Renormalization-group flow equations for U_k and Z_k

Vincenzo Branchina
Laboratoire de Physique Théorique, Université Louis Pasteur, 3-5, rue de l'Université, F-67084, Strasbourg Cedex, France

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By considering the gradient expansion for the Wilsonian effective action S_k of a single component scalar field theory truncated to the first two terms, the potential U_k and the kinetic term Z_k, I show that the recent claim that different expansions of the fluctuation determinant give rise to different renormalization-group equations for Z_k is incorrect. The correct procedure to derive this equation is presented and the set of coupled differential equations for U_k and Z_k is definitely established.

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During recent years there has been growing interest in the so called exact renormalization-group equation(s) (ERGE). Actually several questions in field theory cannot be addressed within the framework of perturbation theory. The entire subject of symmetry breaking and the problem of confinement in QCD are well-known examples of questions still waiting for an explanation. The Wilsonian renormalization-group method [1] seems to provide an interesting nonperturbative approach to these kinds of questions. When the momentum shell of the eliminated modes is chosen to be infinitesimal, it results in an integrodifferential equation for the Wilsonian effective action, the action S_k at the current scale k, the Wegner-Houghton equation [2]. This equation is useless until a specific ansatz is made which allows for a systematic approximation scheme. This can be achieved by considering the derivative expansion, whose lowest order is the so-called local potential approximation (LPA). Let us consider a single component scalar field theory. In the LPA, S_k contains only one function, the local potential $U_k(\phi)$, and the ERGE for S_k becomes a differential equation for U_k. To the next order S_k contains in addition the coefficient $Z_k(\phi)$ of the lowest order derivative term $\partial_\mu\phi\partial_\mu\phi$. While the derivation of the equation for $U_k(\phi)$ in the LPA is straightforward and does not present ambiguities [3,4], the derivation of the coupled differential equations for U_k and Z_k has been plagued by uncertaintes. Actually the authors of [5] have recently applied the expansion of the fluctuation determinant given in [6] to derive these equations. They find an equation for Z_k different from the one that is obtained when the expansion of [7] is applied. They also computed the field anomalous dimension η. As from [6] (but not from [7]) they find the correct two-loop result, they conclude that the expansion introduced in [6] has the correct UV behavior while that of [7] is misleading in the UV region.

By carefully reconsidering the derivation of the equations for U_k and Z_k following the method of [6] I show that the equation for Z_k presented in [5] is incorrect and that actually both methods [6,7] give one and the same equation for Z_k. Concerning the anomalous dimension at two loops we note that being an $O(\hbar^2)$ result it comes from an infinite resummation of terms each coming from a different coefficient function of the gradient expansion. So the fact that η at this order comes out from the two terms U_k and Z_k only should not be used as an argument to decide about the superiority of one expansion with respect to the other. It should rather have been proven that the contributions coming from the infinite terms other than U_k and Z_k add up to zero. Actually we already know that this does not happen. By using a different but equivalent formalism, the computation of η up to two-loop order has been done in [8] where we see that the higher derivatives actually contribute to η.

I will show in the following how to obtain the correct result. Before proceeding to this derivation I review now the functional method of [6] that was intended to provide a way to compute the gradient expansion coefficients of the one loop effective action. Let us consider a single-component scalar field theory. The effective action $\Gamma[\Phi]$ is a highly nonlocal functional. It can be given a quasilocal resemblance through the gradient expansion. Up to second order in the derivatives of the field,

$$\Gamma[\Phi]=\int d^4x\left[U(\Phi)+\frac{1}{2}Z(\Phi)\partial_\mu\Phi\partial_\mu\Phi\right]. \quad (1)$$

For definiteness we work in $d=4$ dimensions. In the loop expansion, on the other hand, up to one-loop order,

$$\Gamma[\Phi]=\Gamma_0[\Phi]+\Gamma_1[\Phi], \quad (2)$$

where $\Gamma_0[\Phi]$ is the tree-level (bare) action, $\Gamma_0[\Phi]=\int d^4x[U(\Phi)+\frac{1}{2}\partial_\mu\Phi\partial_\mu\Phi]$ and $U(\Phi)$ is the classical potential. $\Gamma_1[\Phi]$ is the one-loop contribution to $\Gamma[\Phi]$ and can also be expanded in powers of the field derivatives. Again up to second order,

$$\Gamma_1[\Phi]=\int d^4x\left[U_1(\Phi)+\frac{1}{2}Z_1(\Phi)\partial_\mu\Phi\partial_\mu\Phi\right]. \quad (3)$$

$U_1(\Phi)$ and $Z_1(\Phi)$ are the one-loop contributions to $U(\Phi)$ and to $Z(\Phi)$, respectively.

It is convenient to introduce a Dirac-like notation that will also be very useful in the following. The one-loop contribution to the effective action can be written as (from now on $U^{(n)}$ means the nth derivative with respect to Φ)

$$\Gamma_1[\Phi]=\mathrm{Tr}\ \ln[\hat{P}^2+\hat{U}^{(2)}(\hat{\Phi}(\hat{x}))]. \quad (4)$$

In fact the second functional derivative of the bare action,

$$\frac{\delta^2\Gamma_0[\Phi]}{\delta\Phi(x)\,\delta\Phi(y)}=[-\partial_x^2+U^{(2)}(\Phi(x))]\,\delta(x-y),$$

can be represented as the kernel of the operator $\hat{P}^2 + \hat{U}^{(2)}(\hat{\Phi}(\hat{x}))$ in the "x representation" once we define $\hat{P}_\mu$ and $\hat{U}^{(2)}(\hat{\Phi}(\hat{x}))$ to be, respectively, $-i\partial_\mu$ and $U^{(2)}(\Phi(x))$ in this representation, and introduce the notation $\langle x|y\rangle = \delta(x-y)$:

$$\frac{\delta^2\Gamma_0[\Phi]}{\delta\Phi(x)\,\delta\Phi(y)} = \langle x|[\hat{P}^2 + \hat{U}^{(2)}(\hat{\Phi}(\hat{x}))]|y\rangle. \tag{5}$$

Other representations can also be introduced. We are in particular interested in the "p representation," the transformation function being $\langle x|p\rangle = (1/\sqrt{V})e^{ipx}$ (V is the volume), where traces are conveniently computed:

$$\mathrm{Tr}\,\hat{O} = \sum_p \langle p|\hat{O}|p\rangle = V\int \frac{d^4p}{(2\pi)^4}\langle p|\hat{O}|p\rangle. \tag{6}$$

The notations above allow us to introduce the "completeness relations" in the x and p representations:

$$\hat{I} = \int d^4x|x\rangle\langle x| = \sum_p |p\rangle\langle p|. \tag{7}$$

We can now state the method of [6] in the following way. First we write Eq. (4), a part for a meaningless infinite constant, as [we abbreviate $\hat{U}^{(2)}(\hat{\Phi}(\hat{x}))$ with $\hat{U}^{(2)}$]:

$$\Gamma_1[\Phi] = -\int_0^\infty du\,\mathrm{Tr}[\hat{P}^2 + \hat{U}^{(2)} + u]^{-1}. \tag{8}$$

Second, with the help of Eq. (6), the trace in Eq. (8) is

$$\sum_p \langle p|[\hat{P}^2 + \hat{U}^{(2)} + u]^{-1}|p\rangle. \tag{9}$$

Third, we rewrite Eq. (9) as

$$\sum_p \langle p|[p^2 + \hat{U}^{(2)} + u - (p^2 - \hat{P}^2)]^{-1}|p\rangle. \tag{10}$$

Equation (10), where we have just added and subtracted p^2, contains the essence of the method. For any fixed value of p we want to expand the operator $[p^2 + \hat{U}^{(2)} + u - (p^2 - \hat{P}^2)]^{-1}$ around $[p^2 + \hat{U}^{(2)} + u]^{-1}$.

Up to the second order in the derivatives of the field we get ($\hat{A}_p = p^2 + \hat{U}^{(2)} + u$ and $\hat{B}_p = p^2 - \hat{P}^2$)

$$\mathrm{Tr}[\hat{P}^2 + \hat{U}^{(2)} + u]^{-1} = \sum_p \langle p|[\hat{A}_p^{-1} + \hat{A}_p^{-1}\hat{B}_p\hat{A}_p^{-1} + \hat{A}_p^{-1}\hat{B}_p\hat{A}_p^{-1}\hat{B}_p\hat{A}_p^{-1}]|p\rangle. \tag{11}$$

The first term on the right-hand side (rhs) of Eq. (11), after insertion of Eq. (7) in the x representation and integration in the u variable, gives the one-loop contribution to the effective potential, $U_1(\Phi) = \int d^4x \int [d^4p/(2\pi)^4]\ln[p^2 + U^{(2)}(\Phi(x))]$.

The second and the third term on the rhs of Eq. (11) can be computed by commuting $\hat{A}_p^{-1}$ with $\hat{B}_p$ and applying the relation

$$[\hat{P}_\mu, F(\hat{\Phi}(\hat{x}))] = -i\frac{\partial}{\partial\hat{x}_\mu}F(\hat{\Phi}(\hat{x})). \tag{12}$$

Inserting again Eq. (7) in the x representation and integrating in the u variable, we finally get the coefficient of $\partial_\mu\Phi(x)\partial_\mu\Phi(x)$, i.e., the one-loop contribution $Z_1(\Phi(x))$ to $Z(\Phi(x))$. This result coincides, as it should, with the results of [9] and [7].

Now we want to apply this method to the derivation of the flow equations for U_k and Z_k. Let $S_k[\Phi]$ be the Wilsonian effective action at the scale k. At an infinitesimal lower scale $k-\delta k$ the effective action $S_{k-\delta k}[\Phi]$ is given by

$$e^{-S_{k-\delta k}[\phi]} = \int [D\eta]e^{-S_k[\phi+\eta]} = e^{-S_k[\phi]}\int [D\eta] \times \exp\left[-\left(\int d^4x\frac{\delta S_k[\phi]}{\delta\phi(x)}\eta(x) + \frac{1}{2}\int d^4x d^4y\frac{\delta^2 S_k[\phi]}{\delta\phi(x)\,\delta\phi(y)}\eta(x)\,\eta(y)\right)\right]. \tag{13}$$

In Eq. (13) we have written $\Phi(x) = \phi(x) + \eta(x)$, separating the component $\phi(x)$ with modes from zero up to $k-\delta k$, from $\eta(x)$, the component with modes within the shell $[k-\delta k,k]$. We have also assumed that the expansion around the background field $\phi(x)$ is saturated by the trivial saddle point $\eta=0$. In [10] the spontaneously broken symmetry case, where nontrivial saddle points appear, is treated. Here we limit ourselves to consider the unbroken case. In addition we have kept only terms up to $O(\eta^2)$ as in the infinitesimal shell limit ($\delta k \to 0$) the Gaussian approximation is exact [2].

Let us call $\mathcal{F}$ the subspace of functions with modes within the shell, i.e., $\mathcal{F} = \{\psi(x)|\psi(x) = \Sigma_{\tilde{p}}\psi_{\tilde{p}}e^{i\tilde{p}x},\ |\tilde{p}| \in [k-\delta k,k]\}$. The tilde over the momentum p is used from now on to indicate that $|\tilde{p}| \in [k-\delta k,k]$.

By the help of the Dirac-like notation previously introduced we can write

$$\int d^4x\frac{\delta S_k[\phi]}{\delta\phi(x)}\eta(x) = \langle s_k^{(1)}|\eta\rangle$$
$$\int d^4x d^4y\frac{\delta^2 S_k[\phi]}{\delta\phi(x)\,\delta\phi(y)}\eta(x)\,\eta(y) = \langle\eta|\hat{S}_k^{(2)}|\eta\rangle. \tag{14}$$

Equations (14) define the vector $\langle s_k^{(1)}|$ and the operator $\hat{S}_k^{(2)}$ whose "entries" in the x representation are, respectively, $\delta S_k[\phi]/\delta\phi(x)$ and $\delta^2 S_k[\phi]/\delta\phi(x)\,\delta\phi(y)$. By the help of Eqs. (14), Eq. (13) can be written in the compact form

$$e^{-S_{k-\delta k}[\phi]} = e^{-S_k[\phi]}\int [D\eta]e^{-\langle s_k^{(1)}|\eta\rangle - 1/2\langle\eta|\hat{S}_k^{(2)}|\eta\rangle}. \tag{15}$$

Note that in $\langle s_k^{(1)}|\eta\rangle$ only the Fourier components of $|s_k^{(1)}\rangle$ belonging to the shell $[k-\delta k,k]$ give a contribution. Similarly for $\langle\eta|\hat{S}_k^{(2)}|\eta\rangle$. Performing the Gaussian integration in Eq. (15) we get

$$S_{k-\delta k}[\phi]=S_k[\phi]+\frac{1}{2}\mathrm{Tr}'\ln\tilde{S}_k^{(2)}+\frac{1}{2}\langle s_k^{(1)}|[\tilde{S}_k^{(2)}]^{-1}|s_k^{(1)}\rangle. \tag{16}$$

Few comments are in order. It is clear that the trace has to be taken in the subspace $\mathcal{F}$, and this has been indicated by the label $'$ in Tr$'$. Moreover $\tilde{S}_k^{(2)}$ in Eq. (16) *is not* the operator $\hat{S}_k^{(2)}$ defined in Eq. (14) but rather its *restriction* to the subspace $\mathcal{F}$. This point has been overlooked in the previous literature [11,12,5] and Eq. (16) has always been written as if $\hat{S}_k^{(2)}$ rather than $\tilde{S}_k^{(2)}$ appeared in it. This illegal replacement is at the origin of the incorrect result of [5]. It is easy to write down $\tilde{S}_k^{(2)}$. The projection operator onto $\mathcal{F}$ is $\hat{\mathcal{P}}=\Sigma_{\tilde{p}}|\tilde{p}\rangle\langle\tilde{p}|$ and $\tilde{S}_k^{(2)}=\hat{\mathcal{P}}\hat{S}_k^{(2)}\hat{\mathcal{P}}$.

As for the effective action $\Gamma(\Phi)$ in Eq. (1), we write down now the gradient expansion for the Wilsonian action S_k up to the lowest order derivative term, i.e.,

$$S_k[\Phi]=\int d^4x\left[U_k(\Phi)+\frac{1}{2}Z_k(\Phi)\partial_\mu\Phi\partial_\mu\Phi\right]. \tag{17}$$

From Eq. (16) we obtain then $U_{k-\delta k}$ and $Z_{k-\delta k}$ and finally, sending $\delta k\to 0$, the flow equations for U_k and Z_k. It is not difficult to show (details will be presented in [13]) that with the ansatz (17) $\langle s_k^{(1)}|[\tilde{S}_k^{(2)}]^{-1}|s_k^{(1)}\rangle=0$. Then in Eq. (16) we are only left with the computation of $\mathrm{Tr}'\ln\tilde{S}_k^{(2)}$.

To illustrate the procedure and make our point clear, it will be sufficient to work with a field independent Z_k term. For the complete treatment we only need to follow similar steps starting with a field-dependent Z_k.

Writing the logarithm of the operator as in Eq. (8), we have

$$\mathrm{Tr}'\ln\tilde{S}_k^{(2)}=-\int_0^\infty du\,\mathrm{Tr}'[\mathcal{P}(Z_k\hat{P}^2+\hat{U}_k^{(2)}+u)\mathcal{P}]^{-1}, \tag{18}$$

and expanding as in Eq. (11):

$$\mathrm{Tr}'\ln\tilde{S}_k^{(2)}=-\int_0^\infty du\sum_{\tilde{p}}\langle\tilde{p}|[\tilde{A}_{\tilde{p}}^{-1}+\tilde{A}_{\tilde{p}}^{-1}\tilde{B}_{\tilde{p}}\tilde{A}_{\tilde{p}}^{-1}+\tilde{A}_{\tilde{p}}^{-1}\tilde{B}_{\tilde{p}}\hat{A}_{\tilde{p}}^{-1}\tilde{B}_{\tilde{p}}\tilde{A}_{\tilde{p}}^{-1}]|\tilde{p}\rangle, \tag{19}$$

where $\tilde{A}_{\tilde{p}}=\mathcal{P}\hat{A}_{\tilde{p}}\mathcal{P}$ and $\tilde{B}_{\tilde{p}}=\mathcal{P}\hat{B}_{\tilde{p}}\mathcal{P}$.

It is clear that in Eq. (19) only the first term gives a contribution. In fact as $\tilde{B}_{\tilde{p}}=Z_k\Sigma_{\tilde{p}'}(\tilde{p}^2-\tilde{p}'^2)|\tilde{p}'\rangle\langle\tilde{p}'|$ and both $\tilde{p}$ and $\tilde{p}'$ belong to the shell $[k-\delta k,k]$, the operator $\tilde{B}_{\tilde{p}}$ is $O(\delta k)$. From the sum over $\tilde{p}$ in the shell comes another $O(\delta k)$ and then in Eq. (19) all the terms apart from the first have to be ignored being at least $O(\delta k^2)$.

We are already in a position to compare our result with those of [5]. From Eqs. (7), (8), and (9) of that paper we see that the operators in the full space, i.e., $\hat{A}_{\tilde{p}}$ and $\hat{B}_{\tilde{p}}$, and not their restriction to the $\mathcal{F}$ subspace are considered. If we now replace in our Eq. (19) the operators $\tilde{A}_{\tilde{p}}$ and $\tilde{B}_{\tilde{p}}$ with $\hat{A}_{\tilde{p}}$ and $\hat{B}_{\tilde{p}}$, respectively, we pick up additional contributions from the second and third term of Eq. (19). Performing for instance such a replacement in the second term, by the help of Eqs. (6) and (7) we get immediately $[a_{\tilde{p}}(x)=Z_k\tilde{p}^2+U_k^{(2)}(\phi(x))+u]$:

$$V\int\frac{d^4\tilde{p}}{(2\pi)^4}\int d^4x\langle\tilde{p}|x\rangle\langle x|\hat{A}_{\tilde{p}}^{-1}\hat{B}_{\tilde{p}}\hat{A}_{\tilde{p}}^{-1}|\tilde{p}\rangle =-Z_k\int\frac{d^4\tilde{p}}{(2\pi)^4}\int d^4x\frac{e^{-i\tilde{p}x}}{a_{\tilde{p}}(x)}(\partial_\mu\partial_\mu+\tilde{p}^2)\frac{e^{i\tilde{p}x}}{a_{\tilde{p}}(x)}. \tag{20}$$

From Eq. (20) we can easily see where the mistake originates. The operator $\partial_\mu\partial_\mu$ acting on

$$\frac{e^{i\tilde{p}x}}{Z_k\tilde{p}^2+U^{(2)}(\phi(x))+u}$$

gives rise to three terms. One is proportional to $\tilde{p}_\mu$ and gives zero after the angular integration in the momentum variable $\tilde{p}$. Another is proportional to $-\tilde{p}^2$ and cancels against the $\tilde{p}^2$ term. Finally, a third term is

$$\int\frac{d^4\tilde{p}}{(2\pi)^4}\int d^4x\frac{-Z_k}{Z_k\tilde{p}^2+U_k^{(2)}(\phi(x))+u}\times\partial_\mu\partial_\mu\frac{1}{Z_k\tilde{p}^2+U_k^{(2)}(\phi(x))+u}. \tag{21}$$

This term is not zero and gives additional spurious contributions to $Z_{k-\delta k}$.

We now take a closer look at the first term of Eq. (19):

$$\sum_{\tilde{p}}\langle\tilde{p}|[\mathcal{P}(Z_k\tilde{p}^2+\hat{U}_k^{(2)}+u)\mathcal{P}]^{-1}|\tilde{p}\rangle. \tag{22}$$

In [5] this contribution is written as

$$\int\frac{d^4\tilde{p}}{(2\pi)^4}\int d^4x\frac{1}{Z_k\tilde{p}^2+U_k^{(2)}(\phi(x))+u}. \tag{23}$$

Again this would be right if we could ignore the presence of the projection operator $\mathcal{P}$ in Eq. (22). As $\mathcal{P}|\tilde{p}\rangle=|\tilde{p}\rangle$, this would amount to replacing the inverse $\tilde{A}_{\tilde{p}}^{-1}$in $\mathcal{F}$ of the restricted operator $\tilde{A}_{\tilde{p}}$ with the restriction in $\mathcal{F}$ of the inverse operator, $\mathcal{P}\hat{A}_{\tilde{p}}^{-1}\mathcal{P}$. This replacement would be correct if the operator $\hat{A}_{\tilde{p}}$ was diagonal in the p representation. As we see from its definition, this is certainly not the case.

Actually the projection operator in Eq. (22) is not easy to handle unless we develop $\{\mathcal{P}[Z_k\tilde{p}^2+\hat{U}_k^{(2)}(\hat{\phi}(\hat{x}))+u]\mathcal{P}\}^{-1}$ around the *diagonal* operator $[\mathcal{P}(Z_k\tilde{p}^2+U_k^{(2)}(\phi_0)+u)\mathcal{P}]^{-1}$, obtained expanding $\phi(x)$ around the constant value ϕ_0 $[U_k^{(n)}(\phi_0)=U_{k0}^{(n)}]$:

$$U_k^{(2)}(\phi)=U_{k0}^{(2)}+U_{k0}^{(3)}\partial_\mu\phi+\frac{1}{2}U_{k0}^{(4)}\partial_\mu\phi\partial_\mu\phi+\cdots. \tag{24}$$

If we now insert Eq. (24) in Eq. (22) and then integrate in the u variable, we get the same result we would have obtained if we had started by expanding the logarithm in the fluctuation determinant of Eq. (16) around

$$\ln[\mathcal{P}(Z_k\hat{P}^2+U_{k0}^{(2)})\mathcal{P}], \tag{25}$$

i.e., as if we had used from the very beginning the expansion of [7].

We can now come to our reassuring conclusion. There is no contradiction between the two methods of [6] and [7] that, when applied to the fluctuation determinant in Eq. (16), give *one and the same* result for $Z_{k-\delta k}$, i.e., the same flow equation for Z_k. We have also learned that, due to the constraint imposed by the presence of the projection operator $\mathcal{P}$ the method of [6] trivially turns to the method of [7].

I give now a couple of differential equations for U_k and Z_k that are obtained once the full ϕ dependence of Z_k is taken into account ($A=Z_kk^2+U_k^{(2)}$ and $Z_k^{(n)}$, $A^{(n)}$ are derivatives with respect to the field):

$$k\frac{\partial}{\partial k}U_k=-\frac{k^4}{16\pi^2}\ln A, \tag{26}$$

$$k\frac{\partial}{\partial k}Z_k=-\frac{k^4}{16\pi^2}\left(\frac{Z_k^{(2)}}{A}-\frac{2Z_k^{(1)}A^{(1)}}{A^2}-\frac{Z_k^{(1)^2}k^2}{4A^2}+\frac{Z_kA^{(1)^2}}{A^3}+\frac{Z_k^{(1)}Z_kA^{(1)}k^2}{A^3}-\frac{Z_k^2A^{(1)^2}k^2}{A^4}\right). \tag{27}$$

These equations have already been presented in [14,15] and similar equations in [11,12], but a word of caution has to be said concerning their derivation. In [11,12], where to derive the flow equations the method of [7] was applied, the presence of the projection operator $\mathcal{P}$ was not taken into account. In addition there is one point that I have deliberately avoided mentioning up to now. The presence of the projection operator $\mathcal{P}$, that is due to the choice of a sharp cutoff for the mode elimination, has another effect: it potentially brings additional terms in the derivation of $Z_{k-\delta k}$ from Eq. (16). Again due to the neglect of $\mathcal{P}$, these terms went unnoticed in [11,12]. These derivations are then not on a firm foot. In [14] Eq. (27) is obtained by considering the fluctuation operator directly in the p representation. No ambiguity is then present concerning the restriction of the operator $\hat{S}_k^{(2)}$. Nevertheless the method employed, namely the choice of a particular background field $\phi(x)$, carries ambiguities due to the appearance of these additional nonanalytic terms [16] in contradiction with the gradient expansion itself. Those terms are there neglected without any justification and the whole method seems not to be firmly established.

In [15] the first step beyond the LPA is taken by brute force integration in the space of the Fourier components of the fluctuation field, i.e., without any reference to the functional methods of [6] and [7]. Again a special nonconstant background field is chosen to extract the differential equation for Z_k, namely a field with a single Fourier component $\phi(x)\sim[\varphi_qe^{iqx}+\varphi_{-q}e^{-iqx}]$, with $q\sim0$. As in [14] additional terms nonanalytic in q^2 are found but, for the first time, the method allowed us to compare the magnitude of these terms with the ones that are retained in establishing Eq. (27). In this way it was possible to find the conditions under which these terms can be safely neglected, i.e., the validity conditions of Eq. (27). But at that time it was not yet clear that the result of [5] was incorrect, i.e., if Eq. (27) or the corresponding Eq. (18) of [5] was the correct one. Actually in [15] we incorrectly argued that both equations could be right as being different approximations for different physical situations. Only now it appears clearly that the system of Eqs. (26) and (27) *is* the next order of approximation to the Wegner-Houghton equation in the gradient expansion, after the LPA of [3,4].

In a forthcoming paper [13], I will present a complete analysis of the problems related to the presence of a sharp cutoff. It is sufficient to say here that the results of this work, more general than that of [15], as no reference to a specific background field is done, actually meet in this respect those of [15]: for a sufficiently smooth background field, these additional terms can be neglected and Eqs. (26) and (27) give the correct approximation to the Wegner-Houghton equation at this order.

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