NLO correction to one-jet inclusive production at high energies

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The next-to-leading order correction to the one-jet inclusive cross section in the framework of high energy factorization is calculated. Numerical results for the midrapidity region are compared with predictions of conventional calculations based on collinear factorization.

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I. INTRODUCTION

The recently completed calculation of the next-to-leading order (NLO) correction to Balitskii–Fadin–Kuraev–Lipatov (BFKL) Pomeron [1] involves as its ingredients the formulas for the cross section of tree level two-particle production and one-loop virtual corrections to one-particle production in quasi–multi Regge kinematics (QMRK). These results can be used in studying the properties of the inclusive two-jet production at leading order [2,3] and of the one-jet inclusive cross section in the next-to-leading order at high energies.

The total cross section in the next-to-leading order (NLO BFKL) was calculated in [1] and is now under extensive discussion [4,5]. The total cross section has a complicated structure due to delicate cancellations of singularities in various virtual contributions and infrared divergencies arising in integrations over parameters of real corrections. Cancellation between divergent parts of real and virtual contributions is also needed for the computation of the next-to-leading order one-jet inclusive production cross section, but here the structure of this cancellation is simpler and more transparent than in the case of the total cross section.

The problem of one-jet inclusive production in high energy hadron collisions was investigated, in the leading order (LO), in a number of works [3]. The purpose of this paper is to calculate the one-jet inclusive cross section to the next-to-leading order at high energies proceeding as far as possible with analytical calculations and then turning to numerical estimates.

The outline of the paper is as follows.

In Sec. II we briefly review the results on the particle production cross sections obtained within the high energy factorization scheme and compare them to the expressions obtained using collinear factorization.

In Sec. III we describe a calculation leading to the explicit expression for the one-jet inclusive production cross section in the next-to-leading order.

In Sec. IV some numerical results on jet production in central rapidity region are presented and discussed.

Section V contains a brief conclusion.

II. PARTICLE PRODUCTION AT HIGH ENERGIES

From the theoretical viewpoint the hadron scattering at very high energies is special in the way the hard degrees of freedom (partons) are formed from colliding hadrons. When the ratio of hardness of the process $k_\perp$ (which is the transverse momentum of produced particle) to the invariant energy of colliding hadrons, $\sqrt{S}$, is not too small (up to 10^{-2}) it is possible to describe the structure functions of hadrons by taking into account only processes that contribute logarithms of $k_\perp/\Lambda_{\rm QCD}$ at leading order, i.e., by resummation of the $\alpha_s^n \ln^n(k_\perp/\Lambda_{\rm QCD})$ terms. Such structure function is given by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation [6].

Since the emergence of the combination $\alpha_s \ln(k_\perp/\Lambda_{\rm QCD})$ implies strong ordering of emitted particles in their transverse momenta up until a hard collision block, the transverse momentum of detected particle $k_\perp$ is parametrically bigger than that of any parton involved in the process. Therefore it is possible to calculate the cross section of the hard process using the initial on-shell partons. This prescription is known as collinear factorization [7] and leads to the well-known result for the production rate:

$$\frac{d\sigma}{dk^2\,dy_1}=2\int dy_2\, x_1 f_a(x_1,k^2)\frac{d\hat{\sigma}_{ab}}{dk^2}x_2 f_b(x_2,k^2), \quad (1)$$

where $f_a(x,k^2)$ is a structure function for the parton of type a and $x_{1,2}=k_\perp(e^{\pm y_1}+e^{\pm y_2})/\sqrt{S}$.

At high energies for particles produced with $k_\perp \ll \sqrt{S}$ another big logarithm, $\ln(1/x)$, is important. The resummation of such logarithmic contributions can become more important than of $\ln(k^2/\Lambda_{\rm QCD}^2)$. The resummation of the leading energy logarithms for the structure function is described by the BFKL equation [8]. The domain of validity of the BFKL equation in describing the structure functions is at present not well understood. It is likely that for some kinematical region the correct approach is to continue the logarithms of both types, or at least interpolate between two types of resummation as done, e.g., in the Ciafalon–Catani–Fiorani–Marchesini (CCFM) [9] equation.

The main point is that at high energies the transverse momenta of the incoming parton fluxes can no longer be neglected. To take them into account a new approach called $k_\perp$ or high energy factorization was proposed [10,11]. Extensive description of the method and various applications can be found in [11]. Let us note that this method was *de facto* used earlier in [12].

The method of high energy factorization is based on consideration of ‘‘partons’’ with nonzero transverse momentum

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that are, in contrast with the traditional collinear factorization case, virtual particles. In this case colliding hadrons are described by unintegrated structure function, ϕ, so that

$$\varphi(x,q^2)=q^2\frac{\partial xg(x,q^2)}{\partial q^2}, \tag{2}$$

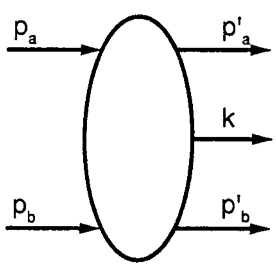
where ϕ/q^2 is proportional to the probability to find the incident parton with the longitudinal momentum component xp_a (p_a is a momentum of the incident particle) and transverse momentum component $q_\perp$.

Note that such an interpretation is somewhat oversimplified, because due to the quantum structure of QCD evolution some form factors may arise changing the form of ϕ. For DGLAP evolution it is a Sudakov form factor [13]. However, when studying the semi-inclusive quantity like one-jet production cross section it is legitimate to use the unintegrated structure function in the simple form of Eq. (2).[1]

Scattering of the off-shell ''partons'' are described by generalized cross sections calculated in quasi–multi Regge kinematics (QMRK) [14,15].

In QMRK one studies the $2\to n+2$ scattering process with two outgoing particles having almost the same momenta as the incident ones and remaining n particles emitted into the central rapidity region separated by large rapidity gaps from incoming particles (the situation, where rapidity gaps between n particles are also large, corresponds to multi Regge kinematics, MRK). Large rapidity gaps allow to distinguish the quantities related to the incident particles from those describing the cross section of the hard process of particle production in the central rapidity region.

Let us, for example, consider the cross section of the process $gg\to ggg$ in the limit of high energy. Two particles with momenta p_a and p_b collide and produce particles with momenta p'_a (almost collinear to p_a), p'_b (almost collinear to p_b), and k (in central rapidity region):



$gg\to ggg$

$$\frac{d\sigma_{gg\to ggg}}{d^2k_\perp dy}=\frac{4N_c^3\alpha_s^3}{\pi^2(N_c^2-1)}\times\int\frac{d^2q_{1\perp}}{q_{1\perp}^2}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k_\perp^2}\frac{d^2q_{2\perp}}{q_{2\perp}^2}, \tag{3}$$

where $q_{1,2}=p_{a,b}-p'_{a,b}$. In Eq. (3) the above-mentioned factorization of the cross section is clearly seen. Indeed, the first, second, and third factors under the integral correspond to $p_a\to p'_a,q_1$ splitting, $q_1,q_2\to k$ scattering, and $p_b\to p'_2,q_2$ splitting, respectively.

The factors related to the splitting of the incident particles should further be transformed to structure functions. This can be done in two steps. First, one assembles incident partons into wave packets describing by form factors [2]. The second step is taking into account additional radiation and virtual corrections summed to give the unintegrated structure functions $\varphi(x,q_\perp)$ with x fixed by the kinematics of the considered process.

The cross sections of producing $n=1,2$ particles in the central region to the lowest perturbative order read (colliding partons are gluons):

$$\frac{d\sigma_1}{d^2k_\perp dy}=\int d^2q_{1\perp}\,d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat\sigma_1}{dk_\perp^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2},$$
$$\frac{d\hat\sigma_1}{dk_\perp^2}=\frac{4N_c\alpha_s}{N_c^2-1}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k_\perp^2}, \tag{4}$$
$$x_{1,0}=k_\perp e^y/\sqrt{S},\quad x_{2,0}=k_\perp e^{-y}/\sqrt{S};$$

$$\frac{d\sigma_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,dy_1dy_2}=\int d^2q_{1\perp}\,d^2q_{2\perp}\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\times\frac{d\hat\sigma_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2},$$
$$\frac{d\hat\sigma_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}=\frac{2N_c^2\alpha_s^2}{(N_c^2-1)\pi^2}\times\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_{1\perp}-k_{2\perp})}{q_{1\perp}^2q_{2\perp}^2}\mathcal{A},$$
$$x_1=k_{1\perp}e^{y_1}(1+k_{2\perp}e^{\Delta y})/\sqrt{S},$$
$$x_2=k_{1\perp}e^{-y_1}(1+k_{2\perp}e^{-\Delta y})/\sqrt{S},$$
$$\Delta y=y_2-y_1. \tag{5}$$

A part of the analytical expression for $\mathcal{A}$ can be found in [15,16] for the subprocesses $gg\to gg$ and in [15,17] for the $gg\to q\bar{q}$ ones. The explicit form of $\mathcal{A}$ was recently derived in [18] and is given in Appendix A. Note that the formula for $gg\to q\bar{q}$ cross section from [17] coincides with the analogous formula in [11] in the limit of massless quarks.

In the leading order of the high energy factorization approach it is natural to consider as colliding partons gluons only. Indeed, resummation of powers of $\alpha_s^n\ln^n 1/x$ implies

[1]I am grateful to Yu. L. Dokshitzer for pointing this out.

that the particle propagating in the t-channel is a gluon, for quarks do not give the factor of $\ln 1/x$ for each α_s. At the next-to-leading order stage the terms resummed are of the order $\alpha_s^{n+1}\ln^n 1/x$ and therefore one should add quarks as colliding partons to the scheme described. However, we have encountered one quark in the t-channel diagrams when adding a contribution for two quarks production from gluon–gluon collision. These are exactly the same diagrams that appear in the one-particle production process where one of the particles is a quark. Therefore, we must not add any further contributions from quark structure functions at NLO calculation (later, an analogous argument will be used for the MRK contribution to the NLO cross section).

Equation (4) gives the rate of one-particle production in the leading order. It was studied in a number of publications [3]. Our aim is to calculate a first correction to it.

III. REAL AND VIRTUAL CONTRIBUTIONS TO NLO ONE-JET PRODUCTION

Particles produced in hard scattering then undergo a fragmentation process and appear in experiment as jets of hadrons. Therefore fixing parameters of a particle produced in a hard block, we are really able to estimate (infrared-safe) characteristics of a jet. Therefore, in the following we will use jet terminology for particles produced in a hard scattering (as it will clear soon in NLO some two-particle states may form a single jet).

The one-jet production in the next-to-leading order includes two contributions, real and virtual. The real contribution comes from the two-particle cross section Eq. (5) integrated over the phase space of one of the particles (considered unobservable) and the fixed four-momentum of the second particle:

$$\frac{d\sigma_r}{d^2k_{1\perp}\,dy_1}=2\int d\Phi\,\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\times\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2},\qquad(6)$$

where

$$d\Phi=d^2q_{1\perp}\,d^2q_{2\perp}\,d^2k_{2\perp}\,d\Delta y.\qquad(7)$$

The factor of 2 in Eq. (6) reflects the identity of outgoing particles.

The virtual contribution has the same form as in Eq. (4), but instead of $\hat{\sigma}_1$ we must use

$$\frac{d\hat{\sigma}_v}{dk_\perp^2}=\frac{4N_c\alpha_s}{N_c^2-1}\,\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k_\perp^2}\mathcal{V}(q_{1\perp},q_{2\perp}).\qquad(8)$$

The virtual correction contains both ultraviolet and infrared divergencies. The ultraviolet one leads, through standard renormalization procedure, to the running coupling constant. The infrared divergence must cancel with the infrared and collinear divergences in the real contribution.

A. Cancellation of collinear and infrared divergences

Let us now outline at the formal level how this cancellation occurs. To deal with the collinear singularity we must introduce a jet-defining algorithm which in the two-particle production case could be expressed through the function $S(k,k_1,k_2)$ (k_1, k_2, and k are on-shell 4-vectors) so that Eq. (6) is replaced by

$$\frac{d\sigma_r}{d^2k_\perp\,dy}=\int d^2k_{1\perp}\,d^2k_{2\perp}\,dy_1dy_2\times\frac{d\sigma_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,dy_1dy_2}S(k,k_1,k_2).\qquad(9)$$

To provide the sought after cancellation between the real and virtual corrections, S should be an infrared safe quantity, which means that the following property should hold (cf. [19]):

$$S(k,\lambda k_1,(1-\lambda)k_1)=\delta^{(2)}(k_\perp-k_{1\perp})\,\delta(y-y_1),\quad 0<\lambda<1.\qquad(10)$$

We choose S in the following form:

$$S(k,k_1,k_2)=\theta(R>R_0)\sum_{i=1,2}\delta^{(2)}(k_\perp-k_{i\perp})\,\delta(y-y_i)+\theta(R<R_0)\,\delta^{(2)}(k_\perp-k_{1\perp}-k_{2\perp})\,\delta\times\left(y-\frac{1}{2}\ln\frac{k_{1\perp}e^{y_1}+k_{2\perp}e^{y_2}}{k_{1\perp}e^{-y_1}+k_{2\perp}e^{-y_2}}\right),\qquad(11)$$

where $R^2=(\phi_1-\phi_2)^2+(y_1-y_2)^2$. Although the last line in Eq. (11) may look artificial, it has the natural meaning. If we claim that jet, initiated by two particles indistinguishable under given resolution, has definite rapidity and that 4-momenta of two particles are added up to form the 4-momentum of the jet, we immediately arrive at Eq. (11). It is straightforward to check that Eq. (11) satisfies Eq. (10).

Grouping together Eqs. (5), (9), and (11) we find

$$\frac{d\sigma_r}{d^2k_\perp\,dy}=2\int d\Phi\,\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2}{d^2k_\perp\,d^2k_{2\perp}\,d\Delta y}\times\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\,\theta(R>R_0)+\int d\Phi\,\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\times\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}\,\theta(R<R_0),\qquad(12)$$

where $k_{1\perp}=k_\perp-k_{2\perp}$ and $\tilde{x}_{1,2}=e^{\pm y}\sqrt{\Sigma/S}$ with $\Sigma=k_1^2+k_2^2+2k_1k_2\,\mathrm{ch}(\Delta y)$ (see Appendix A), and $k_i=|k_{i\perp}|$ which is a notation we shall use from now on. Note that when $R\to 0$ $\tilde{x}_1\to x_{1,0}$ and $\tilde{x}_2\to x_{2,0}$. Let us now rewrite the second term in Eq. (12) as

$$\left[\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\right.$$
$$\left.+\int d\Phi\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right]\theta(R<R_0). \tag{13}$$

The integration in the first line is free from divergencies, while in the second line integrals over $d^2k_{2\perp}$ and $d\Delta y$ do not involve structure functions and could be done analytically. Before doing this integration we make a replacement $\theta(R<R_0)=1-\theta(R>R_0)$ in the last term and then substitute Eq. (13) into Eq. (12) to yield

$$\frac{d\sigma_r}{d^2k_\perp\,dy}=\int d^2q_{1\perp}\,d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\int d^2k_{2\perp}\,d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}$$
$$+\int d\Phi\left(2\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_\perp\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\right.$$
$$\left.\times\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\,\theta(R(k_1,k_2)>R_0)\right)$$
$$+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\theta(R<R_0), \tag{14}$$

where we indicate that $R(k,k_2)=((y-y_2)^2+(\phi-\phi_2)^2)^{1/2}$ and $R(k_1,k_2)=((y_1-y_2)^2+(\phi_1-\phi_2)^2)^{1/2}$ are different in the third and fourth lines of Eq. (14). The third line in Eq. (14) has an infrared singularity when $k_2\to 0$ and the fourth one has singularities when $k_2\to 0$ or $k_1\to 0$. However, it is possible to combine the singularities in the fourth line so that we find only one singular point and also an accompanying factor of 2 thus providing a cancellation of singularities in the third and fourth lines.

Indeed, the expression under the integral in the fourth line in Eq. (14) is symmetric under the simultaneous transformation $k_{1\perp}\leftrightarrow k_{2\perp}$ and $\Delta y\leftrightarrow -\Delta y$ which is nothing else than the permutation of the two produced particles. Therefore we can simply multiply this term at $2\theta(k_{1\perp}>k_{2\perp})$ to obtain

$$\frac{d\sigma_r}{d^2k_\perp\,dy}=\int d^2q_{1\perp}\,d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\int d^2k_{2\perp}\,d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}$$
$$+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_\perp\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)\right.$$
$$\left.-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\theta(R(k_1,k_2)>R_0)\,\theta(k_{1\perp}>k_{2\perp})\right)$$
$$+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\theta(R<R_0). \tag{15}$$

The combination of the third and fourth lines in Eq. (15) is free from singularities [note that for $k_2\to 0$ there is no difference between $R(k,k_2)$ and $R(k_1,k_2)$] and the singularity in the second line cancels with that in $\mathcal{V}$ in Eq. (8). Combining Eq. (15) with Eqs. (4) and (8) we obtain the second order correction to the one-jet inclusive production in the high energy factorization scheme:

$$\frac{d\sigma^{(2)}}{d^2k_\perp\,dy}=\int d^2q_{1\perp}\,d^2q_{2\perp}\,\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\left(\frac{d\hat{\sigma}_2}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp}\right)$$
$$+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2}{d^2k_\perp\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\,\theta(R(k,k_2)>R_0)\right.$$
$$\left.-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\,\theta(R(k_1,k_2)>R_0)\,\theta(k_{1\perp}>k_{2\perp})\right)$$
$$+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\,\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\theta(R<R_0). \tag{16}$$

Let us now analyze the MRK limit of Eq. (16), that is we take the limit $\Delta y\to\infty$ in $\mathcal{A}$ [see Eq. (5) and Eq. (A2) in Appendix A]. In this limit

$$\mathcal{A}\to\mathcal{A}_{\rm MRK}=\frac{q_{1\perp}^2q_{2\perp}^2}{k_{1\perp}^2k_{2\perp}^2} \tag{17}$$

which is precisely the combination of two leading order BFKL kernels responsible for real particles production. However, let us remind that the leading order one-jet production in the high energy factorization, described by Eq. (4), includes MRK contributions to all orders if the unintegrated structure function, $\phi(x,q_\perp)$ includes resummation to all orders of $\alpha_s\ln(1/x)$. It is evidently the case for structure functions undergoing BFKL equation. For other types of structure functions we just assume that the resummed $\alpha_s^n\ln^n(1/x)$ terms are included in some hidden way. Consequently, we must subtract $\mathcal{A}_{\rm MRK}$ from $\mathcal{A}$, and we will imply this subtraction in the following.

To proceed further we must calculate

$$\frac{d\hat{\sigma}_2}{d^2k_\perp}=\int d^2k_{2\perp}\,d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}. \tag{18}$$

Note that the quantity in brackets in the first line of Eq. (16) should coincide (up to the constant factors depending on normalization) with the NLO BFKL kernel written explicitly in [1]. Note however, that when calculating the real contribution in [16] the terms vanishing after integration over $d^2k_\perp$ were dropped which did not change the result for NLO BFKL Pomeron itself. In calculating the one-jet inclusive cross sections these contributions have to be kept.

The integration in Eq. (18) is very difficult. Fortunately, we can do it not for whole $\hat{\sigma}_2$, but only for its singular part $\hat{\sigma}_2^s$. We must also change some other terms in Eq. (16) that emerge when arriving from Eq. (12) at Eq. (16). Finally, the result is

$$\frac{d\sigma^{(2)}}{d^2k_\perp dy}=\int d^2q_{1\perp}\,d^2q_{2\perp}\,\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}$$
$$\times\left(\frac{d\hat{\sigma}_2^s}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp}\right)+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\right.$$
$$\times\frac{d\hat{\sigma}_2}{d^2k_\perp\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\,\theta(R(k,k_2)>R_0)$$
$$-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2^s}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\,\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}$$
$$\left.\times\,\theta(R(k_1,k_2)>R_0)\,\theta(k_{1\perp}>k_{2\perp})\right)$$
$$+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}\right.$$
$$\times\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\,\frac{d\hat{\sigma}_2^s}{d^2k_{1\perp}\,d^2k_{2\perp}\,d\Delta y}$$
$$\left.\times\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\theta(R<R_0). \tag{19}$$

B. Integration

Let us now choose the singular part of $\mathcal{A}$ with contributions from quark and gluon production added up in the form (see the Appendix A, MRK part as it was mentioned above is subtracted):

$$\mathcal{A}^s=-\frac{q_1^2q_2^2}{2k_1^2k_2^2}+\frac{q_1^2q_2^2\,{\rm ch}(\Delta y)}{k_1k_2s}-\left(1-\frac{n_f}{4N_c}\right)\frac{2q_1^2q_2^2}{s\Sigma}$$
$$+\left(\frac{D-2}{2}-\frac{n_f}{N_c}\right)\frac{E^2}{8s^2}. \tag{20}$$

This form of $\mathcal{A}^s$ is chosen to avoid artificial ultraviolet divergency which occurs if one takes $\Sigma=k^2$ as it is in collinear and infrared limits. For the same purpose we take E in the form

$$E=\frac{1}{\Sigma}[k^2(\overline{q_1}-\overline{q_2})(\overline{k_1}-\overline{k_2})-(q_1^2-q_2^2)(k_1^2-k_2^2)+2k_1k_2\,\mathrm{sh}(\Delta y)(k^2-q_1^2-q_2^2)]. \tag{21}$$

Now we integrate the singular part of $\mathcal{A}$ over transverse two-dimensional momentum space analytically continued to $D-2=2+2\varepsilon$ or, strictly speaking, with

$$d^2k_{2\perp}\to\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}} \tag{22}$$

and over rapidity. The results of the calculation of the integral over $\mathcal{A}^s$ are given in Appendix B. The answer reads [see Eq. (5) for relation between $\hat{\sigma}_2$ and $\mathcal{A}$]

$$\frac{d\hat{\sigma}_r^s}{d^2k_\perp}=\frac{2N_c^2\alpha_s^2}{N_c^2-1}\,\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\,\frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}}\,\frac{4\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\times\left(\frac{k^2}{\mu^2}\right)^{\varepsilon}\left(\frac{1}{\varepsilon}+2\psi(1)-2\psi(1+2\varepsilon)-\frac{11+8\varepsilon}{2(1+2\varepsilon)(3+2\varepsilon)}+\frac{n_f}{4N_c}\,\frac{4+6\varepsilon}{(1+2\varepsilon)(3+2\varepsilon)}\right). \tag{23}$$

The result for $\hat{\sigma}_v$ was derived in [20]. Note that the answer depends on the arrangement of different corrections to QMRK amplitude. In this paper the symmetric variant [1] is chosen:

$$\frac{d\hat{\sigma}_v}{d^2k_\perp}=\frac{4N_c^2\alpha_s^2}{N_c^2-1}\,\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\,\frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}}\times\left[-\frac{2}{\varepsilon^2}\left(\frac{k^2}{\mu^2}\right)^{\varepsilon}+\frac{1}{\varepsilon}\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N_c}\right)+\pi^2+\frac{k^2}{3}\times\left\{\left(11-2\frac{n_f}{N_c}\right)\frac{\ln q_1^2/q_2^2}{q_1^2-q_2^2}+\left(1-\frac{n_f}{N_c}\right)\times\left(\left(\frac{q_1^2}{q_2^2}-\frac{q_2^2}{q_1^2}-2\ln\frac{q_1^2}{q_2^2}\right)\times\frac{2q_1^2q_2^2-\overline{q_1}\,\overline{q_2}(q_1^2+q_2^2+4\overline{q_1}\,\overline{q_2})}{(q_1^2-q_2^2)^3}+\frac{\overline{q_1}\,\overline{q_2}}{q_1^2q_2^2}\right)\right\}\right]. \tag{24}$$

From Eqs. (23) and (24) it is easy to see that divergencies of real and virtual parts cancel leaving a finite contribution to the first line in Eq. (19):

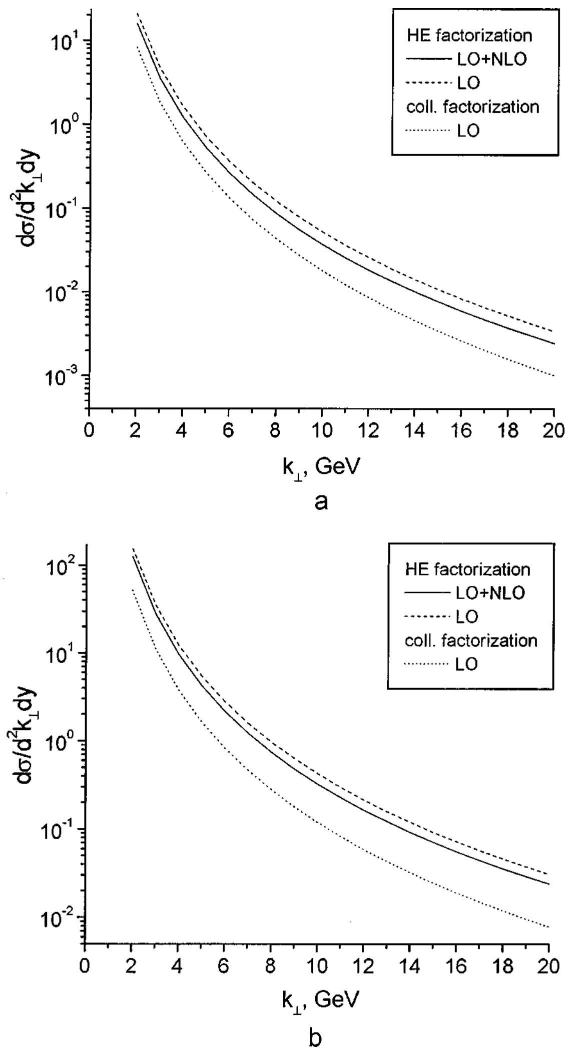


FIG. 1. $d\sigma/d^2k\,dy$ calculated with asymptotic BFKL structure function Eq. (26). 1-loop α_s with $\Lambda_{\rm QCD}=200$ MeV, $R=0.7$. (a) $\sqrt{S}=1.8$ TeV; (b) $\sqrt{S}=14$ TeV.

$$\frac{d\hat{\sigma}_r^s}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp}=\frac{N_c^2\alpha_s^2}{(N_c^2-1)\pi}\,\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\times\left[-\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N_c}\right)\ln\frac{k^2}{\mu^2}-\frac{2\pi^2}{3}+\frac{64}{9}-\frac{7}{9}\frac{n_f}{N_c}+\frac{k^2}{3}\times\left\{\left(11-2\frac{n_f}{N_c}\right)\frac{\ln q_1^2/q_2^2}{q_1^2-q_2^2}+\left(1-\frac{n_f}{N_c}\right)\times\left(\left(\frac{q_1^2}{q_2^2}-\frac{q_2^2}{q_1^2}-2\ln\frac{q_1^2}{q_2^2}\right)\times\frac{2q_1^2q_2^2-\overline{q_1}\,\overline{q_2}(q_1^2+q_2^2+4\overline{q_1}\,\overline{q_2})}{(q_1^2-q_2^2)^3}+\frac{\overline{q_1}\,\overline{q_2}}{q_1^2q_2^2}\right)\right\}\right]. \tag{25}$$

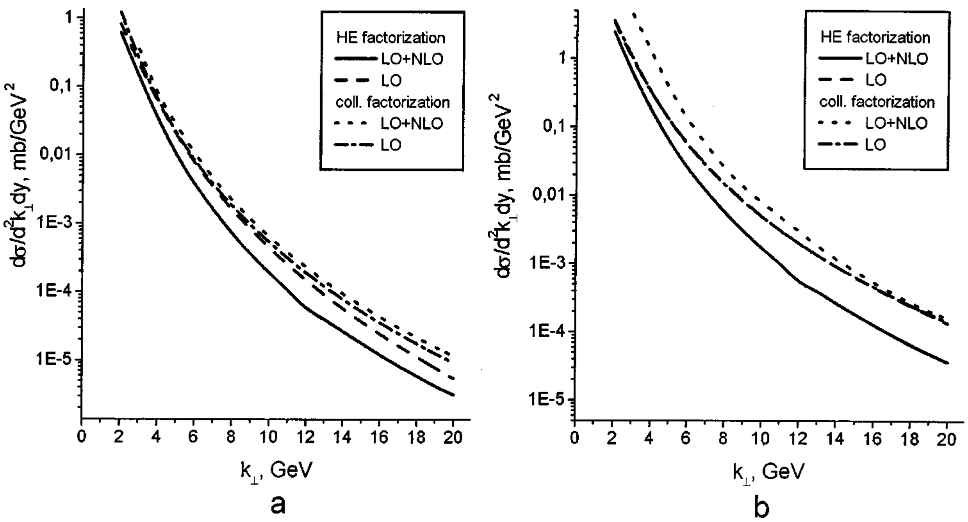


FIG. 2. $d\sigma/d^2k\,dy$ calculated with CTEQ5M structure function [22]. 2-loop α_s with $\Lambda_{\rm QCD}=226$ MeV, $R=0.7$, $n_f=5$. (a) $\sqrt{S}=1.8$ TeV; (b) $\sqrt{S}=14$ TeV.

The first term in Eq. (25), which is proportional to $\ln(k^2/\mu^2)$, is nothing but the well-known contribution corresponding to the running coupling. Therefore, after replacing α_s by the running coupling $\alpha_s(k^2)$, one should drop this term.

Equations (19) and (25) together with Eq. (20) and the formula from Appendix A provide an analytical expression for the one-jet inclusive cross section.

For practical applications one should integrate over the parameters of unintegrated structure functions (disregarding trivial elimination of delta functions). The corresponding numerical calculations numerical studies will be described in the next section.

IV. NUMERICAL ESTIMATES

Let me first mention that numerical results for the collider energies now available (or will be available in the near future) can be made only for transversal momenta of several or as an extreme example several tens GeVs. The reason is that the high energy kinematics is applicable for $x<10^{-2}$. Therefore, jet calculations we are making correspond to the minijet rather than the conventional jet domain. This limits us in the experimental check of the results obtained.

The numerical results strongly depend on the type of structure functions used in the calculation. Let us first consider the asymptotic BFKL structure function [21]

$$\varphi(x,q^2)=C\left(\frac{x_0}{x}\right)^{\lambda}\frac{q}{q_0}\frac{1}{\sqrt{\pi\lambda''\ln(x_0/x)}}\times\exp\left[-\frac{\ln^2(q^2/q_0^2)}{4\lambda''\ln(x_0/x)}\right], \qquad (26)$$

with $\lambda=4\ln 2N_c\alpha_s/\pi$ and $\lambda''=14\zeta(3)N_c\alpha_s/\pi$; α_s is chosen to be equal to 0.2. Because the asymptotic BFKL structure function is a solution of the linear homogeneous equation, it does not have definite normalization and, moreover, the parameters q_0 and x_0 are arbitrary. Since the calculation with this structure function is illustrative only, we simply choose $C=1$, $q_0=1$ GeV, and $x_0=1$. From Eq. (19) it is clear that the NLO correction to the production process depends on the parameter R describing the collinear angle. For this calculation we take $R=0.7$. Further discussion of the R dependence will be given below.

It is important to note that although in asymptotic BFKL structure function the strong coupling constant does not run, to be consistent we should however make it run in a semihard vertices [cf. the discussion after Eq. (19)]. In the actual calculation with 1-loop α_s we choose $\Lambda_{\rm QCD}=200$ MeV.

The one-jet inclusive cross section for $\sqrt{S}=1.8$ TeV and $\sqrt{S}=14$ TeV, $y=0$, and $n_f=4$ is shown in Figs. 1(a) and 1(b) where for collinear factorization Eq. (1) was used with

$$xg(x,k^2)=\int_0^{k^2}\frac{dq^2}{q^2}\varphi(x,q^2).$$

As a second example we consider one-jet production with one of the realistic structure functions, CTEQ5M [22]. In Figs. 2(a) and 2(b) we show differential cross sections of one-jet inclusive production calculated with CTEQ5M structure functions in four different approximations: LO and LO+NLO in high energy and collinear factorizations. CTEQ structure functions satisfy the DGLAP equation not related to BFKL. However, it possibly includes leading MRK contribution through the initial conditions for DGLAP evolution. Note the rapid increase and broadening of the structure function with decreasing *x*, which is the characteristic property of BFKL induced structure functions. NLO collinear factorization calculation was performed making use of the program developed by Elis, Kunst, and Soper [23,24] (for theoretical basis and earlier calculations see [25]). This calculation was done for $\sqrt{S}=1.8$ [Fig. 2(a)] and 14 TeV [Fig. 2(b)], for 2-loop α_s with $\Lambda_{\rm QCD}=226$ MeV (because with these parameters CTEQ5M is calculated), $n_f=5$, $y=0$, and $R=0.7$.

In Figs. 3(a) and 3(b) we take a closer look at the difference between different approximations for the one-jet cross section. Namely, in these figures we show ratios of one-jet cross section calculated in high energy factorization (LO and LO+NLO) and collinear factorization (LO+NLO) schemes to that calculated in the LO of the collinear factorization

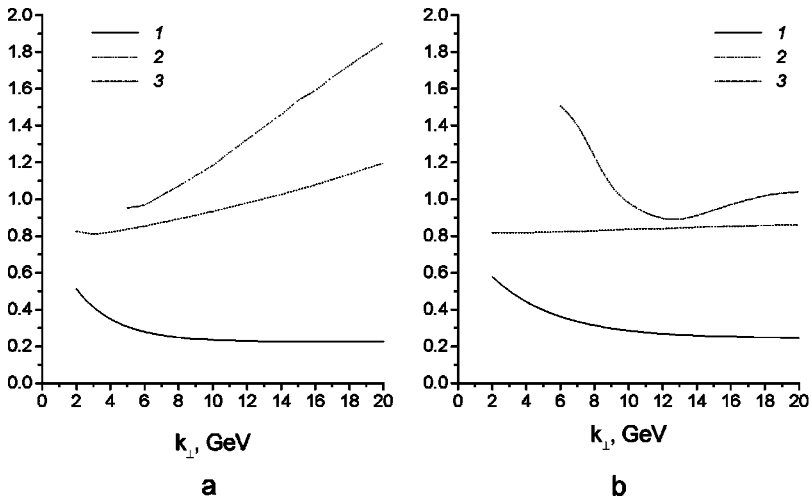


FIG. 3. Ratios of $d\sigma/d^2k\,dy$ calculated with CTEQ5M structure functions [22] in LO and LO +NLO of high energy factorization approach and LO+NLO of collinear factorization approach to the LO of collinear factorization approach. 2-loop α_s with $\Lambda_{\mathrm{QCD}}=226$ MeV, $R=0.7$, $n_f=5$. (a) $\sqrt{S}=1.8$ TeV; (b) $\sqrt{S}=14$ TeV.

scheme. The structure functions and set of parameters are the same as for Figs. 2(a) and 2(b).

From Figs. 1–3 we see that NLO corrections lead to the decrease of the particle production rate at high energies. Correlation between LO and LO+NLO results for collinear factorization is opposite (see also [25] and [26]). For the leading BFKL structure function NLO high energy factorization corrections change cross sections substantially (up to 50% in chosen kinematical interval), for non-BFKL CTEQ5M structure function changes are more dramatic: corrected cross sections are 2 to 5 times smaller than the leading order ones (apparently these results are sensitive to the cone size).

Finally, in Fig. 4 we show the dependence of NLO cross section on the cone size *R*. It may be fitted well by function of the type $A+B\ln R+CR$ and becomes infinitely large (and negative) at $R\to 0$. This is the general property of quantities with canceling virtual and real corrections showing that at small values of *R* the fixed order perturbation theory is not valid (cf. [24]).

V. CONCLUSION

The paper is devoted to the calculation of next-to-leading order correction to one-jet inclusive production in the framework of high energy factorization. High energy factorization scheme allows one to account for the initial transverse momentum of the colliding partons. The natural setup for particle production processes leading to high energy factorization is provided by quasi–multi Regge kinematics.

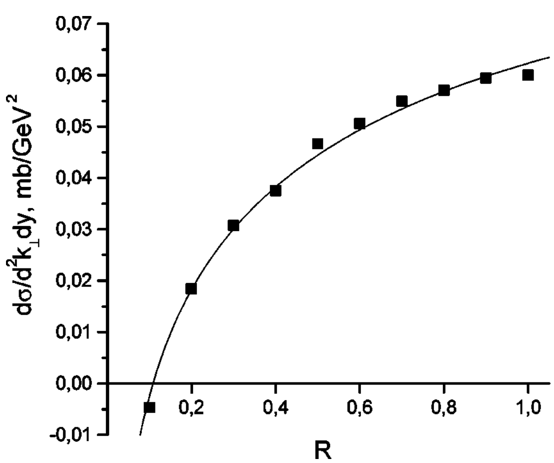


FIG. 4. *R*-dependence of $d\sigma/d^2k\,dy$ calculated with CTEQ5M structure functions. $\sqrt{S}=14$ TeV, 2-loop α_s with $\Lambda_{\mathrm{QCD}}=226$ MeV, $n_f=5$.

The results of computation of NLO contributions to BFKL Pomeron (cf. [1] and references therein) can be used to compute the next-to-leading order corrections to one-jet inclusive production at high energies. This correction includes real and virtual pieces. The infrared singularity in the virtual piece in the NLO contribution cancels the infrared singularity in its real one when an infrared safe jet algorithm is applied. The explicit calculations of the infrared stable one-jet inclusive cross section at the next-to-leading order constitutes the main result of the paper.

Numerical estimates were made to analyze the magnitude of NLO corrections for typical semihard transverse momenta and central rapidity region. Contrary to collinear factorization approach, NLO correction in high energy factorization diminishes the one-jet inclusive cross section. (in the minijet domain).

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APPENDIX A: CROSS SECTIONS OF PAIR PRODUCTION IN HIGH ENERGY FACTORIZATION

We will use the following notation:

$$s=2(k_1k_2\,\mathrm{ch}(\Delta y)-k_{1\perp}k_{2\perp}),$$

$$t=-(q_{1\perp}-k_{1\perp})^2-k_1k_2e^{\Delta y},$$

$$u=-(q_{1\perp}-k_{2\perp})^2-k_1k_2e^{-\Delta y},$$

$$\Sigma=x_1x_2S=k_1^2+k_2^2+2k_1k_2\,\mathrm{ch}(\Delta y),$$

with $k_1=\sqrt{k_{1\perp}^2}$, $k_2=\sqrt{k_{2\perp}^2}$, and $k_{1\perp}k_{2\perp}$ is the dot product with 2D Euclidean metric.

Combined gluons and quarks (fermions) contribution to gg scattering has the form [adopted to Eq. (5)]

$$\mathcal{A}=\mathcal{A}_{\rm gluons}+\frac{n_f}{4N_c^3}\mathcal{A}_{\rm fermions}. \tag{A1}$$

1. $gg\to gg$

Find

$$\mathcal{A}_{\rm gluons}=\mathcal{A}_1+\mathcal{A}_2,$$

$$\begin{aligned}\mathcal{A}_1=q_1^2q_2^2\Bigg\{&-\frac{1}{tu}+\frac{1}{4tu}\frac{q_1^2q_2^2}{k_1^2k_2^2}-\frac{e^{\Delta y}}{4tk_1k_2}-\frac{e^{-\Delta y}}{4uk_1k_2}+\frac{1}{4k_1^2k_2^2}\\&+\frac{1}{\Sigma}\Bigg[-\frac{2}{s}\left(1+k_1k_2\left(\frac{1}{t}-\frac{1}{u}\right)\mathrm{sh}(\Delta y)\right)+\frac{1}{2k_1k_2}\\&\times\left(1+\frac{\Sigma}{s}\right)\mathrm{ch}(\Delta y)-\frac{q_1^2}{4s}\left[\left(1+\frac{k_2}{k_1}e^{-\Delta y}\right)\frac{1}{t}\right.\\&\left.+\left(1+\frac{k_1}{k_2}e^{\Delta y}\right)\frac{1}{u}\right]-\frac{q_2^2}{4s}\left[\left(1+\frac{k_1}{k_2}e^{-\Delta y}\right)\frac{1}{t}\right.\\&\left.+\left(1+\frac{k_2}{k_1}e^{\Delta y}\right)\frac{1}{u}\right]\Bigg]\Bigg\},\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{A}_2=\frac{D-2}{4}\Bigg\{&\left(\frac{(k_{1\perp}-q_{1\perp})^2(k_{2\perp}-q_{1\perp})^2-k_1^2k_2^2}{tu}\right)^2\\&-\frac{1}{4}\left(\frac{(k_{2\perp}-q_{1\perp})^2-k_1k_2e^{-\Delta y}}{(k_{2\perp}-q_{1\perp})^2+k_1k_2e^{-\Delta y}}-\frac{E}{s}\right)\\&\times\left(\frac{(k_{1\perp}-q_{1\perp})^2-k_1k_2e^{\Delta y}}{(k_{1\perp}-q_{1\perp})^2+k_1k_2e^{\Delta y}}+\frac{E}{s}\right)\Bigg\},\end{aligned}$$

$$\begin{aligned}E=&(q_{1\perp}-q_{2\perp})(k_{1\perp}-k_{2\perp})-\frac{1}{\Sigma}(q_1^2-q_2^2)(k_1^2-k_2^2)\\&+2k_1k_2\,\mathrm{sh}(\Delta y)\left(1-\frac{q_1^2+q_2^2}{\Sigma}\right).\end{aligned}$$

2. $gg\to q\bar{q}$

Find

$$\mathcal{A}_{\rm fermions}=N_c^2\mathcal{A}_{1f}+\mathcal{A}_{2f},$$

$$\begin{aligned}\mathcal{A}_{1f}=\Bigg\{&2\frac{q_1^2q_2^2}{s\Sigma}\left(1+k_1k_2\,\mathrm{sh}(\Delta y)\left(\frac{1}{t}-\frac{1}{u}\right)\right)\\&-\left(\frac{(k_{1\perp}-q_{1\perp})^2(k_{2\perp}-q_{1\perp})^2-k_1^2k_2^2}{tu}\right)^2\\&+\frac{1}{2}\left(\frac{(k_{2\perp}-q_{1\perp})^2-k_1k_2e^{-\Delta y}}{(k_{2\perp}-q_{1\perp})^2+k_1k_2e^{-\Delta y}}-\frac{E}{s}\right)\\&\times\left(\frac{(k_{1\perp}-q_{1\perp})^2-k_1k_2e^{\Delta y}}{(k_{1\perp}-q_{1\perp})^2+k_1k_2e^{\Delta y}}+\frac{E}{s}\right)\Bigg\},\end{aligned} \tag{A3}$$

and

$$\mathcal{A}_{2f}=\left\{\left(\frac{(k_{1\perp}-q_{1\perp})^2(k_{2\perp}-q_{1\perp})^2-k_1^2k_2^2}{tu}\right)^2-\frac{q_1^2q_2^2}{tu}\right\},$$

where E is the same as for gluons.

APPENDIX B: INTEGRALS

To make integration easily it is worth to change integration over Δy to integration over $x=k_1/(k_1+k_2e^{\Delta y})$:

$$\int_{-\infty}^{\infty}d\Delta y\cdots=\int_0^1\frac{dx}{x(1-x)}\cdots, \tag{B1}$$

$$\begin{aligned}\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}&\left(-\frac{1}{2}\frac{1}{k_1^2k_2^2}\right)\\&=-\frac{1}{2}\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\frac{1}{(k_\perp-k_{2\perp})^2k_{2\perp}^2}\\&=-\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)},\end{aligned} \tag{B2}$$

$$\begin{aligned}\int&\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\frac{\mathrm{ch}(\Delta y)}{k_1k_2s}\\&=\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\left(\frac{1-x}{2k_2^2sx}+\frac{x}{2k_1^2s(1-x)}\right)\\&=\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\frac{(1-x)^2}{2k_2^2((1-x)k_\perp-k_{2\perp})^2}+(x\leftrightarrow1-x)\\&=\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}[(1-x)^{2\varepsilon}+x^{2\varepsilon}].\end{aligned} \tag{B3}$$

Let's now combine these two contributions and perform an integration over x

$$\begin{aligned}I_{12}=&\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\int\frac{dx}{x(1-x)}\\&\times[(1-x)^{2\varepsilon}+x^{2\varepsilon}-1].\end{aligned} \tag{B4}$$

In order to avoid divergencies we introduce infinitesimal parameter δ ($\delta\ll\varepsilon$) so that

$$\int \frac{dx}{x(1-x)}[(1-x)^{2\varepsilon}+x^{2\varepsilon}-1]$$
$$=\lim_{\delta\to 0}\int \frac{dx}{x^{1-\delta}(1-x)^{1-\delta}}[(1-x)^{2\varepsilon}+x^{2\varepsilon}-1]$$
$$=\lim_{\delta\to 0}\left(2\frac{\Gamma(\delta)\Gamma(2\varepsilon+\delta)}{\Gamma(2\varepsilon+2\delta)}-\frac{\Gamma^2(\delta)}{\Gamma(2\delta)}\right)$$
$$=\frac{1}{\varepsilon}+2\psi(1)-2\psi(1+2\varepsilon) \tag{B5}$$

and

$$I_{12}=\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^{\varepsilon}}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\times\left(\frac{1}{\varepsilon}+2\psi(1)-2\psi(1+2\varepsilon)\right). \tag{B6}$$

For the integrations involving Σ it is useful to change $k_{2\perp}$ on $\kappa=k_{2\perp}-(1-x)k_{\perp}$ so that $s=\kappa^2/x(1-x)$ and $\Sigma=\kappa^2/x(1-x)+k^2$. Now

$$\int \frac{dx}{x(1-x)}\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\frac{1}{s\Sigma}=\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^{\varepsilon}}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\frac{1}{1+2\varepsilon} \tag{B7}$$

and

$$\int \frac{dx}{x(1-x)}\int\frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}}\frac{E^2}{8q_1^2q_2^2s^2}=\pi\frac{\Gamma(1-\varepsilon)}{(4\pi)^{\varepsilon}}\frac{k^{2\varepsilon}}{k^2}\frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\frac{1-\varepsilon}{2(1+2\varepsilon)(3+2\varepsilon)}. \tag{B8}$$

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