that this state satisfies the physicality condition of the Becchi-Rouet-Stora-Tyutin (BRST) procedure in its Kugo-Ojima formulation. The BRST quantization is done for the value $\alpha=1$ of the gauge parameter where the procedure is greatly simplified. Therefore, after assuming that the adiabatic connection of the interaction does not take out the state from the interacting physical space, the predictions of the perturbation expansion, at the value $\alpha=1$, for the physical quantities should have meaning. The validity of this conclusion solves the gauge dependence indeterminacy remaining in the proposed perturbation expansion.

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I. INTRODUCTION

Quantum chromodynamics (QCD) was discovered in the 1970s, and up to this time it has been considered the fundamental theory for strong interactions; as a consequence, it has been extensively investigated [1–8].

In one limit, the smallness of the coupling constant at high momentum (asymptotic freedom) made possible the theoretical investigation of the so-called hard processes by using familiar perturbative language. This so-called perturbative QCD (PQCD) was satisfactorily developed. However, relevant phenomena associated with strong interactions cannot be described by the standard perturbative methods and the development of the so-called nonperturbative QCD is at the moment one of the challenges of this theory.

One of the most peculiar characteristics of strong interactions is color confinement. According to this philosophy, colored objects, such as quarks and gluons, cannot be observed as free particles in contrast with hadrons which are colorless composite states and effectively detected. The physical nature of such a phenomenon remains unclear. Numerous attempts to explain this property have been made, for example, explicit calculations in which the theory is regularized on a spatial lattice [2], and also through the construction of phenomenological models. In the so-called MIT bag model [3], it is assumed that a bag or a bubble is formed around the objects having color in such a manner that they could not escape from it, because their effective mass is smaller inside the bag volume and very high outside. Within the so-called string model [4], which is based on the assumption that the interaction forces between quarks and antiquarks grow when the distance increases, in such a way that the energy increases linearly with the string length $E(L)=kL$.

A fundamental problem in QCD is the nature of the ground state [5–7]. This state is imagined as a very dense state of matter, composed of gluons and quarks interacting in a complicated way. Its properties are not easily accessible in experiments, because quark and gluon fields cannot be directly observed. Furthermore, the interactions between quarks cannot be directly determined.

It is already accepted that in QCD the zero-point oscillations of the coupled modes produce a finite energy density which is determined phenomenologically. The numerical estimate of it is

$$E_{\rm vac}\simeq -f\langle 0|(gG)^2|0\rangle \simeq 0.5\ \text{GeV/fm}^3,$$

where the so-called nonperturbative gluonic condensate $\langle 0|(gG)^2|0\rangle$ was introduced and phenomenologically evaluated by Shifman, Vainshtein, and Zakharov [8]. The negative sign of $E_{\rm vac}$ means that the nonperturbative vacuum energy is lower than the one associated to the perturbative vacuum.

Since a long time ago, one particular kind of model has been shown to be able to predict similar properties. This is the chromomagnetic vacuum approach, in which it is assumed that a vacuum magnetic field is existing at all the points [9]. Concretely a constant magnetic Abelian field H is assumed. A one-loop calculation gives as the result the following energy density:

$$E(H)=\frac{H^2}{2}\left[1+\frac{bg^2}{16\pi^2}\ln\left(\frac{H}{\Lambda^2}\right)\right].$$

This formula predicts negative values for the energy at small fields, so the usual perturbative ground state with $H=0$ is unstable with respect to the formation of a state with a nonvanishing field intensity at which the energy $E(H)$ has a minimum [9]. Many physical problems related with the hadron structure, confinement problem, etc., have been investigated using the Savvidy model. Nevertheless, after some time its intense study was abandoned. The main reasons were (1) The perturbative relation giving $E_{\rm vac}$ would only be valid if the second order in the expansion in powers of the Planck constant is relatively small. (2) The specific spatial direction and the color direction of the magnetic field break the now seemingly indispensable Lorentz and SU(3) invariance of the ground state. (3) The magnetic moment of the vector particle (gluon) is such that its energy in the presence of the field has a negative eigenvalue, which also makes the homogeneous magnetic field H unstable.

Before presenting the objectives of the present work it should be stressed that the perturbative quantization of QCD is realized in the same way as that in QED. The quadratic field terms of the QCD Lagrangian have the same form as the ones corresponding to the electrons and photons in QED. However, in connection with the interaction, there appears a substantial difference due to the coupling of the gluon to itself. In addition, it is a general fact that a perturbative expansion has some freedom in dependence on the initial conditions at $t\rightarrow\pm\infty$ or what is the same, from the states in which the expansion is based. Moreover, as was expressed before, the exact ground state has a nontrivial structure associated with a gluon condensate.

Then, given the above remarks, it is not unreasonable to expect that the true vacuum state could be well described through a modified Feynman expansion perturbatively describing a gluon condensate. Such a perturbative condensate could generate all the low-energy physical observables which in the standard expansion could require an infinite number of terms of the series.

In a previous work, one of the authors (A.C.) [10], following the above idea, proposed a modified perturbation theory for QCD. This expansion retains the main invariance of the theory [the Lorentz and SU(3) ones], and is also able to reproduce the main physical predictions of the chromomagnetic field models. It seems possible to us that this procedure could produce a reasonable if not good description of the low-energy physics. If this is the case, then, the low- and high-energy descriptions of QCD would be unified in a common perturbative framework. In particular, in [10] the results had the interesting outcome of producing a nonvanishing mean value for the relevant quantity $\langle G^2\rangle$. In addition the effective potential for the condensation parameter in the first-order approximation shows a minimum at nonvanishing values of that parameter. Therefore, the procedure is able to reproduce at least some central predictions of the chromomagnetic models and general QCD analysis. The main objective of the present work consists in investigating the foundations of the mentioned perturbation theory. The concrete aim is to find a state in the Fock's space of the noninteracting theory being able to generate that expansion by also satisfying the physicality condition of the Becchi-Rouet-Stora-Tyutin (BRST) quantization approach.

It follows that it is possible to find the sought for state and it turns out to be an exponential of a product of two gluon and ghost creation operators. That is, it can be interpreted as a coherent superposition of states with many zero momentum gluon and ghost pairs. Therefore, this structure gives an explanation for the ability of the expansion being investigated to produce nonzero values of the gluon condensation in the first orders of perturbation theory. The fact that the effective action also shows a minimum for nonvanishing values of the condensation parameter also support the idea that the considered state improves the perturbative expansion. It is also shown that the state satisfies the linear condition which defines the physical subspace in the BRST quantization for the $\alpha=1$ value of the gauge parameter. Thus, the indefiniteness in the appropriate value of this parameter to be used which remained in the former work is resolved opening the way for the study of the predictions of the proposed expansion.

It should mentioned that a similar idea as the one advanced in [10] was afterwards considered in [11]. In that work, gluon and quark condensation in a range of momenta of the order of Λ_{qcd} have been considered with the similar aim of constructiong an alternative perturbative theory for QCD. However, in our view, that construction should break the Lorentz invariance because the condensates are expected to show a nonvanishing energy density. If this limitation is not at work their proposed mechanism could be worth considering as an alternative possibility.

The exposition will be organized as follows: In Sec. II, the BRST operational quantization method for gauge fields developed by Kugo and Ojima is reviewed. Starting from it, in Sec. III the ansatz for the Fock's space state which generates the wanted form of the perturbative expansion is introduced. The proof that the state satisfies the physical state condition is also given in this section. Then, in Sec. IV it is shown that the proposed state can generate the wanted modification of the propagator by a proper selection of the parameters at hand. The modification of the propagator for the ghost particles is also considered in this section. Finally, the evaluation of the gluon condensation parameter done in [10] is reviewed in order to illustrate the ability of the procedure to predict a main property of the real QCD ground state.

II. REVIEW OF THE KO QUANTIZATION PROCEDURE

In the present section the operator formalism developed by Kugo and Ojima (KO) [12–15] is reviewed and, after, specialized to the noninteracting limit of gluodynamics (GD). This formulation makes use of the invariance of the Lagrangian under a global symmetry operation called the BRST transformation [16]. We will consider the construction of a relativistic invariant initial state in the noninteracting limit of QCD. The BRST physical state condition in the noninteracting limit will be also imposed. As explained before, the motivation is that we think that this state has the possibility to furnish the gluodynamics ground state under the adiabatic connection of the interaction. In what follows we will work in Minkowski space with the conventions defined below.

Let G be a compact group and Λ any matrix in the adjoint representation of its associated Lie Algebra. The matrix Λ can be represented as a linear combination of the form

$$\Lambda=\Lambda^a T^a, \tag{1}$$

where T^a are the generators $(a=1,\ldots,\mathrm{Dim}G=n)$ which are chosen as Hermitian ones, satisfying

$$[T^a,T^b]=if^{abc}T^c. \tag{2}$$

The variation of the fields under infinitesimal gauge transformations are given by

$$\delta_\Lambda A_\mu^a(x)=\partial_\mu\Lambda^a(x)+gf^{acb}A_\mu^c(x)\Lambda^b(x)=D_\mu^{ab}(x)\Lambda^b, \tag{3}$$

$$D_\mu^{ab}(x)=\partial_\mu\delta^{ab}+gf^{acb}A_\mu^c(x). \tag{4}$$

The metric $g_{\mu\nu}$ is diagonal and taken in the convention

$$g_{00}=-g_{ii}=1 \quad \text{for} \quad i=1,2,3. \tag{5}$$

The complete GD Lagrangian to be considered is the one employed in the operator quantization approach of [17]. Its explicit form is given by

$$\mathcal{L}=\mathcal{L}_{\mathrm{YM}}+\mathcal{L}_{\mathrm{GF}}+\mathcal{L}_{\mathrm{FP}}, \tag{6}$$

$$\mathcal{L}_{\mathrm{YM}}=-\frac{1}{4}F_{\mu\nu}^a(x)F^{\mu\nu,a}(x), \tag{7}$$

$$\mathcal{L}_{\mathrm{GF}}=-\partial^\mu B^a(x)A_\mu^a(x)+\frac{\alpha}{2}B^a(x)B^a(x), \tag{8}$$

$$\mathcal{L}_{\mathrm{FP}}=-i\partial^\mu\overline{c^a}(x)D_\mu^{ab}(x)c^b(x). \tag{9}$$

The field strength is

$$F_{\mu\nu}^a(x)=\partial_\mu A_\nu^a(x)-\partial_\nu A_\mu^a(x)+gf^{abc}A_\mu^b(x)A_\nu^c(x). \tag{10}$$

Relation (7) defines the Yang-Mills (YM) standard Lagrangian, Eq. (8) is the gauge fixing (GF) term and Eq. (9) is the Lagrangian which describes the dynamics of the unphysical Faddeev-Popov (FP) ghost fields.

The physical state conditions in the BRST procedure [17, 18] are given by

$$Q_B|\Phi\rangle=0, \tag{11}$$

$$Q_C|\Phi\rangle=0, \tag{12}$$

where

$$Q_B=\int d^3x\left[B^a(x)\nabla_0c^a(x)+gB^a(x)f^{abc}A_0^b(x)c^c(x)+\frac{i}{2}g\partial_0(\overline{c^a})f^{abc}c^b(x)c^c(x)\right], \tag{13}$$

with

$$f(x)\nabla_0g(x)\equiv f(x)\partial_0g(x)-\partial_0(f(x))g(x). \tag{14}$$

The BRST charge is conserved as a consequence of the BRST symmetry of the Lagrangian (6). The also conserved charge Q_C is given by

$$Q_C=i\int d^3x[\overline{c^a}(x)\nabla_0c^a(x)+g\overline{c^a}(x)f^{abc}A_0^b(x)c^c(x)], \tag{15}$$

where conservation comes from the Noether theorem due to the invariance of Lagrangian (6) under the phase transformation $c\to e^\theta c,\overline{c}\to e^{-\theta}\overline{c}$. This charge defines the so-called ''ghost number'' as the difference between the number of c and $\overline{c}$ ghosts.

Our interest will be centered here in the Yang-Mills theory without spontaneous breaking of the gauge symmetry in the limit of no interaction. The quantization of the theory defined by the Lagrangian (6), to be considered after in the interaction free limit $g\to 0$, leads to the following commutation relations for the free fields:

$$\begin{aligned}
&[A_\mu^a(x),A_\nu^b(y)]=\delta^{ab}[-ig_{\mu\nu}D(x-y)+i(1-\alpha)\partial_\mu\partial_\nu E(x-y)],\\
&[A_\mu^a(x),B^b(y)]=\delta^{ab}[-i\partial_\mu D(x-y)],\\
&[B^a(x),B^b(y)]=\{\overline{c^a}(x),\overline{c^b}(y)\}=\{c^a(x),c^b(y)\}=0,\\
&\{c^a(x),\overline{c^b}(y)\}=-D(x-y).
\end{aligned} \tag{16}$$

The equations of motion for the noninteracting fields takes the simple form

$$\Box A_\mu^a(x)-(1-\alpha)\partial_\mu B^a(x)=0, \tag{17}$$

$$\partial^\mu A_\mu^a(x)+\alpha B^a(x)=0, \tag{18}$$

$$\Box B^a(x)=\Box c^a(x)=\Box\overline{c^a}(x)=0. \tag{19}$$

These equations can be solved for arbitrary values of α. However, as it was said before, the discussion will be restricted to the case $\alpha=1$ which corresponds to the situation in which all the gluon components satisfy the D'Alambert equation. This selection, as considered in the framework of the usual perturbative expansion, implies that you are not able to check the α independence of the physical quantities. However in the present discussion the aim is to construct a perturbative state that satisfies the BRST physicality condition, in order to connect adiabatically the interaction. Then, the physical character of all the predictions will follow whenever the former assumption that the adiabatic connection does not take the state out of the physical subspace at any intermediate state is valid. Clearly, the consideration of different values of α, would also be a convenient recourse for checking the α independent perturbative expansion. However, at this stage it is preferred to delay the more technical issue of implementing the BRST quantization for any value of α for future work.

In that way the field equations in the $\alpha=1$ gauge will be

$$\Box A_\mu^a(x)=0, \tag{20}$$

$$\partial^\mu A_\mu^a(x)+B^a(x)=0, \tag{21}$$

$$\Box B^a(x)=\Box c^a(x)=\Box \overline{c}^a(x)=0. \tag{22}$$

The solutions of the set (20)–(22) can be written as

$$A_\mu^a(x)=\sum_{k,\sigma}\,[A_{k,\sigma}^a f_{k,\mu}^\sigma(x)+A_{k,\sigma}^{a*}f_{k,\mu}^{\sigma*}(x)],$$

$$B^a(x)=\sum_k\,[B_k^a g_k(x)+B_k^{a*}g_k^*(x)],$$

$$c^a(x)=\sum_k\,[c_k^a g_k(x)+c_k^{a*}g_k^*(x)],$$

$$\overline{c}^a(x)=\sum_k\,[\overline{c}_k^a g_k(x)+\overline{c}_k^{a*}g_k^*(x)]. \tag{23}$$

The wave packets for nonmassive scalar and vector fields are taken as

$$g_k(x)=\frac{1}{\sqrt{2Vk_0}}\exp(-ikx),$$

$$f_{k,\sigma}^\mu(x)=\frac{1}{\sqrt{2Vk_0}}\epsilon_\sigma^\mu(k)\exp(-ikx). \tag{24}$$

As can be seen from Eq. (21) the five $A_{k,\sigma}^a$ and B_k^a modes are not all independent. Indeed, it follows from Eq. (21) that

$$B_k^a=A_k^{S,a}=A_{k,L}^a\,. \tag{25}$$

Then, the expansion of the free Heisenberg fields takes the form

$$A_\mu^a(x)=\sum_k\left(\sum_{\sigma=1,2}A_{k,\sigma}^a f_{k,\mu}^\sigma(x)+A_k^{L,a}f_{k,L,\mu}(x)\right.$$
$$\left.+B_k^a f_{k,S,\mu}(x)+\text{H.c.}\right), \tag{26}$$

where H.c. represents the Hermitian conjugate of the first term. In order to satisfy the commutation relations (16) the creation and annihilation operator associated with the Fourier components of the fields should obey

$$[A_{k,\sigma}^a,A_{k',\sigma'}^{a'+}]=-\delta^{aa'}\delta_{\vec{k}\vec{k}'}\eta_{\sigma\sigma'}\,,$$

$$\{c_k^a,\overline{c}_{k'}^{a'+}\}=i\,\delta^{aa'}\delta_{\vec{k}\vec{k}'}\,,$$

$$\{\overline{c}_k^a,c_{k'}^{a'+}\}=-i\,\delta^{aa'}\delta_{\vec{k}\vec{k}'} \tag{27}$$

and all the others vanish, $\eta_{\sigma\sigma'}=\epsilon_{\sigma',\mu}$.

The above commutation rules and equations of motion define the quantized noninteracting limit of GD. It is possible now to start defining the alternative interaction free ground state to be considered for the adiabatic connection of the interaction. As discussed before, the expectation is that the physics of the perturbation theory being investigated is able to furnish a helpful description of low-energy physical effects.

III. THE ALTERNATIVE INITIAL STATE

After beginning to work in the KO formalism some indications were found, specifically that the appropriate state vector obeying the physical condition in this procedure could have the general structure

$$|\phi\rangle=\exp\sum_a\,[C_1(p)A_{p,1}^{a+}A_{p,1}^{a+}+C_2(p)A_{p,2}^{a+}A_{p,2}^{a+}+C_3(p)$$
$$\times(B_p^{a+}A_p^{L,a+}+i\overline{c}_p^{a+}c_p^{a+})]|0\rangle, \tag{28}$$

where $p=(|\vec{p}|,\vec{p})$ is an auxiliary null four-momentum, and $|\vec{p}|$ is chosen as one of the few smallest values of the modulus of the spatial momentum of the quantized theory in a finite volume V. This value will be taken later in the limit $V\to\infty$ for recovering the Lorentz invariance. From here the sum on the color index a will be explicit. The parameters $C_i(p)$ will be fixed below from the condition that the free propagator associated with a state satisfying the BRST physical state condition, coincides with the one proposed in the previous work [10]. The solution of this problem, would then give a foundation to the physical implications of the discussion in that work.

It should also be noticed that the states defined by Eq. (28) have some similarity with coherent states [19]. However, in the present case, the creation operators appear in squares. Thus, the argument of the exponential creates pairs of physical and nonphysical particles. An important property of this function is that its construction in terms of pairs of creation operators determines that the mean value of an odd number of field operator vanishes. This is at variance with the standard coherent state, in that the mean values of the fields are nonzero. The vanishing of the mean field is a property in common with the standard perturbative vacuum, which Lorentz invariance could be broken by any nonzero expectation value of the four-vector of the gauge field. It should be also stressed that this state, as formed by the superposition of states of pairs of gluons, suggests a connection with some recent work in the literature that consider the formation of gluon pairs due to color interactions.

Let us argue below that the state (28) satisfies the BRST physical conditions

$$Q_B|\Phi\rangle=0, \tag{29}$$

$$Q_C|\Phi\rangle=0. \tag{30}$$

The expressions of the charges in the interaction free limit [17] are

$$Q_B=i\sum_{k,a}\ (c_k^{a+}B_k^a-B_k^{a+}c_k^a), \tag{31}$$

$$Q_C=i\sum_{k,a}\ (\overline{c}_k^{a+}c_k^a+c_k^{a+}\overline{c}_k^a). \tag{32}$$

Consider first the action of Q_B

$$Q_B|\Phi\rangle=i\ \exp\left\{\sum_{\sigma,a}\ C_\sigma(p)A_{p,\sigma}^{a+}A_{p,\sigma}^{a+}\right\}\left(\exp\left\{\sum_a\ C_3(p)i\overline{c}_p^{a+}c_p^{a+}\right\}\sum_{k,b}\ c_k^{b+}B_k^b\exp\left\{\sum_a\ C_3(p)B_p^{a+}A_p^{L,a+}\right\}\right.$$
$$\left.+\ \exp\left\{\sum_a\ C_3(p)B_p^{a+}A_p^{L,a+}\right\}\sum_{k,b}\ -B_k^{b+}c_k^b\exp\left\{\sum_a\ C_3(p)i\overline{c}_p^{a+}c_p^{a+}\right\}\right)|0\rangle=0, \tag{33}$$

where the identity

$$\left[B_k^b,\exp\sum_a\ C_3(p)B_p^{a+}A_p^{L,a+}\right]=-C_3(p)B_p^{b+}\delta_{k,p}\exp\sum_a\ C_3(p)B_p^{a+}A_p^{L,a+} \tag{34}$$

was used. In Eq. (34) the Kronecker delta is defined as

$$\delta_{k,p}=1\quad\text{for}\quad\vec{p}=\vec{k},$$
$$\delta_{k,p}=0\quad\text{for}\quad\vec{p}\neq\vec{k}. \tag{35}$$

For the action of Q_C on the considered state we have

$$Q_C|\Phi\rangle=i\ \exp\left\{\sum_{\sigma,a}\ C_\sigma(p)A_{p,\sigma}^{a+}A_{p,\sigma}^{a+}+\sum_a\ C_3(p)B_p^{a+}A_p^{L,a+}\right\}\left[\sum_{k,b}\ \overline{c}_k^{b+}c_k^b\left(1+\sum_a\ iC_3(p)\overline{c}_p^{a+}c_p^{a+}\right)\right.$$
$$\left.+\sum_{k,b}\ c_k^{b+}\overline{c}_k^b\left(1+\sum_a\ iC_3(p)\overline{c}_p^{a+}c_p^{a+}\right)\right]|0\rangle=0, \tag{36}$$

which vanishes due to the commutation rules of the ghost operators.

Next, the evaluation of the norm of the proposed state is considered. Due to the commutation properties of the operators, it can be written as

$$\langle\Phi|\Phi\rangle=\prod_{a=1,\ldots,8}\left\{\prod_{\sigma=1,2}\langle 0|\exp\{C_\sigma^*(p)A_{p,\sigma}^aA_{p,\sigma}^a\}\exp\{C_\sigma(p)A_{p,\sigma}^{a+}A_{p,\sigma}^{a+}\}|0\rangle\right.$$
$$\left.\times\langle 0|\exp\{C_3^*(p)A_p^{L,a}B_p^a\}\exp\{C_3(p)B_p^{a+}A_p^{L,a+}\}|0\rangle\langle 0|(1-iC_3^*(p)c_p^a\overline{c}_p^a)(1+iC_3(p)\overline{c}_p^{a+}c_p^{a+})|0\rangle\right\}. \tag{37}$$

For the product of the factors associated with transverse modes and the eight values of the color index, after expanding the exponential in series it follows that

$$[\langle 0|\exp\{C_\sigma^*(p)A_{p,\sigma}^aA_{p,\sigma}^a\}\exp\{C_\sigma(p)A_{p,\sigma}^{a+}A_{p,\sigma}^{a+}\}|0\rangle]^8$$
$$=\left[\langle 0|\sum_{m=0}^{\infty}\ C_\sigma^*(p)C_\sigma(p)\frac{(A_{p,\sigma}^a)^{2m}(A_{p,\sigma}^{a+})^{2m}}{(m!)^2}|0\rangle\right]^8$$
$$=\left[\sum_{m=0}^{\infty}\ |C_\sigma(p)|^{2m}\frac{(2m)!}{(m!)^2}\right]^8, \tag{38}$$

where we used the identity

$$\langle 0|(A^{a}_{p,\sigma})^{2m}(A^{a+}_{p,\sigma})^{2m}|0\rangle=(2m)!. \tag{39}$$

The factors linked with the scalar and longitudinal modes can be transformed as follows:

$$\begin{aligned}&[\langle 0|\exp\{C_3^*(p)A_p^{L,a}B_p^a\}\exp\{C_3(p)B_p^{a+}A_p^{L,a+}\}|0\rangle]^8\\&=\left[\langle 0|\sum_{m=0}^{\infty}C_3^*(p)C_3(p)\frac{(A_p^{L,a}B_p^a)^m(B_p^{a+}A_p^{L,a+})^m}{(m!)^2}|0\rangle\right]^8\\&=\left[\sum_{m=0}^{\infty}|C_3(p)|^{2m}\right]^8=\left[\frac{1}{[1-|C_3(p)|^2]}\right]^8\quad\text{for }|C_3(p)|<1,\end{aligned} \tag{40}$$

in which the relation

$$\langle 0|(A_p^{L,a}B_p^a)^m(B_p^{a+}A_p^{L,a+})^m|0\rangle=(m!)^2 \tag{41}$$

was employed.

Finally, the factor connected with the ghost fields can be evaluated to be

$$\begin{aligned}&[\langle 0|(1-iC_3^*(p)c_p^a\overline{c}_p^a)(1+iC_3(p)\overline{c}_p^{a+}c_p^{a+})|0\rangle]^8\\&=[1+|C_3(p)|^2\langle 0|c_p^a\overline{c}_p^a\overline{c}_p^{a+}c_p^{a+}|0\rangle]=[1-|C_3(p)|^2]^8.\end{aligned} \tag{42}$$

After substituting all the calculated factors, the norm of the state can be written as

$$N=\langle\Phi|\Phi\rangle=\prod_{\sigma=1,2}\left[\sum_{m=0}^{\infty}|C_\sigma(p)|^{2m}\frac{(2m)!}{(m!)^2}\right]^8. \tag{43}$$

Therefore, it is possible to define the normalized state

$$|\tilde{\Phi}\rangle=\frac{1}{\sqrt{N}}|\Phi\rangle, \tag{44}$$

$$\langle\tilde{\Phi}|\tilde{\Phi}\rangle=1. \tag{45}$$

Note that, as should be expected, the norm is not dependent on the $C_3(p)$ parameter which defines the nonphysical particle operators entering in the definition of the state.

IV. GLUON AND GHOST MODIFIED PROPAGATORS

Let us consider the determination of the form of the main elements of perturbation theory, which is the free particle propagators. It will be seen that the propagators associated with the considered state have the same form as proposed in [10] under a proper selection of the parameters. Consider the generating functional of the free-particle Green's functions as given by

$$Z(J)\equiv\left\langle\tilde{\Phi}\left|T\left(\exp\left\{i\int d^4xJ(x)A^0(x)\right\}\right)\right|\tilde{\Phi}\right\rangle. \tag{46}$$

As a consequence of Wick's theorem the generating functional can be written in the form [20]

$$\begin{aligned}Z(J)\equiv&\left\langle\tilde{\Phi}\left|\exp\left\{i\int d^4xJ(x)A^{0-}(x)\right\}\exp\left\{i\int d^4yJ(y)A^{0+}(x)\right\}\right|\tilde{\Phi}\right\rangle\\&\times\exp\left\{i\int d^4y\int d^4x\,\theta(y_0-x_0)J(y)J(x)[A^{0-}(x),A^{0+}(y)]\right\}.\end{aligned} \tag{47}$$

Therefore, the sought-for modification to the free propagator is completely determined by the term

$$\prod_{a=1,\ldots,8}\left\langle\tilde{\Phi}\left|\exp\left\{i\int d^4xJ^{\mu,a}(x)A_\mu^{a-}(x)\right\}\exp\left\{i\int d^4xJ^{\mu,a}(x)A_\mu^{a+}(x)\right\}\right|\tilde{\Phi}\right\rangle, \tag{48}$$

where all the color-dependent operators are decoupled thanks to the commutation relations.

The annihilation and creation parts of the field operators in this expression are given by

$$A_{\mu}^{a+}(x)=\sum_{k}\left(\sum_{\sigma=1,2}A_{k,\sigma}^{a}f_{k,\mu}^{\sigma}(x)+A_{k}^{L,a}f_{k,L,\mu}(x)+B_{k}^{a}f_{k,S,\mu}(x)\right), \tag{49}$$

$$A_{\mu}^{a-}(x)=\sum_{k}\left(\sum_{\sigma=1,2}A_{k,\sigma}^{a+}f_{k,\mu}^{\sigma *}(x)+A_{k}^{L,a+}f_{k,L,\mu}^{*}(x)+B_{k}^{a+}f_{k,S,\mu}^{*}(x)\right). \tag{50}$$

For each color the following terms should be calculated:

$$\exp\left\{i\int d^{4}xJ^{\mu,a}(x)A_{\mu}^{a+}(x)\right\}|\Phi\rangle=\exp\left\{i\int d^{4}xJ^{\mu,a}(x)\sum_{k}\left(\sum_{\sigma=1,2}A_{k,\sigma}^{a}f_{k,\mu}^{\sigma}(x)+A_{k}^{L,a}f_{k,L,\mu}(x)+B_{k}^{a}f_{k,S,\mu}(x)\right)\right\}$$
$$\times\exp\{C_{1}(p)A_{p,1}^{a+}A_{p,1}^{a+}+C_{2}(p)A_{p,2}^{a+}A_{p,2}^{a+}+C_{3}(p)(B_{p}^{a+}A_{p}^{L,a+}+i\overline{c}_{p}^{a+}c_{p}^{a+})\}|0\rangle. \tag{51}$$

After a systematic use of the commutation relations among the annihilation and creation operators, the exponential operators can be decomposed in products of the exponential for each space-time mode. This fact allows us to perform the calculation for each kind of wave independently. The transverse components are obtained as

$$\exp\left\{i\int d^{4}xJ^{\mu,a}(x)\sum_{k}A_{k,\sigma}^{a}f_{k,\mu}^{\sigma}(x)\right\}\exp\{C_{\sigma}(p)A_{p,\sigma}^{a+}A_{p,\sigma}^{a+}\}|0\rangle \quad \text{for} \quad \sigma=1,2$$
$$=\exp\left\{C_{\sigma}(p)\left(A_{p,\sigma}^{a+}+i\int d^{4}xJ^{\mu,a}(x)f_{p,\mu}^{\sigma}(x)\right)^{2}\right\}|0\rangle. \tag{52}$$

The expressions for the longitudinal and scalars can be evaluated in a similar manner and are found to be

$$\exp\left\{i\int d^{4}xJ^{\mu,a}(x)\sum_{k}B_{k}^{a}f_{k,S,\mu}(x)\right\}\exp\left\{i\int d^{4}xJ^{\mu,a}(x)\sum_{k}A_{k}^{L,a}f_{k,L,\mu}(x)\right\}\exp\{C_{3}(p)B_{p}^{a+}A_{p}^{L,a+}\}|0\rangle$$
$$=\exp\left\{C_{3}(p)\left(B_{p}^{a+}-i\int d^{4}xJ^{\mu,a}(x)f_{p,L,\mu}(x)\right)\left(A_{p}^{L,a+}-i\int d^{4}xJ^{\mu,a}(x)f_{p,S,\mu}(x)\right)\right\}|0\rangle. \tag{53}$$

Here it should be noticed that the sign difference is produced by the commutation relations.

For the calculation of the total modification (48), we need to evaluate

$$\langle\Phi|\exp\left\{i\int d^{4}xJ^{\mu,a}(x)A_{\mu}^{a-}(x)\right\}=\left(\exp\left\{-i\int d^{4}xJ^{\mu,a}(x)A_{\mu}^{a+}(x)\right\}|\Phi\rangle\right)^{\dagger}, \tag{54}$$

which may be easily obtained by conjugating the result for the left-hand side of Eqs. (51) and (52).

In what follows the following notation will be employed:

$$J_{p,i}^{a}=\int\frac{d^{4}x}{\sqrt{2Vp_{0}}}J^{\mu,a}(x)\epsilon_{i,\mu}(p). \tag{55}$$

After forming the scalar product of Eqs. (52) and (53) the following factors are obtained for each color value:

$$\langle 0|\prod_{\sigma=1,2}\exp\{C_{\sigma}^{*}(p)(A_{p,\sigma}^{a}-iJ_{p,\sigma}^{a})^{2}\}\exp\{C_{\sigma}(p)(A_{p,\sigma}^{a+}-iJ_{p,\sigma}^{a})^{2}\}|0\rangle\langle 0|\exp\{C_{3}^{*}(p)(A_{p}^{L,a}+iJ_{p,S}^{a})(B_{p}^{a}+iJ_{p,L}^{a})\}$$
$$\times\exp\{C_{3}(p)(B_{p}^{a+}-iJ_{p,L}^{a})(A_{p}^{L,a+}-iJ_{p,S}^{a})\}|0\rangle\langle 0|\exp\{C_{3}^{*}(p)(-ic_{p}^{a}\overline{c}_{p}^{a})\}\exp\{C_{3}(p)(i\overline{c}_{p}^{a+}c_{p}^{a+})\}|0\rangle. \tag{56}$$

where the parts of the expression associated with each space-time mode are also decoupled.

In evaluating these matrix elements the idea was the following. First, we expand the exponents of the exponential operators being at the left of the scalar products in Eq. (56) by factorizing the exponential operator having an exponent being linear in the sources J. After that, taking into account that the inverse of this linear operator is annihilating the vacuum, it follows that the net effect of this linear operator is to shift the creation fields entering in the exponential operators at the right of the scalar products in Eq. (56) in a constant being linear in the sources J. Further, the same procedure can be performed to act with the exponential factor which can be also extracted from the new exponential operator acting on the vacuum at the right. Now its action on the operators at the left of the scalar product (56) reduces again to a shift in a constant in the annihilation fields defining this operator. In such a way it is possible to arrive at a recurrence relation which can be proven by mathematical induction.

The recurrence relation obtained after n steps in the case of the transverse modes takes the form

$$\exp\left\{-(J^a_{p,\sigma})^2\left[C^*_\sigma(p)+4C_\sigma(p)\left(C^*_\sigma(p)+\frac{1}{2}\right)^2\sum_{m=0}^{n}\left[4^{2m}(|C_\sigma(p)|^2)^{2m}+4^{2m+1}(|C_\sigma(p)|^2)^{2m+1}\right]\right]\right\}$$
$$\times\langle 0|\exp\{C^*_\sigma(p)(A^a_{p,\sigma})^2\}\exp\left\{-i2^{3+2n}J^a_{p,\sigma}A^a_{p,\sigma}(C^*_\sigma(p))^{n+1}(C_\sigma(p))^{n+1}\left(C^*_\sigma(p)+\frac{1}{2}\right)\right\}$$
$$\times\exp\{C_\sigma(p)(A^{a+}_{p,\sigma})^2\}|0\rangle. \tag{57}$$

After restricting the possible values of C_σ to satisfy $|C_\sigma(p)|<\frac{1}{2}$, the linear part of the operators in the exponent is multiplied by a quantity tending to zero in the limit $n=\infty$ and it can be omitted in such a limit. By also using the formula for the geometrical series the following expression can be obtained for Eq. (57):

$$\exp\left\{-(J^a_{p,\sigma})^2\left[C^*_\sigma(p)+4C_\sigma(p)\right.\right.$$
$$\left.\left.\times\left(C^*_\sigma(p)+\frac{1}{2}\right)^2\frac{1}{[1-(2|C_\sigma(p)|)^2]}\right]\right\}$$
$$\times\langle 0|\exp\{C^*_\sigma(p)(A^a_{p,\sigma})^2\}\exp\{C_\sigma(p)(A^{a+}_{p,\sigma})^2\}|0\rangle. \tag{58}$$

In a similar manner for the factors in Eq. (56) corresponding to longitudinal and scalar modes, the following can be obtained:

$$\exp\left\{-J^a_{p,S}J^a_{p,L}\left(C^*_3(p)\right.\right.$$
$$\left.\left.+C_3(p)(C^*_3(p)+1)^2\frac{1}{[1-|C_3(p)|^2]}\right)\right\}$$
$$\times\langle 0|\exp\{C^*_3(p)A^{L,a}_pB^a_p\}\exp\{C_3(p)B^{a+}_pA^{L,a+}_p\}|0\rangle. \tag{59}$$

Therefore, after collecting the contributions of all the modes and substituting $J^a_{p,i}$, by also assuming $2C_1(p)=2C_2(p)=C_3(p)$ (which follows necessarily in order to obtain the Lorentz invariance), and using the properties of the defined vectors basis, the modification to the propagator becomes

$$\exp\left\{\frac{1}{2}\int\frac{d^4x d^4y}{2p_0V}J^{\mu,a}(x)J^{\nu,a}(y)g_{\mu\nu}\right.$$
$$\left.\times\left[C^*_3(p)+C_3(p)(C^*_3(p)+1)^2\frac{1}{[1-|C_3(p)|^2]}\right]\right\}. \tag{60}$$

Now it is possible to perform the limit process $\vec{p}\to 0$. In doing this limit, it is considered that each component of the linear momentum p is related with the quantization volume by

$$p_x\sim\frac{1}{a},\quad p_y\sim\frac{1}{b},\quad p_z\sim\frac{1}{c},\quad V=abc\sim\frac{1}{p^3}.$$

Since $C_3(p)<1$ then it follows that

$$\lim_{p\to 0}\frac{C^*_3(p)}{4p_0V}\sim\lim_{p\to 0}\frac{C^*_3(p)p^3}{4p_0}=0. \tag{61}$$

For the other limit it follows that

$$\lim_{p\to 0}\frac{C_3(p)(C_3^*(p)+1)^2\frac{1}{[1-|C_3(p)|^2]}}{4p_0V}. \quad (62)$$

Then, after fixing a dependence of the arbitrary constant C_3 of the form $|C_3(p)|\sim(1-\kappa p^2),\kappa>0,y C_3(0)\neq -1$ the limit (62) becomes

$$\lim_{p\to 0}\frac{C_3(p)(C_3^*(p)+1)^2p^3\frac{1}{[1-(1-\kappa p^2)^2)]}}{4p_0}=\frac{C}{2(2\pi)^4} \quad (63)$$

where C is an arbitrary constant determined by the also arbitrary factor κ. An analysis of its properties has been done which shows that C can take only real and positive values.

Therefore, the total modification to the propagator including all color values turns out to be

$$\prod_{a=1,\ldots,8}\left\langle\tilde{\Phi}\left|\exp\left\{i\int d^4xJ^{\mu,a}(x)A_\mu^{a-}(x)\right\}\exp\left\{i\int d^4xJ^{\mu,a}(x)A_\mu^{a+}(x)\right\}\right|\tilde{\Phi}\right\rangle$$
$$=\exp\left\{\sum_{a=1,\ldots,8}\int d^4xd^4yJ^{\mu,a}(x)J^{\nu,a}(y)g_{\mu\nu}\frac{C}{2(2\pi)^4}\right\}. \quad (64)$$

Also, the generating functional associated with the proposed initial state can be written in the form

$$Z[J]=\exp\left\{\frac{i}{2}\sum_{a,b=1,\ldots,8}\int d^4xd^4yJ^{\mu,a}(x)\times\tilde{D}_{\mu\nu}^{ab}(x-y)J^{\nu,b}(y)\right\}, \quad (65)$$

where

$$\tilde{D}_{\mu\nu}^{ab}(x-y)=\int\frac{d^4k}{(2\pi)^4}\delta^{ab}g_{\mu\nu}\left[\frac{1}{k^2}-iC\delta(k)\right]\times\exp\{-ik(x-y)\}, \quad (66)$$

which shows that the gluon propagator has the same form proposed in [10], for the selected gauge parameter value $\alpha=1$ (which corresponds to $\alpha=-1$ in that reference).

Finally, let us consider the possible modifications of the ghost propagator which can be produced by the new initial state. Now, it is necessary to evaluate the expression

$$\prod_{a=1,\ldots,8}\left\langle\tilde{\Phi}\left|\exp\left\{i\int d^4x[\overline{\xi^a}(x)c^{a-}(x)+\overline{c^{a-}}(x)\xi^a(x)]\right\}\right.\right.$$
$$\left.\left.\times\exp\left\{i\int d^4x[\overline{\xi^a}(x)c^{a+}(x)+\overline{c^{a+}}(x)\xi^a(x)]\right\}\right|\tilde{\Phi}\right\rangle, \quad (67)$$

In this case the calculation is simpler because the fermionic character of the ghost makes only two nonvanishing terms exist in the series expansion of the exponential. Therefore, here it is unnecessary to employ recurrence relations. The following result for Eq. (67) is obtained:

$$\exp\left\{-\sum_{a=1,\ldots,8}i\int d^4xd^4y\overline{\xi^a}(x)\xi^a(y)\frac{C_G}{(2\pi)^4}\right\}, \quad (68)$$

where in this case C_G is an arbitrary constant which can be greater or equal than zero. It will be equal to zero if taking $C_3(0)$ real and tending to one. This selection causes the result to coincide with the one in Ref. [10] where the ghost propagator was not modified.

The expression of the generating functional for the ghost particles takes the form

$$Z_G[\overline{\xi},\xi]=\exp\left\{i\sum_{a,b=1,\ldots,8}\int d^4xd^4y\overline{\xi^a}(x)\times\tilde{D}_G^{ab}(x-y)\xi^b(y)\right\}, \quad (69)$$

where

$$\tilde{D}_G^{ab}(x-y)=\int\frac{d^4k}{(2\pi)^4}\delta^{ab}\left[\frac{(-i)}{k^2}-C_G\delta(k)\right]\times\exp\{-ik(x-y)\}. \quad (70)$$

Finally, in order to illustrate one of the main properties of the proposed modified perturbation expansion, let us review here the calculation given in [10] of the gluon condensation parameter, that is, the mean value of G^2 in the ground state. In the simplest approximation, that is, the mean value of G^2 in the interaction free initial state, it corresponds to evaluating

$$\langle 0|S_g[A]|0\rangle=\Bigg\{\Bigg[\frac{1}{2}S^g_{ij}\frac{\delta^2}{\delta j_i\delta j_j}+\frac{1}{3!}S^g_{ijk}\frac{\delta^3}{\delta j_i\delta j_j\delta j_k}+\frac{1}{4!}S^g_{ijkl}\frac{\delta^4}{\delta j_i\delta j_j\delta j_k\delta j_l}\Bigg]\Bigg\}\exp\left(-\frac{i}{2}j_iD_{ij}j_j\right),$$

where, using the DeWitt notation, the symbol $S^g_{ij\,\ldots\,l}$ represents the functional derivative of the action S^g over a number of source arguments $j^{a_i}_{\mu_i}(x_i)$, $j^{a_j}_{\mu j}(x_j)\,\ldots$ and $j^{a_l}_{\mu_l}(x_l)$. As usual in this convention, the equality of two of compact indices $i,j\,\ldots\,l$ means the sum over the color and Lorentz indices and the subsequent integration over the spacetime coordinates. The symbol D_{ij} is just the kernel of the gluon propagator (66). The first and second terms in the squared brackets have zero contribution as evaluated in dimensional regularization at zero value of the sources. On the other hand, the last terms corresponding with quartic gluon self-interaction gives a nonvanishing addition to the gluon condensation parameter precisely due to the condensate term in the propagator.

The contribution can be evaluated to be

$$\langle 0|S_g[\phi]|0\rangle=-\frac{72g^2C^2}{(2\pi)^8}\int dx,$$

which, in the present approximation, corresponds with a gluon condensation parameter given by

$$G^2\equiv\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle=\frac{288g^2C^2}{(2\pi)^8}.$$

Therefore, it turns out that the procedure is able to predict the gluon condensation at the most simple approximation.

V. SUMMARY

By using the operational formulation of the quantum gauge fields theory developed by Kugo and Ojima, a particular state vector of QCD in the noninteracting limit which obeys the BRST physical state condition was constructed. The general motivation for looking for this wave function is to search for a reasonably good description of low-energy QCD properties, through giving a foundation to the perturbative expansion proposed in [10]. The high-energy QCD description should not be affected by the modified perturbative initial state. In addition, it can be expected that the adiabatic connection of the color interaction starting with the initial condition, generates at the end, the true QCD interacting ground state. In the case having the above properties, the analysis would allow one to understand the real vacuum as a superposition of an infinite number of soft gluon pairs.

It has been checked that by properly fixing the free parameters in the constructed state, the perturbation expansion proposed in the former work is reproduced for the special value $\alpha=1$ of the gauge constant. Therefore, the appropriate gauge is determined for which the expansion introduced in that article is produced by an initial state, satisfying the physical state condition for the BRST quantization procedure. The fact that the noninteracting initial state is a physical one, leads one to expect that the final wave function after the adiabatic connection of the interaction will also satisfy the physical state condition for the interacting theory. If this assumption is correct, the results of the calculations of transition amplitudes and the values of physical quantities should also be physically meaningful. In the future, a quantization procedure for arbitrary values of α will be also considered. It is expected that with its help the gauge parameter independence of the physical quantities could be implemented. It seems possible that the results of this generalization will lead to α-dependent polarizations for gluons and ghosts and their respective propagators, which however, could produce α-independent results for the physical quantities. However, this more involved discussion will be delayed for future consideration.

It is important to mention now a result obtained during the calculation of the modification to the gluon propagator, in the chosen regularization. It is that the arbitrary constant C is determined here to be real and positive. This outcome restricts an existing arbitrariness in the discussion given in the previous work. As this quantity C is also determining the square of the generated gluon mass as positive or negative, real or imaginary, therefore it seems very congruent to arrive at a definite prediction of C as real and positive.

The modification to the standard free ghost propagator introduced by the proposed initial state of the noninteracting theory was also calculated. Moreover, after considering the free parameter in the proposed trial state as real, which seems the most natural assumption, the ghost propagator is not modified. This unaltered free one-particle Green's function was assumed in [10], as a possible variant suggested by the fact that the modifications do not contribute in the one-loop calculation of the gluon self-energy.

Some tasks which can be addressed in future works are: The study of the applicability of the Gell-Mann–Low theorem with respect to the adiabatic connection of the interactions, starting from the here-proposed initial state; the investigation of the quantization of zero modes (that is gluon states with exact vanishing four momentum); the ability to consider them with success would allow a formally cleaner definition of the proposed state, by excluding the auxiliary momentum p recursively used in the construction carried out; finally, the application of the proposed perturbation theory in the study of some problems linked with confinement and the hadron structure.

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[1] C. N. Yang and R. Mills, Phys. Rev. **96**, 191 (1954).
[2] M. Creutz, Phys. Rev. D **21**, 2308 (1980).
[3] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
[4] J. L. Gervais and A. Neveu, Phys. Lett. **80B**, 255 (1979); **80B**, 319 (1979).
[5] E. U. Shuryak, Phys. Rep. **115**, 151 (1984).
[6] E. U. Shuryak, *The QCD Vacuum, Hadrons and the Superdense Matter* (World Scientific, Singapore, 1988).
[7] T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[8] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); **B147**, 519 (1979).
[9] G. K. Savvidi, Phys. Lett. **71B**, 133 (1977).
[10] A. Cabo, S. Peñaranda, and R. Martinez, Mod. Phys. Lett. A **10**, 2413 (1995).
[11] P. Hoyer, arXiv: hep-ph/9709444, Talk presented at the APCTP-ICTP Joint International Conference 1997 (AIJIC 97) on Recent Developments in Non-perturbative Methods, 1997 in Seoul, Korea.
[12] T. Kugo and I. Ojima, Prog. Theor. Phys. **60**, 1869 (1978).
[13] T. Kugo and I. Ojima, Prog. Theor. Phys. **61**, 294 (1979).
[14] T. Kugo and I. Ojima, Prog. Theor. Phys. **61**, 644 (1979).
[15] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).
[16] C. Becchi, A. Rouet, and R. Stora, Ann. Phys. (N.Y.) **98**, 287 (1976).
[17] N. Nakanishi and I. Ojima, *Covariant Operator Formalism of Gauge Theories and Quantum Gravity* (World Scientific, Singapore, 1990).
[18] J. Govaerts, *Hamiltonian Quantization and Constrained Dynamics* (Leuven University Press, Leuven, 1991).
[19] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[20] S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).