Axial vector current in an electromagnetic field and low-energy neutrino-photon interactions

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An expression for the axial vector current in a strong, slowly varying electromagnetic field is obtained. We apply this expression to the construction of the effective action for low-energy neutrino-photon interactions.

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The study of neutrino-photon interactions began many years ago when Pontecorvo [1] and Chiu and Morrison [1] suggested that the process $\gamma\gamma\rightarrow\nu\bar{\nu}$ may be important in the analysis of stellar cooling. These interactions are of interest in astrophysics and cosmology.

It is well known that, in the standard model, neutrino-photon interactions appear at the one-loop level. In addition to the heavy gauge bosons which can immediately be integrated out at the desired scales leading to the effective four-fermion interaction, charged fermions (electrons) run in the loop; in particular, the coupling of the photons to these fermions is responsible for the $\gamma\nu$ interactions.

A variety of processes of physical relevance in different situations belong to this class of $\gamma\nu$ interactions, for which an effective-action description involving only neutrino currents and field strength tensors has partly been found. E.g., Dicus and Repko [2] have recently derived an effective action for interactions between two neutrinos and three soft photons, which immediately allows for the calculation of all scattering amplitudes of this type. This effective action can be derived via the expectation value of the electromagnetic vector current $\langle j_\mu\rangle = e\langle\bar{\Psi}(x)\gamma_\mu\Psi(x)\rangle$; the latter can in turn be calculated very easily from the Heisenberg-Euler effective Lagrangian $\mathcal{L}_{\rm HE}$ with the aid of the formula $\langle\bar{\Psi}(x)\gamma_\mu\Psi(x)\rangle = -\delta\mathcal{L}_{\rm HE}/e\,\delta A^\mu(x)$. In [3], the effective action of Dicus and Repko was reproduced by a more direct diagrammatic approach.

Incidentally, processes with two photons, for example, neutrino-photon scattering $\gamma\nu\rightarrow\gamma\nu$, turn out to be highly suppressed in the standard model [4–7] because, according to Yang's theorem [8], two photons cannot couple to the $J=1$ state. As a result, typical cross sections are exceedingly small and suppressed by factors of $1/m_W^2$ (see, e.g., [9]).

The presence of a medium or external electromagnetic field drastically changes the situation. It induces an effective coupling between photons and neutrinos, even enhancing processes such as $\nu\rightarrow\nu\gamma$. Based on the same effective action of Dicus and Repko [2], it was shown in [10] that in the presence of an external magnetic field, cross sections for neutrino-photon processes such as $\gamma\gamma\rightarrow\nu\bar{\nu}$ and $\nu\gamma\rightarrow\nu\gamma$ are amplified by the factor $\sim(m_W/m_e)^4(B/B_c)^2$ for soft photon frequencies $\omega\ll m_e$ and a weak magnetic field $B\ll B_c \equiv m^2/e$ (for extensions to this result, see [11] and [12]). The subject of the electromagnetic properties of neutrinos in media was comprehensively studied in [13] (see also [14] for neutrinos in magnetized media).

In general, an effective action describing low-energy neutrino-photon processes with an arbitrary number of photons and in the presence of strong external electromagnetic fields appears highly desirable. In other words, we are looking for the analogue of the Heisenberg-Euler effective action which turned out to be extremely useful in QED (see, e.g., [15]).

For this, we start from the effective four-fermion interaction Lagrangian valid at energies very much smaller than the W- and Z-boson masses:

$$\mathcal{L}_{4f}=\frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\mu(1+\gamma_5)\nu\bar{E}\gamma_\mu(g_V+g_A\gamma_5)E. \tag{1}$$

Here, E denotes the electron field, $\gamma_5=-i\gamma^0\gamma^1\gamma^2\gamma^3$, $g_V=1-\frac{1}{2}(1-4\sin^2\theta_W)$ and $g_A=1-\frac{1}{2}$ for ν_e, where the first terms in g_V and g_A are the contributions from the W exchange diagram and the second terms are from the Z exchange diagram. Also, $g_V=2\sin^2\theta_W-\frac{1}{2}$ and $g_A=-\frac{1}{2}$ for $\nu_{\mu,\tau}$.

Now concentrating on soft electromagnetic degrees of freedom with energies much smaller than the electron mass, we may integrate out the actual "heaviest" particle, i.e., the electron, and find

$$\mathcal{L}_{\rm eff}=\frac{G_F}{\sqrt{2}}\frac{1}{e}\bar{\nu}\gamma^\mu(1+\gamma_5)\nu\ (g_{\rm V}\langle j^\mu\rangle^A+g_{\rm A}\langle j_5^\mu\rangle^A), \tag{2}$$

where the expectation values of the currents are given in terms of the Green's function in this field: e.g., $\langle j_5^\mu\rangle^A = {\rm i}e\,{\rm tr}[\gamma^\mu\gamma_5 G(x,x|A)]$.

Obviously, in order to obtain this effective Lagrangian, one must calculate the expectation values of vector and axial vector currents in a slowly varying electromagnetic field background. As mentioned above, the vector current expectation value can be easily obtained using the well-known Heisenberg-Euler Lagrangian $\mathcal{L}_{\rm HE}$: $\langle\bar{E}(x)\gamma_\mu E(x)\rangle = -\delta\mathcal{L}_{\rm HE}/e\,\delta A^\mu(x)$. In this way, one obtains a derivative expansion of the vector current around a strong field. For example, the term which is third order in the field and first order in derivatives was used by Dicus and Repko [2] in their study of $\nu\gamma\rightarrow\nu\gamma\gamma$ and cross processes.

With regard to Eq. (2), the derivative expansion of the axial vector current around an arbitrarily strong field is finally required. To our surprise, we could not find such an

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expression in the vast literature on derivative expansions. Therefore, its derivation remains the final task of our present work.

It is convenient to choose a special gauge condition for the potential $A_\mu(x)$ without loss of generality: the so-called Fock-Schwinger gauge[1]

$$(x^\mu - y^\mu) A_\mu(x) = 0, \tag{3}$$

which is satisfied by the series

$$A_\mu(x) = \frac{1}{2}(x^\lambda - y^\lambda) F_{\lambda\mu}(y) + \frac{1}{3}(x^\lambda - y^\lambda) \times (x^\sigma - y^\sigma) \partial_\sigma F_{\lambda\mu}(y) + \cdots . \tag{4}$$

Abbreviating the first term on the right-hand side with $A_{c\mu}$ and the second term with a_μ, we may perform a perturbation expansion for the Green's function with respect to the derivative term a_μ:

$$G(x,x'|A) = G_c(x,x'|A_c) + \int d^4w\, G_c(x,w|A_c)\, e a_\mu(w) \times \gamma^\mu G_c(w,x'|A_c) + \cdots , \tag{5}$$

where $G_c(x,x'|A_c)$ is the Green's function of the electron in the constant field produced by A_c. Inserting the expansion (5) into the definition of $\langle j_5^\mu \rangle^A$, we obtain, to first order in derivatives,

$$\langle j_5^\mu \rangle^A = \frac{1}{3} \partial_\sigma F_{\alpha\beta} \frac{\partial^2}{\partial k^\sigma \partial k^\alpha} [\Pi_5^{\beta\mu}(-k)]|_{k=0}, \tag{6}$$

where we encounter the vector–axial-vector (VA) amplitude $\Pi_5^{\beta\mu}$, i.e., the axial analogue of the polarization tensor. This VA amplitude has been calculated very recently by one of the authors in [18]; an independent confirmation can be found in [19].

Inserting the presentation of [18] into Eq. (6), we finally arrive at the first-order gradient expansion of the axial vector current for arbitrarily strong electromagnetic fields valid for slowly varying fields

$$\begin{aligned}
\langle j_5^\mu \rangle^A &= \mathrm{i}\, e\, \mathrm{tr}[\gamma^\mu \gamma_5 G(x,x|A)] \\
&= \mathrm{i} \frac{e^2}{24\pi^2} \partial_\sigma F_{\lambda\nu} \int_0^\infty \frac{ds}{s} e^{-ism^2} \frac{(eas)(ebs)}{\sin(ebs)\sinh(eas)} \\
&\quad \times \{[C^{\nu\lambda}(C^2)^{\mu\sigma} + C^{\nu\sigma}(C^2)^{\mu\lambda}] N_1 \\
&\quad + [B^{\nu\lambda}(B^2)^{\mu\sigma} + B^{\nu\sigma}(B^2)^{\mu\lambda}] N_2 \\
&\quad - [3F^{*\nu\mu} g^{\lambda\sigma} + F^{*\nu\lambda} g^{\mu\sigma} + F^{*\nu\sigma} g^{\mu\lambda}] N_3\},
\end{aligned} \tag{7}$$

where the N_i are simple functions of the field strength:

$$N_1 = \frac{2\sin(ebs)}{\sinh^2(eas)} \left[\cosh(eas) - \frac{\sinh(eas)}{eas} \right] + 2ebs\, N_3$$

$$N_2 = \frac{2\sinh(eas)}{\sin^2(ebs)} \left[-\cos(ebs) + \frac{\sin(ebs)}{ebs} \right] + 2eas\, N_3$$

$$N_3 = \frac{1}{eas\, ebs(a^2+b^2)} \left[a^2 \frac{\sin(ebs)}{\sinh(eas)} - b^2 \frac{\sinh(eas)}{\sin(ebs)} \right].$$

For the analysis, we employed a decomposition of the field strength tensor in linearly independent subspaces of electric and magnetic parts:

$$C_{\mu\nu} := \frac{1}{a^2+b^2} (a F_{\mu\nu} + b F^*_{\mu\nu}),$$

$$B_{\mu\nu} := \frac{1}{a^2+b^2} (b F_{\mu\nu} - a F^*_{\mu\nu}), \tag{8}$$

where the invariants a and b are defined by

$$a,b = \sqrt{(\mathcal{F}^2 + \mathcal{G}^2)^{1/2} \pm \mathcal{F}}, \quad \mathcal{F} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$\mathcal{G} = -\frac{1}{4} F^*_{\mu\nu} F^{\mu\nu}. \tag{9}$$

As a consequence, we find[2]

$$(C^2)_{\mu\nu} = \frac{1}{a^2+b^2} (F^2_{\mu\nu} + b^2 g_{\mu\nu}),$$

$$(B^2)_{\mu\nu} = \frac{1}{a^2+b^2} (F^2_{\mu\nu} - a^2 g_{\mu\nu}). \tag{10}$$

Concerning our central result for the axial current in Eq. (7), we observe its linearity in the derivative of $F_{\mu\nu}$ (by construction) which is multiplied by a finite proper-time integral containing the complete dependence on the strength of the fields. We would like to point out that our method of performing the derivative expansion in momentum space [cf. Eq. (6)] is much in the spirit of Ref. [20] with the essential difference that we expand around an arbitrary value of field strength.

Our representation of $\langle j_5^\mu \rangle$ in terms of the $C_{\mu\nu}$ and $B_{\mu\nu}$ tensors is very convenient for performing a weak-field expansion; to second order in $1/m^2$, it finally reads

[1]Especially for gradient expansions of the heat kernel or the Heisenberg-Euler Lagrangian, this gauge has proved extremely useful; see, e.g., [16,17].

[2]We always use the metric $g = \mathrm{diag}(+---)$.

$$\langle j_5^\mu\rangle^A = \frac{e^3}{24\pi^2 m^2}[\partial^\mu \mathcal{G} + (\partial^\alpha F_{\alpha\beta})F^{*\beta\mu}] + \frac{e^5}{90\pi^2 m^6} \times \partial_\sigma F_{\alpha\beta}\{\mathcal{G}(F^{\beta\alpha}g^{\mu\sigma} + F^{\beta\sigma}g^{\mu\alpha}) + [F^{*\beta\alpha}(F^2)^{\mu\sigma} + F^{*\beta\sigma}(F^2)^{\mu\alpha}] - \mathcal{F}(3F^{*\mu\beta}g^{\alpha\sigma} + F^{*\beta\alpha}g^{\mu\sigma} + F^{*\beta\sigma}g^{\mu\alpha})\}. \quad (11)$$

As a cross-check, one can indeed prove [21] that the first term can immediately be obtained from the order $1/m^2$ term of the famous triangle graph. As a side remark, it should be mentioned that the axial vector anomaly is not present in our result, since we computed the $1/m^2$-expansion of $\langle j_5^\mu\rangle$, whereas the anomaly is mass independent.

Returning to the quest for the general effective action, we can easily insert Eq. (7) into Eq. (2) and a similar expression for the vector current $\langle j_\mu\rangle$. The well-known latter expression can be derived via the Heisenberg-Euler Lagrangian as mentioned above or, alternatively, via the same method as here proposed for the axial vector current; for this, we would like to stress that Eq. (6) also holds for the vector current with the VA amplitude $\Pi_5^{\mu\nu}$ replaced by the polarization tensor $\Pi^{\mu\nu}$. This tensor in the presence of an arbitrarily constant electromagnetic field was first calculated in [22] (an appropriate representation can be found in [23]). Details will be presented elsewhere. We observe that the axial vector current is of even order in the field, while the vector current is of odd order; this is a direct consequence of the Dirac algebra (Furry's theorem).

To conclude, we have completed the search for a low-energy effective action for neutrino interactions with an arbitrary number of soft photons with wavelengths larger than the Compton wavelength. From a different perspective, we have found the generalization of the Heisenberg-Euler action to the case involving an axial vector coupling.

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[1] B.M. Pontecorvo, Zh. Éksp. Teor. Fiz. **36**, 1615 (1959) [Sov. Phys. JETP **9**, 1148 (1959)]; H.Y. Chiu and P. Morrison, Phys. Rev. Lett. **5**, 573 (1960).
[2] D.A. Dicus and W.W. Repko, Phys. Rev. Lett. **79**, 569 (1997).
[3] A. Abada, J. Matias, and R. Pittau, Phys. Rev. D **59**, 013008 (1999).
[4] M. Gell-Mann, Phys. Rev. Lett. **6**, 70 (1961).
[5] M.J. Levine, Nuovo Cimento A **48**, 67 (1967).
[6] V.K. Cung and M. Yoshimura, Nuovo Cimento A **29**, 557 (1975).
[7] J. Liu, Phys. Rev. D **44**, 2879 (1991).
[8] C.N. Yang, Phys. Rev. **77**, 242 (1950); L.D. Landau, Dokl. Akad. Nauk SSSR **60**, 207 (1948).
[9] D.A. Dicus and W.W. Repko, Phys. Rev. D **48**, 5106 (1993).
[10] R. Shaisultanov, Phys. Rev. Lett. **80**, 1586 (1998).
[11] M.V. Chistyakov, A.V. Kuznetsov, and N.V. Mikheev, *New Trends in Neutrino Physics*, edited by B. A. Kniehl, G. G. Raffelt, and N. Schmitz (World Scientific, Singapore, 1999), hep-ph/9808246.
[12] Tzuu-Kang Chyi *et al.*, Phys. Lett. B **466**, 274 (1999); Tzuu-Kang Chyi *et al.*, hep-ph/9912134.
[13] J.C. D'Olivo, J.F. Nieves, and P.B. Pal, Phys. Rev. D **40**, 3679 (1989); Phys. Rev. Lett. **64**, 1088 (1990); Phys. Lett. B **365**, 178 (1996); J.F. Nieves and P.B. Pal, Phys. Rev. D **49**, 1398 (1994).
[14] P. Elmfors, D. Grasso, and G. Raffelt, Nucl. Phys. **B479**, 3 (1996); D. Grasso, Nucl. Phys. B (Proc. Suppl.) **70**, 267 (1999).
[15] W. Dittrich and M. Reuter, *Effective Lagrangians in Quantum Electrodynamics*, Lecture Notes in Physics Vol. 220 (Springer-Verlag, Berlin, 1985).
[16] J. Hauknes, Ann. Phys. (N.Y.) **156**, 303 (1984).
[17] V.P. Gusynin and I.A. Shovkovy, J. Math. Phys. **40**, 5406 (1999); Can. J. Phys. **74**, 282 (1996).
[18] R. Shaisultanov, hep-th/0002079.
[19] C. Schubert, Phys. Rev. D (to be published), hep-ph/0002276.
[20] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Fortschr. Phys. **32**, 585 (1985).
[21] H. Gies and R. Shaisultanov, Phys. Lett. B **480**, 129 (2000).
[22] I.A. Batalin and A.E. Shabad, Zh. Éksp. Teor. Fiz. **60**, 894 (1971) [Sov. Phys. JETP **33**, 483 (1971)].
[23] L.F. Urrutia, Phys. Rev. D **17**, 1977 (1978).