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Can geodesics in extra dimensions solve the cosmological horizon problem?

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We demonstrate a non-inflationary solution to the cosmological horizon problem in scenarios in which our observable universe is confined to three spatial dimensions (a three-brane) embedded in a higher dimensional space. A signal traveling along an extra-dimensional null geodesic may leave our three-brane, travel into the extra dimensions, and subsequently return to a different place on our three-brane in a shorter time than the time a signal confined to our three-brane would take. Hence, these geodesics may connect distant points which would otherwise be ''outside'' the four dimensional horizon (points not in causal contact with one another).

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I. INTRODUCTION

The universe appears to be homogeneous and isotropic on large scales. According to the Cosmic Background Explorer (COBE) measurements, the cosmic background radiation (CBR) is uniform to a part in 10^4 on large scales (from about $10''$ to $180°$ [1]). Furthermore, the light element abundance measurements seem to indicate that the observable universe (bounded by the last scattering surface) was homogeneous by the time of nucleosynthesis [2]. Hence, we would expect the observable universe today (time t_0) to have been in causal contact by the time of nucleosynthesis t_n; otherwise the initial conditions of the universe would have to be extremely fine-tuned in order for the causally disconnected patches to resemble one another as much as they do. However, in a Friedmann Robertson Walker (FRW) universe [a metric of $ds^2=dt^2-a(t)^2d\mathbf{x}^2$] that is matter or radiation dominated, upon naive extrapolation back to the singularity, one finds that there is a finite horizon length at the time of nucleosynthesis. Hence, for the observable universe to have been in causal contact by the time of nucleosynthesis, the comoving horizon length must have been larger than the comoving distance to the last scattering surface. In other words, our observable universe today (when appropriately scaled back to the time of nucleosynthesis) must have fit inside a causal region at the time of nucleosynthesis.

The comoving size L_o of the observable universe today is

$$L_o=\int_{t_{dec}}^{t_0}\frac{dt}{a(t)} \tag{1}$$

where t_{dec} is the time of the radiation decoupling and t_0 is the time today (subscript 0 refers to today). The comoving size L_n of the horizon at the time of nucleosynthesis is

$$L_n=\int_0^{t_n}\frac{dt}{a(t)}. \tag{2}$$

In order to explain causal contact of all points within our observable universe at the time of nucleosynthesis, we require $L_o<L_n$. However, this condition is not met in a naive FRW cosmology with matter or radiation domination. Even if we take t_n to be the the time of last scattering of CBR and not the nucleosynthesis time, we still have a horizon problem by a factor of 10^5. In both matter or radiation domination cases, the time dependence of the scale factor is a power law with the index less than 1; in a dust (matter) dominated universe, $a\propto t^{2/3}$ and in a radiation dominated universe, $a\propto t^{1/2}$. Hence, in the naive FRW cosmology, $L_0\sim t_0/a_0$ and $L_n\sim t_n/a(t_n)$, such that $L_o>L_n$ while causal connection requires $L_o<L_n$. This is the horizon problem. Inflationary cosmology [3] solves the horizon problem by having a period of accelerated expansion, with $\ddot{a}>0$ (a period of time when the universe was not dust or radiation dominated).

Here we consider a non-inflationary solution to the horizon problem. The argument in the previous paragraph that leads to $L_o>L_n$ has assumed that causal signals travel within the light-cone defined by the null geodesics of a 4-dimensional manifold. If the causal signals can instead travel through higher dimensions, the points that are seemingly causally disconnected from the 4-dimensional point of view may in fact be causally connected. A signal along the geodesic may leave our spatially three dimensional world, travel into the extra dimensions, and subsequently return to a different place in our three dimensional world; the distance between the initial and the final (return) point when measured along the 3-spatial dimensions may be longer than the distance that a light signal confined to our (3+1)-dimensional universe would travel in the same amount of time. Such a geodesic may arise when the curvature allows the path length for a null signal (e.g., light ray) through the higher dimensions to be shorter than any path length in our lower dimensional world alone. Such a possibility has been alluded to before (see for example Ref. [4] and the footnote in Ref. [5]). In this paper, we construct explicit examples of

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such spacetimes. Other papers of interest relating to this topic can be found in Ref. [6], Refs. [7,8], and references therein.

In this paper, we will focus on noncompact 3-branes. We construct an example (in Sec. II A) which is compatible with the cosmology of our universe. In this scenario, there are two separate 3-branes: one is our observable universe and the other is the hidden sector. We take these 3-branes to be ''parallel'' to one another (i.e. each of the branes is located at a constant coordinate value of the extra dimension). A field signal originating on our observable 3-brane travels away from our brane on a path perpendicular to both branes and arrives at the other 3-brane. There, it interacts with fields confined to the hidden sector. Subsequently, due to these interactions, the signal travels along the hidden sector 3-brane. Because of the specific metric we have constructed, the signal can traverse a longer coordinate distance than it could on our brane in the same amount of time. The signal then returns back to our brane through a path perpendicular to the two 3-branes. As a consequence of this path, the signal has traversed an effective distance on our 3-brane that is much longer than any distance it could have covered had it remained on our 3-brane in the same time. Hence points outside of the naive 3-brane ''horizon'' can be connected in this way.

In Sec. II A, we describe a class of metrics which may be used to obtain semirealistic cosmology and for which the geodesic is higher dimensional: as described in the last paragraph, this model requires interactions between our brane and the hidden sector brane. In Sec. II B, we will construct a 2+1 dimensional example of a continuous metric (no interactions required) that allows geodesics to connect seemingly distant points; expansion of the universe is not yet taken into account in this simple case. Then, in Sec. II C, we will generalize such a continuous metric to 4+1 dimensions with expansion. However, the continuous case in Sec. II C does not produce a viable cosmology for our universe: first, the universe is not homogeneous (a special point is required and in this example is singular), and second, in this model both our 3-dimensional world as well as the bulk describing the extra dimensions are expanding with the same scale factor. In Sec. III we discuss some issues of causality violation. Finally, in Sec. IV, we summarize and conclude.

II. HORIZON EVADING METRICS

A. (4+1)-dimensional example with viable cosmology

Here we consider a (4+1)-dimensional case which produces a viable cosmology: our observable universe is homogeneous and expanding. Consider a metric of the form

$$ds^2 = dt^2 - [e^{-2ku} a^2(t) d\mathbf{h}^2 + du^2] \tag{3}$$

with our observable brane located at $u=0$. Here $\mathbf{h}$ is a three dimensional Euclidean vector. (Note that although this metric is similar to the one that was considered by [9], there is a crucial difference in that dt^2 term does not share the conformal factor multiplying $d\mathbf{h}^2$.) Now, consider the null geodesics labeled A, B, C, and D as shown in Fig. 1. Explicitly, the null geodesics can be written as $[u=u_g(t), h=h_g(t)]$ with

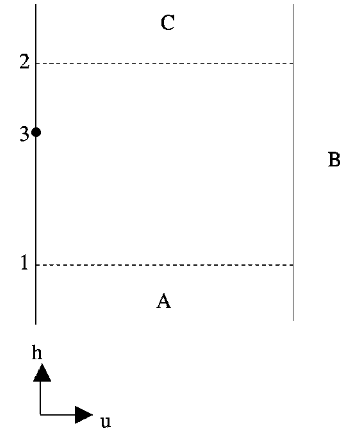

FIG. 1. Branes and geodesics for (4+1)-dimensional example. Our brane is represented by the left hand vertical line with $u=0$; a second brane is represented by the right hand vertical line with $u=L$. The geodesic in the full metric leaves our brane at point 1, travels along A, B, and C, and reenters our brane at point 2. The distance $h_{(1,3)}$ between points 1 and 3 is the horizon distance usually calculated in cosmology in the absence of extra dimensions. Since $h_{(1,2)} > h_{(1,3)}$, points traditionally ''outside the horizon'' are here causally connected.

$$u_g(t) = \begin{cases} A: & t, & 0 \leq t \leq L, \\ B: & L, & L \leq t \leq t_f - L, \\ C: & t_f - t, & t_f - L \leq t \leq t_f, \end{cases} \tag{4}$$

and

$$h_g(t) = \begin{cases} A: & 0, & 0 \leq t \leq L, \\ B: & e^{kL} \int_L^{t-L} \frac{dt'}{a(t')}, & L \leq t \leq t_f - L, \\ C: & L, & t_f - L \leq t \leq t_f, \end{cases} \tag{5}$$

with $\mathbf{h}$ chosen along a particular direction h with an initial value of 0 without any loss of generality.

One can show that, indeed, these paths labeled A, B, and C satisfy the geodesic equations

$$\frac{d^2 x^\mu}{dt^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} = 0 \tag{6}$$

where $\Gamma^\mu_{\alpha\beta}$ is the Christoffel symbol. Here, a signal originating on our observable 3-brane at $u=0$ travels away from our brane on a path A perpendicular to the 3-brane. Once it arrives at $u=L$, it follows a trajectory B with constant u. Subsequently it returns to our brane via trajectory C, again perpendicular to our brane. Hence the effective distance it traverses on our brane is approximately given by the pathlength B (up to small corrections).

The distance traveled by a null signal along the brane between points 1 and 3 in time t_f is

$$h_{(1,3)} = \int_0^{t_f} \frac{dt}{a}. \tag{7}$$

As before, this is the distance that one would naively interpret as the horizon size in Eq. (1).[1] However, for a null signal leaving and reentering the brane via the geodesic A, B, and C, the effective distance traveled on the observable brane in time t_f is the distance between points 1 and 2, i.e., the same as the distance traveled along path B:

$$h_{(1,2)} = e^{kL} \int_L^{t_f - L} \frac{dt}{a(t)}. \tag{8}$$

Clearly for large enough kL,

$$h_{(1,2)} > h_{(1,3)} \tag{9}$$

and regions that ordinarily would be considered out of causal contact can be connected. In the ordinary FRW cosmology, the surface of last scattering of photons encompasses 10^5 causally disconnected patches. Here, however, if we take t_f to be the time of last scattering of photons, as long as $kL \sim \ln(10^5)$, then these patches can have all been in contact with one another and we can solve the horizon problem. Because the induced metric on the brane in this scenario is homogeneous, this scenario is cosmologically viable.

We have performed a numerical exploration of the solutions to the geodesic equations in Eq. (6) resulting from the metric in Eq. (3) for various initial conditions, in particular for initial velocity vectors leaving the brane in a variety of directions. We indeed find that there are geodesics with a continuous path which leave and subsequently reenter our brane, i.e., there are extra-dimensional causal paths which connect points 1 and 2 without a need to jump from one geodesic to another (such as turning the corner from segment A to segment B). However we have not found continuous paths which return to our brane at a point more distant than our naive ''horizon,'' i.e., the effective distance traveled on the observable brane is shorter than $h_{(1,3)}$ in the same time.

On the other hand, as shown above, the scenario of patched causal paths (without a single smooth causal geodesic) can solve the horizon problem. Such a scenario may be effective when there is another ''hidden sector'' brane at $u = L$ and the intersections of segments A and B or segments B and C represent vertices of interactions of the bulk fields with the fields confined on the brane. Hence, since the signal jumps from one geodesic to another through interactions, the bulk fields must interact sufficiently strongly with the fields on each of the branes for this scenario to be viable.

Although naively this requirement may seem problematic in our scenario, in reality, sufficient interactions may be possible. For example, suppose one considers an action of the form

$$S \ni \int d^5x \sqrt{g}\, \partial_\mu \phi \partial^\mu \phi + \int_{\text{brane 1}} \lambda_1 \phi \bar{\psi} \psi + \int_{\text{brane 2}} \lambda_2 \phi \bar{\psi} \psi + \int_{\text{branes 1,2}} \text{K.E.}(\psi) \tag{10}$$

where ϕ is a massless bulk scalar field (with the dimensions properly normalized) and ψ is a four dimensional fermion confined to the boundary. After integrating out the ϕ field we obtain interactions of the form

$$\begin{aligned} S_{\text{eff}} \ni \int_{\text{brane 1}} d^4x_1 s_1 & \left[\frac{\lambda_1^2 s_1}{4} \bar{\psi}(x_1)\psi(x_1) \int_{\text{brane 1}} d^4x_1' G(x_1, x_1') \bar{\psi}(x_1')\psi(x_1') \right. \\ & + \frac{\lambda_1 \lambda_2 s_2}{4} \bar{\psi}(x_1)\psi(x_1) \int_{\text{brane 2}} d^4x_2' G \\ & \left. \times (x_1, x_2') \bar{\psi}(x_2')\psi(x_2') \right] \end{aligned} \tag{11}$$

where $s_1 = a^3(t)$ is the ratio of the d dimensional volume measure to the extra 1 dimensional volume measure evaluated on brane 1 while $s_2 = e^{-3kL} a^3(t)$ is the same ratio on brane 2: i.e.

$$s_i = \left. \frac{\sqrt{|g|}}{\sqrt{|g_{44}|}} \right|_{\text{brane } i}. \tag{12}$$

G is the Green function of a five dimensional Klein Gordon operator: i.e.

$$\frac{1}{2}(\partial_a \{ \sqrt{|g|} g^{ab} \partial_b \} + \epsilon^2) G(x, x') = \delta^{(5)}(x - x') \tag{13}$$

where the derivatives are with respect to x and the infrared regulating mass ϵ is arbitrarily small. Because of the s_2 suppression factor, one would naively expect the couplings connecting the ψs on two different branes to be suppressed. However, because in that case the Green function G contains a $1/s_2$ behavior in the $\epsilon \to 0$ limit, the s_2 factors approximately cancel and the coupling is unsuppressed.[2]

Finally, we note that this patched geodesic model can easily be generalized to spacetimes with dimension greater than just five.

B. Example in 2+1 dimensions

For pedagogical purposes we will here describe a lower dimensional example, in which our observable universe is a one-dimensional surface in a spatially two-dimensional world. We consider a Minkowski spacetime of the form

[1]Note that this does not correspond to a geodesic in the 5-dimensions.

[2]A fuller exploration will be given in a related work [14].

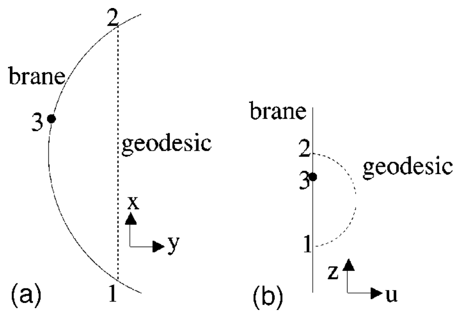

FIG. 2. Brane and geodesics shown in coordinate systems (x,y) and (u,z) in (2+1)-dimensional example. The location of our brane (our observable universe) is shown as $y=\xi(x)$ in (a) and as $u=0$ in (b). The geodesic of the full metric is a straight line between points 1 and 2 in (a) and a curve in (b). The distance $z_{(1,3)}$ between points 1 and 3 in (b) is the horizon distance usually calculated in cosmology in the absence of extra dimensions. Note that $z_{(1,2)}>z_{(1,3)}$, such that points traditionally ''outside the horizon'' are here causally connected.

$$ds^2=dt^2-dx^2-dy^2. \tag{14}$$

Under the coordinate transformation

$$z=\int \sqrt{1+[\xi'(x)]^2}dx \tag{15}$$

and

$$u=y-\xi(x), \tag{16}$$

where the function $\xi(x)$ will be chosen below and the prime operation ($'$) refers to derivatives with respect to x, the line element transforms to

$$ds^2=dt^2-dz^2-du^2-\frac{2\xi'(x(z))dudz}{\sqrt{1+[\xi'(x(z))]^2}}. \tag{17}$$

We will choose the location of the brane (that is our observable universe) to be at $u=0$, such that in the (u,z) coordinate system, the brane is merely a straight line with z as the coordinate on the brane. From Eq. (16), one can see that, in the (x,y) coordinate system, the location of the brane is at $y=\xi(x)$. Hence, in this coordinate system, the brane looks curved. Figures 2(a),(b) show the location of the brane in the (x,y) and (u,z) coordinate systems respectively. Since the (x,y) coordinate system is trivial (Minkowski), it is obvious that a geodesic is simply a straight line. We have hence plotted such a geodesic in Fig. 2(a) between two points 1 and 2. This same geodesic (of the full metric) becomes a curved line in the (u,z) coordinate system, as shown in Fig. 2(b). For comparison, in Fig. 2(b) we have also plotted a third point 3, which is a geodesic of the induced metric on the brane. It is the distance $z_{(1,3)}$ that is the usual horizon that we calculate in cosmology when geodesics off the brane are not considered. The claim is that, in the same amount of time, the distance traveled by a null signal (e.g., light ray) directly along the brane from point 1 to point 3 is shorter than the effective distance on the brane traveled by a light ray traveling from point 1 to point 2. Hence, while we might naively think points 1 and 2 are causally disconnected, in fact they are causally connected by information traveling off the brane at point 1 and reentering the brane at point 2. This behavior can be explained because, in the trivial Minkowski metric of the (x,y) coordinates, the distance between points 1 and 2 can be traversed by a straight line while the distance between points 1 and 3 must involve a curved and hence longer path.

We will illustrate the above quantitatively. In the (u,z) coordinate system, a particle moving along the brane from points 1 to 3 satisfies

$$0=ds^2=dt^2-dz^2 \tag{18}$$

such that the distance $z_{(1,3)}$ traveled along the brane between points 1 and 3 in time t_f is

$$z_{(1,3)}=t_f. \tag{19}$$

This is the distance that one would naively calculate as the horizon size in Eq. (1). However, let us consider a geodesic that leaves and subsequently reenters the brane. For simplicity, we can choose y to be a constant y_1 for the particular geodesic we consider [as drawn in Fig. 2(a)].

Then, in the (y,x) coordinate system, the geodesic is given by

$$y_g=y_1, \quad x_g=-c+t, \tag{20}$$

where c is a positive constant, subscript g refers to the geodesic, and we assume the signal leaves the brane at initial time $t=0$.

Now to proceed we will choose a particular form for $\xi(x)$,

$$\xi'(x)=\tan(kx), \tag{21}$$

i.e.,

$$\xi(x)=-\frac{1}{k}\ln(\cos(kx)), \tag{22}$$

where k is a constant. Then, using Eqs. (15) and (16), we can transform the geodesic in Eq. (20) to the (u,z) coordinate system:

$$u_g(t)=y_1-\xi(t-c)=y+\frac{1}{k}\ln[\cos k(t-c)], \tag{23}$$

$$z_g(t)=\frac{1}{k}\ln[\sec k(t-c)+\tan k(t-c)]. \tag{24}$$

Hence $z_{(1,2)}=z_g(t_f)$ is the z-coordinate distance traversed by the light ray following a geodesic in the full metric from points 1 to 2 in the time t_f. It is clear that the second term in Eq. (24) can blow up, such that it is certainly possible that

$$z_{(1,2)}>z_{(1,3)}, \tag{25}$$

such that seemingly causally disconnected points 1 and 2 have in fact been in causal contact.

In the next subsection, we will generalize the continuous metric of this subsection to an expanding universe in higher dimensions.

C. Generalization of the 2+1 example to 4+1 dimensions

Here we consider a (4+1)-dimensional generalization of the (2+1)-dimensional example and include cosmological expansion. However, as we will see, because the origin becomes a special (even singular) point, this example is not homogeneous and hence does not describe the real universe. Still, it solves the horizon problem in a novel way and as such is instructive.

Consider a metric of the form

$$ds^2 = dt^2 - a^2(t)(dr^2 + f(r)d\Omega^2 + du^2 + 2g(r)du dr) \tag{26}$$

where

$$f(r) = \frac{1}{k^2}(\arccos[\mathrm{sech}(kr)])^2 \tag{27}$$

$$g(r) = \tanh(kr) \tag{28}$$

where k is an arbitrary constant and $d\Omega^2 = \sin^2\theta d\phi^2 + d\theta^2$ is the angular metric of a 2-sphere. Consider the spatial dimensions of the usual observable universe (3-brane) to be at $u=0$. The induced metric on the 3-brane is

$$ds^2 = dt^2 - a^2(t)(dr^2 + f(r)d\Omega^2) \tag{29}$$

which is isotropic only about one particular point in general (i.e. inhomogeneous). Classically, the causal region is bounded by the geodesics of a massless particle satisfying the geodesic equation given in Eq. (6).

Along the brane, the induced metric, Eq. (29), implies a geodesic that is different from the geodesic implied by the higher dimensional embedding metric, Eq. (26). For the embedding metric, the geodesics leave the brane initially and reintersect the brane at a later time. Suppose we consider a light signal starting from the origin at the big bang singularity [when a(t=0)=0]. The geodesic on the brane is then

$$r_b(t) = \int_0^t \frac{dt}{a} \tag{30}$$

and the comoving horizon length (for example at the time of nucleosynthesis) is $L_n = r_b(t_n)$. In the embedding higher dimensional spacetime, the geodesic is given by

$$r_g(t) = \frac{1}{k}\ln\left\{\sec\left[k\frac{r_b(t)}{\sqrt{1+c^2}}\right] + \tan\left[k\frac{r_b(t)}{\sqrt{1+c^2}}\right]\right\} \tag{31}$$

$$u_g(t) = cr(t) + \frac{1}{k}\ln\left\{\cos\left[k\frac{r_b(t)}{\sqrt{1+c^2}}\right]\right\} \tag{32}$$

$$c = \frac{\ln\{\cosh[kr_b(t_f)]\}}{\arccos\{\mathrm{sech}[kr_b(t_f)]\}} \tag{33}$$

where t_f is the time at which the geodesic reintersects the brane after leaving the brane at the initial time $t=0$. If we set $t_f = t_n$ and vary k appropriately, we can make $L_n = r(t_n)$ of Eq. (2) arbitrarily large. Hence, points which from the brane point of view are outside of the causal horizon are actually causally connected.

The metric given by Eq. (26) is not realistic because the cosmology on the brane is not homogeneous. Indeed, the result is not surprising since the spatial curvature on the noncompact brane is positive. The only homogeneous constant curvature 4-dimensional manifolds are the 3 types of FRW metric (positive, zero, and negative intrinsic spatial curvature). The only boundariless positive curvature object of constant curvature in 3+1 dimensions is a 3-sphere, which is compact. Hence, we were doomed to begin with in trying to construct a noncompact homogeneous cosmology by embedding a curved manifold in an Euclidean space. The reason why this approach was successful for the 2+1 embedding of a (1+1)-dimensional manifold was the fact that the intrinsic spatial curvature is always 0 for a one dimensional manifold.

III. PHYSICAL SCENARIO

In this section we discuss a number of issues regarding the apparent causality violation attending the scenarios we have discussed.

First, let us consider whether bulk fields in such higher dimensional spacetimes will contradict any observations. In order for the geodesic in the higher dimensional spacetime to solve the horizon problem, causality must be apparently violated within the 3-brane, at least on cosmological scales during some early time. In order for the apparent causality violation to be hidden today while still solving the horizon problem, the geodesic through the extra dimension must not be accessible today. This is possible, for example, if the form of the cosmological energy density early on in the universe had bulk field coupling, while the form of the cosmological energy density today in the universe has no bulk field coupling.

Secondly, we remark that the apparent causality violation during an early epoch of the universe can be understood in terms of a higher dimensional Green function that falls off less rapidly than one would naively expect in the absence of extra dimensions. The causality violation can manifest in terms of nonlocal interactions of the effective 4-dimensional Lagrangian. As in the last section, consider for example the interaction given by

$$S \ni \int_{\mathrm{brane}} \lambda_1 \phi \bar{\psi}\psi. \tag{34}$$

As before, after integrating out the bulk field ϕ we find the effective interaction given by

$$S_{\mathrm{eff}} \ni \int_{\mathrm{brane}} d^4x_1 s_1 \frac{\lambda_1^2 s_1}{4}\bar{\psi}(x_1)\psi(x_1) \times \int_{\mathrm{brane}} d^4x_1' G(x_1,x_1')\bar{\psi}(x_1')\psi(x_1'). \tag{35}$$

For x_1' that is outside of the 4-dimensional light-cone of x_1, we would generally expect the Green function G to fall off exponentially. However, in the full 5-dimensional spacetime, the point x_1' that is outside of the 4-dimensional light-cone of x_1 will still be within the 5-dimensional light-cone, and hence the interaction will not be exponentially suppressed. Thus at a point x_1 on the brane, one can get a contribution from another point x_1' that is farther away than usually allowed by causality, because the 5-dimensional Green function doesn't fall off as fast outside the 4-dimensional light cone as one would naively expect in the absence of the extra dimensions. One can think of this as a propagator that can leave the brane and hence connect two distant points on the brane.

Thirdly, let us consider what kind of stress energy tensor gives rise to the metric presented in Sec. II A. Note that in light of Ref. [10], because the tt component and the uu component of the metric are identical and time independent, we have a fine tuned solution. More explicitly, to support the brane solutions, the five dimensional metric must satisfy the Israel condition boundary conditions (see for example Refs. [10–12] and references therein):

$$-6\partial_u\alpha=\kappa_5^2\rho\partial_u\nu=\kappa_5^2\left(\frac{1}{2}P+\frac{1}{3}\rho\right) \tag{36}$$

for the metric of the form $ds^2=e^{2\nu}d\tau^2-e^{2\alpha}d\mathbf{h}^2-du^2$ where $\alpha=-ku+\log a(t)$ and $\nu=0$ for the metric of Eq. (3). The pressure P and the energy density ρ are associated with the fields confined on the brane. As it stands, the energy density of the fields confined to the brane is a constant $\rho=-\frac{3}{2}P=6k/\kappa_5^2$. However, as is done in [13], one can add perturbations to the brane energy density ρ such that one can obtain a component of the energy density that dilutes as the universe expands. Note that since we must have $kL\approx\mathcal{O}(10)$ to solve the horizon problem, we must have a brane energy density of $\rho\sim\mathcal{O}(100)M_5^3/L$ where M_5 is the five dimensional Planck's constant defined by $\kappa_5^2=1/M_5^3$. Since we require $\rho<M_5^4$, this scenario is viable only if $L\gg 100/M_5$ which is not unrealistic. We leave a more complete study of viable cosmologies to a future study.

For completeness, we list the rest of the Einstein equation specifying the bulk stress energy tensor for this scenario:

$$T_0^0=-6k^2+3\left(\frac{\dot a}{a}\right)^2$$

$$T_1^1=-3k^2+\left(\frac{\dot a}{a}\right)^2+2\frac{\ddot a}{a}$$

$$T_4^4=-3k^2+3\left(\frac{\dot a}{a}\right)^2+3\frac{\ddot a}{a}$$

$$T_0^4=-3k\frac{\dot a}{a} \tag{37}$$

where we normalize the stress energy as $\mathcal{R}_{MN}-1/2g_{MN}\mathcal{R}=T_{MN}$. There are no fundamental constraints on the bulk stress energy tensor such that these equations cannot be satisfied.

IV. CONCLUSION

In this paper, we have demonstrated that, in theories with extra dimensions in which our "observable" 4-dimensional universe is confined to a submanifold, there may generically exist a non-inflationary solution to the horizon problem. The horizon problem can be stated as follows. If only a finite amount of time has passed subsequent to an initial epoch of our universe (e.g. a singularity), then any causal signal can travel within that time period only a finite distance, referred to as the horizon distance. By contrast, in the context of an FRW universe composed of ordinary matter and radiation, there is experimental evidence that patches of the universe which are separated by a distance longer than the horizon distance seem causally connected.

With the existence of extra dimensions, however, the naive horizon distance calculated by a null geodesic on the 4-dimensional submanifold does not constitute the maximum distance a signal can travel for a given time. A causal signal through the extra dimensions can reach a point in our universe which is many times further away than the naive horizon distance. An example of such a higher dimensional universe is described by Eq. (3) with two 3-branes, where one of the 3-branes is our observable universe and the other 3-brane is a "hidden" universe. The field confined to our brane can interact with the field living on a second "hidden" brane a distance L away from us in the extra dimension via a bulk field. For a given time, a causal signal can travel much further on the "hidden" brane in a direction parallel to brane. Hence, an impulse originating on our brane can take a shortcut through the "hidden" brane and affect our brane at a point outside of the naive "horizon."

Once "equilibration" of the energy density fluctuations is established, the fields confined on the brane may decay to fields that interact less strongly with the bulk fields. Hence, any apparent causality violation occurring through the existence of an extended higher dimensional light-cone may be hidden today.

We have also studied the construction of a metric that solves the horizon problem by embedding a curved 3-brane inside a Minkowskian 5-dimensional spacetime. We find that, for a non-compact 3-brane, the universe thus obtained is inhomogeneous.

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