Reexamination of the constraint on top-color-assisted technicolor models from R_b

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(Received 14 January 2000; published 26 July 2000)

Recent study on the charged top-pion correction to R_b shows that it is negative and large, so that the precision experimental value of R_b gives rise to a severe constraint on the top-color-assisted technicolor models such that the top-pion mass should be of the order of 1 TeV. In this paper, we restudy this constraint by further taking account the extended technicolor gauge boson correction which is positive. With this positive contribution to R_b, the constraint on the top-color-assisted technicolor models from R_b changes significantly. The top-pion mass is allowed to be in the region of a few hundred GeV depending on the models.

PACS number(s): 12.60.Cn, 12.60.Nz, 13.38.Dg

The mechanism of electroweak symmetry breaking remains an open question in current particle physics despite the success of the standard model (SM) tested by the CERN e^+e^- collider LEP and SLAC Large Detector (SLD) precision measurement data. In the SM, an elementary Higgs field is assumed to be responsible to break the electroweak symmetry. So far the Higgs bosons has not been found. Recent investigation shows that the LEP-SLD precision measurement data do not really require the existence of a light Higgs boson [1]. Furthermore, theories with elementary scalar fields suffer from the problems of *triviality* and *unnaturalness*. To completely avoid these problems arising from the elementary Higgs field, various kinds of dynamical electroweak symmetry breaking mechanisms have been proposed, and among which the top-color-assisted technicolor theory [2] is an attractive idea. In the top-color-assisted technicolor theory, there are two kinds of new heavy gauge bosons: (a) the extended technicolor (ETC) gauge bosons including the sideways and diagonal gauge bosons, (b) the top-color gauge bosons including the color-octet colorons C_μ^a and an extra U(1) gauge boson Z'. The technicolor interactions play the major role in breaking the electroweak gauge symmetry and, in addition, give rise to the masses of the ordinary leptons and quarks including a very small portion of the top-quark mass, namely εm_t [5] with a model-dependent parameter $\varepsilon \ll 1$. The top-color interactions also make small contributions to the breaking of the electroweak symmetry, and give rise to the main part of the top-quark mass $(1-\varepsilon)m_t$ similar to the constituent masses of the light quarks in QCD. So that the heaviness of the top-quark emerges naturally in the top-color-assisted technicolor theory. Furthermore, this kind of theory predicts a number of pseudo Goldstone bosons (PGBs) including the technipions in the technicolor sector and the top-pion in the top-color sector. All the new particles in this theory can give corrections to the *Z*-pole observables at LEP and SLC, and thus the LEP-SLD precision data may give constraints on the parameters in the top-color-assisted technicolor theory. These constraints have recently been studied in Refs. [3,4]. Because of the strong coupling between the top-pion and the top and bottom quarks, the top-pion gives rise to a large negative correction to the $Z\to b\bar{b}$ branching ratio R_b. Together with the positive contributions from the colorons and Z', the total top-color correction to R_b is shown to be quite negative which is of the wrong sign when comparing the SM value of R_b to the LEP-SLD data. Since the negative top-pion corrections become smaller when the top-pion is heavier, the LEP-SLD data of R_b give rise to certain lower bound on the top-pion mass. It is shown in Ref. [3,4] that the top-pion mass m_{π_t} should not be lighter than the order of 1 TeV to make the theory consistent with the LEP-SLD data. This implies that the scale of top-color should be much higher than what the original model expected [2]. However, in those analyses, the ETC contributions to R_b are not taken into account. The main ETC corrections to R_b are from the ETC gauge boson contributions. It has been shown in Ref. [6] that the positive diagonal ETC gauge boson contribution is larger than the negative sideways gauge boson contribution, and thus the total ETC correction to R_b is positive. It is the purpose of this short paper to investigate how much the constraint on the top-color-assisted technicolor theory from R_b changes when this positive ETC correction is included.

Since the corrections to R_b in the top-color-assisted technicolor models depends on the values of the parameters in the models, we shall consider the original top-color-assisted technicolor model [2] (it will be referred to as model-I in this paper) and the top-color-assisted multiscale technicolor model [7] (it will be referred to as model-II in this paper) as two typical examples in the investigation. These two models are different only in their ETC parts. For the model-dependent parameter ε, it has been shown that the $b\to s\gamma$ rate is sensitive to the value of ε, and the CLEO data on the $b\to s\gamma$ rate give a constraint on the value of ε, namely $\varepsilon \lesssim 0.1$ [8]. We shall take three values $\varepsilon = 0.05,\ 0.08$, and 0.1 in our calculation to see its effect.

The left-handed and right-handed $Z-b-\bar{b}$ and $Z-t-\bar{t}$ coupling constants g_L^b, g_R^b, g_L^t, and g_R^t in the SM are respectively $g_L^b=-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$, $g_R^b=\frac{1}{3}\sin^2\theta_W$, $g_L^t=\frac{1}{2}$

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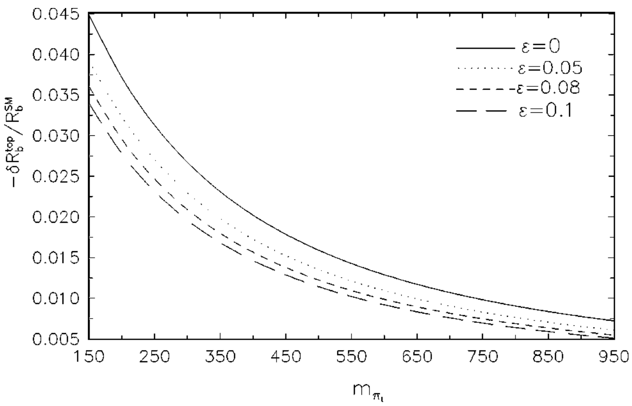

FIG. 1. The top-pion contributed $-\delta R_b/R_b^{SM}$ versus the top-pion mass m_{π_t} (in GeV) for $\varepsilon=0,\ 0.05,\ 0.08,\ 0.1$.

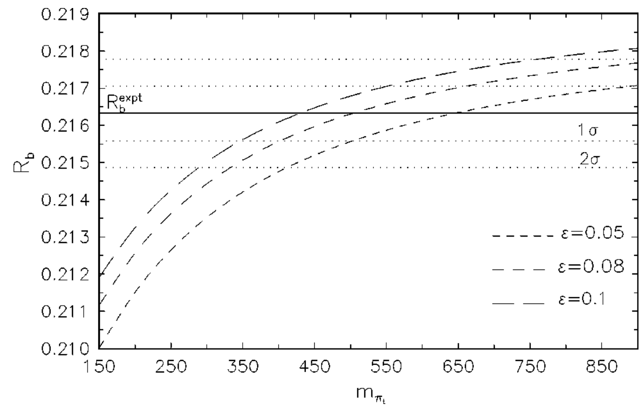

FIG. 2. The predicted R_b in model-I versus the top-pion mass m_{π_t} (in GeV) for $\varepsilon=0.05,\ 0.08,\ 0.1$ together with the experimental value R_b^{expt}. The horizontal solid line denotes the central value of R_b^{expt} and the dotted lines show the 1σ and 2σ bounds.

$-\frac{2}{3}\sin^2\theta_W$, and $g_R^t=-\frac{2}{3}\sin^2\theta_W$.[1] Let δg_L^b and δg_R^b denote, respectively, the corrections to g_L^b and g_R^b from the top-color-assisted technicolor theory. Then the correction to R_b can be expressed as

$$\frac{\delta R_b}{R_b^{SM}}\equiv\frac{R_b-R_b^{SM}}{R_b^{SM}}=(1-R_b^{SM})\frac{2[g_L^b\delta g_L^b+g_R^b\delta g_R^b]}{(g_L^b)^2+(g_R^b)^2}, \quad (1)$$

where $R_b^{SM}=0.2158\pm0.0002$ is the SM prediction for R_b. We shall calculate δg_L^b and δg_R^b from various sectors in model-I and model-II.

We first consider the top-color sector which is the same in model-I and model-II. The Feynman diagrams for the one-loop charged top-pion corrections to the $Z-b-\bar{b}$ vertex and the dependence of $-\delta R_b/R_b^{SM}$ on m_{π_t} have been shown in Figs. 1 and 2 in Ref. [3]. Compared with the charged top-pion contributions, the neutral top-pion contributions are suppressed by a factor of m_b^2/m_t^2 and thus can be ignored. In Ref. [3], the effect of the technicolor contribution to the top-quark mass εm_t is not taken into account (the result in Ref. [3] corresponds to taking $\varepsilon=0$). Taking account of the ε effect, the total one-loop top-pion correction to R_b in the on-shell renormalization scheme reads [9]

$$\delta g_L^{b(\pi_t)}=\left(\frac{v_\pi}{v_w}\right)^2\frac{[(1-\varepsilon)m_t]^2V_{tb}^2}{16\pi^2F_{\pi_t}^2}\{-g_L^b\overline{B}_1(-p_b,m_t,m_{\pi_t})$$
$$+g_R^t[2\overline{C}_{24}^*+\overline{B}_0(-k,m_t,m_t)-m_{\pi_t}^2C_0^*(p_b,$$
$$-k,m_{\pi_t},m_t,m_t)]+g_L^tm_t^2C_0^*(p_b,-k,m_{\pi_t},m_t,m_t)$$
$$+(1-2\sin^2\theta_W)\overline{C}_{24}(-p_b,k,m_t,m_{\pi_t},m_{\pi_t})\}, \quad (2)$$

$$\delta g_R^{b(\pi_t)}=0, \quad (3)$$

where $v_\pi/v_w=(167\ \text{GeV})/(174\ \text{GeV})$ reflects the effect of the mixing between the top-pion and the would-be Goldstone boson [3,4], $F_{\pi_t}=50$ GeV is the top-pion decay constant, p_b and k are respectively the momenta of the external b quark and Z boson, B_i, and C_{ij} are the two-point and three-point scalar integral functions. The factor $[(1-\varepsilon)m_t]^2$ in Eq. (2) comes from the $\pi_t-t-\bar{b}$ coupling when the technicolor contribution to the top-quark mass is considered. This factor causes the ε-dependence of $\delta g_L^{b(\pi_t)}$. The negative correction to R_b from the top-pion decreases with ε. In Fig. 1, we plot the top-pion contributed $-\delta R_b/R_b^{SM}$ versus m_{π_t} with $\varepsilon=0,\ 0.05,\ 0.08,\ 0.1$. The $\varepsilon=0$ curve is just the result given in Ref. [3].

The contributions to δg_L^b and g_R^b from the top-color gauge bosons C_μ^a and Z'_μ have been calculated in Ref. [10,4], which are

$$\delta g_L^{b(C^a)}=g_L^b\frac{\kappa_3}{6\pi}C_2(R)\left[\frac{m_Z^2}{M_C^2}\ln\frac{M_C^2}{m_Z^2}\right],$$
$$\delta g_R^{b(C^a)}=g_R^b\frac{\kappa_3}{6\pi}C_2(R)\left[\frac{m_Z^2}{M_C^2}\ln\frac{M_C^2}{m_Z^2}\right], \quad (4)$$

$$\delta g_L^{b(Z')}=g_L^b\frac{\kappa_1}{6\pi}(Y_L^b)^2\left[\frac{m_Z^2}{M_{Z'}^2}\ln\frac{M_{Z'}^2}{m_Z^2}\right],$$
$$\delta g_R^{b(Z')}=g_R^b\frac{\kappa_1}{6\pi}(Y_R^b)^2\left[\frac{m_Z^2}{M_{Z'}^2}\ln\frac{M_{Z'}^2}{m_Z^2}\right], \quad (5)$$

where κ_3 and κ_1 are respectively the coloron and the Z' couplings [2,4], M_C and $M_{Z'}$ are respectively the masses of C_μ^a and Z', $C_2(R)=\frac{4}{3}$, $Y_L^b=\frac{1}{3}$, and $Y_R^b=-\frac{2}{3}$. We shall take $M_B=M_{Z'}=1$ TeV in the calculation. To obtain proper vacuum tilting (the top-color interactions only condense the top quark but not the bottom quark), the couplings κ_3 and κ_1 should satisfy certain constraint. There is a region of κ_3 and κ_1, namely $\kappa_3=2,\ \kappa_1\leqslant 1$, satisfying the requirement of

[1]Here we have ignored the coupling constant $e/(\sin\theta_W\cos\theta_W)$ which is irrelevant to R_b.

vacuum tilting and the constraints from Z-pole physics and U(1) triviality shown in Refs. [11,12]. We shall take $\kappa_3=2$ and $\kappa_1=1$ in the following calculation.

Next, we consider the ETC sector corrections to R_b. In the top-color-assisted technicolor theory, the technipion-top-bottom coupling is proportional to $\varepsilon m_t/F_\pi$, and the technipion corrections to δg_L^b and δg_R^b are proportional to $(\varepsilon m_t/F_\pi)^2$ which is very small, so that the technipion corrections to R_b is negligible. Therefore, the main contribution is from the ETC gauge bosons. This has been calculated in Ref. [13,14,6] which reads

$$\delta g_L^{b(ETC)}=-\frac{1}{A}\frac{\varepsilon m_t}{16\pi F_\pi}\sqrt{\frac{N_{TC}}{N_C}}\left[\frac{2N_C}{N_{TC}+1}\xi_t(\xi_t^{-1}+\xi_b)-\xi_t^2\right], \tag{6}$$

where N_{TC} and N_C are respectively the number of technicolors and the numbers of ordinary colors, ξ_t and ξ_b are coupling coefficients with $\xi_b=(m_s/m_c)\xi_t^{-1}$ [15], and ξ_t is ETC gauge-group dependent. Following Refs. [13,14], we take $\xi_t=1/\sqrt{2}$. The factor $1/A$ reflects the walking effect in the ETC sector which is taken to be $A=1.7$ in Refs. [15,16]. The decay constant F_π is different in model-I and model-II. In model-I, the ETC sector is the one-family ETC model. Considering the mixing between the top-pion and the would-be Goldstone boson, we have $N_d(F_\pi/\sqrt{2})^2+F_{\Pi_t}^2=v_w^2$ [$N_d=4$ is the number of SU(2) doublets in the one-family technifermion sector], and thus $F_\pi=118$ GeV. In model-II, the ETC sector is the multiscale technicolor model in which $F_\pi=40$ GeV [16]. This difference makes the ETC corrections to R_b very different in model-I and model-II. This positive ETC correction to R_b is larger in model-II than in model-I.

Finally, we add all the corrections together and obtain the total corrections

$$\delta g_L^b=\delta g_L^{b(\pi_t)}+\delta g_L^{b(C^a)}+\delta g_L^{b(Z')}+\delta g_L^{b(ETC)}, \tag{7}$$

$$\delta g_R^b=\delta g_R^{b(C^a)}+\delta g_R^{b(Z')}, \tag{8}$$

in which $\delta g_L^{b(\pi_t)}$, $\delta g_L^{b(C^a)}$, $\delta g_R^{b(C^a)}$, $\delta g_L^{b(Z')}$, $\delta g_R^{b(Z')}$, and $\delta g_L^{b(ETC)}$ are given in Eqs. (2), (4), (5), and (6), respectively. Plugging Eqs. (7) and (8) into (1), we obtain the total correction $\delta R_b/R_b^{SM}$ and the predicted $R_b=R_b^{SM}+\delta R_b$ in model-I and model-II.

Before presenting the numerical results, let us make an examination of the parameter range $\varepsilon=0.05$–0.1 which we take in the calculation. It has been noticed that the ETC sector not only gives rise to a positive contribution to R_b but also contributes positive correction to the oblique correction parameter T (or equivalently $\Delta\rho$, $\Delta\rho=\alpha T$) [17] due to the violation of the custodial symmetry $SU(2)_c$ in the ETC sector [14]. In the original ETC model, the top-quark mass is completely generated by the ETC dynamics, so that the violation of $SU(2)_c$ in the ETC sector is very serious and the positive contribution to T (or $\Delta\rho$) is so large that it can barely be consistent with the experiment [14]. Now we examine this problem in the top-color-assisted technicolor models. In the top-color-assisted technicolor models, the ETC sector only gives rise to a very small portion of the top-quark mass, εm_t, therefore the violation of $SU(2)_c$ in the ETC sector is significantly smaller depending on the values of ε. It has been shown that the most dangerous positive contrbution to T in the ETC sector is from the exchange of the diagonal ETC gauge boson whose couplings to the up-type and down-type techniquarks are different, and this has been studied in Ref. [14]. Since the four-fermion operators contributing the positive correction to T is also related to the ETC generation of the top and bottom quark masses, the formula in Ref. [14] can be further expressed in terms of the parameters εm_t, ξ_t and ξ_b as follows

$$T^{ETC}=\frac{1}{A}\frac{1}{16\sin^2\theta_W\cos^2\theta_W}\frac{N_C^2}{N_{TC}(N_{TC}+1)}\times\frac{\varepsilon m_t F_\pi}{m_Z^2}\sqrt{\frac{N_{TC}}{N_C}}[\xi_t^{-1}-\xi_b]^2. \tag{9}$$

For $\varepsilon=0.05$, 0.08 and 0.1, the values of T^{ETC} are 0.006, 0.009 and 0.01, respectively. These are to be compared with the experimental value $T=0.00\pm0.15$ [18]. We see that, for the parameter range $\varepsilon=0.05$–0.1 which we take in this paper, the ETC contributed positive correction to T is small enough to make the theory consistent with the experiment.[2]

Now we compare our predicted R_b with the experimental value $R_b^{expt}=0.21642\pm0.00073$ [20] to get the new constraints on the two typical top-color-assisted technicolor models. The results of the predicted R_b in model-I with $\varepsilon=0.05$, 0.08, and 0.1 are plotted in Fig. 2 together with the experimental value R_b^{expt}. The horizontal solid line denotes the central value R_b^{expt}, and the horizontal dotted lines indicate the 1σ and 2σ deviations. We see from Fig. 1 that the 2σ constraints on model-I are

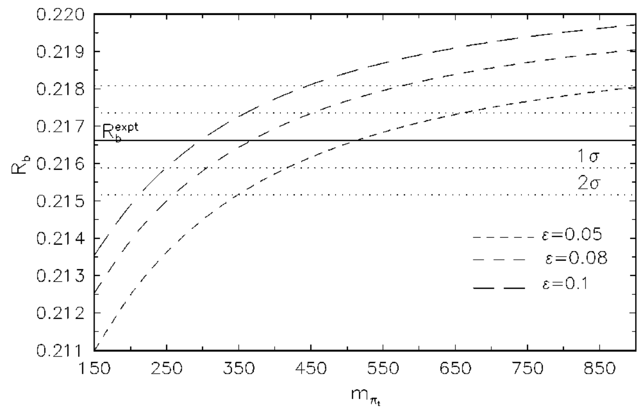

FIG. 3. The predicted R_b in model-II versus the top-pion mass m_{π_t} (in GeV) for $\varepsilon=0.05$, 0.08, 0.1 together with the experimental value R_b^{expt}. The horizontal solid line denotes the central value of R_b^{expt} and the dotted lines show the 1σ and 2σ bounds.

[2]The corrections to T from the exchange of topcolor gauge boson has been studied in Ref. [19].

$$\varepsilon=0.05: \quad 400\ \text{GeV} \lesssim m_{\pi_t},$$
$$\varepsilon=0.08: \quad 340\ \text{GeV} \lesssim m_{\pi_t} \lesssim 900\ \text{GeV}, \tag{10}$$
$$\varepsilon=0.1: \quad 280\ \text{GeV} \lesssim m_{\pi_t} \lesssim 770\ \text{GeV}.$$

The results of the predicted R_b in model-II with $\varepsilon=0.05$, 0.08 and 0.1 are plotted in Fig. 3 together with the experimental value R_b^{expt} and the 1σ and 2σ deviations. Figure 3 shows that the 2σ constraints on model-II are

$$\varepsilon=0.05: \quad 350\ \text{GeV} \lesssim m_{\pi_t} \lesssim 900\ \text{GeV},$$
$$\varepsilon=0.08: \quad 250\ \text{GeV} \lesssim m_{\pi_t} \lesssim 560\ \text{GeV}, \tag{11}$$
$$\varepsilon=0.1: \quad 220\ \text{GeV} \lesssim m_{\pi_t} \lesssim 430\ \text{GeV}.$$

We see that the constraints on model-I and model-II are different due to the different values of F_π in the two models. Since F_π takes a smaller value (causing a larger positive ETC correction to R_b) in model-II, the allowed top-pion mass is lower in model-II than in model-I.

From Fig. 2 and Fig. 3, we see that, when the positive ETC gauge boson correction to R_b is taken into account, the constraints on the two typical top-color-assisted technicolor models are significantly different from that shown in Refs. [3,4]. As is mentioned in Ref. [3], this kind of constraint should only be regarded as a rough estimate since the $\pi_t - t - \bar{b}$ coupling is so strong that higher order corrections from the top-pion are expected to be important. Anyway, the conclusion of the present investigation is that, to be consistent with the experimental value R_b^{expt}, the top-pion mass is roughly in a region of a few hundred GeV, and thus the scale of top-color is likely to be around a couple of TeV which is not much higher than what is expected in the original top-color-assisted technicolor theory [2].

This work is supported by the National Natural Science Foundation of China, the Fundamental Research Foundation of Tsinghua University, a special grant from the Ministry of Education of China, and the Natural Science Foundation of the Henan Scientific Committee.

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