Enlarged quintessence cosmology

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We show that the combination of a fluid with a bulk dissipative pressure and quintessence matter can simultaneously drive an accelerated expansion phase and solve the coincidence problem of our current Universe. We then study some scenarios compatible with the observed cosmic acceleration.

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I. INTRODUCTION

In the wake of the recent measurements of distant exploding stars, supernovae, the existence of negative-pressure dark energy has begun to gain broad consideration. Using type Ia supernovae as standard candles to gauge the expansion of the Universe, observers have found evidence that the Universe is accelerating [1–3]. A new component with significant negative pressure, called quintessence matter (Q matter for short), will in fact cause the cosmic expansion to speed up, so the supernovae observations provide empirical support for a new form of energy with strong negative pressure [1,4–9].

Different forms for the quintessence energy have been proposed. They include a cosmological constant (or more generally a variable cosmological term), a scalar field [7], a frustrated network of non-Abelian cosmic strings, and a frustrated network of domain walls [10,11]. All these proposals assume the Q matter behaves as a perfect fluid with a linear baryotropic equation of state, and so some effort has been invested in determinating its adiabatic index at the present epoch—see e.g., [6,12]. This new energy is to be added to the more familiar components: i.e., normal matter (luminous and dark) plus radiation. The contribution of the radiation component is known to be negligible at the current epoch whereas the main contribution to the former comes from cold dark matter (CDM) [13].

However, QCDM models (including those in which the quintessence energy is just the energy of the quantum vacuum) find difficulties in explaining why the energy densities of the CDM and Q matter should be comparable today. Since both energies redshift at different rates the conditions of the early universe have to be set very carefully for both energy densities to be of the same order today, though one may always invoke some version or other of the anthropic principle to soften the problem a bit. This is the coincidence problem [14]. Recently, a promising solution, for spatially flat metrics, based on the notion of ''tracker fields,'' fields that roll down their potential according to an attractorlike solution to the equations of motion [15,6,9], has been proposed. Unfortunately it is not clear to what extent these fields have the ability to solve the coincidence problem and, at the same time, drive the Universe to the current phase of accelerated expansion.

In any case, all these models overlook the fact that since the cosmic fluid consists in a mixture of different fluids a dissipative pressure may naturally arise which, for expanding universes, is bound to further decrease the total pressure [16]. Recently it has been proposed that the CDM must self-interact to explain the structure of the halos of the galaxies [17]—see however [18]. This self-interaction leads naturally to a viscous pressure whose magnitude will depend on the mean free path of the CDM particles. On the other hand, it has been suggested that dissipative fluids (or equivalently particle production processes) can drive a phase of accelerated expansion—see [19] and references therein.

This paper investigates how the combined action of dissipative normal matter and a quintessence scalar field may lead to the current accelerated expansion stage, and at the same time provide a solution to the coincidence problem different from that relying on a tracker field. It is shown in Sec. II that the flat coincident solution is an attractor. Section III delves into the consequences brought about by a dissipative pressure in the stress-energy tensor, i.e., it explores the dynamics of quintessence-dissipative dark matter (QDDM) models, while Sec. IV briefly outlines some specific models. Finally, Sec. V summarizes our findings. It should be understood that this work mainly deals with the present and late Universe since it is precisely at these stages where the dynamic effects of quintessence and dissipative pressure become important. Units in which $c=8\pi G=k_B=1$ are used throughout.

II. COSMOLOGICAL PROBLEMS

This section shows that the Friedmann-Lamaitre-Robertson-Walker (FLRW) universe filled with perfect normal matter plus quintessence fluid, corresponding to some scalar field governed by Klein-Gordon equation, cannot at the same time drive an accelerated expansion and solve the coincidence problem. To solve it, without abandoning the FLRW geometry, some additional contribution to the stress-energy tensor, such as a bulk dissipative pressure, is needed.

The overall stress-energy tensor of the cosmic fluid without the dissipative pressure reads

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$$T_{ab}=\rho u_a u_b+p h_{ab} \quad (h_{ab}=g_{ab}+u_a u_b, \quad u^a u_a=-1), \tag{1}$$

where $\rho=\rho_m+\rho_\phi$ and $p=p_m+p_\phi$. Here ρ_m and p_m are the energy density and pressure of the matter whose equation of state is $p_m=(\gamma_m-1)\rho_m$ with adiabatic index in the interval $1\leqslant\gamma_m\leqslant 2$. Likewise ρ_ϕ and p_ϕ, the energy density and pressure of the minimally coupled self-interacting Q-matter field ϕ, i.e.,

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi), \quad p_\phi=\frac{1}{2}\dot{\phi}^2-V(\phi), \tag{2}$$

are related by an equation of state similar to that of the matter, viz., $p_\phi=(\gamma_\phi-1)\rho_\phi$, so that its adiabatic index is given by

$$\gamma_\phi=\frac{\dot{\phi}^2}{\dot{\phi}^2/2+V(\phi)}, \tag{3}$$

where for non-negative potentials $V(\phi)$ one has $0\leqslant\gamma_\phi\leqslant 2$. The scalar field can be properly interpreted as Q matter provided $\gamma_\phi<1$—see e.g., [15]. As usual an overdot means derivative with respect to cosmic time. In general γ_ϕ varies as the Universe expands, and the same is true for γ_m since the massive and massless components of the matter fluid redshift at different rates.

The Friedmann equation together with the energy conservation of the normal matter fluid and quintessence (Klein-Gordon equation) are

$$H^2+\frac{k}{a^2}=\frac{1}{3}(\rho_m+\rho_\phi) \quad (k=1,0,-1), \tag{4}$$

$$\dot{\rho}_m+3H\gamma_m\rho_m=0, \tag{5}$$

$$\ddot{\phi}+3H\dot{\phi}+V'=0, \tag{6}$$

where $H\equiv\dot{a}/a$ denotes the Hubble factor and the prime means derivative with respect to ϕ. Introducing $\Omega_m\equiv\rho_m/\rho_c$, $\Omega_\phi,\equiv\rho_\phi/\rho_c$, with $\rho_c\equiv 3H^2$ the critical density, and $\Omega_k\equiv -k/(aH)^2$ plus the definition $\Omega\equiv\Omega_m+\Omega_\phi$ the set of equations (4)–(6) can be recast as (cf. [20])

$$\Omega_m+\Omega_\phi+\Omega_k=1, \tag{7}$$

$$\dot{\Omega}=\Omega(\Omega-1)(3\gamma-2)H, \tag{8}$$

$$\dot{\Omega}_\phi=[2+(3\gamma-2)\Omega-3\gamma_\phi]\Omega_\phi H, \tag{9}$$

where γ is the average adiabatic index given by

$$\gamma\Omega=\gamma_m\Omega_m+\gamma_\phi\Omega_\phi. \tag{10}$$

The following subsections investigate the flatness and coincidence problems. The former will be solved if the solution $\Omega=1$ to Eq. (8) becomes an attractor at late time. In its turn, the coincidence problem will be solved if the ratio Ω_ϕ/Ω_m becomes asymptotically a constant. We shall therefore explore the possibility of constant stable solutions in the $(\Omega,\Omega_m,\Omega_\phi)$ space.

A. The flatness problem and accelerated expansion

The combined measurements of the cosmic microwave background temperature fluctuations and the distribution of galaxies on large scales seem to imply that the Universe may be flat or nearly flat [4,21,22]. Hence the interesting solution at late times of Eq. (8) is $\Omega=1$ (i.e., $k=0$), and so we discard the solution $\Omega=0$ as incompatible with observation. The solution $\Omega=1$ is asymptotically stable for expanding universes ($H>0$) provided that the condition $\partial\dot{\Omega}/\partial\Omega<0$ holds in a neighborhood of $\Omega=1$ and this implies $\gamma<2/3$. Hence the matter stress violates the strong energy condition (SEC) $\rho+3p\geqslant 0$ and as a consequence the Universe accelerates its expansion, i.e., $\ddot{a}/a=-(\rho+3p)/6>0$.

Let us examine more closely the implications of the current accelerated expansion for the QCDM model. Since the mixture of Q matter and perfect dark matter fluid violates the SEC, γ_ϕ must be low enough. Namely, since $\gamma<2/3$, $\gamma_m\geqslant 1$, and $\gamma_\phi<\gamma_m$, Eq. (10) implies $\gamma_\phi<\gamma$. Then, introducing $\Omega=1$ in Eq. (9) we obtain

$$\dot{\Omega}_\phi=3(\gamma-\gamma_\phi)\Omega_\phi H, \tag{11}$$

and therefore $\dot{\Omega}_\phi>0$, i.e., Ω_ϕ will grow until the constraint (7) is saturated, giving $\Omega_\phi=1$ in the asymptotic regime. Thus the matter fluid yields a vanishing contribution to the energy density of the Universe at large time. This implies that a flat FLRW universe driven by a mixture of normal perfect fluid and quintessence matter cannot both drive an accelerated expansion and solve the coincidence problem. Therefore some other contribution must enter the stress-energy tensor of the cosmic fluid, i.e., it must be ''enlarged.''

B. The coincidence problem

As shown above we cannot have both the current accelerated expansion and the coincidence problem solved within a model that assumes a perfect matter fluid and a quintessential scalar field. Things fare differently when a dissipative pressure enters the play. Because of the FLRW metric, velocity gradients causing shear viscosity and temperature gradients leading to heat transport must be absent. Therefore the only admissible dissipative term corresponds to a bulk dissipative pressure π. This quantity is always negative for expanding fluids (i.e., $\pi<0$ so long as $H>0$) and may be understood either as a viscous pressure or as the effect of particle production. On general grounds the former possibility is usually thought to give just a small contribution to the overall pressure, however the impact of the latter is not so much limited. The expression of the bulk stress when interpreted that way is $\pi=-(\rho_m+p_m)\Gamma/3H$ where Γ denotes the particle production rate. This process is dissipative in the sense that the produced particles imply an augmentation of the phase space volume. A recent discussion about the inter-

play between dissipative bulk pressure and cosmological particle production can be found in [23].

So the total stress-energy tensor of the cosmic medium, made up of a dissipative but otherwise normal fluid plus the Q-matter fluid, reads

$$T_{ab}=(\rho_m+\rho_\phi+p_m+p_\phi+\pi)u_a u_b+(p_m+p_\phi+\pi)g_{ab}. \tag{12}$$

A parallel calculation to that of the above leads to the corresponding Einstein-Klein-Gordon field equations

$$\dot{\Omega}=\Omega(\Omega-1)\left[3\left(\gamma+\frac{\pi}{\rho}\right)-2\right]H, \tag{13}$$

and

$$\dot{\Omega}_\phi=\left\{2+\left[3\left(\gamma+\frac{\pi}{\rho}\right)-2\right]\Omega-3\gamma_\phi\right\}H\Omega_\phi, \tag{14}$$

instead of Eqs. (8) and (9). The energy conservation of the normal matter is

$$\dot{\rho}_m+3\left(\gamma_m+\frac{\pi}{\rho_m}\right)\rho_m H=0. \tag{15}$$

Owing to the presence of the dissipative bulk stress the constraint $\gamma<2/3$ no longer has to be fulfilled for the solution $\Omega=1$ of Eq. (13) to be stable. Likewise, inspection of Eq. (14) shows that when $\Omega=1$ one can have $\dot{\Omega}_\phi<0$ just by choosing the ratio π/ρ sufficiently negative. Thereby the constraint (7) allows a nonvanishing Ω_m at large times. By contrast tracker fields based models (valid only when $\Omega_k=0$) predict that $\Omega_m\to 0$ asymptotically [15]. A fixed point solution of Eq. (13) is $\Omega=1$. Note that Eqs. (7) and (14) have fixed point solutions $\Omega_m=\Omega_{m0}$ and $\Omega_\phi=\Omega_{\phi 0}$, respectively, when the partial adiabatic indices and the dissipative pressure are related by

$$\gamma_m+\frac{\pi}{\rho_m}=\gamma_\phi=-\frac{2\dot{H}}{3H^2}. \tag{16}$$

The smaller γ_ϕ, the larger the dissipative effects. Let us investigate the requirements imposed by the stability of these solutions. From Eq. (13) we see that $\gamma+\pi/\rho<2/3$ must be fulfilled if the solution $\Omega=1$ is to be asymptotically stable. This condition, together with Eq. (16), leads to the additional constraint on the viscosity pressure

$$\pi<\left(\frac{2}{3}-\gamma_m\right)\rho_m, \tag{17}$$

which is negative for ordinary matter fluids. Also by virtue of Eq. (11) and the first equality in Eq. (16) we obtain from Eq. (17) that $\gamma_\phi<2/3$.

In the special case of a spatially flat universe ($\Omega=1$), the stability of the solutions Ω_{m0} and $\Omega_{\phi 0}$ may be studied directly from Eq. (14). Namely, setting $\Omega_\phi=\Omega_{\phi 0}+\omega$ and using Eq. (10) it follows that

$$\dot{\omega}=3\Omega_m\left(\gamma_m-\gamma_\phi+\frac{\pi}{\rho_m}\right)H(\Omega_{\phi 0}+\omega). \tag{18}$$

Accordingly the solution $\Omega=1$, $\Omega_\phi=\Omega_{\phi 0}$ is stable for the class of models that satisfies $\psi\equiv\gamma_m-\gamma_\phi+(\pi/\rho_m)<0$ and $\psi\to 0$ for $t\to\infty$. Note that this coincides with the attractor condition (16).

In order to study the stability of the solutions Ω_{m0} and $\Omega_{\phi 0}$ when $k\neq 0$ it is advisable to derive a dynamic equation for the density ratio parameter

$$\epsilon\equiv\frac{\Omega_m}{\Omega_\phi}.$$

For this purpose we combine the logarithmic derivative of ϵ with the definitions of Ω_m and Ω_ϕ and the energy conservation equations (15) and (6)—the latter written in terms of ρ_ϕ. It yields

$$\dot{\epsilon}=3\left(\gamma_\phi-\gamma_m-\frac{\pi}{\rho_m}\right)H\epsilon. \tag{19}$$

To calculate γ_ϕ we use Eq. (15) together with Eqs. (3), (4), and (6), obtaining

$$\gamma_\phi=\gamma_m+\frac{\pi}{\rho_m}-\frac{1}{\Omega_\phi}\left[\frac{2\dot{H}}{3H^2}+\gamma_m+\frac{\pi}{\rho_m}+\left(\frac{2}{3}-\gamma_m-\frac{\pi}{\rho_m}\right)\Omega_k\right]. \tag{20}$$

Introducing Eq. (20) in Eq. (19) we get

$$\dot{\epsilon}=-\frac{3H\epsilon}{\Omega_\phi}\left[\frac{2\dot{H}}{3H^2}+\gamma_m+\frac{\pi}{\rho_m}+\left(\frac{2}{3}-\gamma_m-\frac{\pi}{\rho_m}\right)\Omega_k\right], \tag{21}$$

and perturbating this expression about the solution $\epsilon_0\sim\mathcal{O}(1)$, (i.e., using the ansatz $\epsilon=\epsilon_0+\delta$ with $|\delta|\ll 1$) we obtain with the help of Eq. (16)

$$\dot{\delta}=-\frac{3}{\Omega_\phi}\left(\frac{2}{3}-\gamma_\phi\right)\Omega_k H(\epsilon_0+\delta) \tag{22}$$

near the attractor. For $\Omega_k>0$ (negatively spatially curved universes) it follows that δ decreases, i.e., the ratio $(\Omega_m/\Omega_\phi)_0$ is a stable solution. For $\Omega_k<0$ one has to go beyond the linear perturbative regime and/or restrict the class of models as in the spatially flat case to determine the stability of the solution. We defer this to a future research.

As we mentioned above, recently there have been some claims that CDM must not be a perfect fluid because it ought to self–interact (with a mean free path in the range 1 kpc $\leq l\leq$ 1 Mpc) if one wishes to explain the structure of the halos of galaxies [17]. In this light it is not unreasonable to think that this same interaction is the origin of the dissipative pressure π at cosmological scales. Bearing in mind that $l=1/n\sigma$, with n the number density of CDM particles and σ the interaction cross section, a simple estimation reveals that at such scales l is lower than the Hubble distance H^{-1} and accordingly the fluid approximation we are using is valid.

III. QDDM ASYMPTOTIC ERA

Bulk viscosity arises typically in mixtures—either of different particles species, as in a radiative fluid, or of the same species but with different energies, as in a Maxwell-Boltzmann gas. Physically, we can think of π as the internal ''friction'' that sets in due to the different cooling rates in the expanding mixture.

Any dissipation in exact FLRW universes has to be scalar in nature, and in principle it may be modelled as a bulk viscosity effect within a nonequilibrium thermodynamic theory such as Israel-Stewart's [24,25]. In that formulation, the transport equation for the bulk viscous pressure takes the form

$$\pi+\tau\dot{\pi}=-3\zeta H-\frac{1}{2}\pi\tau\left[3H+\frac{\dot{\tau}}{\tau}-\frac{\dot{\zeta}}{\zeta}-\frac{\dot{T}}{T}\right], \tag{23}$$

where the positive-definite quantity ζ stands for the phenomenological coefficient of bulk viscosity, T the temperature of the fluid, and τ the relaxation time associated to the dissipative pressure—i.e., the time the system would take to reach the thermodynamic equilibrium state if the velocity divergence were suddenly turned off [26]. Usually ζ is given by the kinetic theory of gases or a fluctuation-dissipation theorem or both [27].

Provided the factor within the square bracket in Eq. (23) is small it can be approximated by the more manageable truncated transport equation

$$\pi+\tau\dot{\pi}=-3\zeta H, \tag{24}$$

widely used in the literature. This as well as Eq. (23) meets the requirements of causality and stability to be fulfilled by any physically acceptable transport equation [28].

A. The quasiperfect regime

Here we obtain an explicit expression for the leading behavior of the attractor solution at late time. We begin by writing the equation of motion for the Hubble factor that follows from combining Eqs. (13) and (15) with Eq. (24)

$$\ddot{H}+3\gamma H\dot{H}+\tau^{-1}\left[\dot{H}+\frac{3}{2}(\gamma+\tau\dot{\gamma})H^2-\frac{3}{2}\zeta H\right]+\frac{k}{a^2}\left[\left(1-\frac{3}{2}\gamma\right)(2H-\tau^{-1})+\frac{3}{2}\dot{\gamma}\right]=0. \tag{25}$$

We next evaluate Eq. (10) on the attractor and insert it together with Eq. (16) in Eq. (25) to obtain

$$\nu^{-1}\left(\frac{\ddot{H}}{H}+3\gamma_m\dot{H}\right)+\dot{H}+\frac{3\gamma_m}{2}H^2-\frac{3\zeta}{2\Omega_{m0}}H=0. \tag{26}$$

Observation seems to rule out huge entropy production processes on large scales, otherwise the flux of gamma rays we witness should be much higher [29]. Hence we shall assume that the viscous effects are not as large as that, but however not altogether negligible. If τ is the relaxation time, then $\nu=(\tau H)^{-1}$ is the number of relaxation times in a Hubble time—for quasistatic expansions ν is proportional to the number of particle interactions in a Hubble time. Perfect fluid behavior occurs in the limit $\nu\to\infty$, and a consistent hydrodynamical description of the fluids requires $\nu>1$. Thus we are lead to assume that τH is small and we propose a ''quasiperfect'' expansion in powers of ν^{-1}.

Let us show that the attractor solution of leading behavior $a\simeq t^{\sigma}$ when $t\to\infty$, with σ a positive-definite constant, is consistent in the quasiperfect regime. Indeed, by virtue of Eq. (16) it implies $\gamma_\phi\simeq 2/3\sigma$ and

$$\frac{\pi}{\rho_m}\simeq-\left(\gamma_m-\frac{2}{3\sigma}\right). \tag{27}$$

For approximately constant γ_m ($\gamma_m=1$ for CDM), we get from Eq. (27)

$$\frac{\dot{\pi}}{\pi}\simeq\frac{\dot{\rho}_m}{\rho_m}\simeq-2\frac{H}{\sigma}. \tag{28}$$

Hence Eq. (24) becomes

$$\pi\left(1-\frac{2}{\nu\sigma}\right)\simeq-3\zeta H, \tag{29}$$

and we get to leading order in ν^{-1}

$$\zeta\simeq\Omega_{m0}\left(\gamma_m-\frac{2}{3\sigma}\right)H. \tag{30}$$

Then integration of Eq. (15) yields $\rho_m\simeq a^{-2/\sigma}$, the same scaling law as ρ_ϕ. Finally its insertion in Eq. (4) leads back to $a\simeq t^{\sigma}$, showing the consistency of our assumptions. These results correspond to the lowest order in the quasiperfect expansion.

To go a step further we introduce the expansion of H in powers of ν^{-1}

$$H=H_0(1+h_1\nu^{-1}+\cdots) \tag{31}$$

in Eq. (26), and assuming that $|\dot{\tau}|\ll\nu^{-1}$, it follows the approximated solution

$$H\simeq\frac{\sigma}{t}\left[1+\left(\frac{3\gamma_m\sigma-2}{\sigma}+\frac{\theta}{t}\right)\frac{1}{\nu}\right], \tag{32}$$

where θ is an arbitrary integration constant. This expression reveals that the power law is an attractor solution and that for $\sigma>1$ [deceleration parameter $q=-(\sigma-1)/\sigma<0$], CDM viscosity provides an accelerated expansion scenario that also solves the coincidence problem. It can be shown that γ_ϕ does not pick any correction of order t^{-1} from the subdominant term in Eq. (32). Instead the first correction appears to the order $\nu^{-2}t^{-2}$, and this fact shows the high degree of correction of the approximation that γ_ϕ takes a constant value in the late time regime.

We note that this attractor solution works for any viscosity coefficient with leading behavior (30). In particular the case $\zeta \propto \sqrt{\rho_m}$, investigated in [30,31], satisfies this requirement.

B. Full causal corrections

Here we gauge the changes brought about by the full transport equation (23) on the expansion exponent obtained in the previous section. Using that equation and the viscosity coefficient found in Eq. (30) we get

$$\nu^{-1}\left\{\frac{\ddot{H}}{H}-\frac{1+2r}{2}\frac{\dot{H}^2}{H^2}+\frac{3}{2}\left[\gamma_m\left(\frac{3}{2}-r\right)+1\right]\dot{H}+\frac{9}{4}\gamma_m H^3\right\}+\dot{H}+\frac{1}{\sigma}H^2=0, \tag{33}$$

where, to estimate the corrections, we have assumed that in the asymptotic regime $T\propto\rho^r$, with r a positive-definite constant, and we have used that $\rho_m \simeq \epsilon_0\rho/(1+\epsilon_0)$ in this regime. This power law relationship is the simplest way to guarantee a positive heat capacity. Usually p, ρ, T and the particle number density n are equilibrium magnitudes related by equations of state of the form $\rho=\rho(T,n)$ and $p=p(T,n)$. Further the thermodynamic relation

$$\left(\frac{\partial\rho}{\partial n}\right)_T=\frac{\rho+p}{n}-\frac{T}{n}\left(\frac{\partial p}{\partial T}\right)_n \tag{34}$$

holds. This directly follows from the requirement that the entropy is a state function [32]. In the particular case of a material fluid with $\rho=\rho(T)$ and constant adiabatic index, this relation imposes the constraint $r=(\gamma-1)/\gamma$, so that $0\leqslant r\leqslant 1/2$ for $1\leqslant\gamma\leqslant 2$. Inserting Eq. (31) in Eq. (33), and assuming that $|\dot{\tau}|\ll\nu^{-1}$, we obtain the approximate solution

$$H\simeq\frac{\sigma}{t}\left\{1+\left[-\frac{2}{\sigma}+\frac{3}{2}\left(\gamma_m\left(\frac{1}{2}-r-\frac{3}{2}\sigma\right)+1+\frac{2r+1}{3\sigma}\right)+\frac{C}{t}\right]\frac{1}{\nu}\right\}. \tag{35}$$

Comparison with Eq. (32) shows that except when $\sigma\simeq 1$, the use of the complete transport equation leads to a slighty slower rate of expansion at late time. Also, in this regime, the equilibrium temperature decreases as $T\sim t^{-2r}\sim a^{-2r/\sigma}$.

C. Late Q-matter dynamics

By virtue of Eq. (10) the density parameter ratio can be written in terms of the adiabatic indices

$$\epsilon=\frac{\gamma-\gamma_\phi}{\gamma_m-\gamma}. \tag{36}$$

Since γ_m is approximately constant, if $\gamma\to\gamma_0$ in the asymptotic regime when $\epsilon\to\epsilon_0$, then γ_ϕ must also approach a constant value. Hence from Eq. (3) we get the constraint

$$V(\phi)/\dot{\phi}^2\simeq C \tag{37}$$

with $C>1$ as $\gamma_\phi<2/3$. Potentials that satisfy this constraint have been investigated in [33] and [34]. Then Eq. (6) becomes

$$\frac{\ddot{\phi}}{\dot{\phi}}+\frac{3H}{1+2C}\simeq 0. \tag{38}$$

An interesting potential that meets this constraint is the exponential potential

$$V(\phi)=V_0\exp(-A\phi), \tag{39}$$

where A and V_0 are constants. Now, integrating Eq. (38) and using Eq. (37) we get

$$\phi(t)\simeq\frac{1}{A}\left[\ln\frac{V_0A^2\gamma_\phi}{2(2-\gamma_\phi)}+2\ln t\right]. \tag{40}$$

Hence ϕ slowly rolls down the exponential potential as $\dot{\phi}\propto 1/t$ when $t\to\infty$. Also we find that $C\simeq(3\sigma-1)/2$ with $\gamma_\phi\simeq 2/3\sigma<2/3$ on the attractor, irrespective of V_0 and A.

Perfect fluid QCDM models based on the exponential potential are ruled out by observations [6]. However we shall demonstrate in the next secion that in the realm of QDDM models the exponential potential yields satisfactory results without any fine tuning of the parameters.

IV. QDDM MODELS

This section explores the dynamical evolution of a universe filled with a viscous material fluid and a quintessence scalar field by resorting to models based on simple relationships for the nonequilibrium quantities. This allows us to explore the large dissipative regime where the nonequilibrium pressure has a magnitude comparable with the energy density. Recently tracker-field models with inverse power potentials have attracted much interest [15,35]. Here we will investigate some QDDM models with exponential potentials (39) such that for a wide range of initial conditions the scalar field settles into an attractor solution that depends only upon a few nonequilibrium thermodynamical parameters, addressing the coincidence problem.

A. Linear dissipative regime

The linear regime $\zeta=\alpha H$, with α a constant in the interval $0<\alpha<1$, arises for instance when the coefficient of bulk viscosity takes the form of a radiating fluid. We further assume that the number of interactions of a generic CDM particle in a Hubble time is larger than unity so that the hydrodynamic regime is respected. Here we investigate models with the limiting behavior $\gamma\to 2/3$ in the asymptotic regime. We begin by inserting the ansatz

$$\gamma=\frac{2}{3}(1+\chi) \tag{41}$$

in Eq. (25). It is immediately seen that the latter splits in two equations, namely

$$\ddot{H}+(2+\nu)H\dot{H}+\nu\left(1-\frac{3}{2}\alpha\right)H^3=0, \tag{42}$$

and

$$\left[H^2+\frac{k}{a^2}\right]\dot{\chi}+\left[2H\left(\dot{H}-\frac{k}{a^2}\right)+\tau^{-1}\left(H^2+\frac{k}{a^2}\right)\right]\chi=0. \tag{43}$$

Replacing the solution of Eq. (43) in Eq. (41) it follows

$$\gamma=\frac{2}{3}\left(1+b\frac{a^{2-\nu}}{k+\dot{a}^2}\right), \tag{44}$$

where b is an arbitrary integration constant. This expression for γ will be sensible only if it meets the restriction $0\leqslant\gamma\leqslant 2$. Then Eq. (42) can be transformed into a linear differential equation of second order whose general solution in parametrized form is already known [30,36]. In particular $a\propto t^{\sigma}$, where σ is the largest root of $\nu(1-3\alpha/2)\sigma^2-(2+\nu)\sigma+2=0$, is an asymptotic stable solution in the limit $t\to\infty$. Then, using Eq. (10) together with the attractor conditions $\Omega=1$, $\gamma=2/3$ and Eq. (16), we find

$$\sigma=\frac{2(1-\Omega_{m0})}{2-3\Omega_{m0}\gamma_m}. \tag{45}$$

Hence the quintessence adiabatic index $\gamma_\phi=2/3\sigma$ depends solely on the dark matter parameters Ω_m and γ_m in the asymptotic regime. Moreover, a relationship between the dissipative parameters α and ν follows from Eqs. (29) and (16), namely

$$\alpha=\Omega_{m0}\left(1-\frac{2}{\nu\sigma}\right)\left(\gamma_m-\frac{2}{3\sigma}\right), \tag{46}$$

and the requirement $\alpha>0$ implies $\sigma\nu>2$ or $\nu>\nu_{min}=3\gamma_\phi$. The same condition arises from the requirement that $\gamma\to 2/3$ when $t\to\infty$. Equations (45) and (46) show that α grows with ν attaining $\alpha_{max}=(3\gamma_m-2)\Omega_m/3(1-\Omega_m)$ in the limit $\nu\to\infty$.

As it follows from Eq. (45) there will be accelerated expansion (i.e., $\sigma>1$) if $\Omega_{m0}<2/3\gamma_m<2/3$ and accordingly we obtain a family of exact solutions describing a QDDM scenario that solves the coincidence problem regardless of the value of the spatial curvature. Figure 1 depicts the dependence of γ_ϕ on Ω_m when $\gamma_m=1$. To make a rough estimate of the cosmological parameters in the late time era we assume that our Universe is currently close to the asymptotic attractor regime and use the current observational bounds. After [6] the combination of low redshift, type Ia supernovae and COBE measurements determines (for a spatially flat universe) the range $\Omega_m\sim 0.3-0.4$ and $\gamma_\phi<0.6$. From Fig. 1 it is seen that our linear dissipative model satisfies comfortably these constraints. For $\Omega_m=0.3$ we get from Eq. (45) $\sigma\simeq 1.27$. This is fully consistent with current estimations of Ht today [37], as they provide a lower bound for σ in a universe that started only recently a phase of accelerated expansion and approaches asymptotically the attractor regime.

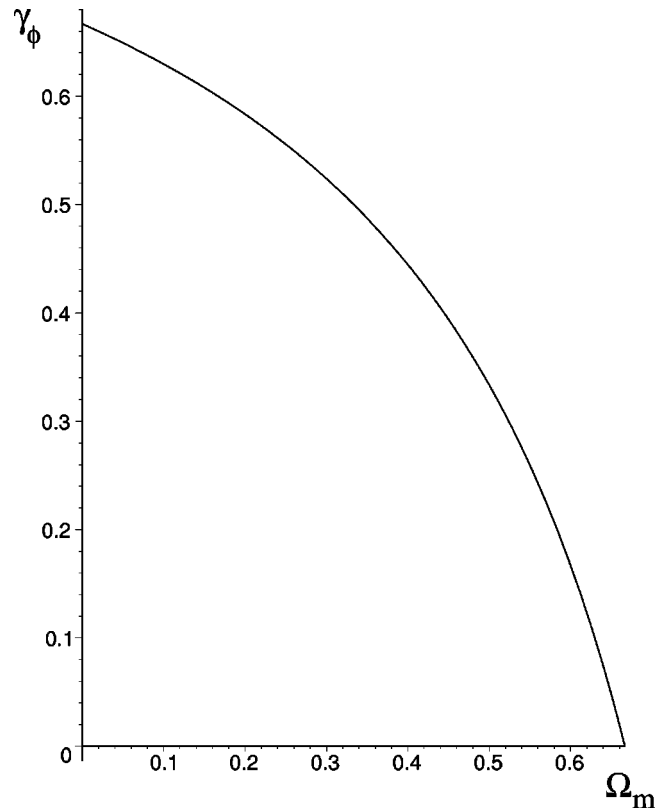

FIG. 1. The adiabatic index γ_ϕ of the quintessence scalar field versus the matter density parameter Ω_m, for CDM ($\gamma_m=1$), in the asymptotic attractor regime for the model presented in Sec. IV A.

B. Viscous speed regime

This scenario is somewhat more general than the previous one. It arises when the bulk viscosity coefficient is given in terms of the speed of the bulk viscous signal v by [25]

$$\frac{\zeta}{\tau}=v^2\gamma_m\rho_m. \tag{47}$$

We further assume τ related to H by the same expression as before, only that to simplify the calculations we now take ν constant. Hence Eq. (25) becomes

$$\nu^{-1}\left[h''+3\gamma h'+3\gamma' h-9\frac{v^2\gamma_m\epsilon}{1+\epsilon}h\right]+h'+3\gamma h=0, \tag{48}$$

where

$$h\equiv H^2+\frac{k}{a^2}, \tag{49}$$

and the prime indicates derivative with respect to $\eta \equiv \ln a$. Here we have used the scale factor $a(t)$ as a coordinate instead of the cosmological time t, i.e., a is assumed to be a monotonic function of t.

Equation (48) is useful to study the asymptotic stability of FLRW expansions at late time because it can be rewritten in terms of the derivative of a Lyapunov function [38]

$$\frac{d}{d\eta}\left\{\frac{1}{2}h'^2+\frac{3}{2}\left[\gamma'+\gamma\nu-3\frac{v^2\gamma_m\epsilon}{1+\epsilon}\right]h^2\right\}$$
$$=-(3\gamma+\nu)h'^2+\frac{3}{2}\left[\gamma'+\gamma\nu-3\frac{v^2\gamma_m\epsilon}{1+\epsilon}\right]'h^2. \quad (50)$$

If the adiabatic index does not decreases too fast in the attractor era, a sufficient condition for the Lyapunov function to have a minimum at the phase space point $(h,h')=(0,0)$ is that $\nu>3v^2$. Within this scenario the parameters of the matter fluid can be taken as quasistatic, yielding an asymptotically stable minimum. The leading behavior of the solutions in this quasistatic regime is given by

$$h^2=c_1a^{\lambda_1}+c_2a^{\lambda_2}, \quad (51)$$

where

$$\lambda_{1,2}=\frac{1}{2}\{-(3\gamma+\nu)\pm[(3\gamma-\nu)^2+36\gamma_m v^2\Omega_m]^{1/2}\} \quad (52)$$

and the parameters are evaluated at the asymptotic attractor era. For large scale factor, Eq. (51) reduces to $h^2\simeq c_1a^{\lambda_1}$ when $-2<\lambda_1<0$, and we have once again a power-law accelerated cosmic expansion with $\sigma=-2/\lambda_1$. Likewise Eq. (51) reduces to $h^2\simeq c_1a^{-2}$ for $\lambda_1=-2$, and $h^2\simeq 0$ for $\lambda_1<-2$. These two latter cases correspond to linear evolutions at late time.

Combining Eqs. (29) and (16) we get

$$\lambda^2+(3\gamma_m+\nu)\lambda+3\gamma_m(\nu-3v^2)=0 \quad (53)$$

and using Eq. (10) together with the attractor constraints $\Omega=1$ and Eq. (16) we find that $\lambda_1=-3\gamma_\phi$ also satisfies Eq. (53). Hence we obtain

$$\gamma_\phi=\frac{1}{6}\{3\gamma_m+\nu-[(3\gamma_m-\nu)^2+36\gamma_m v^2]^{1/2}\}. \quad (54)$$

In this model the quintessence adiabatic index does depend on the parameters ν and v while it is independent of the density parameter Ω_m. We plot γ_ϕ in Fig. 2 for $\gamma_m=1$. As it can be seen a wide range of the parameter space (ν,v) is consistent with a spatially-flat accelerated universe and such that $\gamma_\phi<0.6$ for any value of Ω_m. This shows another solution to the coincidence problem for any value of the spatial curvature and compatible with accelerated expansion. It is also seen that dissipative effects enlarge the parameter space where observational data have to be fitted, but global dynamic information alone cannot determine the specific values of ν and v. Note that the smaller γ_ϕ, the larger the dissipative contribution to the sound speed, and the smaller the interaction rate.

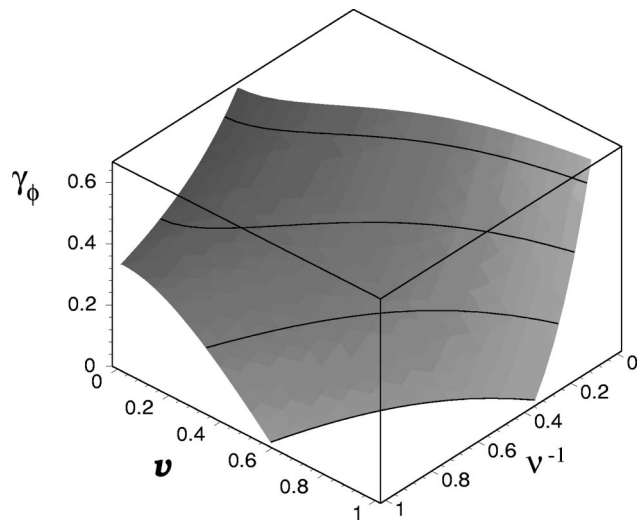

FIG. 2. The adiabatic index γ_ϕ of the quintessence scalar field versus the interaction rate parameter ν^{-1} and the dissipative contribution to the sound speed v, for CDM ($\gamma_m=1$), in the asymptotic attractor regime, for the model presented in Sec. IV B.

V. DISCUSSION

We have proved that the coincidence problem and an accelerated expansion phase of FLRW cosmologies cannot be simultaneously addressed by the combined effect of a perfect fluid and Q matter. Nonetheless, if nonbaryonic dark matter behaves as a dissipative fluid rather than a perfect one, both problems may find a simultaneous solution. This is so because an imperfect (i.e., dissipative) fluid expanding in a FLRW background possesses a negative pressure π that enters the conservation equations of general relativity. The models presented here are compatible with a negative deceleration parameter at present time. In consequence, the quintessence scenario becomes more robust when the dissipative effect of the nonequilibrium pressure arising in the CDM gas is allowed into the picture.

Recently attempts have been made to constrain the state equation of the cosmic fluids (Q matter included) by considering gravitational lensing effects, the mass power spectrum and the anisotropies of the cosmic backgroud radiation [6,39]. We have shown specific models with an ample region in the space of out-of-equilibrium thermodynamic parameters satisfying this constraint in the asymptotic attractor regime which our Universe may well be approaching. We would like to point out that the parameter space should be enlarged by adding these out-of-equilibrium parameters when fitting the observational data. Unfortunately there is some degenerancy in the determination of these parameters from constraints arising from the cosmological dynamics alone. We hope, however, that simulations of structure formation that include dissipative effects will ultimately prove instrumental in discriminating between different models.

In the attractor asymptotic regime the dissipative matter

fluid and the scalar field contribute in a fixed ratio to the pressure and energy density along the QDDM era. This scenario ameliorates the self-adjusting model [40] as it allows for $0<\gamma<2$ for a wide range of initial conditions. It also improves on the tracking models as it solves the coincidence problem in the late accelerated expansion phase. While keeping a finite difference between the adiabatic indices of quintessence and matter fluid, this difference arises in the viscous pressure.

As it has been noted the Q matter proposal may entail some undesirable effects such as the variation of the constant of nature and the presence of unobserved long range forces. Efforts to solve these difficulties by coupling the quintessence field to the electromagnetic field [41], and to the curvature of the metric [42], have been made. Further, a time dependent but otherwise smooth scalar field such as those studied so far, are somehow unphysical as they violate the principle of equivalence—the Q matter must experience clustering in some degree and so it cannot be entirely smooth. Therefore to respect the equivalence principle one should assume that ϕ varies with position as well. Accordingly one should be led to forsake the FLRW metric and take up some inhomogeneous one instead, except that in such a case the computational effort is bound to be enormous and most likely no exact solution will emerge.

Despite that the Q-matter proposal cannot be regarded at this moment with unreserved confidence, we feel this idea is still worth exploring in the hope that the aforesaid difficulties may soon find a satisfactory answer.

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