Weyl invariant formulation of the flux-tube solution in the dual Ginzburg-Landau theory

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The flux-tube solution in the dual Ginzburg-Landau (DGL) theory in the Bogomol'nyi limit is studied by using the manifestly Weyl invariant form of the DGL Lagrangian. The dual gauge symmetry is extended to $[\mathrm{U}(1)]_m^3$, and, accordingly, there appear three different types of flux tube. The string tension for each flux tube is calculated analytically and is found to be the same owing to Weyl symmetry. It is suggested that the manifestly Weyl invariant approach enables us to treat flux tubes of various types in the DGL theory in a framework quite similar to the U(1) dual Abelian Higgs theory.

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I. INTRODUCTION

Recent studies in lattice QCD in the maximally Abelian gauge suggest remarkable properties of the QCD vacuum, such as Abelian dominance [1] and monopole condensation [2], which provide the dual superconductor picture of the QCD vacuum as described by the dual Ginzburg-Landau (DGL) theory [3,4]. The DGL theory is obtained by using Abelian projection [5]. In this scheme, QCD is reduced to $[\mathrm{U}(1)]^2$ gauge theory including color-magnetic monopoles. Based on the dual superconductor picture of the QCD vacuum, we get an intuitive picture of hadrons as the vortex excitation of the color-electric flux [6,7], which we call the color-electric flux tube, or simply the flux tube. In this vacuum, the color-electric flux is squeezed into an almost one-dimensional object such as a string due to the dual Meissner effect caused by monopole condensation. This situation seems to be the same as the appearance of the Abrikosov vortex in the ordinary superconductor system, which is caused by Cooper pair condensation.

We know that the Abrikosov-Nielsen-Olesen (ANO) vortex in an ordinary superconductor can be described by using the Abelian Higgs theory [8], where the key is the breaking of $\mathrm{U}(1)_e$ gauge symmetry through the Higgs mechanism. Moreover, there exists an analytic solution of the ANO vortex at the border of type-I and type-II vacuums, called the Bogomol'nyi limit [9,10]. The analytical solution exhibits interesting features of superconductivity and is useful in understanding the properties of vortex dynamics. Hence, it is considered quite interesting to investigate the flux-tube solution in the dual superconductor QCD vacuum corresponding to the ANO vortex in the Bogomol'nyi limit.

However, the symmetry in the QCD vacuum is not as simple as in an ordinary superconductor system, since now we have to take into account the $[\mathrm{U}(1)]_m^2$ dual gauge symmetry corresponding to the $\mathrm{U}(1)_e$ gauge symmetry in the ordinary superconductor. Note that the symmetry $[\mathrm{U}(1)]^{N-1}$ originates from the maximal torus subgroup of SU(N). Furthermore, we also have *Weyl* symmetry, which is the permutation invariance of the labels among the Abelian color charges. Therefore, the flux tube in the QCD vacuum is expected to have some characteristic aspects beyond being the analog of the ANO vortex in an ordinary superconductor system.

In this paper, we investigate the flux-tube solution in DGL theory in the Bogomol'nyi limit. This study is similar to that given in Ref. [11]. In fact, our result will be shown to be identical. However, we would like to present a useful method to find the Bogomol'nyi limit, and this can be achieved by taking into account the Weyl symmetry in the DGL theory. This idea can be extended straightforwardly to the $[\mathrm{U}(1)]^{N-1}$ dual Abelian Higgs theory that would be reduced from SU(N) gluodynamics [12]. We first write the DGL Lagrangian in a manifestly Weyl invariant form. At the same time, we pay attention to the singular structure in the DGL theory, since it plays a significant role in obtaining the string-like flux-tube solution. Note that the boundary condition of the dual gauge field depends crucially on this singular structure. Second, we consider the Bogomol'nyi limit, the border between the type-I and type-II vacuums. The string tension in this limit is computed analytically. Finally, we discuss the properties of the flux-tube solution in DGL theory.

II. MANIFESTLY WEYL INVARIANT FORM OF THE DGL LAGRANGIAN

The DGL Lagrangian [3,4] is given by[1]

$$\mathcal{L}_{\mathrm{DGL}} = -\frac{1}{4}\left(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu - \frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha \vec{j}^\beta\right)^2 + \sum_{i=1}^{3}\left[|(\partial_\mu + ig\vec{\epsilon}_i\cdot\vec{B}_\mu)\chi_i|^2 - \lambda(|\chi_i|^2 - v^2)^2\right], \tag{2.1}$$

where $\vec{B}_\mu$ and χ_i denote the dual gauge field with two components (B_μ^3, B_μ^8) and the complex scalar monopole field, respectively. The quark field is included in the current $\vec{j}_\mu = e\bar{q}\gamma_\mu \vec{H} q$, $\vec{H} = (T_3, T_8)$. Here, $\vec{\epsilon}_i$ is the root vector of

[1]Throughout this paper, we use the following notation: Latin indices i,j express the labels 1,2,3, which are not to be summed over unless explicitly stated. Boldface letters denote three-vectors.

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SU(3) algebra, $\vec{\epsilon}_1=(-1/2,\sqrt{3}/2)$, $\vec{\epsilon}_2=(-1/2,-\sqrt{3}/2)$, $\vec{\epsilon}_3=(1,0)$, and n^μ denotes an arbitrary constant four-vector,[2] which corresponds to the direction of the Dirac string. The gauge coupling e and the dual gauge coupling g have the relation $eg=4\pi$. This relation guarantees the unobservability of the Dirac string when the dual gauge symmetry is not broken. Note that the DGL Lagrangian (2.1) is invariant under the $[\mathrm{U}(1)]_m^2$ dual gauge transformation:

$$\chi_i\to\chi_i e^{if_i},\quad \chi_i^*\to\chi_i^* e^{-if_i},$$

$$\begin{aligned}\vec{B}_\mu&=(B_\mu^3,B_\mu^8)\\ &\to\left(B_\mu^3-\frac{1}{g}\partial_\mu f_3,B_\mu^8-\frac{1}{\sqrt{3}g}(\partial_\mu f_1-\partial_\mu f_2)\right)\\ &(i=1,2,3),\end{aligned} \tag{2.2}$$

where the phase f_i has the constraint $\Sigma_{i=1}^3 f_i=0$ [3,4].

The nonlocal term in the kinetic term of the dual gauge field is concretely written as

$$\frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha\vec{j}^\beta=\int d^4x'\langle x|\frac{1}{n\cdot\partial}|x'\rangle\varepsilon_{\mu\nu\alpha\beta}n^\alpha\vec{j}^\beta(x'), \tag{2.3}$$

where

$$\langle x|\frac{1}{n\cdot\partial}|x'\rangle=[p\,\theta((x-x')\cdot n)-(1-p)\,\theta((x'-x)\cdot n)]\times\delta^{(3)}(\vec{x}_\perp-\vec{x}'_\perp). \tag{2.4}$$

Here p is an arbitrary real number and $\delta^{(3)}(x)$ is a δ function defined on a three dimensional hypersurface that has the normal vector n_μ, so that $\vec{x}_\perp$ and $\vec{x}'_\perp$ are three-vectors (not necessarily spatial) that are perpendicular to n_μ. It is noted that, in order to define the color-electric charge of the quark in terms of the *dual* gauge field, we need such a nonlocal term, which is a result of the choice of the *one potential approach* [13].

Now, we define an extended dual gauge field to take into account the Weyl invariance in the DGL theory as

$$B_{i\mu}\equiv\sqrt{\frac{2}{3}}\vec{\epsilon}_i\cdot\vec{B}_\mu\quad(i=1,2,3). \tag{2.5}$$

Here, the constraint $\Sigma_{i=1}^3 B_{i\mu}=0$ appears, which has the same structure as the constraint $\Sigma_{i=1}^3 f_i=0$. Furthermore, we divide the dual gauge field into two parts, a regular part and a singular part [14,15],

$$\vec{B}_\mu\equiv\vec{B}_\mu^{\mathrm{reg}}+\vec{B}_\mu^{\mathrm{sing}}. \tag{2.6}$$

The factor $\sqrt{2/3}$ in Eq. (2.5) is a simple normalization to get the factor 1/4 in front of the kinetic term of the dual gauge field [see Eq. (2.8) below]. The singular dual gauge field $\vec{B}_\mu^{\mathrm{sing}}$ is determined so as to cancel the Dirac string in the nonlocal term as

$$\partial_\mu\vec{B}_\nu^{\mathrm{sing}}-\partial_\nu\vec{B}_\mu^{\mathrm{sing}}-\frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha\vec{j}^\beta\equiv\vec{C}_{\mu\nu}. \tag{2.7}$$

In the static q-$\bar{q}$ system, $\vec{C}_{\mu\nu}$ is nothing but the color-electric field originating from the color-electric charge, like the electric field induced by an electric charge, where an explicit form of $\vec{B}_\mu^{\mathrm{sing}}$ is given in Sec. III. It is noted that the cross term of the regular dual field tensor $*\vec{F}_{\mu\nu}^{\mathrm{reg}}\equiv\partial_\mu\vec{B}_\nu^{\mathrm{reg}}-\partial_\nu\vec{B}_\mu^{\mathrm{reg}}$ and $\vec{C}_{\mu\nu}$ can be integrated out; the square of $\vec{C}_{\mu\nu}$ and its integration give the Coulomb energy including the self-energy of the color-electric charge. However, we drop the square of $\vec{C}_{\mu\nu}$ hereafter in order to concentrate on the flux tube itself. Accordingly, we pay attention to the string tension for an ideal flux-tube system which has terminals at infinity.[3]

Thus we obtain

$$\mathcal{L}_{\mathrm{DGL}}=\sum_{i=1}^{3}\left(-\frac{1}{4}{*F_{i\mu\nu}^{\mathrm{reg}}}^2+|[\partial_\mu+ig'(B_{i\mu}^{\mathrm{reg}}+B_{i\mu}^{\mathrm{sing}})]\chi_i|^2-\lambda(|\chi_i|^2-v^2)^2\right), \tag{2.8}$$

$$*F_{i\mu\nu}^{\mathrm{reg}}\equiv\partial_\mu B_{i\nu}^{\mathrm{reg}}-\partial_\nu B_{i\mu}^{\mathrm{reg}}, \tag{2.9}$$

where the dual gauge coupling g is scaled as

$$g'\equiv\sqrt{\frac{3}{2}}g. \tag{2.10}$$

One finds that the dual gauge symmetry becomes very easy to observe, since the dual gauge transformation is defined by

$$\chi_i\to\chi_i e^{if_i},\quad \chi_i^*\to\chi_i^* e^{-if_i},$$

$$B_{i\mu}^{\mathrm{reg}}\to B_{i\mu}^{\mathrm{reg}}-\frac{1}{g'}\partial_\mu f_i\quad(i=1,2,3), \tag{2.11}$$

and accordingly the Lagrangian (2.8) has the extended local symmetry $[\mathrm{U}(1)]_m^3$. However, this does not mean an increase of the gauge degrees of freedom because we have the constraint $\Sigma_{i=1}^3 B_{i\mu}=0$.

The field equations are given by

$$[\partial_\mu+ig'(B_{i\mu}^{\mathrm{reg}}+B_{i\mu}^{\mathrm{sing}})]^2\chi_i=-2\lambda\chi_i(\chi_i^*\chi_i-v^2), \tag{2.12}$$

[2]If the dual gauge symmetry is broken through monopole condensation, n^μ cannot be an arbitrary vector any longer. Instead, this vector describes the dynamics of the string and gives the contribution to the energy of the system.

[3]In order to classify the types of flux tube, we use words such as the q-$\bar{q}$ system.

$$\partial^\nu {}^*F^{\rm reg}_{i\mu\nu} \equiv k_{i\mu} = -ig'(\chi_i^*\partial_\mu\chi_i - \chi_i\partial_\mu\chi_i^*) + 2g'^2(B^{\rm reg}_{i\mu}+B^{\rm sing}_{i\mu})\chi_i^*\chi_i . \tag{2.13}$$

These field equations are to be solved with the proper boundary conditions that quantize the color-electric flux [8]. The flux is given by

$$\Phi_i \equiv \int {}^*F^{\rm reg}_{i\mu\nu} d\sigma^{\mu\nu} = \oint B^{\rm reg}_{i\mu} dx^\mu, \tag{2.14}$$

where $\sigma^{\mu\nu}$ is a two-dimensional surface element in Minkowski space. By using the polar decomposition of the monopole field as $\chi_i = \phi_i e^{i\eta_i}$ ($\phi_i, \eta_i \in \Re$), we get, from the field equation (2.13),

$$B^{\rm reg}_{i\mu} = \frac{k_{i\mu}}{2g'^2\phi_i^2} - B^{\rm sing}_{i\mu} - \frac{1}{g'}\partial_\mu\eta_i . \tag{2.15}$$

We substitute this expression into Eq. (2.14) and integrate out over a large closed loop where the monopole current $k_{i\mu}$ vanishes. Thus we get

$$\Phi_i = -\oint \left(B^{\rm sing}_{i\mu} + \frac{1}{g'}\partial_\mu\eta_i \right) dx^\mu. \tag{2.16}$$

It is suggested from this expression that there are two possibilities for obtaining the flux-tube configuration. One originates from the singularity in $B^{\rm sing}_{i\mu}$ and the other is from the singularity in $\partial_\mu\eta_i$. We find that the former case, as can be seen from the relation (2.7), corresponds to the flux tube that has a quark source. On the other hand, the latter case does not contain any information about the quark, which means there is no terminal; hence, it cannot provide a physical state like the q-$\bar{q}$ system. If one assumes the existence of an external color-electric source or a glueball state as the flux-tube ring [16], it should be taken into account. However, since this is not the case that we discuss in this paper, we assume that there is no singularity in $\partial_\mu\eta_i$. Thus this term can be absorbed into the regular dual gauge field $B^{\rm reg}_{i\mu}$ by the replacement $B^{\rm reg}_{i\mu} + \partial_\mu\eta_i/g' \to B^{\rm reg}_{i\mu}$. In this case, the flux (2.16) just has the meaning of the boundary condition of the regular dual gauge field, which should behave as $B^{\rm reg}_{i\mu} \to -B^{\rm sing}_{i\mu}$ at infinity, where monopoles are condensed.

III. THE STATIC q-$\bar{q}$ SYSTEM

In this section, we consider the static q-$\bar{q}$ system. The quark source is given by the c-number current, which is typical in the heavy quark system,

$$\vec{j}^{\,\mu} \equiv \vec{j}^{\,\mu}{}_j(x) = \vec{Q}_j g^{\mu 0}[\delta^{(3)}(\boldsymbol{x}-\boldsymbol{a}) - \delta^{(3)}(\boldsymbol{x}-\boldsymbol{b})], \tag{3.1}$$

where $\vec{Q}_j \equiv e\vec{w}_j$ is the Abelian color-electric charge of the quark. Here, $\boldsymbol{a}$ and $\boldsymbol{b}$ are position vectors of the quark and the antiquark, respectively, and $\vec{w}_j$ is the weight vector of SU(3) algebra, $\vec{w}_1 = (1/2, \sqrt{3}/6)$, $\vec{w}_2 = (-1/2, \sqrt{3}/6)$, $\vec{w}_3 = (0, -1/\sqrt{3})$. This vector is nothing but the diagonal component of $\vec{H} = (T_3, T_8)$. The label $j = 1,2,3$ can be assigned to the charges red (R), blue (B), and green (G). We assume cylindrical geometry of the system by taking $\boldsymbol{a} = -(r/2)\boldsymbol{e}_z$, $\boldsymbol{b} = (r/2)\boldsymbol{e}_z$, and $n_\mu = \boldsymbol{e}_z$, where the distance between the quark and the antiquark is defined by r. In this system, we get an explicit form of the singular dual gauge field from the relation (2.7) as

$$\boldsymbol{B}^{\rm sing}_i = \sqrt{\frac{2}{3}}\vec{\epsilon}_i \cdot \left[-\frac{\vec{Q}_j}{4\pi\rho}\left(\frac{z+r/2}{\sqrt{\rho^2+(z+r/2)^2}} - \frac{z-r/2}{\sqrt{\rho^2+(z-r/2)^2}} \right)\boldsymbol{e}_\varphi \right], \tag{3.2}$$

where φ is the azimuthal angle around the z axis and ρ denotes the radial coordinate. Since the color-electric charges are defined on the weight vector of SU(3) algebra, there arises the relation

$$\vec{\epsilon}_i \cdot \vec{w}_j = -\frac{1}{2}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = -\frac{1}{2}\sum_{k=1}^{3}\epsilon_{ijk} \equiv -\frac{1}{2}m_{ij}, \tag{3.3}$$

where m_{ij} takes the values 0 or ± 1. The zero of the diagonal component means that one of the monopole fields is decoupled from the system and does not contribute to the energy when we pay attention to one of the color-electric charges, since the color-magnetic charge of the monopole field is defined on the root vector of SU(3) algebra, as $g\vec{\epsilon}_i$.

Here, we investigate the ideal system for the limit $r \to \infty$. That is,

$$\lim_{r\to\infty} \boldsymbol{B}^{\rm sing}_i = \sqrt{\frac{2}{3}}\frac{e m_{ij}}{4\pi\rho}\boldsymbol{e}_\varphi = \frac{m_{ij}}{g'\rho}\boldsymbol{e}_\varphi, \tag{3.4}$$

where we have used $eg = 4\pi$ and $g' = \sqrt{3/2}g$. Thus the fields depend only on the radial coordinate,

$$\phi_i = \phi_i(\rho), \quad \boldsymbol{B}^{\rm reg}_i = B^{\rm reg}_i(\rho)\boldsymbol{e}_\varphi \equiv \frac{\tilde{B}^{\rm reg}_i(\rho)}{\rho}\boldsymbol{e}_\varphi, \tag{3.5}$$

and the field equations (2.12) and (2.13) are reduced to

$$\frac{d^2\phi_i}{d\rho^2} + \frac{1}{\rho}\frac{d\phi_i}{d\rho} - \left(\frac{g'\tilde{B}^{\rm reg}_i + m_{ij}}{\rho} \right)^2 \phi_i - 2\lambda\phi_i(\phi_i^2 - v^2) = 0, \tag{3.6}$$

$$\frac{d^2\tilde{B}^{\rm reg}_i}{d\rho^2} - \frac{1}{\rho}\frac{d\tilde{B}^{\rm reg}_i}{d\rho} - 2g'(g'\tilde{B}^{\rm reg}_i + m_{ij})\phi_i^2 = 0, \tag{3.7}$$

The string tension can be defined by the energy per unit length of the flux tube,

$$\sigma=2\pi\sum_{i=1}^{3}\int_0^\infty \rho d\rho\left[\frac{1}{2}\left(\frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho}\right)^2+\left(\frac{d\phi_i}{d\rho}\right)^2+\left(\frac{g'\tilde{B}_i^{\rm reg}+m_{ij}}{\rho}\right)^2\phi_i^2+\lambda(\phi_i^2-v^2)^2\right], \quad (3.8)$$

and we obtain the flux quantization condition

$$\Phi_i=-\frac{2\pi m_{ij}}{g'}. \quad (3.9)$$

The boundary conditions are given by

$$\tilde{B}_i^{\rm reg}=0,\quad \phi_i=\begin{cases}0 & (i\neq j)\\ v & (i=j)\end{cases}\quad \text{as}\ \ \rho\to 0,$$

$$\tilde{B}_i^{\rm reg}=-\frac{m_{ij}}{g'},\quad \phi_i=v\quad \text{as}\ \ \rho\to\infty. \quad (3.10)$$

Here, we shall confirm the relation (2.7). In this cylindrical system, the nonlocal term can be computed explicitly,

$$\sqrt{\frac{2}{3}}\vec{\epsilon}_i\cdot\frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha\vec{j}^\beta =\sqrt{\frac{2}{3}}\vec{\epsilon}_i\cdot-\vec{Q}_j\delta(x)\delta(y)\boldsymbol{e}_z\quad(\vec{Q}_j\equiv e\vec{w}_j) =\boldsymbol{\nabla}\times\left(\frac{m_{ij}}{g'\rho}\boldsymbol{e}_\varphi\right). \quad (3.11)$$

As can be seen from this expression, one finds that this term exactly cancels the color-electric field that is originates from the singular dual gauge field $\boldsymbol{B}_i^{\rm sing}$ in Eq. (3.4). This shows that the kinetic term of the dual gauge field in the Lagrangian (2.8) can be written with no singular field.

IV. BOGOMOL'NYI LIMIT

In this section, we discuss the properties of the flux tube in the Bogomol'nyi limit. Since now we have the same Lagrangian with U(1) gauge symmetry except only the labels of i and j that classify the kinds of monopole and the quark corresponding to $[\mathrm{U}(1)]_m^3$ dual gauge symmetry, we can use the same strategy to find the Bogomol'nyi limit as given in Ref. [9]. Thus, we can write the string tension (3.8) exactly in the form

$$\sigma=2\pi\sum_{i=1}^{3}|m_{ij}|v^2+2\pi\sum_{i=1}^{3}\int_0^\infty\rho d\rho\left[\frac{1}{2}\left(\frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho}\pm g'(\phi_i^2-v^2)\right)^2+\left(\frac{d\phi_i}{d\rho}\pm(g'\tilde{B}_i^{\rm reg}+m_{ij})\frac{\phi_i}{\rho}\right)^2+\frac{1}{2}(2\lambda-g'^2)(\phi_i^2-v^2)^2\right]. \quad (4.1)$$

From this expression, we find the Bogomol'nyi limit

$$g'^2=2\lambda\quad\text{or}\quad 3g^2=4\lambda. \quad (4.2)$$

In this limit, one finds that the string tension is reduced to

$$\sigma=2\pi\sum_{i=1}^{3}|m_{ij}|v^2=4\pi v^2, \quad (4.3)$$

and the profiles of the dual gauge field and the monopole field are determined by the first order differential equations:

$$\frac{d\phi_i}{d\rho}\pm(g'\tilde{B}_i^{\rm reg}+m_{ij})\frac{\phi_i}{\rho}=0, \quad (4.4)$$

$$\frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho}\pm g'(\phi_i^2-v^2)=0. \quad (4.5)$$

These field equations, of course reproduce the second order differential equations (3.6) and (3.7) when the relation (4.2) is satisfied.

Here, to obtain a string tension of the form (4.1) and the saturated string tension (4.3), we have paid attention to the boundary conditions of the fields (3.10) by taking into account the relation (3.3). For instance, let us consider the R-$\bar{R}$ flux tube, which is given by the label $j=1$. In this system, the monopole field ϕ_1, which has the magnetic charge $g\vec{\epsilon}_1$, is decoupled from the system, since ϕ_1 does not experience any singularity of the flux-tube core, and, accordingly, the regular dual gauge field $B_1^{\rm reg}$ is also decoupled. The behavior of the other fields is interesting; ϕ_2 and ϕ_3 behave as the same monopole field, and $B_2^{\rm reg}$ and $B_3^{\rm reg}$ provide the $\mathrm{U}(1)_{i=2}$ flux tube and $\mathrm{U}(1)_{i=3}$ anti-flux-tube depending on the sign of m_{ij}, which is 1 and -1, respectively. Here, the two dual gauge fields are related to each other through the constraint $\Sigma_{i=1}^3 B_i^{\rm reg}=0$, and the $\mathrm{U}(1)_{i=3}$ anti-flux-tube can be regarded as the $\mathrm{U}(1)_{i=2}$ flux tube, or vice versa. As a result, these flux tubes provide the same string tension $2\pi v^2$, and finally we get twice this string tension, $2\times 2\pi v^2$. This is caused by the $[\mathrm{U}(1)]_m^2$ dual gauge symmetry. We note that this discussion is Weyl symmetric, and thus the final expression for the string tension (4.3) does not depend on the kind of color-electric charge $\vec{Q}_j$. The profiles of the color-electric field can be obtained by solving the first order equations (4.4) and (4.5) by taking into account the above discussion, as is discussed in Ref. [9,10].

Let us consider the meaning of Eq. (4.2). Here, we can define two characteristic scales using three parameters in DGL theory, g, λ, and v. One is the mass of the dual gauge field $m_B=\sqrt{2}g'v=\sqrt{3}gv$ and the other is the mass of the monopole field $m_\chi=2\sqrt{\lambda}v$. These masses are extracted from the Lagrangian (2.8) by taking into account the dual Higgs mechanism. Thus, one finds that the Bogomol'nyi limit in the DGL theory [Eq. (4.2)] is the supersymmetry between the dual gauge field and the monopole field. Since these inverse masses m_B^{-1} and m_χ^{-1} correspond to the penetration depth of the color-electric field and the coherent length of the monopole field, respectively, the Ginzburg-Landau (GL) parameter is defined as

$$\tilde{\kappa} \equiv \frac{m_B^{-1}}{m_\chi^{-1}} = \frac{\sqrt{2\lambda}}{g'} = \frac{2\sqrt{\lambda}}{\sqrt{3}g}. \tag{4.6}$$

Therefore, $\tilde{\kappa}=1$ is regarded as the Bogomol'nyi limit, and the vacuum is classified into two types in terms of the Bogomol'nyi limit: $\tilde{\kappa}<1$ belongs to the type-I vacuum and $\tilde{\kappa}>1$ is the type-II vacuum.

Now we would like to discuss the interaction between two parallel flux tubes of the same type, such as the system R-$\bar{R}$ and R-$\bar{R}$. In general, the flux tubes would interact with each other. However, in the Bogomol'nyi limit, there is no interaction between them. This can be understood through an investigation of the generalized string tension for an exotica that the color-electric charges are given by $n\vec{Q}_j$ and $-n\vec{Q}_j$ for an integer n. In this system, we get the generalized string tension

$$\sigma_n = 4\pi n v^2, \tag{4.7}$$

where m_{ij} is simply replaced by nm_{ij}. One finds that the string tension (4.7) is proportional to n, which implies that the interaction energy is zero. It is considered that this comes from a balance of the propagation ranges of the dual gauge field and the monopole field since $m_B=m_\chi$. In the type-I or type-II vacuum, which is away from the Bogomol'nyi limit, the interaction range of these fields lose its balance, and the flux-tube interaction manifestly appears. The string tension is no longer proportional to n. While an attractive force appears between two parallel flux tubes in the type-I vacuum, the flux tubes repel each other in the type-II vacuum. Numerical investigations of the interaction between two or more parallel flux tubes of the same type in DGL theory are given in Refs. [17,18].

It is interesting to investigate what happens if two parallel flux tubes of different types are placed at a certain distance apart [19]. Here, according to the $[\mathrm{U}(1)]_m^3$ dual gauge symmetry, there appear three different types of flux tube, such as given by R-$\bar{R}$, B-$\bar{B}$, and G-$\bar{G}$, so that these interactions appear very complicated. However, now the system has remarkable aspects owing to the Weyl symmetry. For instance, let us consider the interaction between R-$\bar{R}$ and B-$\bar{B}$. We find that the interaction between them is attractive, since, if we suppose that these flux tubes are unified into one flux tube, it becomes $\bar{G}$-G [see the relation (3.3)]. This means that the energy of the system after unification is reduced to one-half of the initial energy. The same interaction property would be observed in the processes, B-$\bar{B}+G$-$\bar{G}\to\bar{R}$-R and G-$\bar{G}+R$-$\bar{R}\to\bar{B}$-B. These investigations show that, if we pay attention to the Weyl symmetry, we can easily obtain qualitative information about the flux-tube interaction.

V. CONCLUSION

We have studied the flux-tube solution in DGL theory in the Bogomol'nyi limit by using the manifestly Weyl invariant form of the DGL Lagrangian. Here, the original dual gauge symmetry $[\mathrm{U}(1)]_m^2$ is extended to $[\mathrm{U}(1)]_m^3$. This replacement makes further manipulation of the Lagrangian analogous to the U(1) case. We have found that the Bogomol'nyi limit is given by $3g^2=4\lambda$, and the string tension is calculated as $\sigma_n=4\pi n v^2$ for a q-$\bar{q}$ pair with the charge nQ_j and $-nQ_j$ at the two ends. In this limit, the masses of the dual gauge field and the monopole field become *exactly* the same. It should be noted that we could see the same relation with U(1) Abelian Higgs theory except for the three different types of flux tube. To summarize, very similar properties to those of the ANO vortex in Abelian Higgs theory are observed when we see a single flux tube in DGL theory, and the flux-tube solution can easily be obtained if we pay attention to the Weyl symmetry in DGL theory.

Finally, we would like to mention the relation between the work in Ref. [11] and our study. If we replace the monopole field and the parameters used in [11] by $\chi\to\sqrt{2}\chi$, $\eta\to\sqrt{2}v$, and $\lambda\to\lambda/4$, we get the same framework at the starting point, and the Bogomol'nyi limit is replaced by $3g^2=16\lambda\to 3g^2=4\lambda$. The idea of the extension of dual gauge symmetry based on Weyl symmetry in our case, however, seems to be a simple way to reach the final expression for the string tension, which can be applied straightforwardly to the $[\mathrm{U}(1)]^{N-1}$ dual Abelian Higgs theory reduced from SU(N) gluodynamics.

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