# Dynamical evolution of the Universe in the quark-hadron phase transition and nugget formation

Deepak Chandra* and Ashok Goyal†
*Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India*
*and Inter University Center for Astronomy and Astrophysics, Ganeshkhind, Pune-411007, India*
(Received 30 August 1999; published 31 July 2000)

We study the dynamics of a first-order phase transition in the early Universe when it was 10–50 $\mu$s old with quarks and gluons condensing into hadrons. We look at the evolution of the Universe in small as well as large supercooling scenarios, specifically exploring the formation of quark nuggets, their possible survival and identification with the recently observed dark objects in our galactic halo which may account for the dark matter component in the Universe.

PACS number(s): 98.80.Cq, 12.38.Mh, 95.35.+d

## I. INTRODUCTION

It is well known that a phase transition from quark gluon plasma to confined hadronic matter must have occurred at some point in the evolution of the early Universe, typically at around 10–50 $\mu$s after the big bang. This leads to an exciting possibility of the formation of quark nuggets through the cosmic separation of phases [1]. As the temperature of the Universe falls below the critical temperature $T_c$ of the phase transition, the quark gluon plasma supercools and the transition proceeds through the bubble nucleation of the hadron phase. The typical distance between the nucleated bubbles introduces a new distance scale to the Universe which depends critically on the supercooling that takes place. As the hadronic bubbles expand, they heat the surrounding plasma, shutting off further nucleation and the two phases coexist in thermal equilibrium. The hadron phase expands driving the deconfined quark phase into small regions of space and it may happen that the process stops after the quarks reach sufficiently high density to provide enough pressure to balance the surface tension and the pressure of the hadron phase. The quark matter trapped in these regions constitute the quark nuggets. The number of particles trapped in the quark nugget, its size and formation time are dependent sensitively on the degree of super cooling. The duration of the phase transition also depends on the expansion of the Universe and on other parameters such as the bag pressure $B$ and the surface tension $\sigma$.

The quark nuggets formed in the small super cooling scenario are in a hot environment around the critical temperature $T_c$ and are susceptible to evaporation from the surface [2] and to boiling through subsequent hadronic bubble nucleation inside the nuggets [3]. However, in the large super cooling scenario we have the interesting possibility of these nuggets forming at a much lower temperature than $T_c$ due to the long duration of the transition and consequent expansion of the Universe. Alcock and Farhi [2] have shown that the quark nuggets with baryon numbers $\leqslant 10^{52}$ and mass $\leqslant 10^{-5} M_{\odot}$ are unlikely to survive evaporation of hadrons from the surface. Boiling was shown to be an even more efficient mechanism of nugget destruction [3]. These results were somewhat modified by Madsen *et al.* [4] by taking into account the flavor equilibrium near the nugget surface for the case of evaporation and by considering the effect of interactions in the hadronic gas for the case of boiling. In the large supercooling scenario the time of formation of these nuggets can be quite late when the number of baryons in the horizon (of size $\sim 2t$) is large and temperature much lower. These nuggets can easily survive until the present epoch.

Recently there have been studies in the literature [5] where the possibility of these quark nuggets collapsing into primordial black holes (PBHs) has been investigated. These authors have shown that preexisting density fluctuations left over from an early inflationary period of the Universe crossing into the horizon during the first order QCD transition epoch would experience a significant reduction of pressure forces leading to a lower threshold for PBH formation during the QCD epoch than during the early eras. This would facilitate the production of PBHs on approximately the QCD horizon mass scale $\sim 2 M_{\odot}(100 \ \mathrm{MeV}/T_c)^2$. However formation of PBHs even during the QCD epoch is a very rare event involving a high degree of fine tuning and the issue is far from settled.

There have been recent observations by gravitational micro lensing [6] of dark objects in our galactic halo having masses of about 0.01–1 solar mass. If these objects have to be identified with quark nuggets, they could only have been formed at a time later than the time ($\sim$50–100 $\mu$s) when the Universe cooled through $T_c$. Such a possibility of the nuggets forming at a temperature $\sim$0.1 MeV, implying a high degree of super cooling and strongly first order phase transition was recently investigated by Cottingham *et al.* [7] in the Lee-Wick model [8]. Their investigation showed that the time of formation and the baryon content of these nuggets are essentially determined by the rate at which the hadron bubbles nucleate. However, there was still the question of reheating due to the expansion of the bubbles which raises the temperature towards $T_c$. These studies have also been carried out by taking interactions into account in both the phases and by incorporating the effects of curvature [9] energy in the calculations.

*Permanent address: S. G. T. B. Khalsa College, University of Delhi, Delhi-110007, India.
†Email address: agoyal@ducos.ernet.in

 

Studies of quark-hadron phase transitions in the early Universe [10], in heavy ion collision [11], and in high density nuclear matter have been done previously by looking in detail at the dynamics of the phase transition. Kapusta *et al.* have applied a recently computed nucleation rate [11] to a first order phase transition in a set of rate equations to study the time evolution of a quark-gluon plasma in heavy ion collisions. Based on Bjorken hydrodynamics and on current parameter values, they find the transition generates 30% extra entropy and also a time delay of $\sim 11$ fm/$c$ in completion of the transition. Several authors [10] studied how the early Universe during the quark-hadron phase transition evolved through the mixed phase in a scenario with small initial supercooling.

In this paper we calculate the nucleation rate and quantitatively study what happens when the temperature drops to $T_c$. This nucleation rate is used to solve a set of rate equations to study the time evolution of the quark-gluon phase as it converts to hadronic matter in an expanding Universe. A novel feature of this method is that reheating of the plasma during phase transition is included in the calculations along with bubble growth in the expanding Universe in contrast to the Guth-Weinberg formula which has been used in the literature so far. This allows for the completion of the transition and all relevant quantities can be evaluated as a function of time and temperature. In Sec. II we study the bubble nucleation rate and set up the two rate equations to be solved numerically. In Sec. III we discuss the results and finally in Sec. IV we give our conclusions.

## II. BUBBLE NUCLEATION

When the early Universe as a quark-gluon thermodynamic system cools through the critical temperature $T_c$, energetically, the new phase remains unfavorable as there is free energy associated with surface of separation between the phases. Small volumes of new phase are thus unfavorable and all nucleated bubbles with radii less than critical radius collapse and die out. But those with radii greater than the critical radius expand until they coalesce with each other. So supercooling occurs before the new phase actually appears and is then followed by reheating due to release of latent heat. The bubble nucleation rate [10] at temperature $T$ is given by

$$I = I_0 e^{-W_c/T}, \tag{1}$$

where $I_0$ is the prefactor having dimension of $T^4$. The prefactor used traditionally in early Universe studies [10] is given by $I_0=(W_c/2\pi T)^{3/2}T^4$. Csernai and Kapusta [11] have recently computed this prefactor in a coarse-grained effective field theory approximation to QCD and give

$$I_0 = \frac{16}{3\pi}\left(\frac{\sigma}{3T}\right)^{3/2}\frac{\sigma\eta_q R_c}{\xi_q^4(\Delta w)^2}$$

where $\eta_q = 14.4T^3$ is the shear viscosity in the plasma phase, $\xi_q$ is a correlation length of order 0.7 fm in the plasma phase, and $\Delta w$ is the difference in the enthalpy densities of the two phases. In this letter we use both these prefactors for comparison. The critical bubble radius $R_c$ and the critical free energy $W_c$ are obtained by maximizing the thermodynamic work expended to create a bubble and are given by $R_c = 2\sigma/P_h(T) - P_q(T)$ and $W_c = 4\pi\sigma R_c^2(T)/3 = 16\pi\sigma^3/3\Delta P^2$, where $\Delta P = P_h(T) - P_q(T)$ is the pressure difference in hadron and quark phase.

For simplicity we describe the quark matter by a plasma of massless $u,d$ quarks and massless gluons without interaction. The long range nonperturbative effects are parametrized by the bag constant $B$. A possible criterion for choosing the value of the bag constant, for example, could be the one used by Farhi and Jaffe (see Ref. [1]) based on the stability of strange quark matter at $T=0$. However, doubts have been raised in the literature concerning the meaning of $\sigma$ and $B$ at nonzero temperatures and densities [12]. Based on the arguments of entropy conservation at the phase boundary, there are calculations available in the literature [13] where $B$ is given as a function of the chemical potential $\mu$ and temperature $T$ such that $B$ decreases with increase in temperature. Further in a self-consistent relativistic mean field theoretical framework $\sigma$ is generally an increasing function of $B$ [4]. In this work we use a wide parameter range of $B$ and $\sigma$.

The pressure in the QGP phase is given by

$$P_q(T) = \frac{1}{3}g_q\frac{\pi^2}{30}T^4 - B,$$

where $g_q \sim 51.25$ is the effective number of degrees of freedom. In the hadronic phase the pressure is given by

$$P_h(T) = \frac{1}{3}g_h\frac{\pi^2}{30}T^4,$$

where $g_h \sim 17.25$, taking the three pions as massless.

The fraction of the Universe $h(t)$ which has been converted to hadronic phase at the time $t$ was first given by Guth and Weinberg [14] and applied to cosmological first order phase transitions. Csernai and Kapusta [11] gave a kinetic equation for calculating $h(t)$ which takes bubble growth into account. If the early Universe cools to $T_c$ at time $t_c$, then at same later time $t$ the fraction of the Universe in hadronic phase is given by the kinetic equation

$$h(t) = \int_{t_c}^{t} I[T(t')][1-h(t')]V(t',t)\left[\frac{R(t')}{R(t)}\right]^3 dt', \tag{2}$$

where $V(t',t)$ is the volume of a bubble at time $t$ which was nucleated at an earlier time $t'$ and $R(t)$ is the scale factor. This takes bubble growth into account and can be given simply as

$$V(t',t) = \frac{4\pi}{3}\left[R_c[T(t')] + \int_{t'}^{t}\frac{R(t)}{R(t'')}v[T(t'')]dt''\right]^3, \tag{3}$$

where $v(T)$ is the speed of the growing bubble wall and can be taken to be

$$v(T) = v_0\left[1 - \frac{T}{T_c}\right]^{3/2},$$

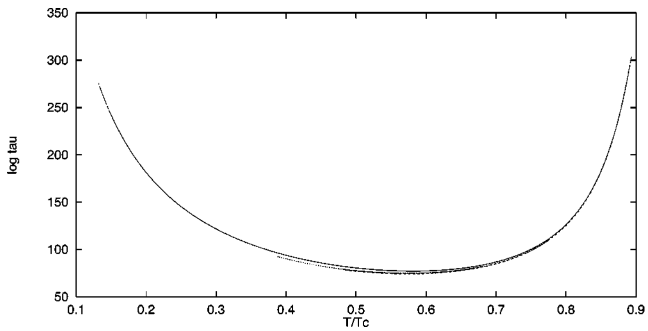


FIG. 1. Log of the nucleation time $\tau$ in units of fm/$c$ as a function of temperature. Solid and dashed curves are for $B^{1/4} = 100$ MeV and $\sigma = 39.5$ MeV fm$^{-2}$ with the standard and the Kapusta prefactors, respectively. Long dashed and dotted curves are for $B^{1/4} = 113$ MeV and $\sigma = 57.1$ MeV fm$^{-2}$ with the standard and the Kapusta prefactors, respectively.

where $v_0 = 3c$. This has the correct behavior in that the closer $T$ is to $T_c$ the slower do the bubbles grow. When $T = (2/3)T_c$ we have $v(T) = 1/\sqrt{3}$ the speed of sound of a massless gas. For $T < (2/3)T_c$ which occurs when there is large super cooling, we use the value $v(T) = 1/\sqrt{3}$. In this analysis collision and fusion of bubbles have not been taken into consideration. This seems to be justified as far as fusion of bubbles is concerned. Witten [1] and Kurki Suonio [10] have shown that for small enough bubbles, surface tension will cause them to coalesce into larger bubbles. This distance scale has been estimated to be given by

$$l_c = 3\left(\frac{\sigma}{T_c^3}\right)^{1/3}\left(\frac{T_c}{200\ \mathrm{MeV}}\right)^{-5/3} \mathrm{mm}.$$

Since the nucleation scale $l_n$ is larger than $l_c$ we are justified in neglecting fusion of bubbles.

The other equation we need is the dynamical equation which couples time evolution of temperature to the hadron fraction $h(t)$. We use the two Einstein's equations as applied to the early Universe neglecting curvature.

$$\frac{\dot{R}}{R} = \sqrt{\frac{8\pi G}{3}}\rho^{1/2}, \tag{4}$$

$$\frac{\dot{R}}{R} = -\frac{1}{3w}\frac{d\rho}{dt}, \tag{5}$$

where $w = \rho + P$ is the enthalpy density of the Universe at time $t$. The energy density in the mixed phase is given by $\rho(T) = h(t)\rho_h(T) + [1 - h(t)]\rho_q(T)$, where $\rho_h$ and $\rho_q$ are the energy densities in the two phases at temperature $T$ and similarly for enthalpy. We numerically integrate the coupled dynamical equations (2), (4), and (5) to study the evolution of the phase transition starting above $T_c$ at some temperature $T$ corresponding to time $t$ obtained by integrating the Einstein's equation (4) and (5) in the quark phase. The number density of nucleated sites at time $t$, is given by

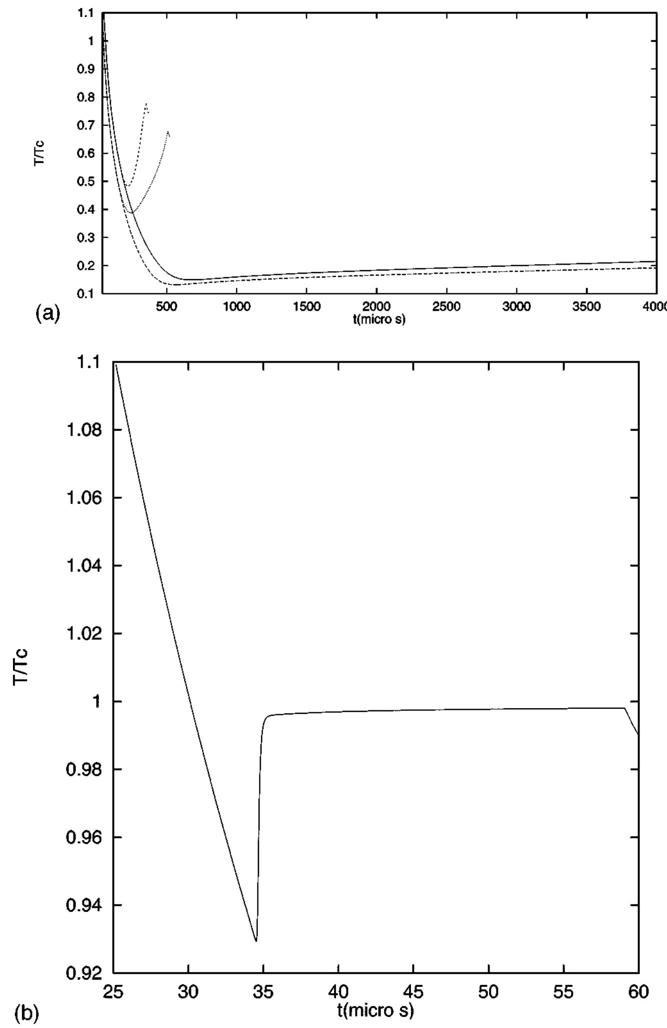


FIG. 2. (a) Temperature as a function of time. Solid, dotted, dashed, and long dashed curves are as in Fig. 1. (b) Temperature as a function of time for the low supercooling case corresponding to $B^{1/4} = 145$ MeV and $\sigma = 57.1$ MeV fm$^{-2}$ for the standard prefactor.

$$N(t) = \int_{t_c}^{t} I(t')[1 - h(t')]\left(\frac{R(t')}{R(t)}\right)^3 dt'. \tag{6}$$

Therefore the typical separation between nucleation sites is $l_n = N(t)^{-1/3}$. This distance scale will eventually determine the number of quarks in a nugget. This scale can be up to $10^{12}$ Km depending on the parameters $B$ and $\sigma$ which correspond to a distance of $\sim$1.4 Mpc today. The nuggets could be points in space around which, later in time, the matter in the Universe may have gravitationally clustered to give the observed large scale structures in the Universe. The observable separation of galaxies in the Universe can be a remnant of this transition with the centers of the galaxies being the quark nuggets. Of course the collision of bubbles and their random nucleation and interaction will also lead to clustering of the nuggets, which can qualitatively explain the clustering of galaxies.

## III. RESULTS

To get an idea about the super cooling before nucleation begins, we can plot the nucleation time as a function of tem-

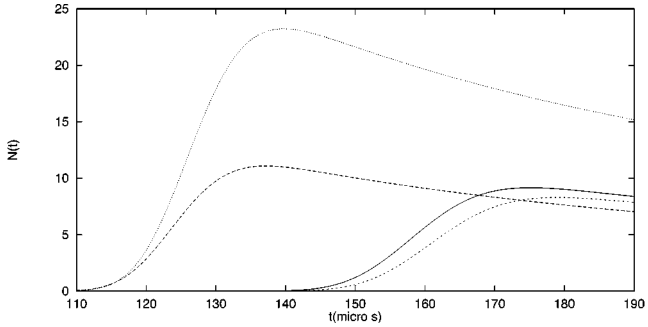


FIG. 3. Average bubble density $N(t)$ as a function of time in units of $(10^5\ \mathrm{km}^3)^{-1}$. The bubble density for the standard prefactor (solid and long dashed curves) is normalized by multiplying with a factor of $10^3$. Curves as in Fig. 1.

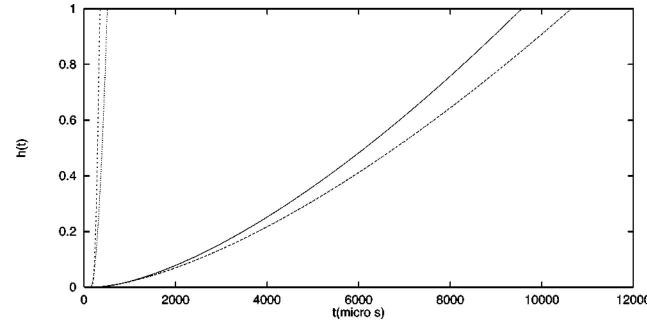


FIG. 4. The hadron fraction as a function of time. Curves as in Fig. 1.

perature, defined by $\tau_{\mathrm{nucleation}}^{-1}=(4\pi R_c^3/3)I$. The quark number density is given by $n_q=(2/\pi^2)\zeta(3)(n_q/n_\gamma)T^3$ where $n_q/n_\gamma$ is the quark to photon ratio estimated from the abundance of luminous matter in the Universe to be roughly equal to $3\times10^{-10}$. The quark nuggets have $N_q$ quarks at time $t$ given by the number of quarks in a volume $1/N(t)$, i.e., $N_q=n_q/N(t)$. The nucleation sites are actually randomly distributed, but we expect a distribution of quark numbers around $N_q$. The average temperature at which nuggets are formed when bubbles coalesce is obtained by finding the average time at which the expanding bubble surfaces meet. Assuming a cubic lattice, we have done this numerically to get the corresponding time $t_f$ and temperature $T_f$ for different values of $B$ and $\sigma$. When the fraction of the space occupied by the bubbles is around 50%, we expect the bubbles to meet in an ideal picture, i.e., if all bubbles are essentially nucleated at one instant which is the maximum nucleation time and they all have the same radius. However, we have a distribution of expanding bubble sizes because of the different points of time at which they were nucleated. Therefore the estimate of the time of nugget formation by treating all bubbles to be of the same size is an underestimate. We find that hadron fraction $h(t)$ is only around 0.12 when bubbles meet by this criteria. However, we do not expect this to change qualitatively the broad picture of the transition and the nugget formation apart from reducing the formation time.

In Fig. 1 we have plotted the $\log_{10}$ of nucleation time $\tau$ as a function of temperature for different values of the bag pressure $B$, surface tension $\sigma$ and the prefactor $I_0$. These curves clearly show when nucleation becomes large and how much super cooling of the Universe occurs. Figure 2 shows the temperature as a function of time both for large [Fig. 2(a)] and small [Fig. 2(b)] supercooling. It is clear from this diagram that reheating takes place as nucleation starts with the release of latent heat. As $\sigma$ increases, the supercooling is larger and reheating is slower. The transition takes much longer to complete with more chance of nugget formation. This allows the nuggets to be formed at a much lower temperature when bubble walls meet. For low supercooling there is rapid reheating, temperature reaches $T_c$ and the phase transition is completed very swiftly with no chance of any nugget formation. From Fig. 3 we see that the average bubble density $N(t)$ is initially zero and then increases with time. As soon as reheating starts, bubble nucleation shuts off at a particular point. The transition now continues only by expansion of the nucleated bubbles. The fall in $N(t)$ beyond this point is due to the expansion of the Universe. Figure 4 shows the fraction $h(t)$ of the Universe in hadron phase as a function of time. For small values of $\sigma$ the transition completes quickly as $h(t)$ goes to 1. But for larger $\sigma$ it takes a larger time for $h(t)$ to become 1. We also notice that in the large super cooling scenario the Kapusta prefactor becomes much bigger than the standard one by many orders of magnitude. This makes the nucleation rate as well as the reheating faster. In the case of low supercooling the two prefactors give almost identical results. The number of quarks in the horizon $N_{qH}$ at time $t$ is $N_{qH}\sim n_q(4/3\pi t^3)\sim(n_q/n_\gamma)[2\zeta(3)/3\pi^2]T^3 4\pi t^3$ and we find that for all interesting cases $N_q\leqslant N_{qH}$ and this number is very sensitively dependent on the surface tension. Physically it is possible to have $N_{qH}\geqslant N_q\geqslant10^{52}$ for some

TABLE I. Some relevant physical quantities for some representative values of $B$ and $\sigma$ using the standard prefactor.

| $B^{1/4}$ MeV | $\sigma$ MeV fm$^{-2}$ | $T_c$ MeV | $t_f$ $\mu$s | $T_f$ MeV | $N_q$ | $N_{qH}$ | $l_n$ m |
|---|---|---|---|---|---|---|---|
| 235 | 50 | 169 | 12.1 | 169 | $2.6\times10^{28}$ | $7\times10^{52}$ | $8\times10^{-3}$ |
| 145 | 57.1 | 104.4 | 34.7 | 99.9 | $9.4\times10^{35}$ | $3.4\times10^{53}$ | 4.6 |
| 125 | 57.1 | 90 | 56.1 | 78.8 | $1.1\times10^{39}$ | $7.3\times10^{53}$ | 63 |
| 125 | 77 | 90 | 1511.8 | 17.9 | $6.3\times10^{48}$ | $1.6\times10^{56}$ | $4.9\times10^5$ |
| 100 | 39.5 | 71.9 | 2595 | 13.9 | $1.2\times10^{50}$ | $3.8\times10^{56}$ | $8.4\times10^5$ |
| 113 | 57.1 | 81.3 | 5138 | 12.3 | $9\times10^{49}$ | $2\times10^{57}$ | $1.7\times10^6$ |

values of the parameters $B$ and $\sigma$. In Table I we list some physical quantities for some representative values of $B$ and $\sigma$.

## IV. CONCLUSIONS

Detailed dynamics of the quark-hadron transition in the early Universe show that the evolution of the Universe does not necessarily follow the small super cooling scenario and certain choices of $B$ and $\sigma$ can have a bearing on the present state of the Universe. As nuggets with $N_q \geqslant 10^{52}$ are expected to survive the transition, they will contribute to the density of the Universe. It may be mentioned that for this scenario to work one has to use a value of the bag constant that is below the two flavor stability bound making the $u$-$d$ matter at $T=0$ the stable state of quark matter rather than the strange matter. This problem can be circumvented by using a somewhat unrealistically large value of the surface tension. We have explored in detail the possibility of nugget formation and also estimated their average separation, time of formation, quark content, and survivability. Clearly, the analysis can be improved by taking interactions into account in both the phases and also bubble interactions may be incorporated in the calculations. This will be reported elsewhere, however, we believe that qualitatively the results given here will hold. If the nuggets studied above are indeed formed in a much cooler environment, they could contribute significantly to the missing mass in the Universe and be candidates for dark matter.

## ACKNOWLEDGMENTS

We would like to thank J. V. Narlikar for providing hospitality at the Inter University Center for Astronomy and Astrophysics, Pune, India where a part of this work was completed.

[1] E. Witten, Phys. Rev. D **30**, 272 (1984); J. Applegate and C. J. Hogan, *ibid.* **31**, 3037 (1985); E. Farhi and R. L. Jaffe, *ibid.* **30**, 2379 (1984).
[2] C. Alcock and E. Farhi, Phys. Rev. D **32**, 1273 (1985).
[3] C. Alcock and A. Olinto, Phys. Rev. D **39**, 1233 (1989); J. A. Frieman *et al.*, *ibid.* **40**, 3241 (1989).
[4] J. Madsen *et al.*, Phys. Rev. D **34**, 2947 (1986); J. Madsen and M. L. Olesen, *ibid.* **43**, 1069 (1991).
[5] J. C. Niemeyer and K. Jedamzik, Phys. Rev. Lett. **80**, 5481 (1998); Phys. Rev. D **59**, 124013 (1999); **59**, 124014 (1999); K. Jedamzik, *ibid.* **55**, R5871 (1997); Phys. Rep. **307**, 155 (1998).
[6] C. Alcock *et al.*, Nature (London) **365**, 621 (1993); E. Aubourg *et al.*, *ibid.* **365**, 623 (1993).
[7] W. N. Cottingham and Vinh Mau, Phys. Rev. Lett. **73**, 1328 (1994).
[8] T. D. Lee and G. C. Wick, Phys. Rev. D **9**, 2291 (1974).
[9] I. Mardor and B. Svetitsky, Phys. Rev. D **44**, 878 (1991); Ashok Goyal and Deepak Chandra, Astron. Astrophys. **330**, 10 (1998).
[10] C. J. Hogan, Phys. Lett. **133B**, 172 (1983); K. Kajantie and M. Kurki-Suonio, Phys. Rev. D **34**, 1719 (1996); G. M. Fuller, G. J. Mathews, and C. R. Alcock, *ibid.* **37**, 1380 (1988).
[11] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992); Phys. Rev. D **46**, 1379 (1992).
[12] J. Madsen, Nucl. Phys. **B24**, 84 (1991).
[13] A. Leonidev *et al.*, Phys. Rev. D **50**, 4657 (1994).
[14] A. H. Guth and E. J. Weinberg, Phys. Rev. D **23**, 876 (1981).