Implications of recent measurements of hadronic charmless B decays

Hai-Yang Cheng
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China

Kwei-Chou Yang
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China
and Department of Physics, Chung-Yuan Christian University, Chung-Li, Taiwan 320, Republic of China

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The implications of recent CLEO measurements of hadronic charmless B decays are discussed. (i) Employing the Bauer-Stech-Wirbel (BSW) model for form factors as a benchmark, the $B\to\pi^+\pi^-$ data indicate that the form factor $F_0^{B\pi}(0)$ is smaller than that predicted by the BSW model, whereas the data of $B\to\omega\pi$, $K^*\eta$ imply that the form factors $A_0^{B\omega}(0)$, $A_0^{BK^*}(0)$ are greater than the BSW model values. (ii) The tree-dominated modes $B\to\pi^+\pi^-$, $\rho^0\pi^\pm$, $\omega\pi^\pm$ imply that the effective number of colors $N_c^{\rm eff}(LL)$ for $(V-A)(V-A)$ operators is preferred to be smaller, while the current limit on $B\to\phi K$ shows that $N_c^{\rm eff}(LR)>3$. The data of $B\to K\eta'$ and $K^*\eta$ clearly indicate that $N_c^{\rm eff}(LR)\gg N_c^{\rm eff}(LL)$. (iii) In order to understand the observed suppression of $\pi^+\pi^-$ and nonsuppression of $K\pi$ modes, both being governed by the form factor $F_0^{B\pi}$, the unitarity angle γ is preferred to be greater than 90°. By contrast, the new measurement of $B^\pm\to\rho^0\pi^\pm$ no longer strongly favors $\cos\gamma<0$. (iv) The observed pattern $K^-\pi^+\sim\overline{K}^0\pi^-\sim\frac{2}{3}K^-\pi^0$ is consistent with the theoretical expectation: The constructive interference between electroweak and QCD penguin diagrams in the $K^-\pi^0$ mode explains why $\mathcal{B}(B^-\to K^-\pi^0)>\frac{1}{2}\mathcal{B}(\overline{B}^0\to K^-\pi^+)$. (v) The observation $N_c^{\rm eff}(LL)<3<N_c^{\rm eff}(LR)$ and our preference for $N_c^{\rm eff}(LL)\sim2$ and $N_c^{\rm eff}(LR)\sim6$ are justified by a recent perturbative QCD calculation of hadronic rare B decays in the heavy quark limit. (vi) The sizable branching ratios of $K^*\eta$ and the enormously large decay rates of $K\eta'$ indicate that it is the constructive interference of two comparable penguin amplitudes rather than the mechanism specific to the η' that accounts for the bulk of $B\to\eta' K$ and ηK^* data. (vii) The new upper limit set for $B^-\to\omega K^-$ no longer imposes a serious problem for the factorization approach. It is anticipated that $\mathcal{B}(B^-\to\omega K^-)\gtrsim2\mathcal{B}(B^-\to\rho^0K^-)\sim2\times10^{-6}$. (viii) An improved and refined measurement of $B\to K^{*-}\pi^+$, $\overline{K}^0\pi^0$ is called for in order to resolve the discrepancy between theory and experiment. Theoretically, it is expected that $\overline{K}^0\pi^0\sim\frac{1}{2}K^-\pi^0$ and $K^-\pi^+\sim3\,K^{*-}\pi^+$.

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I. INTRODUCTION

A number of new hadronic charmless B decay modes have been recently reported by CLEO [1,5,3,2,4],

$$B\to\pi^+\pi^-,\ K_S^0\pi^0,\ \rho^0\pi^\pm,\ \omega\pi^\pm,\ K^{*\pm}\eta,$$
$$K^{*0}\eta,\ \rho^\pm\pi^\mp,\ K^{*\pm}\pi^\mp, \tag{1.1}$$

and several previously observed decays have received improved measurements: $B\to K^\pm\eta'$, $K_S^0\eta'$, $K^\pm\pi^\mp$, $K_S^0\pi^\pm$, $K^\pm\pi^0$, $\omega K^\pm$. Needless to say, these measurements will shed light on the underlying mechanism for charmless B decays and provide important constraints on the phenomenological models under consideration and the parameters involved in the model, such as form factors, unitarity angles, and nonfactorized effects.

Beyond the phenomenological level, the nonleptonic B decays have been studied within the framework of the so-called three-scale perturbative QCD factorization theorem in which nonfactorized and nonspectator contributions can be identified and calculated [6]. Recently, it was shown that, in the heavy quark limit, the hadronic matrix elements for two-body charmless B decays can be computed from first principles and expressed in terms of form factors and meson light-cone distribution amplitudes [7]. Nonfactorizable diagrams in the heavy quark limit are dominated by hard gluon exchange and thus can be calculated as expansion in α_s. As we shall see below, this framework provides a useful guidance on the nonfactorized corrections to the hadronic matrix elements of penguin and nonpenguin operators and gives a justification on the use of generalized factorization in which the effective Wilson coefficients $c_i^{\rm eff}$ are renormalization-scale and -scheme independent, while factorization is applied to the tree-level hadronic matrix elements.

In the present paper we will analyze the data of hadronic charmless B decays within the framework of generalized factorization and see what implications we can learn from the studies of the new measured modes (1.1). This paper is organized as follows. In Sec. II we briefly review the generalized factorization approach relevant to rare B decays. Then we proceed to study $B\to\pi\pi$ and πK decay modes in Sec. III, tree-dominated modes $\rho^0\pi^\pm$ and $\omega\pi^\pm$ in Sec. IV, $B\to K\eta'$, $K^*\eta$ decays in Sec. V and $B^\pm\to\omega K^\pm$ decays in Sec. VI. Comparison of the present paper with the previous work [8] is discussed in Sec. VII. Conclusions are presented in Sec. VIII.

II. FRAMEWORK

In the absence of first-principles calculations for hadronic matrix elements, it is customary to evaluate the matrix elements under the factorization hypothesis so that $\langle O(\mu)\rangle$ is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. However, the naive factorized amplitude is not renormalization scale and γ_5 scheme independent as the scale and scheme dependence of Wilson coefficients are not compensated by that of the factorized hadronic matrix elements. In principle, the scale and scheme problems with naive factorization will not occur in the full amplitude since $\langle O(\mu)\rangle$ involves vertex-type and penguin-type corrections to the hadronic matrix elements of the 4-quark operator renormalized at the scale μ. Schematically,

$$\begin{aligned}&\text{weak decay amplitude}\\ &\quad=\text{naive factorization}+\text{vertex-type corrections}\\ &\qquad+\text{penguin-type corrections}+\text{spectator contributions}\\ &\qquad+\cdots,\end{aligned}\tag{2.1}$$

where the spectator contributions take into account the gluonic interactions between the spectator quark of the B meson and the outgoing light meson. The perturbative part of vertex-type and penguin-type corrections will render the decay amplitude scale and scheme independent. Generally speaking, the Wilson coefficient $c(\mu)$ takes into account the physics evolved from the scale M_W down to μ, while $\langle O(\mu)\rangle$ involves evolution from μ down to the infrared scale. Formally, one can write

$$\langle O(\mu)\rangle=g(\mu,\mu_f)\langle O(\mu_f)\rangle,\tag{2.2}$$

where μ_f is a factorization scale, and $g(\mu,\mu_f)$ is an evolution factor running from the scale μ to μ_f which is calculable because the infrared structure of the amplitude is absorbed into $\langle O(\mu_f)\rangle$. Writing

$$c^{\rm eff}(\mu_f)=c(\mu)g(\mu,\mu_f),\tag{2.3}$$

the effective Wilson coefficient will be scheme and μ-scale independent. Of course, it appears that the μ-scale problem with naive factorization is traded in by the μ_f-scale problem. Nevertheless, once the factorization scale at which we apply the factorization approximation to matrix elements is fixed, the physical amplitude is independent of the choice of μ. More importantly, the effective Wilson coefficients are γ_5-scheme independent. In principle, one can work with any quark configuration, on-shell or off-shell, to compute the full amplitude. Note that if external quarks are off-shell and if the off-shell quark momentum is chosen as the infrared cutoff, $g(\mu,\mu_f)$ will depend on the gauge of the gluon field [9]. But this is not a problem at all as the gauge dependence belongs to the infrared structure of the wave function. However, if factorization is applied to $\langle O(\mu_f)\rangle$, the information of the gauge dependence characterized by the wave function will be lost. Hence, as stressed in [10,8], in order to apply factorization to matrix elements and in the meantime avoid the gauge problem connected with effective Wilson coefficients, one must work in the on-shell scheme to obtain gauge-invariant and infrared finite $c_i^{\rm eff}$ and then apply factorization to $\langle O(\mu_f)\rangle$ afterwards. Of course, the physics should be μ_f independent. In the formalism of the perturbative QCD factorization theorem, the nonperturbative meson wave functions are specified with the dependence of the factorization scale μ_f [10]. These wave functions are universal for all decay processes involving the same mesons. Hence, a consistent evaluation of hadronic matrix elements will eventually resort to the above-mentioned meson wave functions with μ_f dependence.

In general, the scheme- and μ-scale-independent effective Wilson coefficients have the form [11,12]

$$\begin{aligned}c_i^{\rm eff}(\mu_f)=c_i(\mu)+\frac{\alpha_s}{4\pi}\left(\gamma_V^T\ln\frac{\mu_f}{\mu}+\hat{r}_V^T\right)_{ij}c_j(\mu)\\ +\text{penguin-type corrections},\end{aligned}\tag{2.4}$$

where μ_f is the factorization scale arising from the dimensional regularization of infrared divergence [10], and the anomalous dimension matrix γ_V as well as the constant matrix $\hat{r}_V$ arise from the vertex-type corrections to four-quark operators. Note that in the dimensional regularization scheme the matrix $\hat{r}_V$ depends on the definition of γ_5. The infrared pole is consistently absorbed into universal bound-state wave functions. The expressions for the *gauge-invariant* constant matrix $\hat{r}_V$ in the naive dimensional regularization (NDR) and 't Hooft–Veltman (HV) renormalization schemes can be found in Eqs. (2.18) and (2.19), respectively, of [8]. However, the 66 and 88 entries of $\hat{r}_{\rm NDR}$ and $\hat{r}_{\rm HV}$ shown in [8] are erroneous: $(\hat{r}_{\rm NDR})_{66}$ and $(\hat{r}_{\rm NDR})_{88}$ should read 17 instead of 1, while $(\hat{r}_{\rm HV})_{66}$ and $(\hat{r}_{\rm HV})_{88}$ should read 47/3 rather than $-1/3$. This will affect the effective Wilson coefficients $c_6^{\rm eff}$ and $c_8^{\rm eff}$ (see Table I). For example, we have ${\rm Re}\,c_6^{\rm eff}\approx-0.060$ instead of the value -0.048 given in [8]. It should be stressed that the constant matrix $\hat{r}_V$ arising from vertexlike corrections is *not* arbitrary due to the infrared finiteness of vertexlike diagrams: The infrared divergences in individual vertex-type diagrams cancel in their sum.

It is known that the effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations $a_{2i}=c_{2i}^{\rm eff}+(1/N_c)c_{2i-1}^{\rm eff}$ and $a_{2i-1}=c_{2i-1}^{\rm eff}+(1/N_c)c_{2i}^{\rm eff}$ ($i=1,\dots,5$). Phenomenologically, the number of colors N_c is often treated as a free parameter to model the nonfactorizable contribution to hadronic matrix elements and its value can be extracted from the data of two-body nonleptonic decays. As shown in [13–15], nonfactorizable effects in the decay amplitudes of $B\to PP$, VP can be absorbed into the parameters $a_i^{\rm eff}$. This amounts to replacing N_c in $a_i^{\rm eff}$ by $(N_c^{\rm eff})_i$. Explicitly,

$$\begin{aligned}a_{2i}^{\rm eff}&=c_{2i}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i}}c_{2i-1}^{\rm eff},\\ a_{2i-1}^{\rm eff}&=c_{2i-1}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i-1}}c_{2i}^{\rm eff},\quad(i=1,\dots,5),\end{aligned}\tag{2.5}$$

TABLE I. Numerical values of the gauge-invariant effective Wilson coefficients $c_i^{\rm eff}$ for $b\to s$, $b\to d$ and $\bar{b}\to\bar{d}$ transitions evaluated at $\mu_f=m_b$ and $k^2=m_b^2/2$, where use of $|V_{ub}/V_{cb}|=0.09$ has been made. The numerical results are insensitive to the unitarity angle γ.

	$b\to s,\ \bar{b}\to\bar{s}$	$b\to d$	$\bar{b}\to\bar{d}$
$c_1^{\rm eff}$	1.169	1.169	1.169
$c_2^{\rm eff}$	-0.367	-0.367	-0.367
$c_3^{\rm eff}$	$0.0227+i0.0045$	$0.0223+i0.0041$	$0.0225+i0.0050$
$c_4^{\rm eff}$	$-0.0463-i0.0136$	$-0.0450-i0.0122$	$-0.0458-i0.0151$
$c_5^{\rm eff}$	$0.0134+i0.0045$	$0.0130+i0.0041$	$0.0132+i0.0050$
$c_6^{\rm eff}$	$-0.0600-i0.0136$	$-0.0588-i0.0122$	$-0.0595-i0.0151$
$c_7^{\rm eff}/\alpha$	$-0.0311-i0.0367$	$-0.0286-i0.0342$	$-0.0301-i0.0398$
$c_8^{\rm eff}/\alpha$	0.070	0.070	0.070
$c_9^{\rm eff}/\alpha$	$-1.429-i0.0367$	$-1.426-i0.0342$	$-1.428-i0.0398$
$c_{10}^{\rm eff}/\alpha$	0.48	0.48	0.48

where

$$(1/N_c^{\rm eff})_i \equiv (1/N_c)+\chi_i, \tag{2.6}$$

with χ_i being the nonfactorizable terms, which receive contributions from nonfactorized vertex-type, penguin-type, and spectator corrections. In general, χ_i and $(N_c^{\rm eff})_i$ are complex. Recently, it has been shown in [7] that, in the heavy quark limit, all nonfactorizable diagrams are dominated by hard gluon exchange, while soft gluon effects are suppressed by factors of $\Lambda_{\rm QCD}/m_b$. In other words, the nonfactorized term is calculable as the expansion in α_s in the heavy quark limit.

In practical calculations we will set $\mu_f=m_b$ for the effective Wilson coefficients $c^{\rm eff}$. In the generalized factorization approach the μ_f dependence of the effective Wilson coefficients is compensated by that of the nonfactorized terms χ_i introduced above, so that the physical parameters $a_i^{\rm eff}$ are μ_f independent. If $c_i^{\rm eff}$ are fixed at a different factorization scale, say $\mu_f=2m_b$ or $m_b/2$, the corresponding χ_i are accordingly changed, but $a_i^{\rm eff}$ remain intact. Since χ_i or $(N_c^{\rm eff})_i$ are treated as free parameters yet to be extracted from the data and since what we need are the effective parameters $a_i^{\rm eff}$ which are μ_f independent, it becomes not very relevant to extract $(N_c^{\rm eff})_i$ at different factorization scales. By contrast, in order to compute the nonfactorized effects theoretically, say by perturbative QCD (PQCD), one has to know the meson wave functions at the factorization scale μ_f.

To proceed, we assume that χ_i are universal (i.e., process independent) in bottom decays (this amounts to assuming generalized factorization) and that nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators differ from that of $(V-A)(V-A)$ operators; that is, we shall assume that

$$\chi_{LL}\equiv\chi_1=\chi_2=\chi_3=\chi_4=\chi_9=\chi_{10},$$
$$\chi_{LR}\equiv\chi_5=\chi_6=\chi_7=\chi_8, \tag{2.7}$$

and $\chi_{LR}\neq\chi_{LL}$ or equivalently

$$N_c^{\rm eff}(LL)\equiv(N_c^{\rm eff})_1=(N_c^{\rm eff})_2=(N_c^{\rm eff})_3=(N_c^{\rm eff})_4=(N_c^{\rm eff})_9=(N_c^{\rm eff})_{10},$$
$$N_c^{\rm eff}(LR)\equiv(N_c^{\rm eff})_5=(N_c^{\rm eff})_6=(N_c^{\rm eff})_7=(N_c^{\rm eff})_8, \tag{2.8}$$

and $N_c^{\rm eff}(LR)\neq N_c^{\rm eff}(LL)$. As we shall see below, the data analysis and the theoretical study of nonleptonic rare B decays all indicate that $N_c^{\rm eff}(LR)>3>N_c^{\rm eff}(LL)$. In principle, $N_c^{\rm eff}$ can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body B decays, $N_c^{\rm eff}$ is expected to be process insensitive as supported by the data [8].

Although the nonfactorized effects in hadronic charmless B decays are in general small, $\chi\sim\mathcal{O}(0.15)$ [8], they are important for the coefficients a_2, a_3, and a_5. For example, there is a large cancellation between $c_2^{\rm eff}$ and $c_1^{\rm eff}/N_c$, so that even a small amount of χ will modify a_2 dramatically, recalling that $a_2=c_2^{\rm eff}+c_1^{\rm eff}(1/N_c+\chi)$. Consequently, the aforementioned coefficients are very sensitive to the change of $N_c^{\rm eff}$, and moreover a_2 as well as a_5 have a minimum at $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)=3$. Therefore, nonfactorized contributions are important to the class-II modes, e.g., $B^0\to\pi^0\pi^0$, $\rho^0\pi^0$, $\omega\eta$, . . . , and to some decay modes which get contributions from the penguin terms (a_3+a_5), e.g., $B\to\omega K$. It is obvious that the nonfactorized effect in these decays cannot be simply absorbed into form factors. Another example has to do with the decays $B\to\phi K$ and $B\to K\eta'$. In the naive factorization approximation, the form factor F_0^{BK} has to be suppressed in order to accommodate the experimental limit on $B^-\to\phi K^-$. However, the enormously large rate of $B\to K\eta'$ demands a large F_0^{BK}. This difficulty is resolved if $N_c^{\rm eff}(LL)$ and $N_c^{\rm eff}(LR)$ are allowed to deviate from their naive value $N_c^{\rm eff}=3$, for example, $N_c^{\rm eff}(LL)\sim2$ and $N_c^{\rm eff}(LR)\sim6$ (see Sec. V). Hence, it is inevitable to take into account nonfactorized contributions to hadronic matrix elements in order to have a coherent picture for rare hadronic B decays.

III. $B\to\pi\pi$ AND πK DECAYS

Recently CLEO has made the first observation of the decay $B\to\pi^+\pi^-$ with the branching ratio [5]

$$\mathcal{B}(\overline{B}^0\to\pi^+\pi^-)=(4.3^{+1.6}_{-1.4}\pm 0.5)\times 10^{-6}. \tag{3.1}$$

This decay mode puts a stringent constraint on the form factor $F_0^{B\pi}$. Neglecting final-state interactions and employing the Wolfenstein parameters $\rho=0.175$, $\eta=0.370$, corresponding to $\gamma\equiv\mathrm{Arg}(V_{ub}^*)=65°$ and $|V_{ub}/V_{cb}|=0.09$, and the effective number of colors $N_c^{\mathrm{eff}}(LL)=2$ [see Sec. IV for a discussion of $N_c^{\mathrm{eff}}(LL)$], we find $F_0^{B\pi}(0)=0.20\pm 0.04$.[1] This value is substantially smaller than the form factor $F_0^{B\pi}(0)=0.333$ obtained by Bauer, Stech, and Wirbel (BSW) [19]. This has two important implications. First, the predicted decay rates of $B\to K\pi$, which are mainly governed by $F_0^{B\pi}$, will in general be smaller than the central values of experimental measurements [see Eq. (3.2)]. Second, the form factor $F_0^{BK}(0)$ will become smaller too. More specifically, it cannot exceed the value, say 0.33, otherwise the SU(3)-symmetry relation $F_0^{BK}=F_0^{B\pi}$ will be badly broken. Consequently, the predicted $K\eta'$ rates will become too small compared to experiment.

There are several possibilities that the $K\pi$ rates can be enhanced: (i) The unitarity angle γ larger than 90° will lead to a suppresion of $B\to\pi^+\pi^-$ [20,8], which in turn implies an enhancement of $F_0^{B\pi}$ and hence $K\pi$ rates. (ii) A large nonzero isospin $\pi\pi$ phase shift difference of order 70° [8] can yield a substantial suppression of the $\pi^+\pi^-$ mode. However, a large $\pi\pi$ isospin phase difference seems to be very unlikely due to the large energy released in charmless B decays. Indeed, the Regge analysis of [21] indicates $\delta_{\pi\pi}=11°$. (iii) Smaller quark masses, say $m_s(m_b)=65$ MeV, will make the $(S-P)(S+P)$ penguin terms contributing sizably to the $K\pi$ modes but less significantly to $\pi^+\pi^-$ as the penguin effect on the latter is suppressed by the quark mixing angles. Although some of new quenched and unquenched lattice calculations yield smaller m_s (see, e.g., [22]), the value $m_s(m_b)=65$ MeV or equivalently $m_s(1\ \mathrm{GeV})=100$ MeV is barely on the verge of the lower side of lattice results [22]. Therefore, the first possibility appears to be more plausible. Using the values $F_0^{B\pi}(0)=0.28$ and $\gamma=105°$, we find that the $\pi^+\pi^-$ decay is well accommodated (see Table II). As a consequence, the decay rates of $B\to K\pi$ governed by $F_0^{B\pi}$ are enhanced accordingly.

The CLEO collaboration has recently improved the measurements for the decays $B\to K^\pm\pi^\mp$, $B^\pm\to K^0\pi^\pm$, $B^\pm\to K^\pm\pi^0$ and observed for the first time the decay $\overline{B}^0\to\overline{K}^0\pi^0$, thus completing the set of four $K\pi$ branching ratio measurements [5]:

$$\mathcal{B}(\overline{B}^0\to K^-\pi^+)=(17.2^{+2.5}_{-2.4}\pm 1.2)\times 10^{-6},$$

$$\mathcal{B}(B^-\to\overline{K}^0\pi^-)=(18.2^{+4.6}_{-4.0}\pm 1.6)\times 10^{-6},$$

$$\mathcal{B}(B^-\to K^-\pi^0)=(11.6^{+3.0+1.4}_{-2.7-1.3})\times 10^{-6},$$

$$\mathcal{B}(\overline{B}^0\to\overline{K}^0\pi^0)=(14.6^{+5.9+2.4}_{-5.1-3.3})\times 10^{-6}, \tag{3.2}$$

which are to be compared with the 1998 results [23]:

$$\mathcal{B}(\overline{B}^0\to K^-\pi^+)=(14\pm 3\pm 2)\times 10^{-6},$$

$$\mathcal{B}(B^-\to\overline{K}^0\pi^-)=(14\pm 5\pm 2)\times 10^{-6},$$

$$\mathcal{B}(B^-\to K^-\pi^0)=(15\pm 4\pm 3)\times 10^{-6}. \tag{3.3}$$

It is known that $K\pi$ modes are penguin dominated. As far as the QCD penguin contributions are concerned, it will be expected that $\mathcal{B}(\overline{B}^0\to K^-\pi^+)\sim\mathcal{B}(B^-\to\overline{K}^0\pi^-)$ and $\mathcal{B}(B^-\to K^-\pi^0)\sim\mathcal{B}(\overline{B}^0\to\overline{K}^0\pi^0)\sim\frac{1}{2}\mathcal{B}(B\to K\pi^\pm)$. However, as pointed out in [8,20], the electroweak penguin diagram, which can be neglected in $\overline{K}^0\pi^-$ and $K^-\pi^+$, does play an essential role in the modes $K\pi^0$. With a moderate electroweak penguin contribution, the constructive (destructive) interference between electroweak and QCD penguins in $K^-\pi^0$ and $\overline{K}^0\pi^0$ renders the former greater than the latter; that is, $\mathcal{B}(B^-\to K^-\pi^0)>\frac{1}{2}\mathcal{B}(\overline{B}^0\to\overline{K}^0\pi^-)$ and $\mathcal{B}(\overline{B}^0\to\overline{K}^0\pi^0)<\frac{1}{2}\mathcal{B}(\overline{B}^0\to K^-\pi^+)$ are anticipated. For numerical calculations we use the parameters

$$m_u(m_b)=2.13\ \ \mathrm{MeV},\quad m_d(m_b)=4.27\ \ \mathrm{MeV},$$

$$m_s(m_b)=85\ \ \mathrm{MeV},$$

$$F_0^{B\pi}(0)=0.28,\quad F_0^{BK}(0)=0.36,\quad \gamma=105°,$$

$$N_c^{\mathrm{eff}}(LL)=2,\quad N_c^{\mathrm{eff}}(LR)=6. \tag{3.4}$$

We see from Table II that, except for the decay $\overline{K}^0\pi^0$, the agreement of the calculated branching ratios for $K\pi$ modes with experiment is excellent. By contrast, the central value of $\mathcal{B}(\overline{B}^0\to\overline{K}^0\pi^0)$ is much greater than the theoretical expectation. Since its experimental error is large, one has to await the experimental improvement to clarify the issue. The predicted pattern

$$K^-\pi^+\gtrsim\overline{K}^0\pi^-\sim\frac{3}{2}K^-\pi^0\sim 3\,\overline{K}^0\pi^0 \tag{3.5}$$

is in good agreement with experiment for the first three decays.

We would like to make a remark on the trail of having $\cos\gamma<0$. The suggestion of $\gamma>90°$ or a negative Wolfenstein parameter ρ was originally motivated by the 1998 $K\pi$ data which indicated nearly equal branching ratios for the three modes $K^-\pi^+$, $\overline{K}^0\pi^-$ and $K^-\pi^0$. It was pointed out in [24] that $\cos\gamma<0$ as well as a large m_s, say $m_s(m_b)=200$

[1] It was argued in [16] that a small value $|V_{ub}/V_{cb}|\approx 0.06$ is preferred by the $\pi^+\pi^-$ measurement with the form factor $F_0^{B\pi}(0)$ being fixed to be 0.33. However, this Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}/V_{cb}|$ is smaller than the recent LEP average $0.104^{+0.015}_{-0.018}$ [17] and the CLEO result $0.083^{+0.015}_{-0.016}$ [18].

TABLE II. Branching ratios (in units of 10^{-6}) averaged over CP-conjugate modes for charmless $B_{u,d} \to PP$ decays. Predictions are made for $k^2=m_b^2/2$, $\sqrt{\rho^2+\eta^2}=0.41$, $\gamma=105°$, and $N_c^{\rm eff}(LR)=2,3,6,\infty$ with $N_c^{\rm eff}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_c^{\rm eff}(LR)$ in the second case. Classification of decay amplitudes is described in detail in [8]. Results using the improved light-cone sum rule (LCSR′) and the BSW model for heavy-to-light form factors are shown in the upper and lower entries, respectively. Experimental values (in units of 10^{-6}) are taken from [1–5,28]. Our preferred predictions for branching ratios are those using LCSR′ form factors, $N_c^{\rm eff}(LL)=2$ and $N_c^{\rm eff}(LR)=6$.

		$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$				
Decay	Class	2	3	6	∞	2	3	6	∞	Expt.
$\overline{B}_d^0\to\pi^+\pi^-$	I	5.98	5.95	5.92	5.90	5.98	6.79	7.69	8.58	$4.3^{+1.6}_{-1.4}\pm0.5$
		8.31	8.27	8.23	8.20	8.31	9.44	10.6	11.9	
$\overline{B}_d^0\to\pi^0\pi^0$	II,VI	0.54	0.56	0.58	0.60	0.54	0.22	0.18	0.43	<9.3
		0.75	0.78	0.80	0.83	0.75	0.31	0.26	0.60	
$\overline{B}_d^0\to\eta\eta$	II,VI	0.13	0.14	0.16	0.18	0.13	0.18	0.37	0.69	<18
		0.18	0.20	0.23	0.26	0.18	0.25	0.52	0.98	
$\overline{B}_d^0\to\eta\eta'$	II,VI	0.11	0.16	0.24	0.34	0.11	0.12	0.32	0.69	<27
		0.16	0.23	0.34	0.48	0.16	0.18	0.45	0.98	
$\overline{B}_d^0\to\eta'\eta'$	II,VI	0.03	0.05	0.09	0.14	0.03	0.02	0.06	0.17	<47
		0.04	0.07	0.12	0.21	0.04	0.02	0.09	0.24	
$B^-\to\pi^-\pi^0$	III	5.92	5.92	5.92	5.92	5.92	4.68	3.58	2.64	<12.7
		8.23	8.23	8.23	8.24	8.23	6.50	4.98	3.67	
$B^-\to\pi^-\eta$	III	2.64	2.65	2.66	2.68	2.64	2.05	1.60	1.30	<5.7
		3.68	3.69	3.71	3.73	3.68	2.85	2.22	1.80	
$B^-\to\pi^-\eta'$	III	1.79	1.75	1.77	1.86	1.79	1.30	0.91	0.62	<12
		2.51	2.45	2.48	2.62	2.51	1.81	1.26	0.85	
$\overline{B}_d^0\to K^-\pi^+$	IV	16.9	17.7	18.6	19.5	16.9	18.5	20.2	22.0	$17.2^{+2.5}_{-2.4}\pm1.2$
		23.7	24.9	26.1	27.4	23.7	26.0	28.4	30.9	
$B^-\to\overline{K}^0\pi^-$	IV	15.2	16.1	17.0	17.9	15.2	17.5	20.0	22.6	$18.2^{+4.6}_{-4.0}\pm1.6$
		21.4	22.6	23.8	25.1	21.4	24.6	28.0	31.7	
$B^-\to K^-K^0$	IV	1.76	1.86	1.97	2.07	1.76	2.02	2.31	2.61	<5.1
		1.97	2.08	2.19	2.31	1.97	2.26	2.57	2.91	
$\overline{B}_d^0\to\overline{K}^0\pi^0$	VI	5.27	5.63	6.02	6.41	5.27	6.20	7.25	8.39	$14.6^{+5.9+2.4}_{-5.1-3.3}$
		7.66	8.18	8.73	9.28	7.66	9.00	10.5	12.1	
$\overline{B}_d^0\to K^0\overline{K}^0$	VI	1.65	1.75	1.85	1.95	1.65	1.90	2.17	2.45	<17
		1.85	1.95	2.06	2.17	1.85	2.12	2.42	2.73	
$\overline{B}_d^0\to\pi^0\eta$	VI	0.36	0.40	0.45	0.50	0.36	0.41	0.47	0.54	<2.9
		0.50	0.56	0.63	0.69	0.50	0.58	0.66	0.75	
$\overline{B}_d^0\to\pi^0\eta'$	VI	0.11	0.20	0.33	0.48	0.11	0.12	0.14	0.17	<5.7
		0.15	0.29	0.46	0.69	0.15	0.17	0.20	0.24	
$\overline{B}_d^0\to\overline{K}^0\eta$	VI	1.74	1.57	1.40	1.24	1.74	2.38	3.13	4.01	<9.3
		1.30	1.12	0.96	0.81	1.30	1.83	2.45	3.26	
$\overline{B}_d^0\to\overline{K}^0\eta'$	VI	25.8	36.0	47.9	61.6	25.8	27.9	30.1	32.4	$89^{+18}_{-16}\pm9$
		31.4	43.4	57.5	73.5	31.4	34.0	36.8	39.6	
$B^-\to K^-\pi^0$	VI	11.6	12.1	12.6	13.2	11.6	12.4	13.3	14.2	$11.6^{+3.0+1.4}_{-2.7-1.3}$
		15.9	16.6	17.3	18.0	15.9	17.0	18.2	19.5	
$B^-\to K^-\eta$	VI	1.75	1.59	1.43	1.28	1.75	2.44	3.29	4.32	<6.9
		1.37	1.21	1.07	0.94	1.37	1.93	2.67	3.58	
$B^-\to K^-\eta'$	VI	28.8	39.9	52.8	67.5	28.8	30.6	32.5	34.5	$80^{+10}_{-9}\pm7$
		35.2	48.3	63.5	80.8	35.2	37.5	39.8	42.3	

MeV, will allow a substantial rise of $K^-\pi^0$ and a suppression of QCD penguin contributions so that $K^-\pi^0\simeq K^-\pi^+$ can be accounted for. The 1999 data [5] show that $K^-\pi^0 \simeq \frac{2}{3}K^-\pi^+$, in accordance with the theoretical anticipation. The motivation for having a negative cos γ this time is somewhat different: It provides a simply way for accommodating

TABLE III. Form factors at zero momentum transfer for $B\to P$ and $B\to V$ transitions evaluated in the BSW model [19]. The values given in the square brackets are obtained in the light-cone sum rule (LCSR) analysis [25]. We have assumed SU(3) symmetry for the $B\to\omega$ form factors in the LCSR approach. In realistic calculations we use Eq. (3.13) of [8] for $B\to\eta^{(\prime)}$ form factors. For later purposes, we will use the improved LCSR model (LCSR′) for form factors, which is the same as the LCSR of [25] except for the values of $F_0^{B\pi}(0)$ and $F_0^{BK}(0)$ being replaced by those given in Eq. (3.4).

Decay	$F_1=F_0$	V	A_1	A_2	$A_3=A_0$
$B\to\pi^\pm$	0.333 [0.305]				
$B\to K$	0.379 [0.341]				
$B\to\eta$	0.168 [—]				
$B\to\eta'$	0.114 [—]				
$B\to\rho^\pm$		0.329 [0.338]	0.283 [0.261]	0.283 [0.223]	0.281 [0.372]
$B\to\omega$		0.232 [0.239]	0.199 [0.185]	0.199 [0.158]	0.198 [0.263]
$B\to K^*$		0.369 [0.458]	0.328 [0.337]	0.331 [0.283]	0.321 [0.470]

the suppression of $\pi^+\pi^-$ and nonsuppression of $K\pi$ data without having too small light quark masses or too large $\pi\pi$ final-state interactions or too small CKM matrix element V_{ub}.

Finally, as pointed out in [8], the branching ratio of $K^{*-}\pi^+$ predicted to be of order 0.5×10^{-5} is smaller than that of $K^-\pi^+$ owing to the absence of the a_6 penguin term in the former. The observation $\mathcal{B}(\overline{B}^0\to K^{*-}\pi^+)=(22^{+8+4}_{-6-5})\times10^{-6}$ [1] is thus strongly opposite to the theoretical expectation. Clearly, it is important to have a refined measurement of this mode.

IV. TREE-DOMINATED CHARMLESS *B* DECAYS

CLEO has observed several tree-dominated charmless B decays which proceed at the tree level through the b quark decay $b\to u\bar{u}d$ and at the loop level via the $b\to d$ penguin diagrams: $B\to\pi^+\pi^-$, $\rho^0\pi^\pm$, $\omega\pi^\pm$, $\rho^\pm\pi^\mp$. The first three modes have been measured recently for the first time with the branching ratios [3,2]:

$$\mathcal{B}(B^\pm\to\rho^0\pi^\pm)=(10.4^{+3.3}_{-3.4}\pm2.1)\times10^{-6},$$

$$\mathcal{B}(B^\pm\to\omega\pi^\pm)=(11.3^{+3.3}_{-2.9}\pm1.5)\times10^{-6}, \tag{4.1}$$

and Eq. (3.1). These decays are sensitive to the form factors $F_0^{B\pi}$, $A_0^{B\rho}$, $A_0^{B\omega}$, and to the value of $N_c^{\rm eff}(LL)$. To illustrate the sensitivity on form factors, we consider two different form-factor models for heavy-to-light transitions: the BSW model [19] and the light-cone sum rule (LCSR) model [25]. The relevant form factors at zero momentum transfer are listed in Table III. We see from Table IV that the branching ratios of $\rho^0\pi^\pm$ and $\omega\pi^\pm$ decrease with $N_c^{\rm eff}(LL)$ and generally they become too small compared to the data when $N_c^{\rm eff}(LL)>3$, whereas $\mathcal{B}(B\to\pi^+\pi^-)$ increases with $N_c^{\rm eff}(LL)$ and becomes too large when $N_c^{\rm eff}(LL)>3$. More precisely, we obtain $1.1\leqslant N_c^{\rm eff}(LL)\leqslant2.6$ from $\rho^0\pi^\pm$ and $\omega\pi^\pm$ modes. Evidently, $N_c^{\rm eff}(LL)$ in all these tree-dominated decays are preferred to be smaller. This is indeed what is expected since the effective number of colors, $N_c^{\rm eff}(LL)$, inferred from the Cabibbo-allowed decays $B\to(D,D^*)(\pi,\rho)$ is in the vicinity of 2 [27], and since the energy released in the energetic two-body charmless B decays is in general slightly larger than that in $B\to D\pi$ decays, it is thus anticipated that

$$|\chi(\text{two-body rare B decay})|\lesssim|\chi(B\to D\pi)|, \tag{4.2}$$

and hence $N_c^{\rm eff}(LL)\approx N_c^{\rm eff}(B\to D\pi)\sim2$.

Note that the branching ratio of $\rho^0\pi^\pm$ is sensitive to the change of the unitarity angle γ, while $\omega\pi^\pm$ is not. For example, we have $\mathcal{B}(B^\pm\to\rho^0\pi^\pm)\sim\mathcal{B}(B^\pm\to\omega\pi^\pm)$ for $\gamma\sim65°$, and $\mathcal{B}(B^\pm\to\rho^0\pi^\pm)>\mathcal{B}(B^\pm\to\omega\pi^\pm)$ for $\gamma>90°$. It appears that a unitarity angle γ larger than 90°, which is preferred by the previous measurement [2] $\mathcal{B}(B^\pm\to\rho^0\pi^\pm)=(15\pm5\pm4)\times10^{-6}$, is no longer strongly favored by the new data of $\rho^0\pi^\pm$.

It is worth remarking that although the decays $B\to\rho^\pm\pi^\mp$ are sensitive to $N_c^{\rm eff}(LL)$, no useful constraint can be extracted at this moment from the present measurement [3]: $\mathcal{B}(\overline{B}^0\to\rho^+\pi^-+\rho^-\pi^+)=(27.6^{+8.4}_{-7.4}\pm4.2)\times10^{-6}$ due to its large error.

From Tables III and IV it is also clear that the form factor $A_0^{B\omega}$ predicted by the LCSR, which are substantially larger than that of the BSW model, is more favored. Since the form factor $F_0^{B\pi}$ in the light cone sum rule (LCSR) model is slightly big, we will employ the improved LCSR model (called LCSR′), which is the same as the LCSR model of [25] except that the values of $F_0^{B\pi}(0)$ and $F_0^{BK}(0)$ are replaced by those given in Eq. (3.4), in ensuing calculations.

V. $B\to K\eta'$ AND $K^*\eta$ DECAYS

The improved measurements of the decays $B\to\eta' K$ by CLEO [4]

$$\mathcal{B}(B^\pm\to\eta' K^\pm)=(80^{+10}_{-\ 9}\pm7)\times10^{-6},$$

$$\mathcal{B}(B^0\to\eta' K^0)=(89^{+18}_{-16}\pm9)\times10^{-6}, \tag{5.1}$$

are larger than the previously published results [28]:

TABLE IV. Same as Table II except for charmless $B_{u,d}\to VP$ decays.

		$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$				
Decay	Class	2	3	6	∞	2	3	6	∞	Expt.
$\overline{B}_d^0\to\rho^-\pi^+$	I	18.5	18.5	18.5	18.5	18.5	21.0	23.6	26.4	
		26.5	26.5	26.5	26.5	26.5	30.0	33.8	37.7 }	$27.6^{+8.4}_{-7.4}\pm4.2$
$\overline{B}_d^0\to\rho^+\pi^-$	I	14.8	14.6	14.8	14.9	14.8	16.3	18.3	20.3 }	
		8.54	8.34	8.43	8.51	8.54	9.32	10.4	11.6	
$\overline{B}_d^0\to\rho^+K^-$	I,IV	2.27	2.59	2.96	3.36	2.27	2.45	2.64	2.85	<25
		1.31	1.49	1.70	1.93	1.31	1.41	1.52	1.64	
$\overline{B}_d^0\to\rho^0\pi^0$	II	0.73	0.72	0.71	0.70	0.73	0.01	0.54	2.33	<5.1
		0.80	0.79	0.78	0.77	0.80	0.02	0.50	2.22	
$\overline{B}_d^0\to\omega\pi^0$	II	0.16	0.07	0.02	0.003	0.16	0.07	0.02	0.01	<5.8
		0.19	0.10	0.05	0.04	0.19	0.09	0.07	0.14	
$\overline{B}_d^0\to\omega\eta$	II	0.32	0.35	0.39	0.44	0.32	0.004	0.28	1.14	<12
		0.30	0.32	0.35	0.41	0.30	0.02	0.29	1.13	
$\overline{B}_d^0\to\omega\eta'$	II	0.30	0.28	0.25	0.23	0.30	0.04	0.15	0.66	<60
		0.23	0.24	0.23	0.23	0.23	0.01	0.15	0.66	
$\overline{B}_d^0\to\rho^0\eta$	II	0.002	0.002	0.003	0.003	0.002	0.002	0.003	0.006	<10
		0.05	0.05	0.05	0.05	0.05	0.01	0.02	0.10	
$\overline{B}_d^0\to\rho^0\eta'$	II,VI	0.04	0.01	0.01	0.05	0.04	0.05	0.06	0.08	<23
		0.02	0.02	0.04	0.08	0.02	0.02	0.05	0.12	
$B^-\to\rho^-\pi^0$	III	13.2	13.3	13.3	13.3	13.2	12.0	10.9	9.77	<77
		17.1	17.1	17.2	17.2	17.1	16.8	16.5	16.2	
$B^-\to\rho^0\pi^-$	III	13.1	13.2	13.3	13.4	13.1	9.98	7.29	5.04	$10.4^{+3.3}_{-3.4}\pm2.1$
		9.64	9.69	9.76	9.82	9.64	6.14	3.46	1.61	
$B^-\to\omega\pi^-$	III	9.56	10.1	10.8	11.5	9.56	7.80	6.31	5.09	$11.3^{+3.3}_{-2.9}\pm1.5$
		6.25	6.71	7.27	7.93	6.25	4.20	2.64	1.57	
$B^-\to\rho^-\eta$	III	9.46	9.70	10.0	10.3	9.46	7.95	6.62	5.41	<15
		11.9	11.8	12.1	12.3	11.9	10.8	10.0	9.25	
$B^-\to\rho^-\eta'$	III	5.49	4.88	4.45	4.20	5.49	4.27	3.21	2.31	<47
		6.97	6.43	6.00	5.67	6.97	6.16	5.40	4.69	
$B^-\to\rho^0K^-$	III,VI	0.86	0.90	0.97	1.05	0.86	0.63	0.46	0.30	<22
		0.57	0.56	0.57	0.59	0.57	0.34	0.18	0.07	
$\overline{B}_d^0\to K^{*-}\pi^+$	IV	5.63	5.63	5.63	5.63	5.63	6.27	6.95	7.66	22^{+8+4}_{-6-5}
		8.13	8.13	8.13	8.13	8.13	9.06	10.3	11.1	
$B^-\to\overline{K}^{*0}\pi^-$	IV	3.41	3.41	3.41	3.41	3.41	4.34	5.38	6.54	<27
		4.93	4.93	4.93	4.93	4.93	6.27	7.78	9.45	
$B^-\to K^{*0}K^-$	IV	0.38	0.39	0.39	0.39	0.38	0.49	0.61	0.74	<12
		0.43	0.44	0.44	0.44	0.43	0.56	0.69	0.84	
$B^-\to K^{*-}K^0$	IV	0.26	0.30	0.35	0.41	0.26	0.23	0.20	0.18	–
		0.12	0.14	0.17	0.19	0.12	0.11	0.09	0.08	
$\overline{B}_d^0\to\phi\pi^0$	V	0.02	0.001	0.01	0.03	0.02	0.002	0.05	0.17	<5.4
		0.03	0.002	0.01	0.04	0.03	0.003	0.08	0.25	
$\overline{B}_d^0\to\phi\eta$	V	0.01	0.001	0.003	0.02	0.01	0.001	0.03	0.09	<9
		0.02	0.001	0.004	0.02	0.02	0.002	0.04	0.14	
$\overline{B}_d^0\to\phi\eta'$	V	0.006	0.0005	0.002	0.01	0.006	0.001	0.02	0.06	<31
		0.01	0.001	0.002	0.01	0.01	0.001	0.03	0.09	
$B^-\to\phi\pi^-$	V	0.04	0.003	0.01	0.06	0.04	0.005	0.11	0.37	<4.0
		0.06	0.005	0.01	0.09	0.06	0.01	0.17	0.54	
$\overline{B}_d^0\to K^{*0}\overline{K}^0$	VI	0.36	0.36	0.36	0.36	0.36	0.46	0.57	0.69	–
		0.41	0.41	0.41	0.41	0.41	0.52	0.64	0.78	

TABLE IV. (*Continued.*)

Decay	Class	$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$				Expt.
		2	3	6	∞	2	3	6	∞	
$\overline{B}_d^0\to\overline{K}^{*0}K^0$	VI	0.24	0.28	0.33	0.38	0.24	0.21	0.19	0.16	–
		0.11	0.13	0.15	0.18	0.11	0.10	0.09	0.08	
$\overline{B}_d^0\to\overline{K}^{*0}\pi^0$	VI	0.67	0.67	0.68	0.69	0.67	0.93	1.28	1.74	<4.2
		1.43	1.45	1.45	1.46	1.43	1.93	2.52	3.21	
$\overline{B}_d^0\to\rho^0\overline{K}^0$	VI	2.92	3.32	3.68	4.07	2.92	2.74	2.58	2.50	<27
		2.23	2.50	2.75	3.01	2.23	2.10	2.01	2.00	
$\overline{B}_d^0\to\omega\overline{K}^0$	VI	0.33	0.16	1.55	4.55	0.33	0.71	5.01	13.3	<21
		0.83	0.04	0.89	3.45	0.83	0.34	4.37	13.0	
$\overline{B}_d^0\to\overline{K}^{*0}\eta$	VI	7.28	8.84	10.5	12.4	7.28	6.90	6.53	6.20	$13.8^{+5.5}_{-4.6}\pm1.6$
		5.49	6.42	7.40	8.45	5.49	5.64	5.78	5.94	
$\overline{B}_d^0\to\overline{K}^{*0}\eta'$	VI	4.18	1.91	0.52	0.41	4.18	4.98	5.41	5.88	<24
		1.10	0.43	0.19	0.56	1.10	1.28	1.30	1.33	
$\overline{B}_d^0\to\phi\overline{K}^0$	VI	11.3	7.15	3.97	1.74	11.3	5.87	2.22	0.32	<28
		13.0	8.23	4.57	2.00	13.0	6.76	2.56	0.37	
$B^-\to K^{*-}\pi^0$	VI	4.46	4.45	4.43	4.42	4.46	4.68	4.92	5.17	<800
		5.50	5.50	5.48	5.47	5.50	5.90	6.32	6.76	
$B^-\to\rho^-\overline{K}^0$	VI	2.53	2.99	3.49	4.02	2.53	2.27	2.02	1.78	<140
		1.46	1.72	2.01	2.32	1.46	1.31	1.16	1.03	
$B^-\to\phi K^-$	VI	11.6	7.71	4.28	1.87	11.6	6.33	2.39	0.35	<5.9
		13.2	8.87	4.92	2.16	13.2	7.29	2.75	0.40	
$B^-\to K^{*-}\eta$	VI	9.60	11.4	13.3	15.4	9.60	8.51	7.50	6.57	$26.4^{+9.6}_{-8.2}\pm3.3$
		7.93	9.01	10.2	11.4	7.93	7.45	6.99	6.55	
$B^-\to K^{*-}\eta'$	VI,III	4.93	2.16	0.76	0.71	4.93	5.61	6.36	7.18	<35
		1.43	0.67	0.57	1.12	1.43	1.58	1.75	1.95	
$B^-\to\omega K^-$	VI,III	1.50	0.78	1.79	4.53	1.50	1.04	5.30	14.3	<8.0
		1.87	0.56	1.09	3.46	1.87	0.55	4.53	13.8	

$$\mathcal{B}(B^\pm\to\eta' K^\pm)=(65^{+15}_{-14}\pm9)\times10^{-6},$$
$$\mathcal{B}(B^0\to\eta' K^0)=(47^{+27}_{-20}\pm9)\times10^{-6}. \qquad (5.2)$$

This year CLEO has also reported the new measurement of $B\to K^*\eta$ with the branching ratios [4]

$$\mathcal{B}(B^\pm\to\eta K^{*\pm})=(26.4^{+9.6}_{-8.2}\pm3.3)\times10^{-6},$$
$$\mathcal{B}(B^0\to\eta K^{*0})=(13.8^{+5.5}_{-4.4}\pm1.6)\times10^{-6}. \qquad (5.3)$$

Theoretically, the branching ratios of $K\eta'$ ($K^*\eta$) are anticipated to be much greater than $K\pi$ ($K^*\eta'$) modes owing to the presence of constructive interference between two penguin amplitudes arising from nonstrange and strange quarks of the η' or η.[2] In general, the decay rates of $K\eta'$ increase slowly with $N_c^{\rm eff}(LR)$ if $N_c^{\rm eff}(LL)$ is treated to be the same as $N_c^{\rm eff}(LR)$, but fast enough with $N_c^{\rm eff}(LR)$ if $N_c^{\rm eff}(LL)$ is fixed at the value of 2. Evidently, the data much favor the latter case (see Table II).[3] On the contrary, the branching ratios of $K^*\eta$ in general decrease with $N_c^{\rm eff}(LR)$ when $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$ but increase with $N_c^{\rm eff}(LR)$ when $N_c^{\rm eff}(LL)=2$. Again, the latter is preferred by experiment. Hence, the data of both $K\eta'$ and $K^*\eta$ provide another strong support for a small $N_c^{\rm eff}(LL)$ and for the relation $N_c^{\rm eff}(LR)>N_c^{\rm eff}(LL)$. In other words, the nonfactorized effects due to $(V-A)(V-A)$ and $(V-A)(V+A)$ operators should be treated differently.

It appears from Tables II and IV that the data of $K^*\eta$ and in particular $K\eta'$ are well accommodated by $N_c^{\rm eff}(LR)=\infty$. However, we have argued in [8] that $N_c^{\rm eff}(LR)\lesssim6$. In principle, the value of $N_c^{\rm eff}(LR)$ can be extracted from the decays $B\to\phi K$ and ϕK^*. The present limit [1,2]

$$\mathcal{B}(B^\pm\to\phi K^\pm)<0.59\times10^{-5} \qquad (5.4)$$

[2]In a recent analysis [26], the branching ratio of $K^*\eta'$ is predicted to be similar to that of $K^*\eta$, whereas it is found not to exceed 1×10^{-6} according to [11] and the present paper.

[3]As stressed in [8], the contribution from the η' charm content will make the theoretical prediction even worse at the small values of $1/N_c^{\rm eff}$ if $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$! On the contrary, if $N_c^{\rm eff}(LL)\approx2$, the $c\bar{c}$ admixture in the η' will always lead to a constructive interference irrespective of the value of $N_c^{\rm eff}(LR)$.

at 90% C.L. implies that

$$N_c^{\rm eff}(LR)\geqslant\begin{cases}5.0 & \text{BSW},\\ 4.2 & \text{LCSR}',\end{cases}\tag{5.5}$$

with $N_c^{\rm eff}(LL)$ being fixed at the value of 2. Note that this constraint is subject to the corrections from spacelike penguin and W-annihilation contributions. At any rate, it is safe to conclude that $N_c^{\rm eff}(LR)>3>N_c^{\rm eff}(LL)$.

Since the penguin matrix elements of scalar and pseudoscalar densities are sensitive to the strange quark mass, the discrepancy between theory and experiment, especially for $K\eta'$, can be further improved by using an even smaller m_s, say $m_s(m_b)=65$ MeV. However, as remarked in Sec. III, this small strange quark mass is not favored by lattice calculations. Moreover, it will lead to too large $B\to K\pi$ rates. For example, the predicted $\mathcal{B}(\overline{B}^0\to K^-\pi^+)=28\times10^{-6}$ using $m_s(m_b)=65$ MeV is too large compared to the observed branching ratio $(17.2^{+2.5}_{-2.4}\pm1.2)\times10^{-6}$.

Several new mechanisms have been proposed in the past few years to explain the observed enormously large rate of $K\eta'$, for example, the large charm content of the η' [29] or the two-gluon fusion mechanism via the anomaly coupling of the η' with two gluons [30,31]. These mechanisms will in general predict a large rate for $K^*\eta'$ comparable to or even greater than $K\eta'$ and a very small rate for $K^*\eta$ and $K\eta$. The fact that the $K^*\eta$ modes are observed with sizeable branching ratios indicates that it is the constructive interference of two comparable penguin amplitudes rather than the mechanism specific to the η' that accounts for the bulk of $B\to\eta' K$ and ηK^* branching ratios.

Two remarks are in order. First, as shown in [8], the charged $\eta' K^-$ mode gets enhanced when $\cos\gamma$ becomes negative, while the neutral $\eta' K^0$ mode remains steady. Therefore, it is important to see if the disparity between $\eta' K^\pm$ and $\eta' K^0$ is confirmed when experimental errors are improved and refined in the future. Second, we see from Table IV that the form factor $A_0^{BK^*}(0)$ entering the decay amplitude of $B\to K^*\eta$ is preferred to be larger than the value predicted by the BSW model.

The observation $N_c^{\rm eff}(LL)<3<N_c^{\rm eff}(LR)$ is consistent with a recent perturbative QCD calculation of $B\to\pi\pi$ decays in the heavy quark limit. Following the notation of [7], we find the nonfactorized terms:[4]

$$\chi_1=\chi_2=\chi_3=\chi_4=-\chi_5=-\chi_6=\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}(f^{\rm I}+f^{\rm II}).\tag{5.8}$$

It follows from Eq. (2.7) that

$$\chi_{LR}=-\chi_{LL}=-\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}(f^{\rm I}+f^{\rm II}).\tag{5.9}$$

Several remarks are in order. (i) Since $f^{\rm I}$ is complex due to final-state interactions via hard gluon exchange [7], so are χ_i

[4]Note that Eqs. (4)–(8) in [7] can be reproduced from Eqs. (2.12)–(2.19) and (4.1) in [8] with the nonfactorized terms given by Eq. (5.8). For example, from [8] we obtain in the NDR scheme (the superscript ''eff'' of a_i is dropped for convenience) that

$$\begin{aligned}
a_2&=c_2+\frac{c_1}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_1\left(12\ln\frac{m_b}{\mu}-18\right)+c_1^{\rm eff}\chi_2,\\
a_4&=c_4+\frac{c_3}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_3\left(12\ln\frac{m_b}{\mu}-18\right)+c_3^{\rm eff}\chi_4+\frac{\alpha_s}{9\pi}(C_t+C_p+C_g),\\
a_5&=c_5+\frac{c_6}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_6\left(-12\ln\frac{m_b}{\mu}+6\right)+c_6^{\rm eff}\chi_5,
\end{aligned}\tag{5.6}$$

with C_t, C_p, C_g being defined in [11] and $C_F=(N_c^2-1)/(2N_c)$, while Eqs. (6) and (8) of [7] lead to

$$\begin{aligned}
a_2&=c_2+\frac{c_1}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_1\left(12\ln\frac{m_b}{\mu}-18+f^{\rm I}+f^{\rm II}\right),\\
a_4&=c_4+\frac{c_3}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_3\left(12\ln\frac{m_b}{\mu}-18+f^{\rm I}+f^{\rm II}\right)+\frac{\alpha_s}{9\pi}(C_t+C_p+C_g),\\
a_5&=c_5+\frac{c_6}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_6\left(-12\ln\frac{m_b}{\mu}+6-f^{\rm I}-f^{\rm II}\right),
\end{aligned}\tag{5.7}$$

where the hard scattering function $f^{\rm I}$ corresponds to the hard gluon exchange between the two outgoing light mesons and $f^{\rm II}$ describes the hard nonfactorized effect involving the spectator quark of the B meson. The expressions for the hard scattering functions $f^{\rm I}$ and $f^{\rm II}$ can be found in [7]. Comparing Eqs. (5.6) with (5.7) yields

$$\chi_2=\chi_4=-\chi_5=\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}(f^{\rm I}+f^{\rm II}).$$

Note that the quark mass entering into the penguin matrix elements of scalar and pseudoscalar densities via equations of motion is fixed at the scale μ_f.

and $N_c^{\rm eff}(LL)$ and $N_c^{\rm eff}(LR)$. (ii) Contrary to the common assertion, the nonfactorized term is dominated by hard gluon exchange in the heavy quark limit as soft gluon contributions to χ_i are suppressed by orders of $\Lambda_{\rm QCD}/m_b$ [7]. However, it is nontrivial to calculate the power corrections to χ_i since the hard scattering function $f^{\rm II}$, which convolutes with the B meson wave function, is a function of the momentum fraction of the spectator quark which is of order $\Lambda_{\rm QCD}/m_b$. In order to have a good estimation, one has to know the B meson wave function first. Therefore, nonfactorized contributions in the PQCD calculation is subject to the uncertainty due to the unknown $\Lambda_{\rm QCD}/m_b$ corrections. (iii) Because Re $\chi_{LL}>0$, it is obvious that $|N_c^{\rm eff}(LL)|<3$ and $|N_c^{\rm eff}(LR)|>3$ [see Eq. (2.6)]. Furthermore, $N_c^{\rm eff}(LL)\sim 2$ implies $N_c^{\rm eff}(LR)\sim 6$. Therefore, the assumption (2.7) and the empirical observation $N_c^{\rm eff}(LR)>3>N_c^{\rm eff}(LL)$ are consistent with the perturbative QCD calculation performed in the heavy quark limit.

VI. $B\to\omega K$ AND ρK DECAYS

The previous CLEO observation [32] of a large branching ratio for $B^\pm\to\omega K^\pm$

$$\mathcal{B}(B^\pm\to\omega K^\pm)=(15^{+7}_{-6}\pm 2)\times 10^{-6}, \tag{6.1}$$

imposes a serious problem to the generalized factorization approach: The observed rate is enormously large compared to naive expectation [8]. Since the ωK^- amplitude differs from that of $\rho^0 K^-$ only in the QCD penguin term proportional to (a_3+a_5) and in the electroweak penguin term governed by a_9, it is naively anticipated that their branching ratios are similar as the contributions from a_3, a_5, a_9 are not expected to be large. While the branching ratio of $B^\pm\to\rho^0 K^\pm$ is estimated to be of order 1×10^{-6} (see Table IV), the prediction of $\mathcal{B}(B^\pm\to\omega K^\pm)$ is less certain because the penguin contribution proportional to (a_3+a_5) depends sensitively on $N_c^{\rm eff}(LR)$. At any rate, it is reasonable to assert that $\mathcal{B}(B^-\to\omega K^-)\gtrsim 2\mathcal{B}(B^-\to\rho^0 K^-)\sim 2\times 10^{-6}$.

As pointed out recently in [2], the additional data and reanalysis of old CLEO data did not support the previously reported observation (6.1). Therefore, the new measurement of $B^-\to\omega K^-$ no longer imposes a serious difficulty to the factorization approach. The theoretical prediction $\mathcal{B}(B^-\to\omega K^-)\sim(2.1-5.5)\times 10^{-6}$ for $N_c^{\rm eff}(LL)=2$ and $N_c^{\rm eff}(LR)$ ranging from 6 to ∞ is consistent with the current limit 8.0×10^{-6} [2]. It is important to measure the branching ratios of ωK and ρK modes in order to understand their underlying mechanism. From Table IV we see that $\rho^0 K^0\sim\rho^+ K^0>\rho^0 K^+$ is expected in the factorization approach.

VII. COMPARISON WITH REF. [7]

Although we have followed the framework of [8] to study nonleptonic charmless B decays, it is useful at this point to summarize the differences between the present work and Ref. [8].

The 66 and 88 entries of the constant matrix $\hat{r}_V$ in NDR and HV γ_5 schemes given in [8] are erroneous and have been corrected here. As a result, the magnitude of the effective penguin Wilson coefficient $c_6^{\rm eff}$ is enhanced. The decay rates of the penguin-dominated modes governed by the a_6 penguin term are thus enhanced. For example, the branching ratios of $\overline{K}^{*0}\eta$ and $K^{*-}\eta$ are enhanced by almost a factor of 2.

In order to accommodate the new data of $\pi^+\pi^-$ and $K\pi$ decays, we have fixed the relevant form factors and the unitarity angle to be $F_0^{B\pi}(0)=0.28$, $F_0^{BK}(0)=0.36$, and $\gamma=105°$.

While the strange quark mass is slightly changed to $m_s(m_b)=85$ MeV, the u and d quark masses are modified to $m_u(m_b)=2.13$ MeV, $m_d(m_b)=4.27$ MeV in order to respect the chiral-symmetry relation $m_{\pi^\pm}^2/(m_u+m_d)=m_{K^+}^2/(m_u+m_s)$.

Branching ratios of all $B_{u,d}\to PP, VP, VV$ modes are tabulated in Tables II, IV, and V in BSW and LCSR′ models for form factors. Our preference for heavy-to-light form factors is that given by the LCSR′ model.

VIII. CONCLUSIONS

Implications inferred from recent CLEO measurements of hadronic charmless two-body decays of B mesons are discussed in the present paper. Our main conclusions are as follows.

(1) Employing the Bauer-Stech-Wirbel (BSW) model for form factors as a benchmark, the $B\to\pi^+\pi^-$ data indicate that the form factor $F_0^{B\pi}(0)=0.20\pm 0.04$ is much smaller than that predicted by the BSW model, whereas the data of $B\to\omega\pi$, $K^*\eta$ imply that the form factors $A_0^{B\omega}(0)$, $A_0^{BK^*}(0)$ are greater than the values obtained in the BSW model.

(2) The tree-dominated modes $B\to\pi^+\pi^-$, $\rho^0\pi^\pm$, $\omega\pi^\pm$ imply that the effective number of colors $N_c^{\rm eff}(LL)$ for $(V-A)(V-A)$ operators is preferred to be smaller $1.1\leqslant N_c^{\rm eff}(LL)\leqslant 2.6$, while the current limit on $B\to\phi K$ shows that $N_c^{\rm eff}(LR)>3$. The data of $B\to K\eta'$ and $K^*\eta$ clearly support the observation $N_c^{\rm eff}(LR)\gg N_c^{\rm eff}(LL)$.

(3) The decay rates of $\pi^+\pi^-$ and $K\pi$ are governed by the form factor $F_0^{B\pi}$. In order to explain the observed suppression of $\pi^+\pi^-$ and nonsuppression of $K\pi$ modes, the unitarity angle γ is favored to be greater than 90° (see, also, [26,33]). By contrast, the new measurement of $B^\pm\to\rho^0\pi^\pm$ no longer strongly favors cos $\gamma<0$.

(4) The observed pattern $K^-\pi^+\sim\overline{K}^0\pi^-\sim\frac{3}{2}K^-\pi^0$ is consistent with the theoretical expectation: The constructive interference between electroweak and QCD penguin diagrams in the $K^-\pi^0$ mode explains why $\mathcal{B}(B^-\to K^-\pi^0)>\frac{1}{2}\mathcal{B}(\overline{B}^0\to K^-\pi^+)$.

(5) We found that, except for the decays $K^{*-}\pi^+$ and $\overline{K}^0\pi^0$, all the measured charmless B decays can be well accommodated by the LCSR′ form factors and the parameters $m_s(m_b)=85$ MeV, $F_0^{B\pi}(0)=0.28$, $F_0^{BK}(0)=0.36$, $\gamma=105°$, $N_c^{\rm eff}(LL)=2$, $N_c^{\rm eff}(LR)=6$.

(6) The observation $N_c^{\rm eff}(LL)<3<N_c^{\rm eff}(LR)$ and our preference for $N_c^{\rm eff}(LL)\sim 2$ and $N_c^{\rm eff}(LR)\sim 6$ are theoretically consistent with a recent perturbative QCD calculation of

TABLE V. Same as Table II except for charmless $B_{u,d} \to VV$ decays.

		$N_c^{\text{eff}}(LL)=2$				$N_c^{\text{eff}}(LL)=N_c^{\text{eff}}(LR)$				
Decay	Class	2	3	6	∞	2	3	6	∞	Expt.
$\overline{B}_d^0 \to \rho^- \rho^+$	I	32.0	32.0	32.0	32.0	32.0	36.1	40.5	45.1	<2200
		19.6	19.6	19.6	19.6	19.6	22.1	24.8	27.6	
$\overline{B}_d^0 \to \rho^0 \rho^0$	II	1.17	1.17	1.17	1.17	1.17	0.14	0.43	2.02	<40
		0.72	0.72	0.72	0.72	0.72	0.09	0.26	1.23	
$\overline{B}_d^0 \to \omega \omega$	II	0.71	0.58	0.54	0.60	0.71	0.20	0.53	1.73	<19
		0.44	0.36	0.33	0.37	0.44	0.12	0.33	1.07	
$B^- \to \rho^- \rho^0$	III	27.0	27.0	27.0	27.0	27.0	21.3	16.3	12.0	<120
		16.5	16.5	16.5	16.5	16.5	13.0	9.97	7.32	
$B^- \to \rho^- \omega$	III	21.9	22.5	23.2	24.0	21.9	18.3	15.1	12.3	<47
		13.5	13.9	14.3	14.8	13.5	11.3	9.34	7.61	
$\overline{B}_d^0 \to K^{*-} \rho^+$	IV	9.89	9.89	9.89	9.89	9.89	11.1	12.3	13.6	
		6.16	6.16	6.16	6.16	6.16	6.89	7.65	8.46	
$\overline{B}_d^0 \to \overline{K}^{*0} \rho^0$	IV	1.00	0.99	0.98	0.96	1.00	1.35	1.89	2.62	<460
		0.73	0.71	0.71	0.70	0.73	0.97	1.33	1.80	
$\overline{B}_d^0 \to \overline{K}^{*0} K^{*0}$	IV	0.64	0.64	0.64	0.64	0.64	0.81	1.01	1.23	
		0.32	0.32	0.32	0.32	0.32	0.41	0.51	0.62	
$B^- \to K^{*-} \rho^0$	IV	9.98	10.0	10.1	10.1	9.98	10.4	10.9	11.5	<900
		5.80	5.87	5.90	5.92	5.80	6.12	6.44	6.82	
$B^- \to \overline{K}^{*0} \rho^-$	IV	6.42	6.42	6.42	6.42	6.42	8.20	10.2	12.4	
		4.00	4.00	4.00	4.00	4.00	5.11	6.36	7.75	
$B^- \to K^{*-} K^{*0}$	IV	0.68	0.68	0.68	0.68	0.68	0.87	1.08	1.31	
		0.34	0.34	0.34	0.34	0.34	0.43	0.54	0.66	
$\overline{B}_d^0 \to \rho^0 \phi$	V	0.03	0.002	0.01	0.06	0.03	0.01	0.10	0.33	<13
		0.02	0.001	0.006	0.04	0.02	0.003	0.07	0.21	
$\overline{B}_d^0 \to \omega \phi$	V	0.03	0.002	0.01	0.06	0.03	0.01	0.10	0.33	<21
		0.02	0.001	0.006	0.04	0.02	0.003	0.07	0.21	
$B^- \to \rho^- \phi$	V	0.07	0.004	0.02	0.12	0.07	0.01	0.22	0.70	<16
		0.05	0.003	0.01	0.08	0.05	0.01	0.14	0.44	
$\overline{B}_d^0 \to \rho^0 \omega$	VI	0.65	0.42	0.24	0.11	0.65	0.32	0.11	0.02	<11
		0.40	0.26	0.15	0.07	0.40	0.20	0.07	0.01	
$\overline{B}_d^0 \to \overline{K}^{*0} \omega$	VI	15.0	7.80	2.91	0.43	15.0	4.54	0.19	2.01	<19
		8.17	4.48	1.84	0.37	8.17	2.81	0.25	0.62	
$\overline{B}_d^0 \to \overline{K}^{*0} \phi$	VI	21.3	13.3	7.28	3.08	21.3	11.0	4.05	0.53	<21
		11.1	6.79	3.70	1.56	11.1	5.58	2.06	0.27	
$B^- \to K^{*-} \omega$	VI	19.7	11.4	5.58	2.31	19.7	6.70	1.04	2.79	<52
		10.9	6.64	3.48	1.58	10.9	4.15	0.83	1.09	
$B^- \to K^{*-} \phi$	VI	22.7	14.2	7.74	3.27	22.7	11.7	4.31	0.57	<41
		11.8	7.22	3.94	1.66	11.8	5.93	2.19	0.29	

charmless B decays performed in the heavy quark limit.

(7) The new upper limit set for $B^- \to \omega K^-$ no longer imposes a serious problem to the factorization approach. It is anticipated that $\mathcal{B}(B^- \to \omega K^-) \gtrsim 2\mathcal{B}(B^- \to \rho^0 K^-) \sim 2 \times 10^{-6}$.

(8) An improved and refined measurement of $B \to K^{*-}\pi^+$, $\overline{K}^0\pi^0$ is called for in order to resolve the discrepancy between theory and experiment. Theoretically, it is expected that $\overline{K}^0\pi^0 \sim \frac{1}{2} K^-\pi^0$ and $K^-\pi^+ \sim 3\, K^{*-}\pi^+$.

(9) Theoretical calculations suggest that the following 18 decay modes of B_u^- and $\overline{B}_d^0$ have branching ratios of order 10^{-5} or above (in sequence of their decay rate strength): $\eta' K^-$, $\eta' K^0$, $\rho^+\rho^-$, $\rho^-\rho^0$, $\rho^-\omega$, $K^-\pi^+$, $\overline{K}^0\pi^-$, $\rho^-\pi^+$, $K^{*-}\eta$, $\rho^+\pi^-$, $\rho^-\pi^0$, $K^-\pi^0$, $\rho^0\pi^-$, $\overline{K}^{*0}\eta$, $\omega\pi^-$, $K^{*-}\rho^0$, $K^{*-}\rho^+$, $\rho^-\eta$. Many of them have been observed and the rest will have a good chance to be seen soon.

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