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Classical Nambu-Goldstone fields

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It is shown that a Nambu-Goldstone (NG) field may be coherently produced by a large number of particles in spite of the fact that the NG bosons do not couple to flavor conserving scalar densities such as $\bar{\psi}\psi$. If a flavor oscillation process takes place, the phases of the pseudoscalar or flavor-violating densities of different particles do not necessarily cancel each other. The NG boson gets a macroscopic source whenever the total (spontaneously broken) quantum number carried by the source particles suffers a net increase or decrease in time. If the lepton numbers are spontaneously broken such classical NG (Majoron) fields may significantly change the neutrino oscillation processes in stars, pushing the observational capabilities of neutrino-Majoron couplings down to $m_\nu/300$ GeV.

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As is well known, spontaneous violation of global symmetries leads to the appearance of massless Nambu-Goldstone (NG) bosons in the particle spectrum. The NG bosons not only have zero mass but they also have no scalar potential terms such as ϕ^4. Nevertheless, they do not mediate long range forces because they only have derivative couplings [1]. This can be understood as follows. A NG boson ϕ associated with a quantum number Λ broken by the vacuum transforms under $U(1)_\Lambda$ as $\phi \rightarrow \phi + \alpha$, $\alpha = \text{const}$, and all the other fields are made invariant in the unitary gauge. The Λ conservation law takes the form, ignoring for simplicity possible mixing with other NG bosons,

$$V_\Lambda \partial_\mu \partial^\mu \phi + \partial_\mu J^\mu_\Lambda = 0, \tag{1}$$

which is nothing but the equation of motion of the NG boson. V_Λ is the scale of symmetry breaking and J^μ_Λ is the current of the other fermion and boson particles. In other words, this equation reads $\partial_\mu J^\mu = 0$, where

$$J^\mu = J^\mu_\Lambda + V_\Lambda \partial^\mu \phi \tag{2}$$

is an exactly conserved current.

The source of a scalar field is of course a scalar density. The possible fermion bilinears are the scalar and pseudoscalar densities such as $\overline{f}_i f_j$ and $\overline{f}_i \gamma_5 f_j$ but they have to be derived from the divergence of some current J^μ_Λ. The actual current depends on the particular theory and quantum number Λ. J^μ_Λ is determined in leading order by the quantum numbers of the existing particles but receives also higher order corrections with a more general flavor structure. In any case, it suffices to apply the Dirac equation to arbitrary vector and axial-vector bilinears, written in terms of the fermion mass eigenstates, to conclude, as follows from

$$\partial_\mu \overline{f}_i \gamma^\mu \gamma_5 f_j = i(m_i + m_j) \overline{f}_i \gamma_5 f_j, \tag{3}$$

$$\partial_\mu \overline{f}_i \gamma^\mu f_j = i(m_i - m_j) \overline{f}_i f_j, \tag{4}$$

that the NG bosons couple to pseudoscalar densities (generally these can be both flavor-diagonal and nondiagonal) or to off-diagonal scalar densities. However couplings to flavor-diagonal scalar densities are not possible [2,3]. The pseudoscalars that are diagonal in flavor vanish for free particle states and the flavor violating densities depend on the relative phase of distinct flavors. The natural conclusion has been that these relative phases cancel each other when summed over a large number of particles and therefore long range "$1/r$" NG fields are not possible. Only spin-dependent "$1/r^3$" interaction potentials can exist [2,3].

However, there may be cases where certain quantum numbers are not conserved in a very large scale. For example, the total lepton number is violated if the neutrinos have Majorana masses, while the partial lepton numbers L_e, L_μ, or L_τ are violated if they are also mixed (i.e., if the nondiagonal elements of the neutrino mass matrix are nonzero in the flavor basis). In other words, this means that in physical processes the individual lepton numbers are violated by the neutrino oscillation phenomena, e.g., in stars or other astrophysical objects. If the unconserved quantum number Λ is spontaneously broken, it implies the existence of a massless NG boson - a Majoron [2].[1] As it was pointed out by one of us [4], in this case the nonvanishing divergency $\partial_\mu J^\mu_\Lambda$ generates a classical field for the respective NG boson (Majoron). The prototype of such system is a flux of neutrinos undergoing a flavor oscillation process. The key idea can be phrased as follows: as far as the neutrino oscillation process implies a nonconservation of the currents like $J^\mu_e = \overline{\nu}_e \gamma^\mu \nu_e$, etc. associated with the partial lepton numbers, then the conservation of the full current (2) should imply the existence of a classical configuration of the corresponding Majoron fields

[1] We call such NG bosons Majoron independently of whether the quantum number Λ is the total lepton number L or a partial lepton number $L_{e,\mu,\tau}$.

ϕ_e, etc., with nonvanishing $\partial_\mu \phi_e$ [4]. (Clearly, a constant NG field does not carry much physical sense.) This seems in contradiction with what was said about the interaction of a NG boson with fermion scalar densities. It is the aim of this paper to show that it is not so because the phases of wave functions of different flavors are not independent from each other if transition processes take place between them. In that event, the wave functions of such flavors interfere constructively with each other and potentially form a macroscopic source of the NG boson built out of nondiagonal densities like $\overline{f_i}\gamma_5 f_j$ or $\overline{f_i} f_j$, in the basis of the mass eigenstates. To see this one has to be more specific about the nature of the fermion current and equations of motion.

Consider a NG boson associated with the spontaneous breaking of a partial lepton number Λ, which could be L_e, L_μ, and L_τ, or any combination of these. Denoting the neutrinos by f^a, with quantum numbers Λ_a, and restricting ourselves to the fermion current, the equation of motion (1) of the corresponding NG boson (Majoron) is given in leading order by

$$\partial_\mu \partial^\mu \phi = -\frac{1}{V_\Lambda} \sum_a \partial_\mu (\overline{f^a} \gamma^\mu \Lambda_a f^a). \tag{5}$$

After applying the fermion equations of motion, the interactions that violate the lepton number Λ emerge in the second term. They vary from model to model but include in general neutrino Majorana masses. These are the lowest dimension terms that violate Λ and we confine to them here neglecting any higher order flavor-violating gauge or Yukawa interactions. For simplicity, the neutrino mass matrix, m, is assumed to be real in the weak basis. Denoting by ν_L^a and ν_L^{aC} the left-handed neutrino fields and their charge conjugates, the equations of motion are of the form

$$i \partial\!\!\!/ \nu_L^a = m_{ab} \nu_L^{aC} + V_\mu^a \gamma^\mu \nu_L^a. \tag{6}$$

The potentials V_μ^a account for the local neutral and (Fierz-transformed) charge current interactions with the medium [5] and couplings to the NG boson namely,

$$\mathcal{L}_{\nu\nu\phi} = \frac{1}{V_\Lambda} \partial_\mu \phi \;\; \overline{\nu_L^a} \, \gamma^\mu \Lambda_a \, \nu_L^a. \tag{7}$$

The result is

$$\partial_\mu \partial^\mu \phi = \frac{i}{V_\Lambda} [\overline{\nu_L^a} (\Lambda m)_{ab} \nu_L^{bC} - \overline{\nu_L^{aC}} (m\Lambda)_{ab} \nu_L^b], \tag{8}$$

where the quantum number Λ is written in matrix form. For example, one can consider the anomaly free quantum number $L_e - L_\mu$, in which case the (two-flavor) matrices of leptonic charge and neutrino mass have the form

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad m = \begin{pmatrix} M_1 & M \\ M & M_2 \end{pmatrix}, \tag{9}$$

respectively, where M is a $\Lambda = L_e - L_\mu$ conserving entry and M_1, M_2 are the Λ violating ones. The result above illustrates also that in scattering processes the effective coupling constants of neutrinos to Majorons are essentially the neutrino masses divided by the scale of global symmetry breaking, V_Λ. This point will be called later when discussing the observational implications.

In order to evaluate the source terms over a system of particles one needs to relate the field operators with the wave functions. The chiral fields ν_L and ν_L^C are the left and right-handed projections of the Majorana fields

$$\nu_L + \nu_L^C = \int a\psi + a^\dagger \psi^C, \tag{10}$$

where a and $a^\dagger$ are annihilation and creation operators and ψ single particle wave functions. The expectation value of the operators in Eq. (8) gives then a sum over all the existing ν particles in terms of their wave functions ($\gamma_5 = -1$ for left-handed spinors):

$$\partial_\mu \partial^\mu \phi = \frac{i}{V_\Lambda} \sum_\nu \overline{\psi^a} (m\Lambda + \Lambda m)_{ab} \gamma_5 \psi^b. \tag{11}$$

The second member contains precisely the pseudoscalar densities present in Eq. (3). It remains to establish the ψ equations of motion. They are nonlinear at the operator level due to Majorana mass terms but the wave functions obey linear equations as follows from Eqs. (6) and (10):

$$i \partial\!\!\!/ \psi^a = m_{ab} \psi^b - V_\mu^a \gamma^\mu \gamma_5 \psi^a. \tag{12}$$

These are the equations of motion relevant for the most common cases of neutrino propagation in matter or vacuum. What is usually done is to separate the spin and flavor degrees of freedom by expressing the wave function as a product of a left-handed spinor ψ_0^α ($\gamma_5 \psi_0 = -\psi_0$), solution of the zero mass Dirac equation

$$i \partial\!\!\!/ \psi_0 = 0, \tag{13}$$

and a flavor-valued wave function φ^a that obeys a well known evolution equation as a function of the distance travelled by each neutrino. Here, one has to go beyond that approximation because the pseudoscalar densities in Eq. (11) vanish for spinors with well defined chirality. An approximate solution of Eq. (12) is

$$\psi^{a\alpha}(x) \cong \psi_0^\alpha \varphi^a + \gamma_{\alpha\beta}^0 \frac{m_{ab}}{2E} \psi_0^\beta \varphi^b, \tag{14}$$

which applies for the most common cases where only the scalar components V_0^a exist and even for vector potentials $\vec{V}^a$ that are parallel to the neutrino velocity $\vec{v}$. The wave function φ^a is considered as a function of the distance travelled by the neutrino to the point $\vec{x}$, $s = \vec{x} \cdot \vec{v} - \vec{x}_0 \cdot \vec{v}$, while the dependence on the variable $t - s$ (constant along the neutrino trajectory) is totally absorbed in $\psi_0 \cdot \varphi^a$ obeys the familiar evolution equation [5–7]

$$i\frac{d\varphi^a}{ds}=\left(\frac{m^2}{2E}+V_\mu v^\mu\right)_{ab}\varphi^b, \tag{15}$$

where $v^\mu=p^\mu/E$ is the neutrino velocity and $V_\mu^{ab}=\delta_{ab}V_\mu^a$.

Using these wave functions, the Majoron equation of motion (11) reads in leading order

$$\partial_\mu\partial^\mu\phi=\frac{i}{V_\Lambda}\sum_\nu \varphi^{a\dagger}\left[\Lambda,\frac{m^2}{2E}\right]_{ab}\varphi^b\psi_0^\dagger\psi_0. \tag{16}$$

It is now clear that no flavor conserving terms contribute to the Majoron source either in the weak basis where Λ is diagonal or in the mass eigenstate basis where m is a diagonal matrix. The existence of a Majoron source relies on the interference between different flavor components of the neutrino wave functions. But that simply means the existence of neutrino oscillations. Another necessary condition is that the flavor dynamics Hamiltonian does not conserve the quantum number Λ. Here, as a result of our particular specification of the neutrino interactions, Λ is only violated by the mass matrix. In general there could be also some additional nonstandard neutrino flavor-violating interactions with matter constituents whose effects can be comprised in the potential V_μ in Eq. (15). The remarkable feature is the absence of cancellation due to the arbitrariness of the wave function initial phases: each neutrino contributes with a term that only depends on the phase invariants $\psi_0^\dagger\psi_0$ and $\varphi^{a\dagger}\varphi^b$. This crucial fact permits that in certain circumstances the contributions from a large number of particles add to each other with a definite sign making so a source of macroscopic dimensions. The precise understanding of the nature of those circumstances is the subject of next discussion.

Making use of Eq. (15) one can write the last equation as

$$\partial_\mu\partial^\mu\phi=-\frac{1}{V_\Lambda}\sum_\nu \psi_0^\dagger\psi_0\frac{d}{ds}\varphi^{a\dagger}\Lambda_a\varphi^a. \tag{17}$$

Again, the second member does not depend on the initial phases of the individual particles. It rather depends on whether there is a net increase or decrease of the Λ number carried by the neutrinos. Take the example of a large system where only ν_e are produced out of electrons captured in nuclear reactions. The oscillations $\nu_e\to\nu_\mu$ necessarily lead to a total decrease of the partial lepton number $\Lambda=L_e-L_\mu$ that was initially carried by electrons. If Λ is a spontaneously broken quantum number, then the equation of motion of the corresponding NG boson ϕ_Λ is (we ignore a possible mixing with other NG bosons [4]):

$$\partial_\mu\partial^\mu\phi_\Lambda=-\frac{1}{V_\Lambda}\sum_\nu \psi^\dagger\psi\frac{d(P_{\nu_e}-P_{\nu_\mu})}{ds}$$
$$=-\frac{2}{V_\Lambda}\sum_\nu \psi^\dagger\psi\frac{dP_{\nu_e}}{ds}, \tag{18}$$

where for simplicity we have assumed that only $\nu_e\leftrightarrow\nu_\mu$ takes place and thus $P_{\nu_e}+P_{\nu_\mu}=1$. Therefore, the shape of the Majoron field ϕ_Λ is determined in terms of the probability $P_{\nu_e}(s)=\varphi^{e\dagger}\varphi^e/\varphi^\dagger\varphi$ of observing the ν_e flavor. Since in the presence of oscillations this probability is essentially smaller than $P_{\nu_e}(0)=1$, the total ϕ_Λ source charge is positive. The consequence is a long range ϕ_Λ classical field that propagates at the velocity of light. The ϕ_Λ sources exist where L_e-L_μ is either created or destroyed and the size of these regions is determined by the neutrino oscillation length, or resonance length in the case of resonant oscillations in a medium.

An explicit example is the well known case of two-flavor oscillations in vacuum [5]. With a mixing angle θ and Δm^2 mass difference the ν_e flavor probability is

$$P_{\nu_e}(s)=1-\sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4E}s\right), \tag{19}$$

as a function of the distance s travelled by the neutrino from its production point. The equation of motion of the NG field becomes

$$\partial_\mu\partial^\mu\phi_\Lambda=\frac{2}{V_e}\sum_\nu \psi^\dagger\psi\sin^2 2\theta\frac{\Delta m^2}{4E}\sin\left(\frac{\Delta m^2}{4E}s\right). \tag{20}$$

The source term changes sign as a function of s, but the scale of spatial variation is the flavor oscillation wavelength, $4E/\Delta m^2$, not any scale related to the number density or linear momentum of the particles. The neutrinos produced too far away give a vanishing contribution because the coherence is lost, but the ones produced within a sphere of the scale of the oscillation wavelength give a nonzero source term. Its sign depends on the distance to the point where each ν_e was produced, but if one integrates over all neutrinos, the result is finite as shown in the following example. If the reactor or star produces ν_e neutrinos in a stationary basis, the NG field obeys a Poisson equation and the total ''charge'', the volume integral of the second member, is positive. This can be seen from Eqs. (18) and (19): the volume integration is trivial if one considers neutrinos propagating in plane waves and the average value of the probability P_{ν_e} over the neutrino spectrum converges to $1-\sin^2 2\theta/2$ at distances much larger than the oscillation length. For a total ν luminosity equal to $\dot{N}_\nu$, the total ''charge'' is $2/V_\Lambda$ times the number of neutrinos that undergo the transition $\nu_e\to\nu_\mu$ per unity of time,

$$\frac{2}{V_\Lambda}\frac{dN}{dt}(\nu_e\to\nu_\mu)=\frac{2}{V_\Lambda}\dot{N}_\nu(1-\langle P_{\nu_e}(\infty)\rangle)=\frac{1}{V_\Lambda}\dot{N}_\nu\sin^2 2\theta. \tag{21}$$

This quantity has a definite sign, regardless of the size of the reactor, the only assumption is the nuclear reactions feed the system with ν_e at a constant rate in time. That means that the NG field is just like the electrostatic potential due to an electric charge distribution: it exits at distances much larger than the size of the source (the place where neutrinos oscillate) with a Coulombian ''$1/r$'' shape. What is really different is the nature of the charge. The ''charge'' of this NG field is the time rate of decreasing of the total L_e-L_μ number carried by leptons.

In the case of neutrino propagation in the medium, the neutrino oscillation is typically suppressed by the effects of the coherent interaction with the matter constituents. However, there can be a resonant neutrino conversion, so called MSW (Mikheyev-Smirnov-Wolfenstein) oscillation [6,7], where almost complete $\nu_e \rightarrow \nu_\mu$ conversion takes place within the length of the resonance area. Then, the source of the NG field lies in the resonance shell, spheric in the case of a star, and the total NG charge is finite (provided that the $\nu_e \rightarrow \nu_\mu$ conversions are not compensated by an exactly equal number of $\overline{\nu}_e \rightarrow \overline{\nu}_\mu$ or $\nu_\mu \rightarrow \nu_e$ conversions, which might happen if $\overline{\nu}_e$ or ν_μ are also produced in the reactor or star). Consequently, a Coulombian field is produced with

$$\phi_\Lambda = \frac{2}{V_\Lambda} \frac{1}{4\pi r} \frac{dN}{dt} (\nu_e \rightarrow \nu_\mu) \tag{22}$$

at radius r larger than the radius of the resonance shell. The field is constant in the region inside that shell.

The fermions interact with the long range NG field through its gradient. As shown in Eq. (7), $\partial_\mu \phi_\Lambda$ acts as a vector potential on the neutrinos (and charged leptons). In the case illustrated above a radially moving neutrino ''feels'' a potential energy equal to

$$V_{NG} = \mp \frac{1}{V_\Lambda} \vec{v}_\nu \cdot \vec{\nabla} \phi_\Lambda = \pm \frac{2}{V_\Lambda^2} \frac{1}{4\pi r^2} \frac{dN}{dt} (\nu_e \rightarrow \nu_\mu), \tag{23}$$

where the upper sign is for ν_e, $\overline{\nu}_\mu$ and the lower sign for ν_μ, $\overline{\nu}_e$. The derivative nature of the NG boson couplings gives an effective $\nu\nu\phi$ coupling constant equal to V_Λ^{-1} over the distance to the ϕ_Λ source. This is typically an extremely small number. However neutrino oscillations are sensitive to very tiny external potentials. In addition the NG boson couplings depend on the particle quantum numbers and such a nonuniversality makes them potentially important for the neutrino oscillations themselves [see Eq. (15)]. Because these are long-range fields one can conceive that a NG field generated in some region of a star may affect the propagation of other neutrinos or flavors outside that region or even out of the star.

Some possible effects can be devised concerning the supernova neutrinos [4]. The NG potentials are proportional to the ν luminosity over $(rV_\Lambda)^2$. In the case of supernovas this number is comparable to the values of $\Delta m^2/2E$ that are interesting for the solar ν solutions for scales V_Λ as large as the weak breaking scale. It was shown [4] that the NG fields may then have a significant role in resonant ν oscillations. A special feature is the dependence of the effects on the absolute magnitude of the ν fluxes. The signature is a surprise, i.e., oscillation patterns of supernova neutrinos that are in contradiction with the solar, atmospheric, and laboratory neutrino observations and/or that turn off as the ν fluxes decay after the first instants of supernova ν emission. This may be observed with detectors capable of detecting supernova neutrinos and measuring their energy spectra or time evolution. Then, one could go beyond present experimental and astrophysical limits on the Majoron effective couplings to neutrinos [8] [these are typically of the order of the neutrino masses over the scale V_Λ, see Eq. (8)], provided that the scale of symmetry breaking is under 1 TeV [9]. The NG field could have some impact on solar neutrinos too if the scale V_Λ is below KeV. Though such a situation is not very plausible, there exist some models [10] in which V_Λ can be very low and thus the effective Majoron-neutrino coupling constants rather large, up to order 10^{-2}–10^{-3}, still evading the present limits.

The following remark is in order. The physical features of generating the classical Majoron field due to neutrino oscillation is very much different from the mechanism of Majoron production due to neutrino coherent scattering on the matter components of a medium—the so called matter induced neutrino decay with Majoron emission [11]. In the latter case Majorons are produced as particles since the medium provides an energy splitting between the neutrino and antineutrino states and so the transitions either $\nu \rightarrow \overline{\nu} + \phi$ or $\overline{\nu} \rightarrow \nu + \phi$ are possible, depending on the neutrino flavor and on the chemical content of the medium (i.e., which of two states becomes ''heavier'' in the matter background). Very low-momentum Majorons could also be produced as a result of (stimulated) neutrino decays in the early Universe [12], forming a sort of Bose condensate with large occupation numbers in the low momemtum part of the spectrum, but still made of individual Majoron particles. On the opposite, the production of the classical Majoron field rather resembles the situation with classical electromagnetic field produced by the electric current.

Concluding, we have shown that if a process of flavor oscillation takes place a pseudoscalar density like the one of Eq. (11) may give rise to a long-range NG field as a result of the constructive interference between the wave functions of different mass eigenstates. Furthermore, the source of the NG boson is the time rate of decreasing of the quantum number associated with it, which is nothing but the term $-\partial_\mu J^\mu_\Lambda$ in Eq. (1) [4]. If the lepton numbers are spontaneously broken at the Fermi energy scale or below, and that means effective neutrino-Majoron coupling constants as low as $m_\nu/300$ GeV, the associated Majoron fields are still significant enough for neutrino oscillations in supernovas whose spectra may then show evidence of their existence beyond the limits [8] that can be reached from laboratory or astrophysical scattering processes.

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