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Gauge invariance of the deeply virtual Compton scattering amplitude

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We analyze in detail the problem of gauge invariance of the deeply virtual Compton scattering (DVCS) amplitude. Using twist-3 one-gluon exchange diagram contributions and the QCD equations of motion, we derive the general gauge invariant expression of the DVCS amplitude on a (pseudo)scalar particle (pion, He^4). Similarly to the case of deep inelastic scattering, the amplitude does not depend on the twist-3 quark-gluon correlations at the Born level. The contribution of the derived amplitude to the single-spin asymmetry with longitudinally polarized lepton is calculated.

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Deeply virtual Compton scattering (DVCS) has recently attracted much attention. One of the main reasons for this interest is the fact that the DVCS process gives information about a new type of parton distribution, called skewed parton distribution (see, for example, [1–4] and references therein). The process

$$\gamma^*(q)N(p)\rightarrow\gamma(q')N(p') \tag{1}$$

has been shown to factorize in the Bjorken region with $(q')^2=0$, $-q^2$ large and small transfer $t=(p-p')^2$, as the product of a perturbatively calculable coefficient function and a long distance object, the skewed parton distribution, which generalizes the notion of parton distributions.

The fact that there is a problem with the photon gauge invariance of the DVCS amplitude in leading order in the Bjorken limit is fairly well known (see, for instance, [4]). The relevant terms are proportional to the transverse component of the momentum transfer and provide the leading contribution to some observables, and in particular, to the single spin asymmetry.

As was shown in [5], a fruitful analogy between the transverse spin case of the deep inelastic scattering (DIS) and the DVCS process can be used to derive a general solution of this problem. We elaborate on this approach in the current paper.

For simplicity, we concentrate here on the DVCS process of (pseudo)scalar hadrons, which may be pions or helium-4 nuclei, but our calculation may be generalized to any hadrons. In particular, the obtained DVCS amplitude presents the helicity-independent part of the nucleon DVCS amplitude. We also neglect hadron masses effects, i.e., kinematical power corrections, which may be studied independently.

The Lorentz structure of the hard subgraph of the leading order DVCS diagram [Fig. 1(a)] has the form of a transverse projector $g^{\mu\nu}-P^\mu n^\nu-n^\mu P^\nu$ [2]. All 4-vectors may be presented in the form of the Sudakov decomposition over two light-cone vectors P,n and one component transversal to the given light-cone vectors. Hence, if the virtual photon momentum, which has a transverse component, is convoluted with the hard part of the leading order DVCS amplitude, one obtains a term directly proportional to the transverse component of the virtual photon momentum. In other words, the measure of the photon gauge invariance violation is the nonzero transverse component of the virtual photon momentum. The recent general analysis [6] confirmed that violating terms are indeed kinematically subleading.

In this paper we generalize the Ellis-furmanski-Petronzio (EFP) factorization scheme [7] to the nonforward case and calculate the complete expression for the DVCS amplitude up to twist-3 order. While this scheme was originally applied to the case of the twist-4 power corrections to DIS, our analysis will be closer to the subsequent treatment of the transverse polarization in DIS at the twist-3 level [8,9]. Analogously to that case, process (1) is described by the diagrams of Fig. 1, which contain leading and next-to-leading twist contributions. Namely, the diagram 1(b), in the case of the transverse gluon field, is entirely at twist-3 level, while the handbag diagram 1(a) contains, besides the standard twist-2 term (produced by the good component of quark fields and collinear parton momenta), a twist-3 term, related to the quark gluon contribution of diagram 1(b) by the equations of motion. The latter play a crucial role in DIS, guaranteeing that the sum of diagrams 1(a) and 1(b) is gauge invariant and, moreover, not explicitly dependent of the quark-gluon correlations. We will show that the situation in DVCS is quite similar.

The sum of the $T^{(a)}_{\mu\nu}$ amplitude from diagram 1(a) and $T^{(b)}_{\mu\nu}$ from diagram 1(b), has the following form, such as in [8]:

$$T^{(a)}_{\mu\nu}+T^{(b)}_{\mu\nu}=\int dk\,\mathrm{tr}\{E_{\mu\nu}(k)\Gamma(k)\}+\int dk_1dk_2\,\mathrm{tr}\{E_{\mu\rho\nu}(k_1,k_2)\Gamma_\rho(k_1,k_2)\}, \tag{2}$$

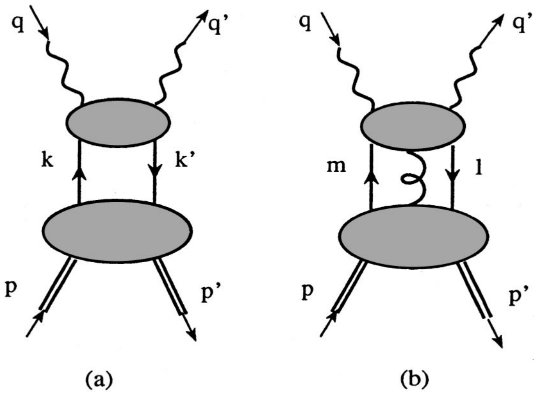

FIG. 1. The DVCS diagrams (notation: $k=xP-\Delta/2+k_T$, $k'=xP+\Delta/2+k'_T$, $m=x_1P-\Delta/2$, and $l=x_2P+\Delta/2$).

where $E_{\mu\nu}$ and $E_{\mu\rho\nu}$ are the coefficient functions with two quark legs and two quark and one gluon legs, respectively. For simplicity, we restrict to the Born diagrams for the coefficient functions. In Eq. (2), the following notation is introduced:

$$\Gamma_{\alpha\beta}(k)=-\int dze^{i(k-\Delta/2)z}\langle p'|\psi_\alpha(z)\bar{\psi}_\beta(0)|p\rangle,$$

$$\Gamma^{\rho}_{\alpha\beta}(k_1,k_2)=-\int dz_1dz_2e^{i(k_1-\Delta/2)z_1+i(k_2-k_1)z_2}\times\langle p'|\psi_\alpha(z_1)gA^\rho(z_2)\bar{\psi}_\beta(0)|p\rangle,$$

$$p'=P+\frac{\Delta}{2},\quad p=P-\frac{\Delta}{2},\quad \Delta=q-q'. \tag{3}$$

Here p' and p are the final and initial hadron momenta, q' and q are the final and initial photon momenta. For the sake of convenience, we neglect all the kinematical power corrections and put P^2 and $t\equiv\Delta^2$ both equal to zero, keeping only the terms linear in Δ_T. Also, we choose the axial gauge condition for gluons, i.e., $n\cdot A=0$, where n is a light-cone vector, normalized by the condition $n\cdot P=1$. It is convenient to assume that $n=q'/P\cdot q'$, although our result will not depend explicitly on the choice of n. We carry out a decomposition of k in the basis defined by the $P-$ and $n-$light-cone vectors

$$k=xP+(k\cdot P)n+k_T,\quad x=k\cdot n. \tag{4}$$

Apart from this,

$$\Delta=-2\xi P+\Delta_T,\quad -2\xi=\Delta\cdot n. \tag{5}$$

Further, we carry out the following replacement for the integration momentum in Eq. (2)

$$dk_i\rightarrow dk_idx_i\delta(x_i-k_i\cdot n). \tag{6}$$

Expanding the two-quark coefficient function $E_{\mu\nu}$ (see Eq. (2)] in a Taylor series and, next, using the following Ward identity [7,8]

$$\frac{\partial E_{\mu\nu}(k)}{\partial k^\rho}=E_{\mu\rho\nu}(k,k) \tag{7}$$

we can write the DVCS amplitude as

$$T^{(a)}_{\mu\nu}+T^{(b)}_{\mu\nu}=\int dx\,\mathrm{tr}\{E_{\mu\nu}(xP)\Gamma(x)\}+\int dx_1dx_2\times\mathrm{tr}\{E_{\mu\rho\nu}(x_1P,x_2P)\omega_{\rho\rho'}\Gamma_{\rho'}(x_1,x_2)\}, \tag{8}$$

where $w_{\rho\rho'}=\delta_{\rho\rho'}-n_{\rho'}P_\rho$, and

$$\Gamma_{\alpha\beta}(x)=-\int d\lambda e^{i(x+\xi)\lambda}\langle p'|\psi_\alpha(\lambda n)\bar{\psi}_\beta(0)|p\rangle,$$

$$\Gamma^{\rho'}_{\alpha\beta}(x_1,x_2)=\frac{1}{2}\int d\lambda_1d\lambda_2e^{i(x_1+\xi)\lambda_1+i(x_2-x_1)\lambda_2}\times\langle p'|\bar{\psi}_\beta(0)(\overrightarrow{D^{\rho'}}-\overleftarrow{D^{\rho'}})(\lambda_2n)\psi_\alpha(\lambda_1n)|p\rangle, \tag{9}$$

where

$$\overrightarrow{D_\mu}=i\overrightarrow{\partial^\mu}+gA^\mu,\quad \overleftarrow{D^\mu}=i\overleftarrow{\partial^\mu}-gA^\mu.$$

Let us now focus on the QCD equations of motion both for incoming and outcoming quarks. From these equations, we deduce integral relations for structure functions, parametrizing quark and quark-gluon correlations. So, let us to start from the QCD equations of motion (we consider massless quarks)

$$\langle\overrightarrow{\hat{D}(z)}\psi(z)\bar{\psi}(0)\rangle=0,\quad\langle\bar{\psi}(z)\psi(0)\overleftarrow{\hat{D}(0)}\rangle=0, \tag{10}$$

where $\langle\,\rangle$ denote the asymmetrical matrix elements. Keeping only vector and axial projections (since for massless quarks all other structures do not contribute), we decompose the quark and quark-gluon correlators in the γ basis. We have

$$-4\langle\psi(z)\bar{\psi}(0)\rangle=\langle\bar{\psi}(0)\gamma_\alpha\psi(z)\rangle\gamma_\alpha-\langle\bar{\psi}(0)\gamma_\alpha\gamma_5\psi(z)\rangle\gamma_\alpha\gamma_5, \tag{11}$$

$$-4\langle gA^\rho(y)\psi(z)\bar{\psi}(0)\rangle=\langle\bar{\psi}(0)\gamma_\alpha gA^\rho(y)\psi(z)\rangle\gamma_\alpha-\langle\bar{\psi}(0)\gamma_\alpha\gamma_5gA^\rho(y)\psi(z)\rangle\gamma_\alpha\gamma_5. \tag{12}$$

In terms of P, Δ^T, n vectors, we introduce the parametrization of relevant vector and axial correlators [see Eqs. (11) and (12)] in the following forms, where terms proportional to n, contributing only at twist-4 level, have been omitted, and the axial gauge condition $n\cdot A=0$ has been taken into account:

$$\langle \overline{\psi}(0)\gamma_\mu \psi(z)\rangle \overset{\mathcal{F}}{=} H_1(x)P_\mu + H_3(x)\Delta^T_\mu, \quad (13)$$

$$\tfrac{1}{2}\langle \overline{\psi}(0)\gamma_\mu (i\overrightarrow{\partial}^T_\rho - i\overleftarrow{\partial}^T_\rho)\psi(z)\rangle \overset{\mathcal{F}}{=} H_1^T(x)P_\mu \Delta^T_\rho, \quad (14)$$

$$\langle \overline{\psi}(0)\gamma_5\gamma_\mu \psi(z)\rangle \overset{\mathcal{F}}{=} iH_A(x)\epsilon_{\mu\Delta^T P_n}, \quad (15)$$

$$\tfrac{1}{2}\langle \overline{\psi}(0)\gamma_5\gamma_\mu (i\overrightarrow{\partial}^T_\rho - i\overleftarrow{\partial}^T_\rho)\psi(z)\rangle \overset{\mathcal{F}}{=} iH_A^T(x)P_\mu \epsilon_{\rho\Delta^T P_n}, \quad (16)$$

$$\langle \overline{\psi}(0)\gamma_\mu g A^T_\rho(y)\psi(z)\rangle \overset{\mathcal{F}}{=} B(x_1,x_2)P_\mu \Delta^T_\rho, \quad (17)$$

$$\langle \overline{\psi}(0)\gamma_5\gamma_\mu g A^T_\rho(y)\psi(z)\rangle \overset{\mathcal{F}}{=} iD(x_1,x_2)P_\mu \epsilon_{\rho\Delta^T P_n}, \quad (18)$$

here $\varepsilon_{\rho\Delta^T Pn} \equiv \varepsilon_{\rho\alpha\beta\gamma}\Delta^{T\alpha}P^\beta n^\gamma$; $\overset{\mathcal{F}}{=}$ denotes the Fourier transformation with measure ($z=\lambda n,\ z'=0$)

$$dx\, e^{-i(xP-\Delta/2)z + i(xP+\Delta/2)z'}$$

for quark correlators, and

$$dx_1 dx_2\, e^{-i(x_1P-\Delta/2)z - i(x_2-x_1)Py + i(x_2P+\Delta/2)z'}$$

for quark-gluon correlators. Note that for the nonforward case the latter are actually new objects. We kept only the argument x for all the correlators, dropping for brevity the dependence of the distributions of the skewedness parameter ξ, recovering it below where it is necessary. Their dependence on $t=\Delta^2$ is beyond our scope. Finally, their dependence on the factorization scale parameter μ^2 requires an extended separate investigation.

The dependence on ξ plays a crucial role in the disappearance of the H_3 term, when a local current, related to its integral in x, is considered. Although this is required by the conservation of local vector current, this effect is proportional to t and is therefore beyond the scope of our approximation. At the same time, this is also required by T invariance [10]. It is therefore natural, that symmetry of H (c.f. [11,4]), resulting also from T invariance,

$$H_3(x,\xi) = -H_3(x,-\xi)$$

is relevant. Calculating the integral in x and implying the polynomiality condition [2], one gets a function which is independent on ξ, and hence vanishes since the only odd constant is zero. A similar argument is also applicable to the function H_A, so that

$$\int dx\, H_3(x) = 0, \quad \int dx\, H_A(x) = 0. \quad (19)$$

Using $i\hat{\partial}$ acting on Eq. (11) and γ on Eq. (12) from the left or right side, the QCD equations of motion yield the following integral relations for structure functions:

$$\int dy[B^{(A)}(x,y) - D^{(S)}(x,y) + \delta(x-y)H_A^T(y)] = -\xi H_3(x) - \tfrac{1}{2}H_1(x) - xH_A(x),$$

$$\int dy[B^{(S)}(x,y) + \delta(x-y)H_1^T(y) - D^{(A)}(x,y)] = xH_3(x) + \xi H_A(x), \quad (20)$$

where symmetrical and antisymmetrical functions are defined as

$$B^{(S,A)}(x,y) = \tfrac{1}{2}[B(x,y) \pm B(y,x)]. \quad (21)$$

Note again the important difference with DIS, where the axial correlator is symmetric and the vector one is antisymmetric [8]. The latter property is based on T invariance, just like the symmetry properties in ξ discussed above. To see the relation between $x\leftrightarrow y$ and ξ symmetry, it is instructive to write the general T invariance relations

$$B(x,y,\xi) = B(y,x,-\xi),$$

$$D(x,y,\xi) = -D(y,x,-\xi). \quad (22)$$

So the ''unnatural'' symmetry in x,y results from the antisymmetrical in the ξ part, clearly absent in the forward case. It is worthy to note that a similar unnatural symmetry may appear due to the final state interaction phases in the case of T-odd fragmentation functions [12].

We can thus write the DVCS amplitude in a gauge invariant manner. Let us first write the contribution from the pure quark amplitude:

$$T^{(a)}_{\mu\nu} = \int dx \frac{1}{(xP+Q)^2}[H_1(x)S_{\nu(xP+Q)\mu P} + H_3(x)S_{\nu(xP+Q))\mu\Delta^T} + H_A(x)\varepsilon_{\alpha\Delta^T Pn}\varepsilon_{\nu(xP+Q)\mu\alpha}] + (\mu\to\nu, Q\to -Q), \quad (23)$$

where the following notations were introduced

$$S_{\mu_1\mu_2\mu_3\mu_4} = g_{\mu_1\mu_2}g_{\mu_3\mu_4} + g_{\mu_1\mu_4}g_{\mu_2\mu_3} - g_{\mu_1\mu_3}g_{\mu_2\mu_4},$$

$$Q = (q+q')/2.$$

As a corollary, the contribution of the amplitude corresponding to the one-gluon exchange diagram, has the form

$$T^{(b)}_{\mu\nu}=\frac{1}{4}\int dx_1 dx_2 \frac{1}{(x_1P+Q)^2(x_2P+Q)^2}\{[B(x_1,x_2)+\delta(x_1-x_2)H_1^T(x_2)]\times \mathrm{tr}(\gamma_\nu(x_2\hat{P}+\hat{Q})\hat{\Delta}^T(x_1\hat{P}+\hat{Q})\gamma_\mu\hat{P})+i[D(x_1,x_2)-\delta(x_1-x_2)H_A^T(x_2)]\varepsilon_{\alpha\Delta^T Pn}\times \mathrm{tr}(\gamma_\nu(x_2\hat{P}+\hat{Q})\gamma_\alpha(x_1\hat{P}+\hat{Q})\gamma_\mu\hat{P}\gamma_5)\}+\text{``crossed.''} \tag{24}$$

Calculating all traces in Eq. (24), using the following obvious identities

$$\frac{\pm(P\cdot Q)(x_1+x_2)+Q_2}{(x_1P\pm Q)^2(x_2P\pm Q)^2)}=\frac{1}{2}\left(\frac{1}{(x_1P\pm Q)^2}+\frac{1}{(x_2P\pm Q)^2}\right),$$

$$\frac{\pm(P\cdot Q)(x_1-x_2)}{(x_1P\pm Q)^2(x_2P\pm Q)^2)}=\frac{1}{2}\left(\frac{1}{(x_2P\pm Q)^2}-\frac{1}{(x_1P\pm Q)^2}\right),$$

and the equations of motion (20) in terms of symmetric and antisymmetric functions, we add contributions of Eqs. (23) and (24), taking into account the crossed diagrams. The gauge invariant expression of the DVCS amplitude is thus

$$T_{\mu\nu}=-\frac{1}{2P\cdot Q}\int dx\left(\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right)\mathcal{T}_{\mu\nu}, \tag{25}$$

where

$$\mathcal{T}_{\mu\nu}=H_1(x)[-2\xi P_\mu P_\nu-P_\mu Q_\nu-P_\nu Q_\mu+g_{\mu\nu}(P\cdot Q)-\tfrac{1}{2}P_\mu\Delta^T_\nu+\tfrac{1}{2}P_\nu\Delta^T_\mu]-H_3(x)(\xi P_\nu\Delta^T_\mu+3\xi P_\mu\Delta^T_\nu+\Delta^T_\mu Q_\nu+\Delta^T_\nu Q_\mu)-\frac{\xi}{x}H_A(x)(3\xi P_\mu\Delta^T_\nu-\xi P_\nu\Delta^T_\mu-\Delta^T_\mu Q_\nu+\Delta^T_\nu Q_\mu).$$

We can see that the first term of Eq. (25), proportional to the H_1 function, completely coincides with the improved DVCS amplitude, proposed by Guichon and Vanderhaegen (GV) in [4]. Indeed, if we proceed, in such terms, to GV basis [4], where the n vector is expressed via the virtual photon momentum q and decompose the Q vector in this basis, then we derive the following (omitting the terms with H_3 and iH_A functions):

$$T_{\mu\nu}=\int dx\frac{1}{2}\left(\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right)H_1(x)\times\left(P_\mu n_\nu+P_\nu n_\mu-g_{\mu\nu}-\frac{P_\nu\Delta^T_\mu}{P\cdot q}\right)\equiv T^{\mathrm{LO}}_{\mu\nu}+\frac{P_\nu}{P\cdot q}\Delta^T_\lambda T^{\mathrm{LO}}_{\mu\lambda}, \tag{26}$$

where the definition of $T^{LO}_{\mu\nu}$ amplitude is related with the transverse direction projector, which is $(P_\mu n_\nu+P_\nu n_\mu-g_{\mu\nu})$.

In complete analogy to DIS, the answer does not depend explicitly on quark-gluon correlations. However, in contrast to DIS, it contains two additional new functions, instead of the single function g_2 in the DIS case. While H_3 may be considered as an analog of g_2 (the coefficients of them are Δ_T and s_T, respectively), the DIS analog of the function H_A is excluded by T invariance and may be present only for fragmentation.

We emphasize that the deduced gauge invariant expression of the DVCS amplitude has a significant meaning for the sequential application of QCD to the investigation of any observable values. To demonstrate this, let us consider the single (electron) spin asymmetry (SSA), which arises in the collision of the longitudinally polarized electron beams with the unpolarized scalar target. The SSA parameter (for details see, for instance, [2]) is

$$\mathcal{A}_L=\frac{d\sigma(\rightarrow)-d\sigma(\leftarrow)}{d\sigma(\rightarrow)+d\sigma(\leftarrow)}, \tag{27}$$

where

$$d\sigma(\rightarrow)-d\sigma(\leftarrow)\sim\frac{e^6F_+(t)2\xi}{q^2t(k-\Delta)^2(k'+\Delta)^2}\varepsilon_{kk'P\Delta}\int dx[\delta(x+\xi)-\delta(x-\xi)]\cdot\Bigg(H_1(x)[(k+k')\cdot P]+2H_3(x)(k'\cdot\Delta)-\frac{2\varepsilon}{x(P\cdot Q)}H_A(x)[(k\cdot\Delta)(k'\cdot P)-(k'\cdot\Delta)(k\cdot P)]\Bigg)+\frac{e^6}{q^4}\epsilon_{kk'P\Delta}\frac{2\xi}{(P\cdot Q)}\times\int dx dx'\left([\delta(x+\xi)-\delta(x-\xi)]\left[\frac{P}{x'-\xi}+\frac{P}{x'+\xi}\right]-[\delta(x'+\xi)-\delta(x'-\xi)]\times\left[\frac{P}{x-\xi}+\frac{P}{x+\xi}\right]\right)\cdot\left(H_1(x)H_3(x')-H_1(x')H_3(x)-\left[H_1(x')\frac{H_A(x)}{x}-H_1(x)\frac{H_A(x')}{x'}\right]\xi\right), \tag{28}$$

where $F_+(t)$ is the target electromagnetic form factor, emanating from the Bethe-Heitler diagrams, and k and k' denote the momenta of the initial and final electron $(k-k'=q)$.

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[1] D. Müller *et al.*, Fortschr. Phys. **42**, 101 (1994).
[2] X. Ji, Phys. Rev. D **55**, 7114 (1997).
[3] A. V. Radyushkin, Phys. Rev. D **56**, 5524 (1997).
[4] P. A. M. Guichon and M. Vanderhaegen, Prog. Part. Nucl. Phys. **41**, 125 (1998); X. Ji, J. Phys. G **24**, 1181 (1998).
[5] B. Pire and O. V. Teryaev, in Proceedings of the 13th International Symposium on High Energy Spin Physics ''Spin98,'' 1998, Protvino, Russia.
[6] J. Blümlein and D. Robaschik, hep-ph/0002071.
[7] R. K. Ellis, W. Furmanski, and R. Petronzio, Nucl. Phys. **B207**, 1 (1982).
[8] A. V. Efremov and O. V. Teryaev, Yad. Phys. **39**, 1517 (1984) [Sov. J. Nucl. Phys. **39**, 962 (1984)].
[9] M. Anselmino, A. Efremov, and E. Leader, Phys. Rep. **261**, 1 (1995).
[10] M. V. Polyakov and C. Weiss, Phys. Rev. D **60**, 114017 (1999).
[11] L. Mankiewicz, G. Piller, and T. Weigl, Eur. Phys. J. C **5**, 119 (1998).
[12] O. V. Teryaev, in *SPIN-96 Proceedings*, edited by C. W. de Jager, T. J. Ketal, P. J. Mulders, J. E. Oberski, and M. Oskam-Tamboezer (World Scientific, Singapore, 1997), p. 594.