

New and standard physics contributions to anomalous Z and γ self-couplings

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We examine the standard and the new physics (NP) contributions to the ZZZ , $ZZ\gamma$, and $Z\gamma\gamma$ neutral gauge couplings. At the one-loop level, if we assume that there is no CP violation contained in NP beyond the standard model (SM) one, we find that only CP conserving neutral gauge couplings are generated, either from the standard quarks and leptons, or from possible new physics fermions. Bosonic one-loop diagrams never contribute to these couplings, while the aforementioned fermionic contributions satisfy $h_3^Z \approx -f_5^\gamma$, $h_4^Z = h_4^\gamma = 0$. We also study examples of two-loop NP effects that could generate nonvanishing $h_4^{\gamma Z}$ couplings. We compare quantitative estimates from SM, minimal supersymmetric standard model (MSSM), and some specific examples of NP contributions, and we discuss their observability at future colliders.

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I. INTRODUCTION

Recently there has been a renewed interest in the possible existence of anomalous neutral gauge boson self-couplings. This is due to the acquisition of new experimental results at the CERN e^+e^- collider LEP2 [1] which, together with the Fermilab Tevatron results [2], begin to produce interesting constraints on such couplings, which should further improve in the future at the next colliders [3].

This has led to a reexamination of the phenomenological description commonly used for these couplings. The necessity of certain corrections was discovered and their implications for ZZ and $Z\gamma$ production at e^-e^+ and hadron colliders were discussed [4].

For what concerns the quantitative theoretical predictions for each of these neutral couplings, very little has been said up to now [5–8], in contrast to the charged (ZWW and γWW) self-couplings for which several types of predictions have been given for a long time [9–11]. A reappraisal of the theoretical expectations for these neutral couplings is still lacking. Therefore, the purpose of this paper is to fill this lack and study the standard model (SM) predictions for these couplings, as well as the predictions arising from possible new physics beyond it.

In Sec. II we first recall some general properties following from Bose statistics, Lorentz symmetry, and $SU(2) \times U(1)$ gauge invariant effective Lagrangians. The most notable of them is that the neutral gauge couplings vanish whenever all three gauge bosons are on-shell. Thus, at least one of the gauge bosons need to be off-shell, for such couplings to appear. Then, in Sec. III we consider the perturbative contributions to these couplings arising at one loop. When standard vertices for the gauge boson interactions are used, and in particular no CP violation in the photon- and Z couplings to fermions is considered, then of course only CP conserving neutral gauge self-couplings can arise. At the one-loop level, such couplings can only be induced by a fermionic triangle diagram, involving either new or SM fermions. We give the exact expression of these contributions to the real and imagi-

nary parts of the neutral gauge couplings, in terms of the fermionic ones (g_{vj} , g_{aj}) and the fermion masses M_j , as well as the squared mass s of the off-shell vector boson. To elucidate the remarkable properties of these results, we study both the high s behavior of these gauge couplings at fixed fermion mass M_j , as well as their high fermion-mass limit ($M_j^2 \gg s, m_Z^2$). As we will see, the behavior in these limits is intimately related to the way the anomaly cancellation is realized.

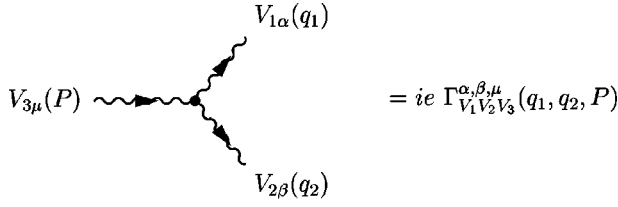
The quantitative aspects of these fermionic contributions are discussed in Sec. IV, where we consider the SM contributions to the real and imaginary parts of the couplings, as well as the relative magnitude of the lepton, light quark, and the top quark contributions. We observe that the anomaly cancellation is intimately accompanied by considerable cancellation between the lepton and quark contribution to the physical neutral gauge boson couplings at high energy. We then consider the supersymmetric contributions due to the charginos and neutralinos in the minimal supersymmetric standard model (MSSM). And finally we discuss the possibility of heavy fermions associated with some form of new physics (NP) with a high intrinsic scale Λ_{NP} .

Section V is devoted to contributions that could arise beyond the fermionic one-loop level, either through higher-order perturbative diagrams or through nonperturbative effects. Finally, in Sec. VI, we summarize our results and their consequences for the observability of neutral self-boson couplings at present and future colliders.

II. GENERAL PROPERTIES OF NEUTRAL SELF-BOSON COUPLINGS

Because of Bose statistics, the Z and γ self-couplings vanish identically, when all three particles are on-shell. The general form of the couplings of one off-shell boson ($V = Z, \gamma$) to a final pair of on-shell ZZ or $Z\gamma$ bosons, is

$$\Gamma_{ZZV}^{\alpha\beta\mu}(q_1, q_2, P) = \frac{i(s - m_V^2)}{m_Z^2} [f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) - f_5^V \epsilon^{\mu\alpha\beta\rho} (q_1 - q_2)_\rho], \quad (1)$$

FIG. 1. The general neutral gauge boson vertex $V_1 V_2 V_3$.

$$\Gamma_{Z\gamma V}^{\alpha\beta\mu}(q_1, q_2, P) = \frac{i(s - m_V^2)}{m_Z^2} \left\{ h_1^V (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\mu\beta}) \right. \\ \left. + \frac{h_2^V}{m_Z^2} P^\alpha [(P q_2) g^{\mu\beta} - q_2^\mu P^\beta] \right. \\ \left. - h_3^V \epsilon^{\mu\alpha\beta\rho} q_{2\rho} - \frac{h_4^V}{m_Z^2} P^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_{2\sigma} \right\}, \quad (2)$$

where the momenta are defined as in Fig. 1 and $s \equiv P^2$, is¹ used. The expressions (1), (2) follow from the general forms written in [12,13] and the corrections made in [4]. The forms associated with f_4^V , h_1^V , h_2^V are CP violating, whereas the ones associated with f_5^V , h_3^V , h_4^V are CP conserving.

The CP conserving forms in Eqs. (1), (2) are C and P violating and in this respect they are analogous to the anapole ZW^+W^- and γW^+W^- vertices [13,12,10],

$$\Gamma_{W^+W^-V}^{\alpha\beta\mu}(q_1, q_2, P) = i \frac{z_V}{m_W} \left\{ \epsilon^{\mu\alpha\rho\sigma} P_\sigma (q_1 - q_2)_\rho P^\beta \right. \\ \left. - \epsilon^{\mu\beta\rho\sigma} P_\sigma (q_1 - q_2)_\rho P^\alpha \right\}, \quad (3)$$

as well as to the corresponding gauge boson-fermion anomalous anapole coupling. None of these couplings exist at the tree level in the standard model (SM). At the one-loop SM level though, as we will see below, such couplings do appear and tend to be strongly decreasing with s .

Since the CP -violating couplings in Eqs. (1), (2) can never be generated, if the NP interactions of Z and photon conserve CP , we concentrate below on the CP conserving couplings f_5^V , h_3^V , h_4^V . As already observed these are analogous to the anapole ones. But the situation in this neutral anapole sector is rather different from the one in the sector of the general charged ZWW and γWW couplings. This can be seen by comparing the results of the calculation of the triangular graph of Fig. 2, with the generic expectations from a dimensional analysis in the effective Lagrangian framework. More explicitly, the contribution of a heavy fermion of mass Λ_{NP} to the aforementioned one-loop triangular graph results in an f_5^V or h_3^V coupling, which may occasionally behave like $(m_W/\Lambda_{NP})^2$. On the other hand, when one writes the effective Lagrangian in terms of $SU(2) \times U(1)$ gauge-

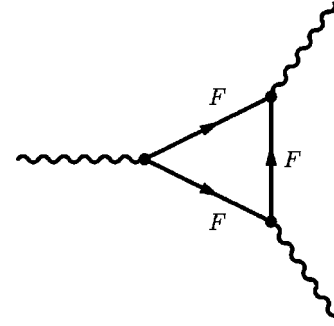


FIG. 2. The fermionic triangle.

invariant operators in the linear representation [14], then at the lowest nontrivial level of $\dim=6$ operators several anomalous ZWW and γWW couplings are generated [15,16]. However, at this level, neither the anapole ZWW coupling of Eq. (3), nor any neutral gauge couplings ever appear. These couplings require higher dimensional $\dim \geq 8$ operators, which means that their magnitude should be depressed by at least one more power of m_W^2/Λ_{NP}^2 and behave like² $(m_W/\Lambda_{NP})^4$.

It is, therefore, interesting to examine more precisely the conditions under which such couplings can be generated and what type of NP effects determine their magnitude.

III. FERMION LOOP CONTRIBUTIONS

We have first looked at the perturbative ways in which the neutral couplings in Eqs. (1), (2) could be generated. One immediately observes that at the one-loop level the relevant graphs are triangular ones of the type of Fig. 2. For scalars or W^\pm bosons running along the loop in such graphs, with standard ZWW and γWW couplings, we always get identically vanishing contributions. In particular for the CP -conserving couplings, the reason is that the $\epsilon^{\mu\nu\rho\sigma}$ tensor can never be generated from them. Only a fermionic loop (either with a single fermion F_j running along the loop, or with mixed F_1, F_2, \dots fermionic contributions), can generate such $\epsilon^{\mu\nu\rho\sigma}$ terms, through the axial Z coupling (see Fig. 2). To describe them, we use the standard definitions

$$\mathcal{L} = -e Q_j A^\mu \bar{F}_j \gamma_\mu F_j - \frac{e}{2s_W c_W} Z^\mu \bar{F}_j (\gamma_\mu g_{vj} - \gamma_\mu \gamma_5 g_{aj}) F_j \\ - \frac{e}{2s_W c_W} Z^\mu \bar{F}_1 (\gamma_\mu g_{v12} - \gamma_\mu \gamma_5 g_{a12}) F_2, \quad (4)$$

where Q_j is the F_j charge, while g_{vj} , g_{aj} and the mixed couplings g_{v12} , g_{a12} determine the Z -fermion interactions. If there are no CP -violating NP sources, then all these couplings must be real, and hermiticity requires $g_{v12} = g_{v21}$,

²The same conclusion should also be valid if the nonlinear Higgs representation is used. In this later case the ZWW anapole coupling can be generated at the dominant $D_{chiral}=4$ level; but the generation of neutral self-couplings still requires higher dimensional operators [17].

¹ $\epsilon^{0123} = 1$.

$g_{a12} = g_{a21}$. As already stated, in such a case only the CP -conserving neutral gauge boson couplings in Eqs. (1), (2) can, in principle, be generated.

Using standard techniques for preserving conserved vector current and Bose symmetry (or equivalently for isolating the anomaly contribution), we get the one-loop fermionic contributions to the CP -conserving couplings $h_3^{Z,\gamma}$ and $f_5^{Z,\gamma}$ presented in Appendix A in terms of Passarino-Veltman functions, and in Appendix B in terms of the Feynman parametrization. These expressions are used below for computing the precise predictions of the standard model and of the MSSM. Before presenting these though, the following remarks are in order.

We first note that at the one-loop level, we can never generate h_4^Z and h_4^γ : i.e.,

$$h_4^Z \equiv h_4^\gamma \equiv 0. \quad (5)$$

Therefore, the neutral couplings most likely to appear are $f_5^{\gamma,Z}$ and $h_3^{\gamma,Z}$. From the expressions given in the Appendixes A or B, it is easy to obtain their behavior in the high- and low-energy limit. At *high energy* $s \gg M_j^2, m_Z^2$, we get

$$h_3^Z \simeq -f_5^\gamma \simeq N_F \frac{e^2 Q_j g_{vj} g_{aj}}{8\pi^2 s_W^2 c_W^2} \left(\frac{m_Z^2}{s} \right), \quad (6)$$

$$f_5^Z \simeq -N_F \frac{e^2 g_{aj} [g_{aj}^2 + 3g_{vj}^2]}{48\pi^2 s_W^3 c_W^3} \left(\frac{m_Z^2}{s} \right), \quad (7)$$

$$h_3^\gamma \simeq N_F \frac{e^2 Q_j g_{vj} g_{aj}}{4\pi^2 s_W c_W} \left(\frac{m_Z^2}{s} \right), \quad (8)$$

where M_j is the mass of the single fermion F_j assumed to run along the loop in Fig. 2 and N_F is a counting factor for color and/or hypercolor.

In the opposite *heavy fermion* F_j limit where $M_j^2 \gg s, m_Z^2$, we get

$$f_5^Z \simeq \frac{e^2 g_{aj} N_F}{960\pi^2 s_W^3 c_W^3} \left(\frac{m_Z^2}{M_j^2} \right) \left[5g_{vj}^2 + g_{aj}^2 + \frac{(2s + 3m_Z^2)(7g_{vj}^2 + g_{aj}^2)}{21M_j^2} \right], \quad (9)$$

$$h_3^Z \simeq -f_3^\gamma \simeq -\frac{e^2 Q_j g_{vj} g_{aj} N_F}{96\pi^2 s_W^2 c_W^2} \left(\frac{m_Z^2}{M_j^2} \right) \times \left[1 + \frac{2(s + m_Z^2)}{15M_j^2} \right], \quad (10)$$

$$h_3^\gamma \simeq -\frac{e^2 Q_j^2 g_{aj} N_F}{48\pi^2 s_W c_W} \left(\frac{m_Z^2}{M_j^2} \right) \left[1 + \frac{2s + m_Z^2}{15M_j^2} \right], \quad (11)$$

which is the situation applying to NP contributions characterized by high scale $\Lambda_{NP} \simeq M_j$.

In both these limits, the relation

$$h_3^Z \simeq -f_3^\gamma \quad (12)$$

holds *independently* of the fermion couplings. Nevertheless, this relation cannot be exact. Indeed, as one can see from the expressions of the Feynman integrals (B4),(B1), the approximate equality (12) is violated by the different threshold effects associated with the $Z\gamma$ and ZZ final pairs, in the triangular graph in Fig. 2. It turns out though that these threshold effects are rather small and moreover, they rapidly diminish as soon as s goes away from threshold. In this respect, we have checked that in a high M_j expansion, the violation of the equality (12) is very tiny and of the order of m_Z^6/M_j^6 . The overall conclusion is therefore, that Eq. (12) is approximately correct for most of the range of the s and M_j values. This is also shown by the numerical applications presented in Sec. IV below.

We next mention the fermion contribution to the anapole ZWW and γWW couplings [18,19], since they are of the same nature as the above neutral couplings. Indeed, when one computes the triangle loop with a doublet of fermions F, F' of masses $M_F, M_{F'}$, one obtains the result given in Eqs. (B21)–(B23) in Appendix B. Using then Eqs. (B24)–(B26) for the case of standard couplings and a degenerate fermion isodoublet pair satisfying $M_F = M_{F'} \gg m_W$, that result simplifies to

$$z_{ZZ} \simeq -\frac{s_W}{c_W} z_\gamma = N_F \frac{e^2}{1152\pi^2 s_W c_W} \left(\frac{m_W^2}{M_F^2} \right) \left(1 + \frac{4s + 2m_W^2}{15M_F^2} \right), \quad (13)$$

for quarks, and

$$z_{ZZ} \simeq -\frac{s_W}{c_W} z_\gamma = -N_F \frac{e^2}{384\pi^2 s_W c_W} \left(\frac{m_W^2}{M_F^2} \right) \left(1 + \frac{4s + 2m_W^2}{15M_F^2} \right), \quad (14)$$

for leptons.

Note however that, contrary to the neutral case in which only anapole type of couplings arise, fermion loops (and also boson loops) contribute to all types of charged ZWW and γWW couplings. Complete one-loop calculations for these later couplings have been presented in the SM case [18] and in the MSSM case [19].

The numerical predictions for the fermion contributions to f_5^V and h_3^V , are strongly affected by the way the anomalies are (presumably) cancelled in the complete theory. Consequently, in the final part of this section we discuss their effect. Such anomalies are generated by triangular fermion loop diagrams. In SM they are cancelled whenever one or more complete families of leptons and quarks are considered. In the MSSM the total chargino or neutralino contributions are separately anomaly free. However, since these anomalies are proportional to the quantities

$$\sum_j g_{vj} g_{aj} Q_j, \quad \sum_j g_{aj} Q_j^2, \quad \sum_j Q_j,$$

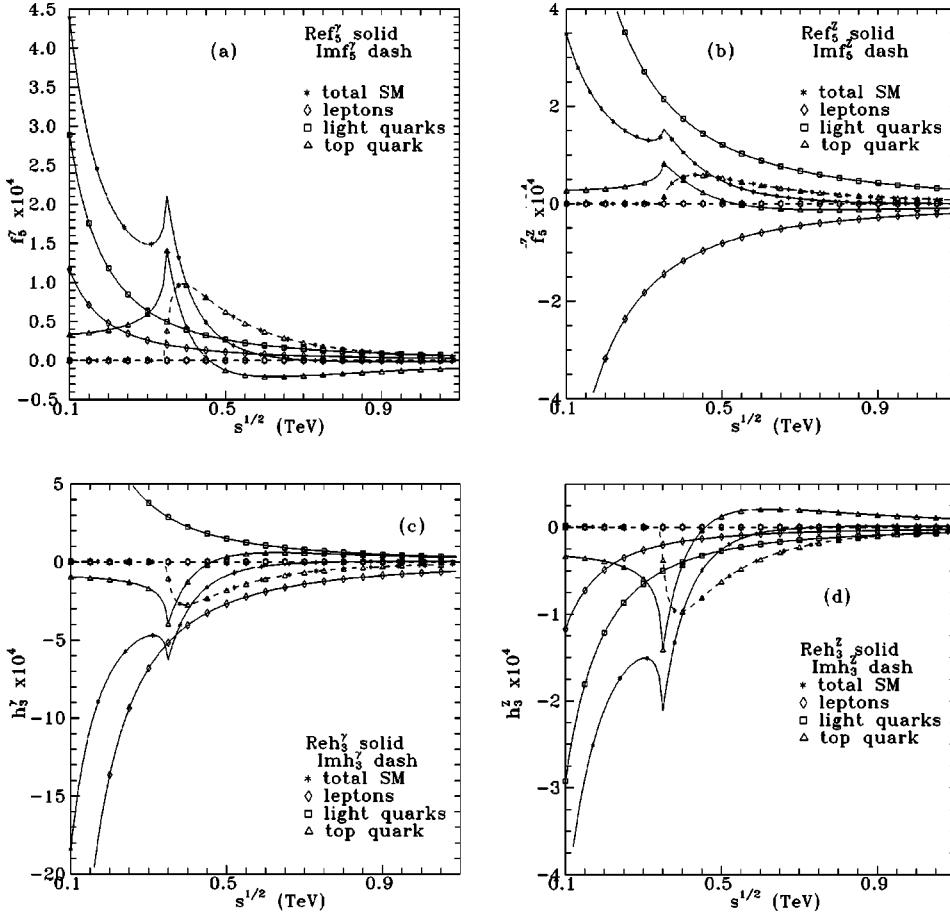


FIG. 3. The SM contributions to the neutral gauge boson self-interactions. The separate contributions from the SM leptons, the top quark and the other quarks are also given.

and independent of the masses of the fermions running along the loop, their cancellation may involve fermions with very large mass differences. Consequently, three distinct possibilities arise. Either all participating fermions are almost degenerate at scale Λ_{NP} ; or they have mass differences of the electroweak size; or finally certain fermions are much lighter than others, the contributions of the heavier ones to the neutral couplings being then negligible.

These different situations lead to very different predictions for the size of the neutral couplings. In the first case, a complete family of exactly degenerate heavy fermions (for example degenerate heavy leptons and quarks with the SM structure) would lead to the vanishing of all the NP couplings. This arises because, in this case, the combination of the heavy fermion contributions is the same as in the mass independent cancelling triangle anomaly. This is the unbroken $SU(2) \times U(1)$ situation.

If instead, one introduces mass splittings of the electroweak size (i.e., $\approx m_Z^2$) among the multiplets; like, e.g., between the heavy lepton and the heavy quark doublets, then the resulting couplings are of the order m_Z^4/M_F^4 , which means that they are suppressed by an extra power m_Z^2/M_F^2 as compared to Eqs. (9)–(11). This case corresponds to a spontaneous broken $SU(2) \times U(1)$ situation. Identifying M_F with Λ_{NP} , we then indeed get a contribution of the order $1/\Lambda_{NP}^4$, similar to what is predicted by dim=8 gauge-invariant operators.

Finally, if a single heavy fermion (or a partial set of heavy

fermions) is much lighter than all the other fermions in the family, then the couplings are directly given by the leading terms in Eqs. (9)–(11); i.e., just proportional to (m_Z^2/M_F^2) . This is obviously the most favorable situation for their observability, but it would essentially mean that $SU(2) \times U(1)$ gauge symmetry is strongly broken in the NP sector.

In the next section we give some quantitative illustrations.

IV. CONTRIBUTIONS FROM STANDARD MODEL AND BEYOND

A. The standard model contributions

The SM contribution arises from the three families of leptons and quarks, and the anomaly cancellation occurs separately inside each family. To the couplings $h_3^{Z,\gamma}$ and f_5^Z only the charged fermions contribute; while f_5^Z receives contributions from the neutrinos also. In Fig. 3 we have drawn the SM contributions to the real and the imaginary parts of these couplings. The respective magnitudes of the contributions of the leptons, the five light quarks, and the top quark are also shown. The consequences of the anomaly cancellation are clearly reflected in the behavior of the predicted couplings at high energy, i.e., for $s \gg m_Z^2, M_F^2$. Indeed it can be seen in Fig. 3 that although each fermionic contribution decreases like $\sim 1/s$ in agreement with Eqs. (6)–(8), their sum decreases like $\sim \ln s/s^2$.

The neutral couplings get imaginary parts as soon as $s > 4M_F^2$ or $M_Z^2 > 4M_F^2$. After a spectacular threshold en-

TABLE I. SM contributions in units of 10^{-4} .

\sqrt{s}	f_5^γ	f_5^Z	h_3^γ	h_3^Z
200	$2.05 - 0.15 \times 10^{-4} i$	$1.85 - 0.48 \times 10^{-4} i$	$-7.21 + 0.81 \times 10^{-2} i$	$-2.10 + 0.40 \times 10^{-2} i$
500	$0.25 + 0.62 i$	$0.46 + 0.53 i$	$-0.88 - 1.82 i$	$-0.26 - 0.62 i$

hancement though, the imaginary contribution of each fermion behaves like $(m_Z^2 M_F^2 / s^2) \ln(s/M_F^2)$ for $s \gg (M_Z^2, 4M_F^2)$. Therefore, the light fermion contribution to the imaginary parts of the couplings is strongly suppressed, and only the top quark effect is visible on Fig. 3.

Summarizing, the SM neutral couplings are in general complex but the relative importance of the real and imaginary parts is strongly energy dependent. Below the $2m_t$ threshold, the imaginary part is negligible. Above $2m_t$, real and imaginary parts have a comparable magnitude; with the imaginary part being somewhat larger. In Table I, we collect these SM contributions at LEP2 ($\sqrt{s} = 200$ GeV) and at a 500 GeV linear collider. The results in Fig. 3 and Table I indicate also the extent to which Eq. (12) is approximately correct for any s and M_j values.

B. The MSSM contributions

As a first example of new physics effect, we have computed the additional one-loop contributions arising in the minimal supersymmetric standard model (MSSM). They are rather simple, as the only new fermions are charginos and neutralinos. The two charginos $\chi_{1,2}^\pm$ contribute to the four $h_3^{Z,\gamma}$ and $f_5^{Z,\gamma}$ couplings; while the four neutralinos χ_{1-4}^0 contribute only to f_5^Z . Charginos couple to the gauge bosons through both their gaugino and higgsino components, whereas neutralinos only contribute through their higgsino components. The new feature, as compared to SM contributions, is that there now exist triangle loops with mixed contributions, for example (F_1, F_2, F_2) in the chargino case. This is caused by the nondiagonal g_{v12}, g_{a12} Z couplings in Eq. (4). The explicit expressions of these new contributions are also given in Appendixes A and B. However, because the nondiagonal Z couplings are generally weaker than the diagonal ones (see below), these mixed contributions turn out to be notably smaller than the unmixed contributions.

Let us first discuss the chargino contributions. If μ and the soft breaking supersymmetric (SUSY) parameters are taken to be real, as would be the case if no new CP violation source, beyond the one contained in the Yukawa couplings exists; then the chargino masses are given by [20]

$$M_{\chi_{1,2}}^2 = \frac{1}{2} \{ M_2^2 + \mu^2 + 2m_W^2 \mp \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4[M_2\mu - m_W^2 \sin(2\beta)]^2} \}. \quad (15)$$

The photon couplings in Eq. (4) are then fixed by $Q_{\chi_{1,2}} = +1$.

For the $Z\chi_1\chi_1$ couplings in Eq. (4) we have [20]

$$g_{v1} = \frac{3}{2} - 2s_W^2 + \frac{1}{4} [\cos 2\phi_L + \cos 2\phi_R],$$

$$g_{a1} = -\frac{1}{4} [\cos 2\phi_L - \cos 2\phi_R], \quad (16)$$

while for the $Z\chi_2\chi_2$ couplings

$$g_{v2} = \frac{3}{2} - 2s_W^2 - \frac{1}{4} [\cos 2\phi_L + \cos 2\phi_R],$$

$$g_{a2} = \frac{1}{4} [\cos 2\phi_L - \cos 2\phi_R], \quad (17)$$

where

$$\sin 2\phi_R = -\frac{2\sqrt{2}m_W(\mu \cos \beta + M_2 \sin \beta)}{D},$$

$$\sin 2\phi_L = -\frac{2\sqrt{2}m_W(\mu \sin \beta + M_2 \cos \beta)}{D}$$

$$\cos 2\phi_R = -\frac{M_2^2 - \mu^2 + 2m_W^2 \cos 2\beta}{D},$$

$$\cos 2\phi_L = -\frac{M_2^2 - \mu^2 - 2m_W^2 \cos 2\beta}{D}, \quad (18)$$

with

$$D = \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2}. \quad (19)$$

Finally, the mixed $Z\chi_1\chi_2$ couplings are

$$g_{v12} = -\frac{\text{sgn}(M_2)}{4}$$

$$\times [\text{sgn}(M_2\mu - m_W^2 \sin 2\beta) \sin 2\phi_R + \sin 2\phi_L],$$

$$g_{a12} = -\frac{\text{sgn}(M_2)}{4}$$

$$\times [\text{sgn}(M_2\mu - m_W^2 \sin 2\beta) \sin 2\phi_R - \sin 2\phi_L], \quad (20)$$

where $\text{sgn}(x)$ means the sign of x . As expected, the anomaly cancellation in the chargino sector arises when one sums the mixed and unmixed contributions of the two charginos.

For the numerical applications we used the sets of parameters at the electroweak scale presented in Table II [21].

TABLE II. Sets of MSSM chargino parameters at the electroweak scale.

Set	M_2	μ	$\tan \beta$	$M_{\chi_1^+}$	$M_{\chi_2^+}$
(1)	81	-215	2	94.71	237.94
(2)	215	-81	2	94.71	237.94
(3)	120	300	2.5	96.13	328.57
(4)	200	800	4	193.95	809.43
(5)	152	316	3	127.89	345.55
(6)	150	263	30	132.34	294.88

The main feature of the data in Table II is that almost always one of the charginos is considerably lighter than the other. Thus, at energies around $2M_{\chi_1}$ but considerably below $2M_{\chi_2}$, the dominant contribution comes from the lighter chargino, while the heavier one gives a weaker contribution of opposite sign. At energies higher than both chargino thresholds, these two contributions tend to cancel; the cancellation being a relic of the anomaly cancellation in the chargino sector.

More quantitatively, as the chargino couplings are of electroweak size, when one chargino is light enough (around 100 GeV or less), then their contribution is similar to the SM ones, for what concerns the real parts of the couplings. Similarly to the top quark SM case, the imaginary parts get

threshold enhancements just above $2M_{\chi}$, where their magnitude is comparable to that of the real part. These features can be seen in Figs. 4 and 5 where we have illustrated the cases of the Sets 5 and 6. The results for the Sets 1, 2, 3 are quite similar; although it may be remarked that the chargino contribution, particularly to f_5^Z or f_5^γ , is somewhat more pronounced there. In Set 4 the chargino contribution is very small, because there, the lightest chargino is relatively heavy, and the Z-axial couplings are very small. The latter is due to $|\mu| \gg |M_2|$ in this model, which forces ϕ_L, ϕ_R to have similar values and thereby the Z-axial couplings to have very low values. The cases of Sets 5 and 6 are identical to the low and high $\tan \beta$ scenarios suggested in [22] as a benchmark for SUSY studies. In Table III we give the precise predictions for these sets at $\sqrt{s} = 200, 500$ GeV.

In addition there is a neutralino contribution to f_5^Z . It only arises from the Z couplings to the Higgsino components. In order to have an estimate of the largest possible neutralino effect, we have taken the case in which the lightest neutralinos are of Higgsino type. As the contribution to f_5^Z is proportional to the cubic power of the Z-neutralino coupling, it would become rapidly negligible if the neutralinos were not dominantly Higgsino-like. So we have taken pure axial Z-Higgsino couplings

$$g_{\nu\chi_1^0} = g_{\nu\chi_2^0} = 0, \quad g_{a\chi_1^0} = -g_{a\chi_2^0} = 1, \quad (21)$$

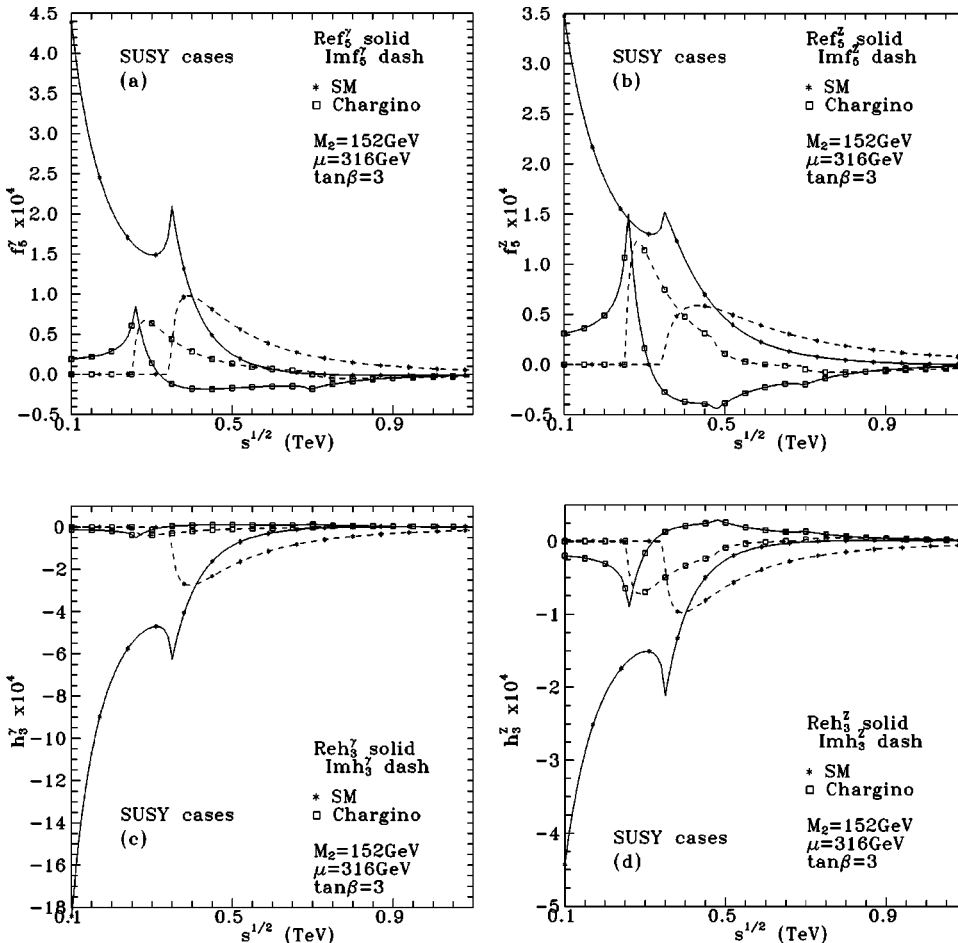


FIG. 4. Chargino contribution for the Set 5 SUSY scenario. For comparison the SM contribution is also shown. The relevant parameters are indicated in the figure.

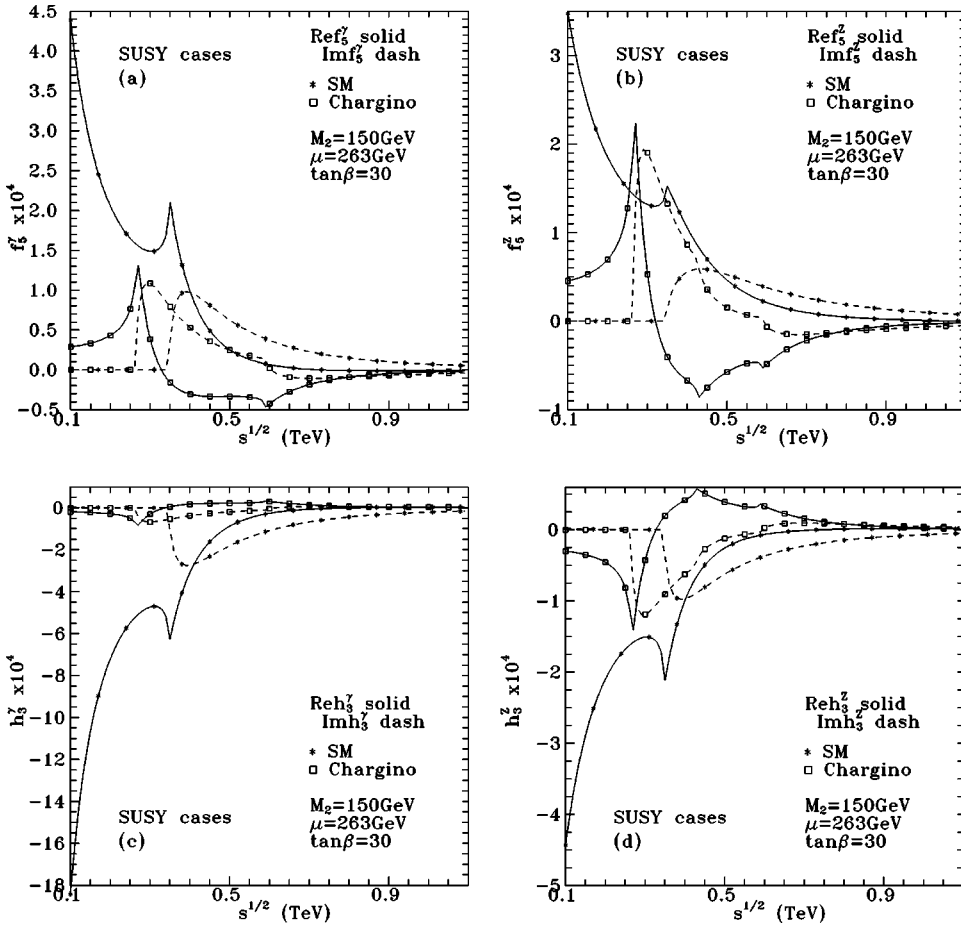


FIG. 5. Chargino contribution for the **Set 6** SUSY scenario. For comparison the SM contribution is also shown. The relevant parameters are indicated in the figure.

and only considered the contribution from two neutralinos. The results for the masses (in GeV) [23]

$$(M_{\chi_1^0}, M_{\chi_2^0}) = (71, 130), (78, 165), (93, 165), (170, 195), \quad (22)$$

are shown in Fig. 6. They are somewhat smaller than those due to charginos for comparable masses, this being due to the difference in the Z couplings. Remarkable threshold effects nevertheless appear when one neutralino is light enough. The largest effects are obtained for the couple (71, 130) GeV.

For this couple, at 200 and at 500 GeV, the contributions to f_5^Z in units of 10^{-4} are

$$-0.26 + 3.3 i \quad \text{and} \quad -0.32 - 0.22 i, \quad (23)$$

respectively; i.e., comparable to the SM and to chargino contributions. The same type of comment about the behavior of the real and of the imaginary parts can be made.

Let us finally note that, if there is nearly degeneracy of the lightest chargino and neutralino, their cumulative effect in f_5^Z can be occasionally important and increase the SM contribution by a factor of 2 to 3.

C. Other NP possibilities

The above study of the MSSM contributions gives already a good feeling of what can arise from any other perturbative NP contributions at one loop. We now want to extend the discussion by considering quantitatively, in a model-independent way, the contribution of a new fermion F . Such new fermionic states arise in many extensions of the SM, as in grand unified theory or in technicolor (TC) models.

TABLE III. Chargino contributions for Set (5,6) in units of 10^{-4} .

Set	\sqrt{s}	f_5^Z	f_5^Z	h_3^Z	h_3^Z
(5)	200 GeV	0.29	0.49	-0.19	-0.31
(5)	500 GeV	$-0.17 + 0.13 i$	$-0.39 + 0.11 i$	$0.11 - 0.10 i$	$0.26 - 0.09 i$
(6)	200 GeV	0.43	0.70	-0.29	-0.46
(6)	500 GeV	$-0.34 + 0.25 i$	$-0.57 + 0.15 i$	$0.22 - 0.20 i$	$0.39 - 0.12 i$

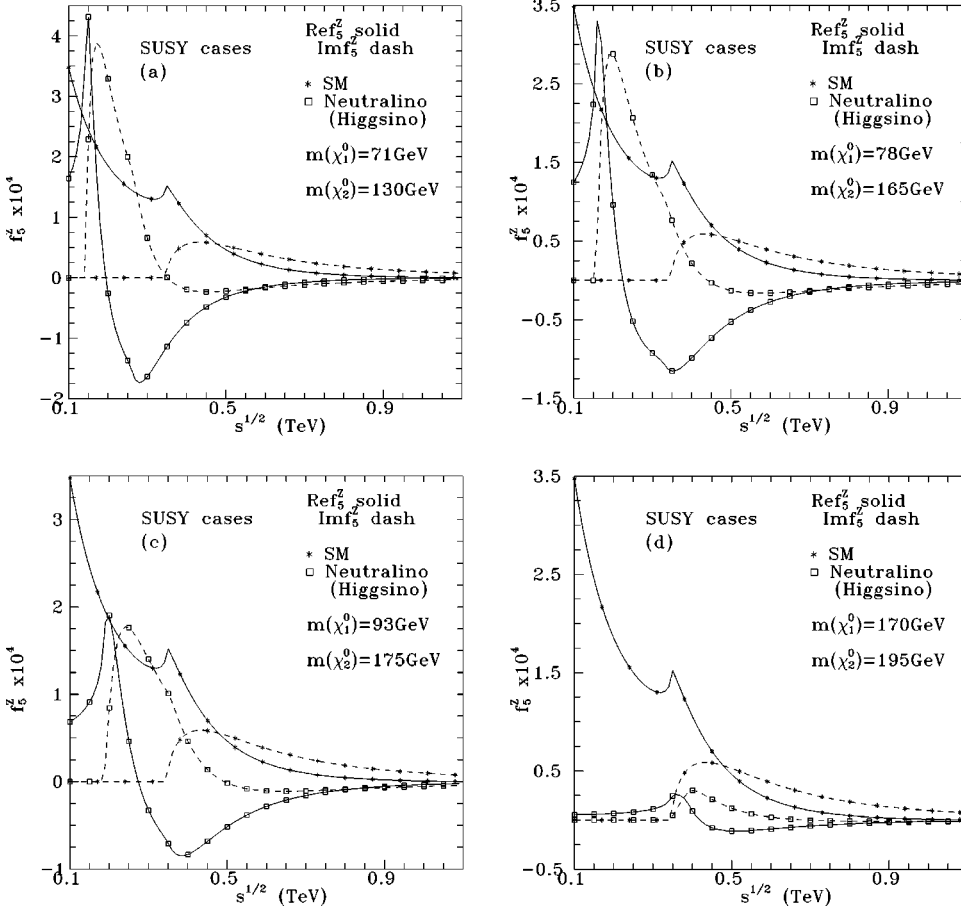


FIG. 6. Contribution to f_5^Z from two higgsino-like neutralinos with the indicated masses.

We take couplings of electroweak size, which means typical values of the order of $g_{vF} = g_{aF} = \frac{1}{2}$, $Q_F = 1$, and keep the color-hypercolor factor $N_F = 1$. In Fig. 7 we show how the resulting couplings depend on the fermion mass M_F , at fixed energies $\sqrt{s} = 200, 500$ GeV. The cusps in the real parts and the peaks in the imaginary parts that occur around $M_F = \sqrt{s}/2$, are clearly visible there. They have the same structure as in the previous SM and MSSM cases and with the chosen couplings, their magnitude is similar, i.e., a few 10^{-4} .

However, for $M_F \gg \sqrt{s}/2$, which is the case that we now want to discuss, the M_F dependence is smooth and the resulting neutral couplings are *purely real*. Thus in this limit, they can be well approximated by the energy-independent empirical formulas

$$h_3^\gamma = -0.02 \times 10^{-4} \left(\frac{1 \text{ TeV}}{M_F} \right)^2, \quad (24)$$

$$h_3^Z = -f_5^\gamma = -0.01 \times 10^{-4} \left(\frac{1 \text{ TeV}}{M_F} \right)^2, \quad (25)$$

$$f_5^Z = 0.009 \times 10^{-4} \left(\frac{1 \text{ TeV}}{M_F} \right)^2, \quad (26)$$

[compare with Eqs. (9)–(11)].

As they stand, these results would describe the most favorable NP case in which a single fermion is lighter than the other ones. With the chosen electroweak couplings and taking $N_F = 1$, such a contribution becomes nevertheless quickly unobservable when the mass M_F reaches the level of a few hundred of GeV. It could only be sizeable if the heavy fermions have enhanced couplings to the photon or to the Z (see Sec. V), or if the color-hypercolor factor N_F is large. Consequently, it is unlikely that such a new contribution (even without the depressing effect of the anomaly cancellation), can appreciably modify the SM prediction.

V. HIGHER ORDERS AND NONPERTURBATIVE EFFECTS

At the one-loop level we have exploited so far, only the fermionic triangle diagram can contribute to the generation of neutral gauge couplings. In such a case, the CP -conserving $h_4^{\gamma,Z}$ couplings are never generated. This is a direct consequence of the symmetries of the fermionic trace of the triangular diagram and of Shouten's relation. It is, therefore, interesting to examine if there is any other way to generate such $h_4^{\gamma,Z}$ couplings and enhance its magnitude; compare Eq. (2).

Perturbatively, this may happen at a higher-loop level. We have thus explored diagrams of the form of Fig. 8(a), where the hatched blob denotes a fermion one-loop diagram generating an anapole ZWW coupling. We have found that

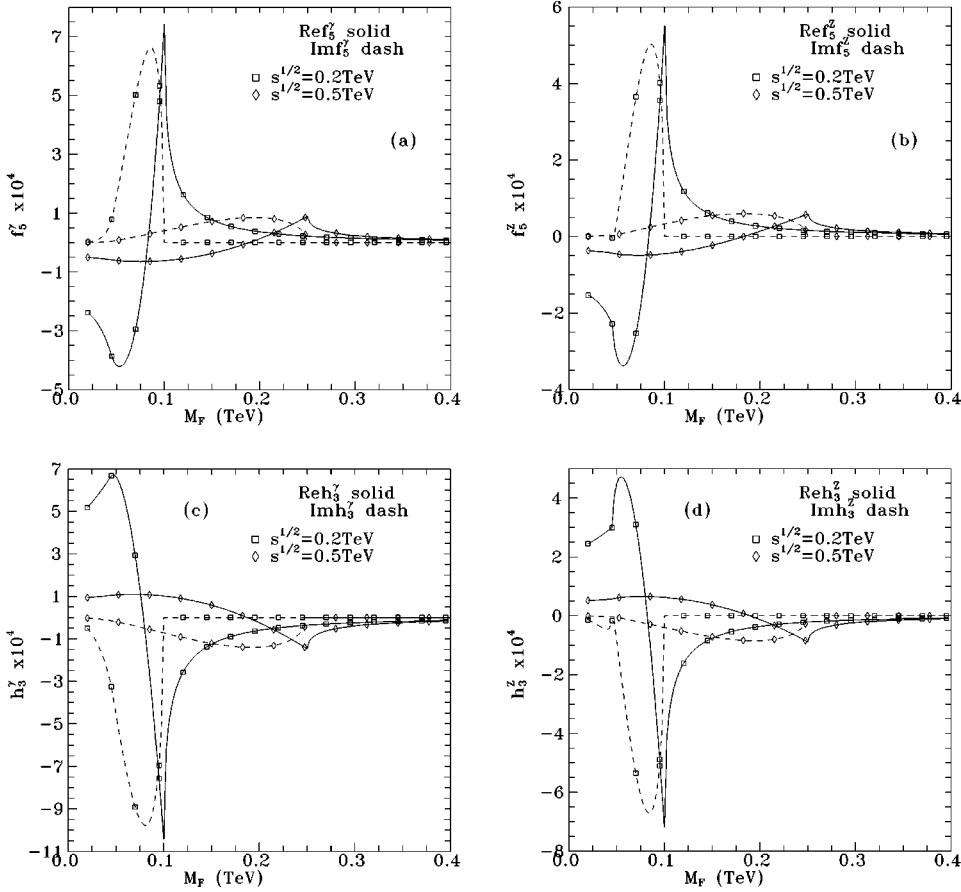


FIG. 7. Single heavy fermion contribution versus M_F at $\sqrt{s_{e^-e^+}} = 200$ and 500 GeV.

such an anapole ZWW vertex, inside the W loop of Fig. 8(a), generates an $h_4^{\gamma,Z}$ coupling of the size

$$h_4^{\gamma,Z} \sim \frac{\alpha}{4\pi} h_3^{\gamma,Z}; \quad (27)$$

i.e., the size of a typical electroweak correction to the NP prediction to $h_3^{\gamma,Z}$. Of course the result (27) should only be considered as a rough order of magnitude expectation, since a complete model prediction would require the computation of other diagrams appearing at the same two-loop order, like those depicted in Fig. 8(b).

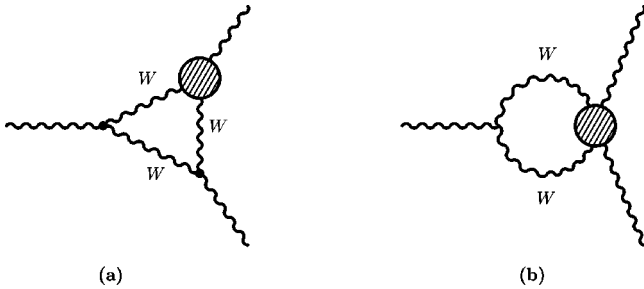


FIG. 8. Additional bosonic contribution to the fermionic triangle. The W lines represent both W^\pm and Goldstone Φ^\pm contributions. (a) Bosonic triangle and fermionic triangle (represented by the hatched blob). (b) Bosonic bubble and fermionic box (represented by the hatched blob).

It is conceivable that *nonperturbative* effects could enhance the above neutral gauge boson couplings. In this respect we may consider strong vector \mathcal{V} (ρ -like) and axial \mathcal{A} (A_1 -like) resonances coupled to the photon and Z , such as in technicolor (TC) models [24], through the junctions

$$e g_{\mathcal{V}\gamma} = e F_{\mathcal{V}} M_{\mathcal{V}}, \quad e g_{\mathcal{V}Z} = e \frac{(1-2s_W^2)}{2s_W c_W} F_{\mathcal{V}} M_{\mathcal{V}},$$

$$e g_{\mathcal{A}Z} = \frac{e}{2s_W c_W} F_{\mathcal{A}} M_{\mathcal{A}}, \quad (28)$$

where we expect that in the strong-coupling regime [24]

$$\frac{F_{\mathcal{V},\mathcal{A}}}{M_{\mathcal{V},\mathcal{A}}} \simeq O\left(\frac{1}{\sqrt{2}\pi}\right), \quad (29)$$

should hold. New strong interactions can generate nonperturbative couplings among the \mathcal{V} and \mathcal{A} vector bosons; like, e.g., those expected in the vector dominance model (VDM) of hadron physics for the ρ and A_1 vector mesons [25]. Such couplings, depicted by the central bubble in Fig. 9, could have the same Lorentz decomposition as in Eq. (2), but with strengths determined by

$$h_3^S/\Lambda_{NP}^2, \quad h_4^S/\Lambda_{NP}^4,$$

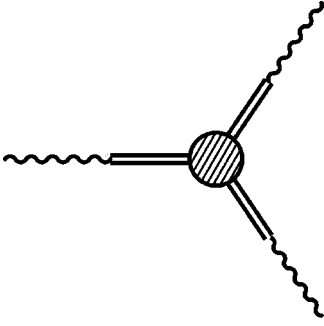


FIG. 9. Contributions from strongly interacting vector mesons. Double lines denote the heavy vector bosons (\mathcal{V} , \mathcal{A}), while the wavy lines describe Z , or γ .

where $h_{3,4}^S \sim \sqrt{4\pi}$. By multiplying these strengths by the junctions given in Eq. (28) and using the corresponding \mathcal{V} or \mathcal{A} propagators according to Fig. 9, one then obtains the corresponding predictions for neutral gauge couplings. For s , $m_Z^2 \ll M_{\mathcal{V},\mathcal{A}}^2 \approx \Lambda_{NP}^2$ we then get

$$h_4^{Z,\gamma} \approx \frac{m_Z^2}{M_{\mathcal{V},\mathcal{A}}^2} h_3^{Z,\gamma} \approx \alpha \left(\frac{m_Z^2}{M_{\mathcal{V},\mathcal{A}}^2} \right)^2. \quad (30)$$

For vector meson masses $M_{\mathcal{V},\mathcal{A}}$ not too far in the TeV range, these values may be somewhat higher than those predicted by the previous perturbative computations.

VI. CONCLUDING REMARKS

We have studied various ways to generate neutral triple gauge boson couplings among the photon and Z . Since these couplings are actually form factors involving at least one off-shell vector boson, they depend on the corresponding energy-squared variable s . The simplest way to generate them is through a fermionic triangle loop, involving fermions with arbitrary vector and axial gauge couplings. Such diagrams only generate CP -conserving couplings satisfying

$$h_3^Z \approx -f_5^\gamma, \quad h_4^Z \equiv h_4^\gamma \equiv 0, \quad (31)$$

for almost any s and fermion mass.

We have then studied the high-energy behavior of these couplings at a fixed fermion mass; as well as the high fermion mass limit at current energies. This last case allows us to illustrate different possible situations, that depend on the NP mass spectrum and on the way the anomaly cancellation takes place. The most favorable situation arises whenever one of the states needed to cancel the anomaly is much lighter than the rest.

To acquire a feeling of the expected magnitudes we have presented in Fig. 3 the SM prediction for these coupling as a function of the energy \sqrt{s} . These SM predictions are found to be at the level of a few 10^{-4} for LEP (200 GeV), while for a linear collider (LC) (500 GeV) they reduce to a few 10^{-5} ; see Table I. Just above the $2m_t$ threshold, an imaginary part of the order of 10^{-4} appears due to top quark contribution.

According to a previous study [4], such values should be unobservable at LEP2, but marginally observable at a high luminosity LC [26].

Subsequently, we have computed the supersymmetric (chargino and neutralino) contributions in MSSM, varying the input masses inside the currently reasonable ranges appearing in Table II and Eq. (22). The results strongly depend on the mass of the lightest SUSY state (chargino or neutralino). If this is close to 100 GeV, then the SUSY contribution can be comparable to or even larger (if the chargino and neutralino effects cumulate) than the contribution from the top quark, or even the total SM one; compare Figs. 4,5 with Fig. 3, and the results in Table III. As in the SM top case, a non-negligible imaginary part is again generated just above the chargino or neutralino thresholds.

To summarize, the ‘‘low mass’’ contributions, both in SM and in MSSM, predict complex and strongly energy-dependent values for these couplings, with remarkable cusp and peak effects which could, however, only be observed at a high luminosity LC. Note also that for such small values of the couplings no effect from the imaginary parts should be observable in ZZ or $Z\gamma$ production cross sections, because there is no imaginary tree-level contribution with which they could interfere, see [4].

To acquire a somewhat more model-independent feeling, we have also considered the contributions of a single heavy fermion, supposed to be the lightest one of a new physics spectrum. Keeping its gauge couplings to photon and Z as standard, we have studied its contribution versus its mass M_F . Results are presented in Fig. 7 for medium values of M_F , while empirical formulas have been established for higher values. Obviously, as soon as the fermion is too heavy to be produced in pairs, no imaginary part is generated by such NP. The real parts decrease like $1/M_F^2$ and become of the order of 10^{-6} for a mass M_F in the TeV range; compare Eqs. (24)–(26). So one would need either a large color-hypercolor factor or enhanced couplings of the heavy fermion to the photon or to the Z , in order to get observable NP contributions of this type.

We next briefly comment on the possibilities offered by e^+e^- colliders with much higher energies, like, e.g., an LC [26] at a center-of-mass energy of 1 or 1.5 TeV, or the CERN Linear Collider (CLIC) [29] at 4 or 5 TeV. The observability limit for a coupling constant like $f_5^{Z,\gamma}$ or $h_3^{Z,\gamma}$ scales with the luminosity and the center-of-mass energy squared, like $\sim 1/(\mathcal{L}s)^{1/2}$. However, a heavy NP fermion with a mass M_F comparable or higher than \sqrt{s} , gives contributions which should scale like $1/M_F^2 \sim 1/s$, so that our conclusions are the same as for a lower energy collider.

Finally, we have discussed how high-order effects could feed the couplings h_4^V , which are still vanishing at one loop. However, such contributions are always accompanied by an additional $\alpha/4\pi$ factor, which make them totally unobservable. Thus the only chance to generate h_4^V is through some kind of nonperturbative contributions. An example of such an effect based on analogy with low-energy hadronic physics and the vector dominance model has been considered. But even such a rather extreme model only leads to

$$\frac{h_4^V}{h_3^V} \sim \frac{m_Z^2}{\Lambda_{NP}^2}, \quad (32)$$

which for reasonable values of Λ_{NP} should render h_4^V unobservable.

Our overall conclusion is that it is very unlikely that the neutral gauge couplings would depart *in an observable way* from SM prediction; provided of course that the new states couple to the photon and Z through standard gauge couplings. Exceptions to this statement could come, either from low-lying (order 100 GeV) new states (for example, light charginos or neutralinos); or from enhanced photon-new fermion or Z -new fermion couplings [for example, due to some resonant states of the vector dominance model (VDM) type].

ACKNOWLEDGMENT

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APPENDIX A: FERMION LOOP CONTRIBUTIONS IN TERMS OF PASSARINO-VELTMAN FUNCTIONS

Using the short-hand notation of the Passarino-Veltman functions [27]

$$B_0(s;ij) = B_0(s;M_i,M_j), \quad (A1)$$

$$B_Z(s;ij) \equiv B_0(s;M_i,M_j) - B_0(m_Z^2 + i\epsilon;M_i,M_j), \quad (A2)$$

$$C_{ZZ}(s;ijk) \equiv C_0(m_Z^2, m_Z^2, s;M_i, M_j, M_k), \quad (A3)$$

$$C_{Z\gamma}(s;ijk) \equiv C_0(m_Z^2, 0, s;M_i, M_j, M_k), \quad (A4)$$

where $s \equiv P^2$, and observing the symmetry relations

$$B(s;ij) = B(s;ji), \quad (A5)$$

$$C_{ZZ}(s;ijk) = C_{ZZ}(s;kji), \quad (A6)$$

we get for the contribution of F_j -fermion loop

$$f_5^\gamma(j) = -\frac{N_F Q_j e^2 m_Z^2 g_{aj} g_{vj}}{8\pi^2 s_W^2 c_W^2 (s - 4m_Z^2)^2 s} \left\{ 2B_Z(s;jj) m_Z^2 (s + 2m_Z^2) + (s - 2m_Z^2)(s - 4m_Z^2) \right. \\ \left. + 2C_{ZZ}(s;jjj) [M_j^2 (s - 4m_Z^2)(s - 2m_Z^2) + 2m_Z^4 (s - m_Z^2)] \right\}, \quad (A7)$$

$$f_5^Z(j) = -\frac{N_F e^2 m_Z^2 g_{aj}}{16\pi^2 s_W^3 c_W^3 (s - 4m_Z^2)^2} \left\{ \frac{(3g_{vj}^2 + g_{aj}^2)(s - 4m_Z^2)}{3} + \frac{B_Z(s;jj)}{s - m_Z^2} [4g_{aj}^2 M_j^2 (s - 4m_Z^2) + (3g_{vj}^2 + g_{aj}^2) m_Z^2 (s + 2m_Z^2)] \right. \\ \left. + 2C_{ZZ}(s;jjj) [(g_{vj}^2 + g_{aj}^2) M_j^2 (s - 4m_Z^2) + m_Z^4 (3g_{vj}^2 + g_{aj}^2)] \right\}, \quad (A8)$$

$$h_3^\gamma(j) = \frac{N_F e^2 Q_j^2 g_{aj} m_Z^2}{4\pi^2 c_W s_W (s - m_Z^2)} \left[1 + 2M_j^2 C_{Z\gamma}(s;jjj) + \frac{m_Z^2}{s - m_Z^2} B_Z(s;jj) \right], \quad (A9)$$

$$h_3^Z(j) = \frac{N_F e^2 Q_j g_{vj} g_{aj} m_Z^2}{4\pi^2 s_W^2 c_W^2 (s - m_Z^2)^2} \left\{ \frac{sm_Z^2}{s - m_Z^2} B_Z(s;jj) + \frac{s + m_Z^2}{2} [2C_{Z\gamma}(s;jjj) M_j^2 + 1] \right\}, \quad (A10)$$

where M_j is the F_j mass, and N_F denotes any (color hypercolor) counting factor.

The mixed term contribution arises when two different fermions, having the same charge but mixed ZF_1F_2 couplings, are running along the loop in Fig. 2. We consider mixed ZF_1F_2 couplings of the type appearing in the second line of Eq. (4), which arise, e.g., in the chargino and neutralino cases.³ Thus, for the $ZZ\gamma^*$ case we obtain

$$f_5^\gamma(12) = -\frac{N_F Q_1 e^2 m_Z^2 g_{a12} g_{v12}}{8\pi^2 s_W^2 c_W^2 (s - 4m_Z^2)^2 s} R_{ZZ\gamma^*}, \quad (A11)$$

where

³For simplicity we disregard the case where three different fermions run along the loop in Fig. 2, which could, in principle, only arise in the case of three light neutralino states.

$$\begin{aligned}
R_{ZZ\gamma^*} = & 4(s-m_Z^2)(M_2^2-M_1^2)[B_0(s;11)-B_0(s;22)]+2(s-2m_Z^2)(s-4m_Z^2)+2m_Z^2(s+2m_Z^2) \\
& \times [B_0(s;11)+B_0(s;22)-2B_0(m_Z^2+i\epsilon;12)]+4(s-m_Z^2)(M_1^4+M_2^4+m_Z^4-2M_1^2M_2^2-2M_1^2m_Z^2-2M_2^2m_Z^2) \\
& \times [C_{ZZ}(s;121)+C_{ZZ}(s;212)]+2s(s+2m_Z^2)[M_2^2C_{ZZ}(s;121)+M_1^2C_{ZZ}(s;212)], \tag{A12}
\end{aligned}$$

and $Q_1=Q_2$ is the common charge of F_1, F_2 .

Correspondingly for the ZZZ^* case we get

$$\begin{aligned}
f_5^Z(12) = & -\frac{N_F e^2 m_Z^2}{16\pi^2 s_W^3 c_W^3 (s-4m_Z^2)} \{ [g_{a1}(g_{a12}^2+g_{v12}^2)+2g_{a12}g_{v12}g_{v1}]R_a \\
& + [g_{a1}(g_{a12}^2+g_{v12}^2)-2g_{a12}g_{v12}g_{v1}]R_b + g_{a1}(g_{a12}^2-g_{v12}^2)R_c + (1\leftrightarrow 2) \}, \tag{A13}
\end{aligned}$$

where

$$\begin{aligned}
R_a = & 1 + \frac{2(M_1^2-M_2^2)(s+2m_Z^2)}{s(s-4m_Z^2)} [B_0(m_Z^2;12)-B_0(m_Z^2;11)] - \frac{1}{(s-m_Z^2)(s-4m_Z^2)} \\
& \times \left\{ \frac{C_{ZZ}(s;112)}{s} [-2M_1^2s^3-s^2(M_1^2(M_1^2-M_2^2)-m_Z^2(7M_1^2+3M_2^2-4m_Z^2))+4sm_Z^2(M_1^2M_2^2-2M_1^2m_Z^2-M_2^4+m_Z^4) \right. \\
& + 4m_Z^4(M_1^2-M_2^2)^2] - C_{ZZ}(s;121)[s^2M_2^2+s(M_1^4-3M_1^2M_2^2-3M_1^2m_Z^2+2M_2^4-2M_2^2m_Z^2+2m_Z^4) \\
& + 2m_Z^2(M_1^4-M_2^4+2M_2^2m_Z^2-m_Z^4)] + B_Z(s;11)[(M_1^2-m_Z^2)(s+2m_Z^2)-2M_2^2(s-m_Z^2)] \\
& \left. + B_Z(s;12)[-3M_1^2(s-2m_Z^2)+(M_2^2-2m_Z^2)(s+2m_Z^2)] \right\}, \tag{A14}
\end{aligned}$$

$$R_b = \frac{M_1^2}{s-m_Z^2} \{ (M_1^2-M_2^2-m_Z^2)[C_{ZZ}(s;112)-C_{ZZ}(s;121)] + (s-m_Z^2)C_{ZZ}(s;112) + B_Z(s;11) + 2B_Z(s;12) \}, \tag{A15}$$

$$R_c = \frac{M_1M_2}{s-m_Z^2} \{ (2M_1^2-2M_2^2+s)[C_{ZZ}(s;112)-C_{ZZ}(s;121)] + 2(s-m_Z^2)C_{ZZ}(s;121) + 2B_Z(s;11) + 4B_Z(s;12) \}. \tag{A16}$$

Finally, for the $Z\gamma Z^*$ mixed case we have

$$h_3^Z(12) = \frac{N_F e^2 Q_1 g_{v12} g_{a12} m_Z^2}{4\pi^2 s_W^2 c_W^2 (s-m_Z^2)^2} \left\{ \frac{2sm_Z^2}{s-m_Z^2} B_Z(s;12) + (s+m_Z^2)[M_2^2 C_{Z\gamma}(s;122) + M_1^2 C_{Z\gamma}(s;211) + 1] \right\}. \tag{A17}$$

We note that there is no mixed contribution for h_3^γ .

APPENDIX B: FERMION LOOP CONTRIBUTIONS IN TERMS OF FEYNMAN INTEGRALS

The contributions of a single F_j -fermion loop in terms of Feynman integrals are

$$f_5^\gamma(j) = N_F \frac{e^2 Q_j g_{vj} g_{aj}}{4\pi^2 s_W^2 c_W^2} I_5^\gamma, \tag{B1}$$

$$f_5^Z(j) = -N_F \frac{e^2 g_{aj}}{96\pi^2 s_W^3 c_W^3} ([g_{aj}^2+3g_{vj}^2]I_5^{Z1} + [g_{vj}^2-g_{aj}^2]I_5^{Z2}), \tag{B2}$$

$$h_3^\gamma(j) = -N_F \frac{e^2 Q_j^2 g_{aj}}{2\pi^2 s_W c_W} I_3^\gamma, \tag{B3}$$

$$h_3^Z(j) = -N_F \frac{e^2 Q_j g_{vj} g_{aj}}{4\pi^2 s_W^2 c_W^2} I_3^Z, \quad (\text{B4})$$

with

$$I_5^\gamma = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1(1-x_1-x_2)m_Z^2}{D_{ZZ}(s)}, \quad (\text{B5})$$

$$I_5^{Z1} = \frac{6m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ \frac{[(1-x_1-x_2)(m_Z^2 x_2^2 - s x_1^2) + x_2(1-3x_1)(s x_1 + m_Z^2(2x_2-1))]}{D_{ZZ}(s)} - \frac{m_Z^2[(1-x_1-x_2)(x_2^2-x_1^2) + x_2(1-3x_1)(x_1+2x_2-1)]}{D_{ZZ}(m_Z^2)} \right\}, \quad (\text{B6})$$

$$I_5^{Z2} = \frac{6M_F^2 m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{(1-3x_1)}{D_{ZZ}(s)}, \quad (\text{B7})$$

where

$$D_{ZZ}(s) \equiv M_j^2 + s x_1(x_1+x_2-1) + m_Z^2 x_2(x_2-1), \quad (\text{B8})$$

and

$$I_3^\gamma = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1(1-x_1-x_2)m_Z^2}{D_{Z\gamma}(s)}, \quad (\text{B9})$$

$$I_3^Z = \frac{m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{(s x_1 - m_Z^2 x_2)(1-x_1-x_2)}{D_{Z\gamma}(s)}, \quad (\text{B10})$$

where

$$D_{Z\gamma}(s) \equiv M_j^2 + s x_1(x_1+x_2-1) + m_Z^2 x_2(x_1+x_2-1). \quad (\text{B11})$$

The mixed contributions due to two different fermions (with the same charge) around the loop are

$$h_3^Z(12) = -N_F \frac{e^2 Q_1 g_{v12} g_{a12}}{4\pi^2 s_W^2 c_W^2} I_3^Z + (1 \leftrightarrow 2), \quad (\text{B12})$$

where I_3^Z is given by Eq. (B10) with $D_{Z\gamma}(s)$ replaced by

$$D'_{Z\gamma}(s) \equiv M_1^2 + (M_2^2 - M_1^2)(1-x_1-x_2) + s x_1(x_1+x_2-1) + m_Z^2 x_2(x_1+x_2-1). \quad (\text{B13})$$

Correspondingly,

$$f_5^\gamma(12) = N_F \frac{e^2 Q_1 g_{v12} g_{a12}}{4\pi^2 s_W^2 c_W^2} I_5^\gamma + (1 \leftrightarrow 2), \quad (\text{B14})$$

where I_5^γ is given by the same expression as in Eq. (B5) with $D_{ZZ}(s)$ replaced by

$$D'_{ZZ}(s) \equiv M_1^2 + (M_2^2 - M_1^2)x_2 + s x_1(x_1+x_2-1) + m_Z^2 x_2(x_2-1). \quad (\text{B15})$$

Finally,

$$f_5^Z(12) = -N_F \frac{e^2}{96\pi^2 s_W^3 c_W^3} \{ [g_{a1}(g_{a12}^2 + g_{v12}^2) + 2g_{v1}g_{a12}g_{a12}] I_5^{Za} + M_1^2 [g_{a1}(g_{a12}^2 + g_{v12}^2) - 2g_{v1}g_{a12}g_{v12}] I_5^{Zb} + 2M_1 M_2 g_{a1}(g_{v12}^2 - g_{a12}^2) I_5^{Zc} \} + (1 \leftrightarrow 2), \quad (\text{B16})$$

with

$$I_5^{Za} = \frac{3m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ [s x_1(1-x_1-x_2)(x_2-1) + m_Z^2 x_2^2(1-x_2)] \left(\frac{1}{D_1} + \frac{2}{D_2} \right) + (1-3x_2)(\ln D_1 + 2 \ln D_2) \right\}, \quad (\text{B17})$$

$$I_5^{Zb} = \frac{3m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ \frac{1-x_1}{D_1} - \frac{2x_2}{D_2} \right\}, \quad (\text{B18})$$

$$I_5^{Zc} = \frac{3m_Z^2}{s-m_Z^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ \frac{2x_2}{D_1} - \frac{2(1-2x_2)}{D_2} \right\}, \quad (\text{B19})$$

and

$$D_1 = M_1^2 + (M_2^2 - M_1^2)x_2 + sx_1(x_1 + x_2 - 1) + m_Z^2 x_2(x_2 - 1),$$

$$D_2 = M_1^2 + (M_2^2 - M_1^2)x_1 + sx_1(x_1 + x_2 - 1) + m_Z^2 x_2(x_2 - 1). \quad (\text{B20})$$

Some of the unmixed parts of these expressions can be derived from the results obtained in [6]. We have checked by doing a direct numerical integration that they agree with the numerical results obtained from the Passarino-Veltman expressions and the FF package [28].

It is easy, from the above analytic expressions, to derive the asymptotic expressions given in Eqs. (6)–(8) and (9)–(11).

For comparison we also give the fermion doublet contribution to the γWW and ZWW anapole couplings defined in Eq. (3). Denoting the “up” and “down” fermions as F and F' and defining their Z couplings as in Eq. (4), we get

$$z_Z = -N_F \frac{e^2}{32\pi^2 s_W c_W} ([g_{vF} + g_{aF}]I_F + [g_{vF'} + g_{aF'}]I_{F'}), \quad (\text{B21})$$

$$z_\gamma = -N_F \frac{e^2}{16\pi^2 s_W^2} (Q_F I_F + Q_{F'} I_{F'}), \quad (\text{B22})$$

where

$$I_F = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1(1-x_1-x_2)m_W^2}{M_F^2 + sx_1(x_1+x_2-1) + m_W^2 x_2(x_1+x_2-1) + (M_{F'}^2 - M_F^2)x_2}, \quad (\text{B23})$$

and $I_{F'} = I_F(M_F^2 \rightarrow M_{F'}^2)$. In the case of a degenerate doublet $M_F = M_{F'}$, with standard couplings, this result simplifies to

$$z_Z = -\frac{s_W}{c_W} z_\gamma = N_F \frac{e^2}{48\pi^2 s_W c_W} I_W, \quad (\text{B24})$$

for a doublet of quarks, and

$$z_Z = -\frac{s_W}{c_W} z_\gamma = -N_F \frac{e^2}{16\pi^2 s_W c_W} I_W, \quad (\text{B25})$$

for a doublet of leptons, with

$$I_W = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1(1-x_1-x_2)m_W^2}{M_F^2 + sx_1(x_1+x_2-1) + m_W^2 x_2(x_1+x_2-1)}, \quad (\text{B26})$$

which, for $M_F \gg m_W$, leads to Eqs. (13) and (14).

Note that for a completely degenerate standard family (a doublet of lepton and a doublet of colored quarks) the total contribution vanishes.

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