Quasilocal calculation of tidal heating

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We present a method for computing the flux of energy through a closed surface containing a gravitating system. This method, which is based on the quasilocal formalism of Brown and York, is illustrated by two applications: a calculation of (i) the energy flux, via gravitational waves, through a surface near infinity and (ii) the tidal heating in the local asymptotic frame of a body interacting with an external tidal field. The second application represents the first use of the quasilocal formalism to study a non-stationary spacetime and shows how such methods can be used to study tidal effects in isolated gravitating systems.

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I. INTRODUCTION

In many physical problems in gravitation, one is interested in the interaction of a nearly isolated gravitating system with an external universe. The interaction effects are computed in a ''buffer zone'' (see Sec. 20.6 of Ref. [1] and Sec. I B of Ref. [2]) surrounding the gravitating system, in which the radius of curvature, scale of inhomogeneity, and rate of change of curvature are much smaller than the size of the body. The formalism of Thorne and Hartle [2] and Zhang [3] has been used recently by Purdue [4] and Favata [5] to compute the gauge-invariant heating of a body interacting with an external tidal field.

Until now, calculations of the sort described in these references have made use of pseudotensors to compute energy and momentum fluxes. However, quasilocal methods should be equally applicable in situations with a reasonably well defined buffer zone—in this case, the quasilocal surface can be conveniently located in the buffer zone. While quasilocal methods are not fundamentally different than pseudotensor methods [6], an advantage of quasilocal method is that all quantities (e.g., energy fluxes) can be computed in terms of real tensors on the quasilocal surface. Gauge ambiguities in the total amount of energy and energy flux such as those reported in Ref. [2] and discussed in Ref. [4] still exist for the quasilocal methods, but now the ambiguities can be understood in terms of distortions of the quasilocal surface and so their geometric origin is identified.

In this paper we present a quasilocal formalism for computing the work done on a gravitating system by an external universe. Our formalism is based on the quasilocal mass of Brown and York [7]—the on-shell value of the gravitational Hamiltonian—which coincides with the Arnowitt-Deser-Misner energy at spatial infinity and the Trautman-Bondi-Sachs energy at null infinity [7,8]. It is complementary to but independent of [9] which studied how motion of the observers affects the Brown-York energy. We use our expression for energy flux to compute (i) the energy lost in gravitational radiation from a gravitational system and (ii) the heating of a body through interactions with an external tidal field. Problem (i) demonstrates that the formula for the work reproduces the known gravitational radiation flux formula when the quasilocal surface is located in the wave-zone. Problem (ii) reproduces the calculation of Purdue [4] using quasilocal methods and shows how these methods are applicable for problems in which the quasilocal surface is located in a buffer zone.

II. QUASILOCAL ENERGY FLUX

In this section, we derive an expression for the energy flux through a closed two-surface surrounding a gravitating system. Our analysis closely follows Sec. V of Ref. [7], which derives a conserved measure of mass for stationary systems. We relax the requirement that the quasilocal two-surface time evolution vector be a Killing vector of the spacetime and thereby obtain an expression for the rate of change in the mass of the system.

Consider a gravitating system separated from the external universe by a $(2+1)$-dimensional timelike boundary B. This boundary has an outward ''radial'' normal vector n^a, a metric $\gamma_{ab}=g_{ab}-n_a n_b$ induced by its embedding in the spacetime with metric g_{ab}, and an extrinsic curvature $\Theta_{ab}=-\frac{1}{2}\pounds_n\gamma_{ab}$ (with trace $\Theta=\gamma^{ab}\Theta_{ab}$). Let $\triangle_a$ be the derivative operator compatible with the metric γ_{ab}. Foliate the boundary B into closed two-surfaces Ω_t of constant time t; then the time evolution vector t^a on B satisfies $t^a\triangle_a t=1$ and can be decomposed into a lapse function N and a shift vector V^a on Ω_t via $t^a=Nu^a+V^a$, where u^a is the timelike normal to Ω_t embedded in B. The closed, spacelike, two-surface Ω_t has an induced metric $\sigma_{ab}=\gamma_{ab}+u_a u_b$ and, viewed as a two-surface embedded in a three-dimensional spacelike hypersurface Σ locally defined such that $n^a\in T\Sigma$, the extrinsic curvature of Ω_t is $k_{ab}=-\frac{1}{2}\pounds_n\sigma_{ab}$. A full discussion of the geometry of the boundary B and its foliation (including a diagram) may be found in [9]. The notation there is substantially the same as here though u^a is written as $\tilde{u}^a$.

The Codazzi identity,

$$\triangle_a\tau^{ab}=\gamma^{bc}n^d R_{cd}/8\pi, \tag{1}$$

where $\tau^{ab}=(\Theta\gamma^{ab}-\Theta^{ab})/8\pi$, relates the extrinsic curvature of B to the spacetime Ricci curvature R_{ab}. It then follows from the Einstein field equations that

$$\triangle_a(t_b\tau^{ab}) = t^a n^b T_{ab} + \tfrac{1}{2}\tau^{ab}\pounds_t\gamma_{ab}. \tag{2}$$

We restrict our attention to a vacuum spacetime in which the stress-energy tensor T_{ab} vanishes. Then, if t^a is a Killing vector field of the boundary metric γ_{ab}, Eq. (2) is a conservation equation and the quantity

$$M = \int_{\Omega_t} d^2x\,\sqrt{\sigma}u_a t_b \tau^{ab} \tag{3}$$

is a conserved measure of the total mass contained within the boundary Ω_t. It is the ‘‘non-orthogonal’’ Brown-York mass [7,9], up to a subtraction term that is required for it to be bounded for large surfaces in asymptotically flat spacetimes (see, e.g., Lau or Mann [10]).

When t^a is *not* a Killing vector of the boundary, then Eq. (2) represents an energy flow from the system. Between two times t_1 and t_2 one can integrate to find that $\Delta M = -\frac{1}{2}\int_B d^3x\sqrt{-\gamma}\tau^{ab}\pounds_t\gamma_{ab}$ is the change in the mass contained by Ω_t. Subtraction terms from a reference spacetime do not need to be included here as it expresses the *change* in the mass of the system. The rate at which this work is done is

$$\frac{dW}{dt} = -\frac{1}{2}\int_{\Omega_t} d^2x\,\sqrt{-\gamma}\tau^{ab}\pounds_t\gamma_{ab} \tag{4}$$

which describes the rate of change of the system's mass due to the purely gravitational interaction between it and the surrounding environment.

It is illustrative to decompose the expression for the work into terms involving projections of $\pounds_t\gamma_{ab}$ normal to and into the spatial two-surfaces Ω_t. We find

$$\frac{dW}{dt} = \int_{\Omega_t} d^2x\,\sqrt{\sigma}\{\tfrac{1}{2}s^{ab}\pounds_t\sigma_{ab} - \varepsilon\pounds_t N + j_a\pounds_t V^a\} \tag{5}$$

where $\varepsilon = \sigma^{ab}k_{ab}/8\pi$, $j_a = \sigma_{ab}u_c\Theta^{bc}/8\pi$, and $s^{ab} = [k^{ab} + \sigma^{ab}(n^c u^d \triangle_d u_c - \sigma^{cd}k_{cd})]/8\pi$ are the quasilocal surface energy, momentum, and stress densities. The first two are potentials conjugate to changes in the lapse function and shift vector respectively while the surface stress density is a work potential conjugate to changes in the size and shape of the surface Ω_t.

The stress density can be further decomposed as follows. A change in the two-metric, $\delta\sigma_{ab} = \mathrm{s}_{ab}\delta\sqrt{\sigma} + \sqrt{\sigma}\delta\mathrm{s}_{ab}$, is written as a change in the ‘‘size’’ $\sqrt{\sigma}$ of the surface plus a change in the conformally-invariant part of the metric (the ‘‘shape’’ of the surface) $\mathrm{s}_{ab} = \sigma_{ab}/\sqrt{\sigma}$. Correspondingly, the surface stress density is decomposed into a surface tension $s = s^{ab}\mathrm{s}_{ab}$ and a shear $\eta^{ab} = s^{ab}/\sqrt{\sigma}$. Then we rewrite the work term as $\frac{1}{2}s^{ab}\pounds_t\sigma_{ab} = \frac{1}{2}(s\pounds_t\sqrt{\sigma} + \eta^{ab}\pounds_t\mathrm{s}_{ab})$.

The above has a particularly nice application in the physics of thin shells. Israel [11] first showed that a thin shell of matter can be described in general relativity by matching two spacetimes along a timelike boundary B such that even though they induce the same surface metric on B, the extrinsic curvature in each spacetime is different. If Θ^+_{ab} and Θ^-_{ab} are those curvatures this (mild) singularity can be accounted for if there is a (distributional) stress energy tensor $S_{ab} = \tau^+_{ab} - \tau^-_{ab}$ over B. A set of observers dwelling on the surfaces Ω_t (which foliate B) measures the shell to have matter-energy $M^+ - M^-$ [Eq. (3)]. A more detailed discussion of this may be found in [9] but here we note that the above analysis for the quasilocal energy also shows that the set of observers dwelling on Ω_t measures the matter-energy to change with rate dW/dt [Eq. (5)]. Then the quasilocal densities defined above are the energy, angular momentum, and stress tensor of the *matter shell*. A set of observers being evolved by $t^a = u^a$ see work being done on the shell at a rate equal to the integral of the stress tensor contracted with the time rate of change of the area—exactly as one would expect from classical physics.

III. GRAVITATIONAL RADIATION

Equation (4) purportedly measures the change in the mass of a system. In this section we apply our work formula to obtain the correct mass loss for a system radiating gravitational waves. For this we suppose that the quasilocal surface is in the wave-zone, far away from the radiating system. Although this is not a very interesting application of a quasilocal method (since an asymptotic method, such as the Bondi-Sachs mass loss formula could as well be used), it is useful to confirm that Eq. (4) does recover the correct result.

Gravitational radiation far from the generating source can be described as a transverse-traceless perturbation to the flat-space metric. In spherical-polar coordinates, the metric is given by $ds^2 = -dt^2 + dr^2 + (r\,d\theta)^2 + (r\sin\theta d\phi)^2 + h_{\mu\nu}dx^\mu dx^\nu$ where $h_{\mu\nu}dx^\mu dx^\nu = h_+[(r\,d\theta)^2 - (r\sin\theta d\phi)^2] + 2h_\times(rd\theta)(r\sin\theta d\phi)$ is the transverse, trace free perturbation. The ‘‘plus,’’ h_+, and ‘‘cross,’’ $h_\times$, polarizations represent outgoing, spherical waves, and have the form $h_+(t,r,\theta,\phi) = s_+(t-r,\theta,\phi)/r$ and $h_\times(t,r,\theta,\phi) = s_\times(t-r,\theta,\phi)/r$. We then find the energy lost by the radiating system by inserting this metric into Eq. (4) while taking the boundary to be a sphere of constant r in the wave-zone (very large r). The integrand of Eq. (4) is

$$\frac{dE}{dt\,d\Omega_t} = -\frac{r^2}{16\pi}[(\partial h_+/\partial t)^2 + (\partial h_\times/\partial t)^2] \tag{6}$$

to leading order in the perturbation and in r. This is the standard expression for the flux of gravitational radiation—see, e.g., Eq. (10) of Ref. [12].

By inspection of the form of the perturbation, it is clear that the energy loss arises due to the shearing of the bounding two-surface Ω_t since, to leading order, the perturbation does not affect the volume element on that two-surface. Thus the entire energy loss (in the transverse, trace-free gauge) arises from the ‘‘$\eta^{ab}\pounds_t\mathrm{s}_{ab}$’’ work term.

As a simple example, consider two point-particles, each of mass $m = M/2$, orbiting each other in the xy-plane with angular frequency ω and constant separation a. The quadrupole moment tensor $\mathcal{I}_{jk}$ in Cartesian coordinates is $\mathcal{I}_{xx} = -\mathcal{I}_{yy} = \frac{1}{8}Ma^3\cos 2\omega t$ and $\mathcal{I}_{xy} = \frac{1}{8}Ma^2\sin 2\omega t$ (constant terms omitted). The far-field metric perturbation is $h_{jk} = 2(\partial^2\mathcal{I}_{jk}/\partial t^2)/r$, so $h_{yy} = -h_{xx} = (Ma^2\omega^2/r)\cos 2\omega(t-r)$ and $h_{xy} = -(Ma^2\omega^2/r)\sin 2\omega(t-r)$.

Using Eqs. (4.3) and (4.4) of Ref. [13], we find

$$h_+ = -\tfrac{1}{2} M a^2 \omega^2 r^{-1} (1+\cos^2\theta) \cos 2[\omega(t-r)-\phi] \tag{7a}$$

$$h_\times = -M a^2 \omega^2 r^{-1} \cos\theta \sin 2[\omega(t-r)-\phi]. \tag{7b}$$

We then integrate Eq. (6) over the sphere at large r to obtain the loss of energy from the system:

$$-dE/dt = \tfrac{2}{5} M^2 a^4 \omega^6 = \tfrac{2}{5}(M/a)^5 \tag{8}$$

where we have used Kepler's law $a^3\omega^2 = M$ for particles in a circular orbit.

IV. TIDAL HEATING

We now calculate the work done by an external gravitational field to deform a self-gravitating body. The canonical example of this effect in the solar system is the tidal heating of Io by Jupiter. In this instance, the gradient of Jupiter's gravitational field distorts Io from being a perfect sphere and then tidally locks it in its orbit so that it always presents the same face to Jupiter. That orbit is strongly perturbed by the other Gallilean moons and so its radial distance from Jupiter varies with time. With this variation comes a corresponding one in the gradient of the field and so Io is gradually stretched and then allowed to relax. The energy transferred by this pumping is largely dispersed as heat and it is this heat that produces the volcanic activity on Io. The same type of process occurs for any two bodies in non-circular orbits about each other.

First from a Newtonian perspective, we may mathematically describe the gravitational fields in this situation as follows. We assume that the self-gravitating body is far enough away from the source of the external field that that field is nearly uniform close to the body. Then in a rectangular coordinate system that orbits with the body with its origin at the center of mass, the Newtonian potential of the external field may be written as $\Phi_{\rm ext} = \frac{1}{2}\mathcal{E}_{ij}x^i x^j$ where $\mathcal{E}_{ij}$ is the (time-dependent but symmetric and trace-free) quadrupole moment of the field and x^i is the position vector based at the body's center of mass. At the same time, to quadrupolar order the Newtonian potential of the body is $\Phi_{\rm o} = -M/r - \frac{3}{2} r^{-3}\mathcal{I}_{ij}n^i n^j$, where M is the mass of the body, r is the radial distance from the center of mass, $\mathcal{I}_{ij}$ is its (time-dependent but symmetric and trace-free) quadrupole moment, and n^i is the unit normal radial vector.

With this in mind the techniques of Thorne and Hartle [2] can be used to construct a metric that describes these situations in the slow moving, nearly Newtonian limit. First, define an annulus surrounding the body whose inner boundary is chosen so that the gravitational field of the body is weak throughout and whose outer boundary is chosen so that the external field is nearly uniform. This region is termed the buffer zone. The rectangular coordinate system is replaced with one that is chosen so that the metric is as close to Minkowskian as possible over the buffer zone [4]. Then to first order in perturbations from Minkowski and first order in time derivatives the metric can be written as

$$\begin{aligned} ds^2 = &-(1+2\Phi)dt^2 + 2(A_j + \partial_t \xi_j) dx^j dt \\ &+ [(1-2\Phi)\delta_{ij} + \partial_i \xi_j + \partial_j \xi_i] dx^i dx^j \end{aligned} \tag{9}$$

where the indices run from one to three and δ_{ij} is the Cartesian metric diag[1,1,1] on a spacelike slice. The Newtonian potential is $\Phi = -M/r - \frac{1}{2}(3r^{-3}\mathcal{I}_{ij} - r^2\mathcal{E}_{ij})n^i n^j$ and $A_j = -2r^{-2}n^k d\mathcal{I}_{jk}/dt - \frac{2}{21}r^3(5n_j n^k - 2\delta_j^k)n^l d\mathcal{E}_{kl}/dt$ is a vector potential that must be added so that the metric is a solution to the first order Einstein equations. Here, n^i is the radial normal with respect to the flat spatial metric δ_{ij} and $r^2 = x^2 + y^2 + z^2$. The diffeomorphism generating vector field ξ_j represents the gauge ambiguity in setting up a nearly Minkowski coordinate system. In order that the metric be slowly evolving and nearly Minkowski, ξ_j must be of the form $\xi_j = \alpha r^{-2}\mathcal{I}_{jk}n^k + \beta r^3 \mathcal{E}_{jk}n^k + \gamma r^3 \mathcal{E}_{kl}n^k n^l n_j$, where α, β, and γ are free constants of order one.

We set up a constant r timelike quasilocal surface B in the buffer zone and foliate with constant t spacelike two-surfaces Ω_t. Then the time vector t^a is $\partial/\partial t$. In calculating the rate of change of the mass contained within Ω_t it is most convenient to switch to spherical coordinates. We make the standard transformation to spherical coordinates $x^i = r[\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta]$; in these coordinates, the metric is

$$\begin{aligned} ds^2 = &-(1+2\Phi)dt^2 + (1-2\Phi)[dr^2 + (r\,d\theta)^2 \\ &+ (r\sin\theta d\phi)^2] + 2\overline{A}_r dr dt + 2\overline{A}_\theta (r\,d\theta) dt \\ &+ 2\overline{A}_\phi (r\sin\theta d\phi) dt + H_{rr}dr^2 + H_{\theta\theta}(r\,d\theta)^2 \\ &+ H_{\phi\phi}(r\sin\theta d\phi)^2 + 2H_{r\theta}dr(r\,d\theta) \\ &+ 2H_{r\phi}dr(r\sin\theta d\phi) + 2H_{\theta\phi}(r\,d\theta)(r\sin\theta d\phi)^2 \end{aligned} \tag{10}$$

where $H_{rr} = -4\alpha r^{-3}\mathcal{I}_{rr} + 6(\beta+\gamma)r^2\mathcal{E}_{rr}$, $H_{\theta\theta} = 2\alpha r^{-3}\mathcal{I}_{\theta\theta} + 2\beta r^2\mathcal{E}_{\theta\theta} + 2\gamma r^2\mathcal{E}_{rr}$, $H_{\phi\phi} = 2\alpha r^{-3}\mathcal{I}_{\phi\phi} + 2\beta r^2\mathcal{E}_{\phi\phi} + 2\gamma r^2\mathcal{E}_{rr}$, $H_{r\theta} = -\alpha r^{-3}\mathcal{I}_{r\theta} + (4\beta + 2\gamma)r^2\mathcal{E}_{r\theta}$, $H_{r\phi} = -\alpha r^{-3}\mathcal{I}_{r\phi} + (4\beta+2\gamma)r^2\mathcal{E}_{r\phi}$, and $H_{\theta\phi} = 2\alpha r^{-3}\mathcal{I}_{\theta\phi} + 2\beta r^2\mathcal{E}_{\theta\phi}$. In these expressions $\mathcal{E}_{rr} = \mathcal{E}_{ij}e^i_r e^j_r$, $\mathcal{E}_{r\theta} = \mathcal{E}_{ij}e^i_r e^j_\theta$, etc., with $e^i_r = n^i$, $e^i_\theta = \partial_\theta e^i_r$ and $e^i_\phi = (1/\sin\theta)\partial_\phi e^i_r$. Also, $\overline{A}_r = (A_j + \partial_t \xi_j)e^j_r$, etc., but we do not need their expanded forms since only time derivatives of them show up in later calculations and we are ignoring second order time derivatives.

As might be expected, the subsequent calculations are quite involved and we did them partially with GRTensor [14]. To lowest order

$$\begin{aligned} \frac{dW}{dt} &= -\frac{1}{2}\int_{\Omega_t} d^2x \sqrt{-\gamma}\,\tau^{ab}\pounds_t \gamma_{ab} \\ &= \frac{1}{2}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} + \frac{1}{60}\frac{d}{dt}[2(-3-2\beta-2\beta^2+4\gamma+4\gamma^2 \\ &\quad + 8\beta\gamma)r^5\mathcal{E}_{ij}\mathcal{E}_{ij} + 2(3-2\alpha+6\beta-12\gamma+8\alpha\gamma)\mathcal{E}_{ij}\mathcal{I}_{ij} \\ &\quad - (-9+12\alpha+4\alpha^2)r^{-5}\mathcal{I}_{ij}\mathcal{I}_{ij}]. \end{aligned} \tag{11}$$

The calculations used the identities $\int_{\Omega_t} d\theta d\phi \sin\theta A_{rr}B_{rr} = (8\pi/15)A_{ij}B_{ij}$ and $\int_{\Omega_t} d\theta d\phi \sin\theta(2A_{\theta\phi}B_{\theta\phi} - A_{\theta\theta}B_{\phi\phi} - A_{\phi\phi}B_{\theta\theta}) = (4\pi/3)A_{ij}B_{ij}$ where the integrations are over the unit sphere.

This result requires some interpretation. As the external field changes with time and thereby forces the self-gravitating body to change configuration, the work done by the external field can be split into time reversible and irreversible parts [as seen in Eq. (11)]. The reversible work represents work being done to increase the potential energy of the system and is recoverable. On the other hand the irreversible part represents work done to deform and/or heat up the system. This is the tidal heating that we are interested in. Further, from the quasilocal perspective, we expect to see an energy flow arising from fluctuations of the quasilocal surface within otherwise static fields. Of course this work would also be reversible. Thus, it is only the irreversible part that we are interested in and we have calculated that to be $\frac{1}{2}\mathcal{E}_{ij} d\mathcal{I}_{ij}/dt$ above. This is the same leading term obtained when one does the corresponding calculation in Newtonian gravity or with pseudotensors [4] and it is independent of diffeomorphisms generated by ξ_j which correspond to fluctuations of the quasilocal surface. Note however, that the time reversible and gauge dependent terms of Eq. (11) are dependent on those fluctuations and furthermore that dependence is different from that found in Ref. [4] using pseudotensor methods. Similarly other pseudo-tensor or quasilocal methods would obtain a different gauge dependence for these terms. What is important is that the physically relevant time irreversible term does not depend on the ξ_j-generated diffeomorphisms.

Finally for completeness let us consider how this energy flow splits up into its components parts as considered in Eq. (5). Then to the order that we are interested the angular momentum term is zero and we are left with two terms $dW_N/dt = -\int d\theta d\phi \sqrt{\sigma}\varepsilon \pounds_t N$ and $dW_\sigma/dt = \frac{1}{2}\int d\theta d\phi \sqrt{\sigma} N s^{ab} \pounds_t \sigma_{ab}$. We find

$$\frac{dW_N}{dt} = \frac{1}{2}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} + \frac{\alpha}{15}\frac{d\mathcal{E}_{ij}}{dt}\mathcal{I}_{ij} - \frac{\beta}{5}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} - \frac{4\gamma}{5}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} + \frac{1}{60}\frac{d}{dt}[2(4\gamma+\beta-2)r^5\mathcal{E}_{ij}\mathcal{E}_{ij} - 6\mathcal{E}_{ij}\mathcal{I}_{ij} - 3(2\alpha-3)r^{-5}\mathcal{I}_{ij}\mathcal{I}_{ij}]. \tag{12}$$

The second term is a bit more complicated. It is

$$\frac{dW_\sigma}{dt} = -\frac{\alpha}{15}\frac{d\mathcal{E}_{ij}}{dt}\mathcal{I}_{ij} + \frac{\beta}{5}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} + \frac{4\gamma}{5}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt} + \frac{1}{30}\frac{d}{dt}[(-1-3\beta-2\beta^2+4\gamma^2+8\beta\gamma)r^5\mathcal{E}_{ij}\mathcal{E}_{ij} + 2(3-\alpha+3\beta-6\gamma+4\alpha\gamma)\mathcal{E}_{ij}\mathcal{I}_{ij} - (2\alpha^2-9\alpha+9)r^{-5}\mathcal{I}_{ij}\mathcal{I}_{ij}]. \tag{13}$$

Thus part of the work done is measured by deformations of the surface and part is measured by changes in how observers choose to measure the rate of passage of time. Note that individually the time irreversible sections of the two parts are gauge dependent, but when we combine them we reobtain Eq. (11) and the gauge dependence vanishes back into the reversible part where we would expect it.

V. CONCLUSIONS

We have modified the quasilocal energy formalism of Brown and York so that it may be used to study nonstationary spacetimes where energy flows in and out through the quasilocal surface. As applications of this extension we have examined implications for the physics of relativistic thin shells of matter, the energy carried from a source to infinity by gravitational waves, and the transfer of energy to a body during gravitational tidal heating. The success of the formalism in all three applications provides further evidence that the Brown-York energy has physical content. Furthermore, in the tidal heating application we have seen how the quasilocal formalism provides a geometrical explanation of the gauge ambiguities that are also found in the Newtonian and pseudotensor approaches.

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