Spectroscopy of doubly heavy baryons

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Spectra of masses are calculated for the families of doubly heavy baryons in the framework of the nonrelativistic quark model with the QCD potential of Buchmüller and Tye. We suppose the quark-diquark structure for the wave functions and take into account the spin-dependent splittings. The physical reasons causing the existence of quasistable excited states in the subsystem of heavy diquark are considered for the heavy quarks of identical flavors.

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I. INTRODUCTION

The first observation of the B_c^+ meson by the Collider Detector at Fermilab (CDF) Collaboration [1] opens a new direction in the physics of hadrons containing heavy quarks. This particle completes the list of heavy quarkonia $(Q\overline{Q'})$ and heavy flavored mesons accessible for the experimental investigations. It begins another list of long-lived hadrons containing two heavy quarks. So, in addition to the B_c^+ meson this class of hadrons would be continued by the doubly heavy baryons Ξ_{cc}, Ξ_{bc}, and Ξ_{bb} [see the notation in the framework of the quark model of the Particle Data Group (PDG) [2]]. The experimental discovery of B_c^+ was prepared by theoretical studies of the meson spectroscopy as well as the mechanisms of its production and decay (see review in Ref. [3]). To observe the doubly heavy baryons it is necessary to give reliable theoretical predictions of their properties. The initial steps forward towards such a goal were done.

(1) In Ref. [4] the estimates for the lifetimes of Ξ_{cc}^+ and Ξ_{cc}^{++} baryons were obtained in the framework of operator product expansion in the inverse heavy quark mass.

(2) Papers [5] were devoted to the investigation of differential and total cross sections for the production of $\Xi_{QQ'}$ baryons in various interactions in the model of fragmentation, in the model of intrinsic charm [6] (for the hadronic production of Ξ_{cc}) and in the framework of perturbative QCD calculations up to $O(\alpha_s^4)$ contributions, taking into account the hard nonfragmentational regime in addition to the fragmentation, which dominates at high transverse momenta $p_T \gg M$.

(3) In Refs. [7,8] the masses of ground states of doubly heavy baryons were estimated, and the excitations of Ξ_{cc} were considered in Ref. [11].

In the present paper we analyze the basic spectroscopic characteristics for the families of doubly heavy baryons $\Xi_{QQ'} = (QQ'q)$, where $q = u,d$ and $\Omega_{QQ'} = (QQ's)$. A general approach of potential models to calculate the masses of baryons containing two heavy quarks was considered in Ref. [9]. The physical motivation used the pair interactions between the quarks composing the baryon, that was explored in the three-body problem. Clear implications for the mass spectra of doubly heavy baryons were derived. So, for the given masses of heavy charmed and beauty quarks, the approximation of factorization for the motion of doubly heavy diquark and light quark is not accurate. It results in the ground state mass and excitation levels, essentially deviating from the estimates in the framework of appropriate three-body problem. For example, we can easily find that in the oscillator potential of pair interactions an evident introduction of Jacobi variables leads to the change of vibration energy $\omega \to \sqrt{3/2}\,\omega$ in comparison with naive expectations of diquark factorization.

However, the stringlike picture of doubly heavy baryon shown in Fig. 1, certainly destroys the above conclusions based on the pair interactions. Indeed, to the moment we have to introduce the center of string, which is very close to the center of mass for the doubly heavy diquark. Furthermore, the light quark interacts with the doubly heavy diquark as a whole, i.e., with the string tension identical to that in the heavy-light mesons. Therefore, two different assumptions on the nature of interactions inside the doubly heavy baryons. Pair interactions or a stringlike picture result in a distinct variation of predictions on the mass spectra of these baryons: the ground states and excitation levels. The only criterion testing the assumptions is provided by an experimental observation and measurements.

In this paper we follow the approximation of double heavy diquark, which is quite reasonable as we have clarified in the discussion given above. To enforce this point we refer to the consideration of doubly heavy baryon masses in the framework of QCD sum rules [8], that recently was essen-

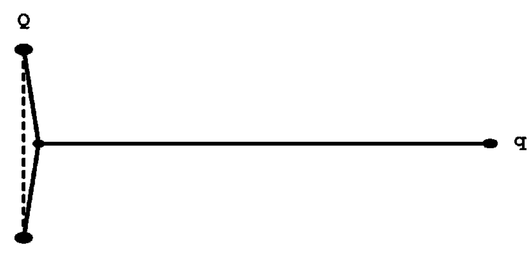

FIG. 1. The picture of doubly heavy baryon QQq with the colored fields forming the strings between the heavy and light quarks, that destroys the pair interactions and involves the additional ''center-of-mass'' point close to the center of mass for the heavy-heavy system.

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tially improved in paper [10], exploring the nonrelativistic QCD (NRQCD) version of sum rule method. The conclusion drawn in the sum rules is the ground state mass is in a good agreement with the estimates obtained in the potential approach with the factorization of doubly heavy diquark.

The qualitative picture for the forming of bound states in the system of $(QQ'q)$ is determined by the presence of two scales of distances, which are given by the size of the QQ'-diquark subsystem, $r_{QQ'}$, in the antitriplet color state as well as by the confinement scale $\Lambda_{\rm QCD}$ for the light quark q, so that

$$r_{QQ'}\cdot\Lambda_{\rm QCD}\ll 1, \quad \Lambda_{\rm QCD}\ll m_Q.$$

Under such conditions, the compact diquark QQ' looks similar to a static source approximated by the local colored QCD field interacting with the light quark. Therefore, we may use a set of reliable results in models of mesons with a single heavy quark, i.e., with a local static source belonging to the antitriplet representation of the $SU(3)_c$ group. The successful approaches are the potential models [12] and the heavy quark effective theory (HQET) [13] in the framework of expansion in the inverse heavy quark mass. We apply the nonrelativistic quark model with the potential by Buchmüller and Tye [14]. Then *theoretically* we can talk on the rough approximation for the light quark. Indeed, since $m_q^{\rm QCD}\ll\Lambda_{\rm QCD}$ the light quark is relativistic. Nevertheless, we introduce the system with a finite number of degrees of freedom and an instantaneous interaction $V(\mathbf{r})$. This fact is a disadvantage because the confinement supposes the following: (a) the generation of sea around the light quark, i.e., the presence of infinite number of gluons and quark-antiquark pairs and (b) the nonperturbative effects with the correlation time $\tau_{\rm QCD}\sim 1/\Lambda_{\rm QCD}$, that is beyond the potential approach. However, *phenomenologically* the introduction of constituent mass $m_q^{NP}\sim\Lambda_{\rm QCD}$ as a basic parameter determining the interaction with the QCD condensates, allows us successfully to adjust the nonrelativistic potential model with a high accuracy ($\delta M\approx 30-40$ MeV) by fitting the existing experimental data, that makes the approach to be quite a reliable tool for the prediction of masses for the hadrons, containing the heavy and light quarks.

As for the diquark QQ', it is completely analogous to the heavy quarkonium $Q\bar{Q}'$ except the very essential peculiarities: (1) $(QQ')_{\bar{3}_c}$ is a system with the nonzero color charge; (2) for the quarks of the same flavor $Q=Q'$ it is necessary to take into account the Pauli principle for the identical fermions. The second item turns out to forbid the sum of quark spins S=0 for the symmetric, spatial parity P-even wave functions of diquark $\Psi_d(\mathbf{r})$ (the orbital momentum equals $L_d=2n$, where $n=0,1,2\ldots$), as well as $S=1$ is forbidden for the antisymmetric, antiodd functions $\Psi_d(\mathbf{r})$ (i.e., $L_d=2n+1$). The nonzero color charge leads to two problems.

First, we cannot generally apply the confinement hypothesis on the form of potential (an infinite growth of energy with the increase of the system size) for the object under consideration. However, it is unpossible to imagine a situation, when a big colored object with a size $r>1/\Lambda_{\rm QCD}$ has a finite energy of self-action, and, to the same moment, it is confined inside a white hadron [the singlet over $SU(3)_c$] with $r\sim 1/\Lambda_{\rm QCD}$ due to the interaction with another colored source. In the framework of the well-justified picture of the hadronic string, the tension of such string in the diquark with the external leg inside the baryons is only two times less than in the quark-antiquark pair inside the meson $q\overline{q'}$ and, hence, the energy of diquark linearly grows with the increase of its size. So, the effect analogous to the confinement of quarks takes place in the similar way. In the potential models we can suppose that the quark binding appears due to the effective single exchange by a colored object in the adjoint representation of $SU(3)_c$ (the sum of scalar and vector exchanges is usually taken). Then, the potentials in the singlet $(q\overline{q'})$ and antitriplet (qq') states differ by the factor of 1/2, that means the confining potential with the linear term in the QCD-motivated models for the heavy diquark $(QQ')_{\bar{3}_c}$. In the present work we use the nonrelativistic model with the Buchmüleer-Tye potential for the diquark, too.

Second, in the singlet color state $(Q\overline{Q'})$ there are separate conservations of the summed spin S and the orbital momentum L, since the QCD operators for the transitions between the levels determined by these quantum numbers, are suppressed. Indeed, in the framework of multipole expansion in QCD [15], the amplitudes of chromomagnetic and chromoelectric dipole transitions are suppressed by the inverse heavy quark mass, but in addition, the major reason is provided by the following: (a) the necessity to emit a white object, i.e., at least two gluons, which results in the higher order in $1/m_Q$ and (b) the projection to a real phase space in a physical spectrum of massive hadrons in contrast to the case of massless gluon. Furthermore, the probability of a hybrid state, say, the octet subsystem $(Q\overline{Q'})$ and the additional gluon, i.e., the Fock state $|Q\overline{Q'}_{8_c}g\rangle$, is suppressed due to both the small size of system and the nonrelativistic motion of quarks (for a more strict consideration see Ref. [16]). In the antitriplet color state, the emission of a soft nonperturbative gluon between the levels determined by the spin S_d and the orbital momentum L_d of diquark, is not forbidden, if there are no some other no-go rules or small order parameters. For the quarks of identical flavors inside the diquark, the Pauli principle leads to that the transitions are possible only between the levels, which either differ by the spin $(\Delta S_d=1)$ and the orbital momentum $(\Delta L_d=2n+1)$, instantaneously, or belong to the same set of radial and orbital excitations with $\Delta L_d=2n$. Therefore, the transition amplitudes are suppressed by a small recoil momentum of diquark in comparison with its mass. The transition operator changing the diquark spin as well as its orbital momentum, has the higher order of smallness because of either the additional factor of $1/m_Q$ or the small size of diquark. These suppressions lead to the existence of quasistable states with the quantum numbers of S_d and L_d. In the diquark composed by the quarks of different flavors, bc, the QCD operators of dipole transitions with the single emission of soft gluon are not forbidden, so that the lifetimes of levels can be comparable with the times for the forming of bound states or with the inverse distances between the levels themselves. Then, we cannot insist on the appearance of excitation system for such the diquark with definite quantum numbers of the spin and orbital momentum.[1]

Thus, in the present work we explore the presence of two physical scales in the form of factorization for the wave functions of the heavy diquark and light constituent quark. So, in the framework of the nonrelativistic quark model the problem of the calculation of mass spectrum and characteristics of bound states in the system of doubly heavy baryons is reduced to two standard problems on the study of stationary levels of energy in the system of two bodies. After that, we take into account the relativistic corrections dependent of the quark spins in two subsystems under consideration. The natural boundary for the region of stable states in the doubly heavy system can be assigned to the threshold energy for the decay into a heavy baryon and a heavy meson. As was shown in Ref. [17], the appearance of such a threshold in different systems can be provided by the existence of universal characteristics in QCD, a critical distance between the quarks. At distances greater than the critical separation, the quark-gluon fields become unstable; i.e., the generation of valence quark-antiquark pairs from the sea takes place. In other words, hadronic strings having a length greater than the critical one, decay into strings of smaller sizes with a high probability close to unity. In the framework of the potential approach this effect can be taken into account by restricting the consideration of excited diquark levels by the region, wherein the size of the diquark is less than the critical distance $r_{QQ'} < r_c \approx 1.4-1.5$ fm. Furthermore, the model with the isolated structure of diquark looks to be reliable, just if the size of the diquark is less than the distance to the light quark $r_{QQ'} < r_l$.

The peculiarity of the quark-diquark picture for the doubly heavy baryon is the possibility of mixing between the states of higher diquark excitations, possessing different quantum numbers, because of the interaction with the light quark. Then it is difficult to assign some definite quantum numbers to such excitations. We will discuss the mechanism of this effect.

The paper is organized as follows. In Sec. II we describe a general procedure for the calculation of masses for the doubly heavy baryons in the framework of assumptions drawn above. We take into account the spin-dependent corrections to the potential motivated in QCD. The results of numerical estimates are presented in Sec. III and, finally, our conclusions are given in Sec. IV.

II. NONRELATIVISTIC POTENTIAL MODEL

As we mentioned in the Introduction, we solve the problem of the calculation of mass spectra of baryons containing two heavy quarks in two steps. First, we compute the energy levels of the diquark. Second, we consider the two body problem for the light quark interacting with the pointlike diquark having the mass obtained in the first step. In accordance with the effective expansion of QCD in the inverse heavy quark mass, we separate two stages of such calculations. So, the nonrelativistic Schrödinger equation with the model potential motivated by QCD is solved numerically. After that, the spin-dependent corrections are introduced as perturbations suppressed by the quark masses.

A. Potential

We use the Buchmüller-Tye potential, which takes into account the asymptotic freedom of QCD at short distances. So, the effective coupling constant in the exchange by the octet color state between the quarks is approximated by the QCD running coupling constant up to two-loop accuracy. At long distances, there is the linear term of interaction energy, which leads to the confinement. These two regimes are the limits for the effective β function by Gell-Mann–Low. It was given explicitly in Ref. [14]. In the antitriplet quark state we introduce the factor of 1/2 because of the color structure of bound quark-quark state. For the interaction of diquark with the light constituent quark, the corresponding factor is equal to unity.

As was shown in Ref. [18], the nonperturbative constituent term introduced into the mass of the nonrelativistic quark, exactly coincides with the additive constant, subtracted from the Coulomb potential. We extract the masses of heavy quarks by fitting the real spectra of charmonium and bottomonium,

$$m_c = 1.486 \text{ GeV}, \quad m_b = 4.88 \text{ GeV}, \tag{1}$$

so that the mass of the level in the heavy quarkonium has been calculated as, say, $M(\bar{c}c) = 2m_c + E$, where E is the energy of the stationary Schrödinger equation with the model potential V. Then, we have supposed that the mass of the meson with a single heavy quark is equal to $M(Q\bar{q}) = m_Q + m_q + E$ and $E = \langle T \rangle + \langle V - \delta V \rangle$, whereas the additive term in the potential is introduced because the constituent mass of light quark is determined as a part of interaction energy $\delta V = m_q$. In accordance with fitting the masses of heavy mesons, we get $m_q = 0.385$ GeV.

The results of calculations for the eigenenergies in the Schrödinger equation with the Buchmüller-Tye potential are presented in Tables I–III. The characteristics of corresponding wave functions are given in Tables IV–VI.

We have checked that the binding energy and the wave function of the light quark hardly depend on the flavors of heavy quarks. Indeed, large values of diquark masses give small contributions into the reduced masses. This fact leads to small corrections to the wave functions in the Schrödinger equation. So, for the states lying below the threshold of doubly heavy baryon decay into the heavy baryon and heavy meson, the energies of levels of light constituent quark are equal to

[1] In other words, the presence of gluon field inside the baryon Ξ_{bc} leads to the transitions between the states with the different excitations of a diquark, such as $|bc\rangle \to |bcg\rangle$ with $\Delta S_d = 1$ or $\Delta L_d = 1$, which are not suppressed.

TABLE I. The spectrum of bb diquark levels without spin-dependent splittings: masses and mean-squared radii.

Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	9.74	0.33	2P	9.95	0.54
2S	10.02	0.69	3P	10.15	0.86
3S	10.22	1.06	4P	10.31	1.14
4S	10.37	1.26	5P	10.45	1.39
5S	10.50	1.50	6P	10.58	1.61
3D	10.08	0.72	4D	10.25	1.01
5D	10.39	1.28	6D	10.53	1.51
4F	10.19	0.87	5F	10.34	1.15
6F	10.47	1.40	5G	10.28	1.01
6G	10.42	1.28	6M	10.37	1.15

$$E(1s)=0.38\ \text{GeV},\quad E(2s)=1.09\ \text{GeV},$$

$$E(2p)=0.83\ \text{GeV},$$

where the energy has been defined as the sum of light quark constituent mass and eigenvalue of the Schrödinger equation. In HQET the value of $\bar{\Lambda}=E(1s)$ is generally introduced. Then we can draw a conclusion that our estimate of $\bar{\Lambda}$ is in a good agreement with calculations in other approaches [13]. This fact confirms the reliability of such phenomenological predictions. For the light quark radial wave functions at the origin we find

$$R_{1s}(0)=0.527\ \text{GeV}^{3/2},$$

$$R_{2s}(0)=0.278\ \text{GeV}^{3/2},$$

$$R'_{2p}(0)=0.127\ \text{GeV}^{5/2}.$$

The analogous characteristics of bound states of the c quark interacting with the bb diquark are equal to

$$E(1s)=1.42\ \text{GeV},\quad E(2s)=1.99\ \text{GeV},$$

$$E(2p)=1.84\ \text{GeV},$$

with the wave functions

TABLE II. The spectrum of bc diquark levels without spin-dependent splittings: masses and mean-squared radii.

Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	6.48	0.48	3P	6.93	1.16
2S	6.79	0.95	4P	7.13	1.51
3S	7.01	1.33	3D	6.85	0.96
2P	6.69	0.74	4D	7.05	1.35
4F	6.97	1.16	5F	7.16	1.52
5G	7.09	1.34	6H	7.19	1.50

TABLE III. The spectrum of cc-diquark levels without spin-dependent splittings: masses and mean-squared radii.

Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	3.16	0.58	3P	3.66	1.36
2S	3.50	1.12	4P	3.90	1.86
3S	3.76	1.58	3D	3.56	1.13
2P	3.39	0.88	4D	3.80	1.59

$$R_{1s}(0)=1.41\ \text{GeV}^{3/2},\quad R_{2s}(0)=1.07\ \text{GeV}^{3/2},$$

$$R'_{2p}(0)=0.511\ \text{GeV}^{5/2}.$$

For the binding energy of strange constituent quark we add the current mass $m_s\approx 100-150$ MeV.

B. Spin-dependent corrections

According to Ref. [19], we introduce the spin-dependent corrections causing the splitting of nL levels of diquark as well as in the system of light constituent quark and diquark ($n=n_r+L+1$ is the principal number, n_r is the number of radial excitation, L is the orbital momentum). For the heavy diquark containing the identical quarks we have

$$\begin{aligned}V_{SD}^{(d)}(\mathbf{r})=&\frac{1}{2}\left(\frac{\mathbf{L}_d\cdot\mathbf{S}_d}{2m_Q^2}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right)\\&+\frac{2}{3}\alpha_s\frac{1}{m_Q^2}\frac{\mathbf{L}_d\cdot\mathbf{S}_d}{r^3}+\frac{4}{3}\alpha_s\frac{1}{3m_Q^2}\\&\times\mathbf{S}_{Q1}\cdot\mathbf{S}_{Q2}[4\pi\delta(\mathbf{r})]-\frac{1}{3}\alpha_s\frac{1}{m_Q^2}\frac{1}{4\mathbf{L}_d^2-3}\\&\times[6(\mathbf{L}_d\cdot\mathbf{S}_d)^2+3(\mathbf{L}_d\cdot\mathbf{S}_d)-2\mathbf{L}_d^2\mathbf{S}_d^2]\frac{1}{r^3},\end{aligned}\tag{2}$$

where $\mathbf{L}_d$, $\mathbf{S}_d$ are the orbital momentum in the diquark system and the summed spin of quarks composing the diquark, respectively. Taking into account the interaction with the light constituent quark gives ($\mathbf{S}=\mathbf{S}_d+\mathbf{S}_l$)

TABLE IV. The characteristics of radial wave function for the bb diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV$^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	1.346	2P	0.479
2S	1.027	3P	0.539
3S	0.782	4P	0.585
4S	0.681	5P	0.343

TABLE V. The characteristics of radial wave function for the bc diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV$^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
$1S$	0.726	$2P$	0.202
$2S$	0.601	$3P$	0.240
$3S$	0.561	$4P$	

$$
\begin{aligned}
V_{SD}^{(l)}(\mathbf{r}) = & \frac{1}{4}\left(\frac{\mathbf{L}\cdot\mathbf{S}_d}{2m_Q^2}+\frac{2\mathbf{L}\cdot\mathbf{S}_l}{2m_l^2}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right) \\
& +\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{(\mathbf{L}\cdot\mathbf{S}_d+2\mathbf{L}\cdot\mathbf{S}_l)}{r^3} \\
& +\frac{4}{3}\alpha_s\frac{1}{3m_Q m_l}(\mathbf{S}_d+\mathbf{L}_d)\cdot\mathbf{S}_l[4\pi\delta(\mathbf{r})] \\
& -\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{1}{4\mathbf{L}^2-3}[6(\mathbf{L}\cdot\mathbf{S})^2+3(\mathbf{L}\cdot\mathbf{S}) \\
& -2\mathbf{L}^2\mathbf{S}^2-6(\mathbf{L}\cdot\mathbf{S}_d)^2-3(\mathbf{L}\cdot\mathbf{S}_d)+2\mathbf{L}^2\mathbf{S}_d^2]\frac{1}{r^3},
\end{aligned}
\quad (3)
$$

where the first term corresponds to the relativistic correction to the effective scalar exchange, and other terms appear because of corrections to the effective single-gluon exchange with the coupling constant α_s.

The value of effective parameter α_s can be determined in the following way. The splitting in the S-wave heavy quarkonium $(Q_1\bar{Q}_2)$ is given by the expression

$$\Delta M(ns)=\frac{8}{9}\alpha_s\frac{1}{m_1 m_2}|R_{nS}(0)|^2, \quad (4)$$

where $R_{nS}(r)$ is the radial wave function of quarkonium. From the experimental data on the system of $c\bar{c}$

$$\Delta M(1S,c\bar{c})=117\pm 2 \ \ \text{MeV}, \quad (5)$$

and $R_{1S}(0)$ calculated in the model, we can determine $\alpha_s(\Psi)$. Let us take into account the dependence of this parameter on the reduced mass of the system μ. In the framework of one-loop approximation for the running coupling constant of QCD we have

$$\alpha_s(p^2)=\frac{4\pi}{b\ln(p^2/\Lambda_{\text{QCD}}^2)}, \quad (6)$$

TABLE VI. The characteristics of radial wave function for the cc diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV$^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
$1S$	0.530	$2P$	0.128
$2S$	0.452	$3P$	0.158

whereas $b=11-2n_f/3$ and $n_f=3$ at $p^2<m_c^2$. From the phenomenology of potential models [12] we know that the average kinetic energy of quarks in the bound state practically does not depend on the flavors of quarks, and it is given by the values

$$\langle T_d\rangle \approx 0.2 \ \ \text{GeV} \quad (7)$$

and

$$\langle T_l\rangle \approx 0.4 \ \ \text{GeV}, \quad (8)$$

for the antitriplet and singlet color states, correspondingly. Substituting the definition of the nonrelativistic kinetic energy

$$\langle T\rangle=\frac{\langle p^2\rangle}{2\mu}, \quad (9)$$

we get

$$\alpha_s(p^2)=\frac{4\pi}{b\ln(2\langle T\rangle\mu/\Lambda_{\text{QCD}}^2)}, \quad (10)$$

whereas numerically $\Lambda_{\text{QCD}}\approx 113$ MeV.

For the identical quarks inside the diquark, the scheme of LS coupling well known for the corrections in the heavy quarkonium, is applicable. Otherwise, for the interaction with the light quark we use the scheme of jj coupling [here, $\mathbf{LS}_l$ is diagonal at the given $\mathbf{J}_l$, $(\mathbf{J}_l=\mathbf{L}+\mathbf{S}_l, \mathbf{J}=\mathbf{J}_l+\bar{\mathbf{J}})$, where $\mathbf{J}$ denotes the total spin of baryon, and $\bar{\mathbf{J}}$ is the total spin of diquark, $\bar{\mathbf{J}}=\mathbf{S}_d+\mathbf{L}_d$].

Then, to estimate various terms and mixings of states, we use the transformations of bases (in what follows $\mathbf{S}=\mathbf{S}_l+\bar{\mathbf{J}}$)

$$
\begin{aligned}
|J;J_l\rangle = & \sum_S (-1)^{(\bar{J}+S_l+L+J)}\sqrt{(2S+1)(2J_l+1)} \\
& \times\begin{Bmatrix} \bar{J} & S_l & S \\ L & J & J_l \end{Bmatrix}|J;S\rangle
\end{aligned}
\quad (11)
$$

and

$$
\begin{aligned}
|J;J_l\rangle = & \sum_{J_d} (-1)^{(\bar{J}+S_l+L+J)}\sqrt{(2J_d+1)(2J_l+1)} \\
& \times\begin{Bmatrix} \bar{J} & L & J_d \\ S_l & J & J_l \end{Bmatrix}|J;J_d\rangle.
\end{aligned}
\quad (12)
$$

Thus, we have defined the procedure of calculations for the mass spectra of doubly heavy baryons. This procedure leads to results presented in the next section.

III. NUMERICAL RESULTS

In this section we present the results on the mass spectra which account for the spin-dependent splitting of levels. As we have clarified in the Introduction, the doubly heavy baryons with identical heavy quarks allow quite a reliable interpretation in terms of diquark quantum numbers (the summed spin and the orbital momentum). Dealing with the excitations of the bc diquark, we show the results on the spin-dependent splitting of the ground $1S$ state, since the emission of soft gluon breaks the simple classification of levels for the higher excitations of such diquark. For the doubly heavy baryons, the quark-diquark model of bound states obviously leads to the most reliable results for the system with the larger mass of heavy quark, i.e., for Ξ_{bb}.

A. Ξ_{bb} baryons

For the quantum numbers of levels, we use the notations $n_d L_d n_l l_l$, i.e., we show the value of principal quantum number of diquark, its orbital momentum by a capital letter and the principal quantum number for the excitations of light quark and its orbital momentum by a lowercase letter. We denote the shift of level by $\Delta^{(J)}$ as dependent on the total spin of baryon J. So, for $1S2p$ we have

$$\Delta^{(5/2)} = 10.3 \ \text{MeV}. \tag{13}$$

The states with the total spin $J=\frac{3}{2}$ (or $\frac{1}{2}$), can have different values of J_l, and, hence, they have a nonzero mixing, when we perform the calculations in the perturbation theory built over the states with the definite total momentum J_l of the light constituent quark. For $J=\frac{3}{2}$, the mixing matrix equals

$$\begin{pmatrix} -3.0 & -0.5 \\ -0.5 & 11.4 \end{pmatrix} \ \text{MeV}, \tag{14}$$

so that the mixing practically can be neglected, and the level shifts are determined by the values

$$\Delta'^{(3/2)} = \lambda_1' = -3.0 \ \text{MeV},$$

$$\Delta^{(3/2)} = \lambda_1 = 11.4 \ \text{MeV}. \tag{15}$$

For $J=\frac{1}{2}$, the mixing matrix has the form

$$\begin{pmatrix} -5.7 & -17.8 \\ -17.8 & -14.9 \end{pmatrix} \ \text{MeV}, \tag{16}$$

with the eigenvectors given by

$$|1S2p(\tfrac{1}{2}')\rangle = 0.790|J_l=\tfrac{3}{2}\rangle - 0.613|J_l=\tfrac{1}{2}\rangle,$$

$$|1S2p(\tfrac{1}{2})\rangle = 0.613|J_l=\tfrac{3}{2}\rangle + 0.790|J_l=\tfrac{1}{2}\rangle, \tag{17}$$

and the eigenvalues equal

$$\Delta'^{(1/2)} = \lambda_2' = 8.1 \ \text{MeV},$$

$$\Delta^{(1/2)} = \lambda_2 = -28.7 \ \text{MeV}. \tag{18}$$

For the $2S2p$ level, the corresponding quantities are equal to

$$\Delta^{(5/2)} = 10.3 \ \text{MeV}, \tag{19}$$

and for $J=\frac{3}{2}$, the mixing matrix is equal to

$$\begin{pmatrix} -3.6 & -0.5 \\ -0.5 & 12.4 \end{pmatrix} \ \text{MeV}, \tag{20}$$

so that

$$\Delta'^{(3/2)} = \lambda_1' = -3.6 \ \text{MeV},$$

$$\Delta^{(3/2)} = \lambda_1 = 12.4 \ \text{MeV}. \tag{21}$$

For $J=\frac{1}{2}$, the matrix has the form

$$\begin{pmatrix} -6.1 & -17.6 \\ -17.6 & -13.5 \end{pmatrix} \ \text{MeV}, \tag{22}$$

with the eigenvectors

$$|1S2p(\tfrac{1}{2}')\rangle = 0.776|J_l=\tfrac{3}{2}\rangle - 0.631|J_l=\tfrac{1}{2}\rangle,$$

$$|1S2p(\tfrac{1}{2})\rangle = 0.631|J_l=\tfrac{3}{2}\rangle + 0.776|J_l=\tfrac{1}{2}\rangle, \tag{23}$$

possessing the eigenvalues

$$\Delta'^{(1/2)} = \lambda_2' = 8.2 \ \text{MeV},$$

$$\Delta^{(1/2)} = \lambda_2 = -27.8 \ \text{MeV}. \tag{24}$$

We can straightforwardly see, that the difference between the wave functions as caused by the different masses of diquark subsystem, is unessential.

The splitting of diquark $\Delta^{(J_d)}$ has the form

$3D1s$:

$$\Delta^{(3)} = -0.06 \ \text{MeV},$$

$$\Delta^{(2)} = 0.2 \ \text{MeV},$$

$$\Delta^{(1)} = -0.2 \ \text{MeV}. \tag{25}$$

$4D1s$:

$$\Delta^{(3)} = -2.6 \ \text{MeV},$$

$$\Delta^{(2)} = -0.8 \ \text{MeV},$$

$$\Delta^{(1)} = -4.6 \ \text{MeV}. \tag{26}$$

$5D1s$:

$$\Delta^{(3)} = 2.6 \ \text{MeV},$$

$$\Delta^{(2)} = -0.9 \ \text{MeV},$$

$$\Delta^{(1)} = -4.7 \ \text{MeV}. \tag{27}$$

$5G1s$:

$$\Delta^{(5)} = -0.3 \ \text{MeV},$$

$$\Delta^{(4)} = 0.3 \ \text{MeV},$$

$$\Delta^{(3)} = 1.1 \text{ MeV},$$

$$\Delta^{(2)} = 1.7 \text{ MeV},$$

$$\Delta^{(1)} = 2.0 \text{ MeV}. \quad (28)$$

$6G1s$:

$$\Delta^{(5)} = 3.2 \text{ MeV},$$

$$\Delta^{(4)} = -0.5 \text{ MeV},$$

$$\Delta^{(3)} = -4.4 \text{ MeV},$$

$$\Delta^{(2)} = -7.9 \text{ MeV},$$

$$\Delta^{(1)} = -10.5 \text{ MeV}. \quad (29)$$

Such corrections are unessential up to the current accuracy of method ($\delta M \approx 30-40$ MeV). They can be neglected for the excitations, whose sizes are less than the distance to the light quark, i.e., for diquarks with small values of principal quantum number.

For the hyperfine spin-spin splitting in the system of quark-diquark, we have

$$\Delta_{\rm hf}^{(l)} = \frac{2}{9}\left[J(J+1) - \overline{J}(\overline{J}+1) - \frac{3}{4}\right]\alpha_s(2\mu T)\frac{1}{m_c m_l}|R_l(0)|^2, \quad (30)$$

where $R_l(0)$ is the radial wave function at the origin for the light constituent quark, and for the analogous shift of diquark level, we find

$$\Delta_{\rm hf}^{(d)} = \frac{1}{9}\alpha_s(2\mu T)\frac{1}{m_c^2}|R_d(0)|^2. \quad (31)$$

The mass spectra of Ξ_{bb}^{+} and Ξ_{bb}^{0} baryons are shown in Fig. 2 and in Table VII, wherein we restrict ourselves by the presentation of *S*-, *P*-, and *D*-wave levels.

We can see in Fig. 2, that the most reliable predictions are the masses of baryons $1S1s$ ($J^P = 3/2^+,\ 1/2^+$), $2P1s$ ($J^P = 3/2^-,\ 1/2^-$), and $3D1s$ ($J^P = 7/2^+, \dots, 1/2^+$). The $2P1s$ level is quasistable, because the transition into the ground state requires the instantaneous change of both the orbital momentum and the summed spin of quarks inside the diquark. The analogous kind of transitions seems to be the transition between the states of orthohydrogen and parahydrogen in the molecule of H_2. This transition take place in a nonhomogeneous external field due to the magnetic moments of other molecules. For the transition of $2P1s \to 1S1s$, the role of such external field is played by the nonhomogeneous chromomagnetic field of the light quark. The corresponding perturbation has the form

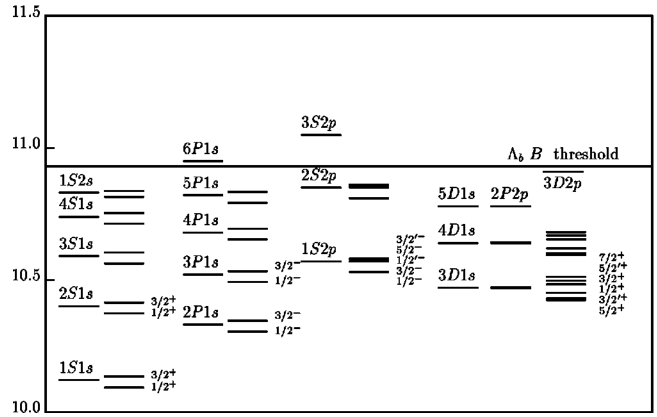

FIG. 2. The spectrum of baryons, containing two *b* quarks Ξ_{bb}^{-} and Ξ_{bb}^{0}, which account for the spin-dependent splittings of low-lying excitations. The masses are given in GeV.

$$\delta V \sim \frac{1}{m_Q}[\mathbf{S}_1\cdot\mathbf{H}_1 + \mathbf{S}_2\cdot\mathbf{H}_2 - (\mathbf{S}_1+\mathbf{S}_2)\cdot\langle\mathbf{H}\rangle]$$
$$= \frac{1}{2m_Q}(\nabla\cdot\mathbf{r}_d)(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{H}$$
$$\sim \frac{1}{m_Q}\frac{\mathbf{r}_l\cdot\mathbf{r}_d}{m_q r_l^5}(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{J}_l f(r_l),$$

where $f(r_l)$ is a dimensionless nonperturbative function depending on the distance between the light quark and diquark. The δV operator changes the orbital momentum of light quark, too. It results in the mixing between the states with the same values of J^P. If the splitting is not small (for instance, $2P1s-1S2p$, where $\Delta E \sim \Lambda_{\rm QCD}$), then the mixing is suppressed as

$$\delta V/\Delta E \sim \frac{1}{m_Q m_q}\frac{r_d}{r_l^4}\frac{1}{\Delta E} \ll 1.$$

Since the admixture of $1S2p$ in the $2P1s$ state is low, the $2P1s$ levels are quasistable, i.e., their hadronic transitions into the ground state with the emission of π mesons are suppressed as we have derived, though an additional sup-

TABLE VII. The mass spectrum of Ξ_{bb}^{-} and Ξ_{bb}^{0} baryons.

$(n_d L_d n_l L_l),\ J^P$	Mass (GeV)	$(n_d L_d n_l L_l),\ J^P$	Mass (GeV)
$(1S1s)1/2^+$	10.093	$(3P1s)1/2^-$	10.493
$(1S1s)3/2^+$	10.133	$(3D1s)5/2'^+$	10.497
$(2P1s)1/2^-$	10.310	$(3D1s)7/2^+$	10.510
$(2P1s)3/2^-$	10.343	$(3P1s)3/2^-$	10.533
$(2S1s)1/2^+$	10.373	$(1S2p)1/2^-$	10.541
$(2S1s)3/2^+$	10.413	$(1S2p)3/2^-$	10.567
$(3D1s)5/2^+$	10.416	$(1S2p)1/2'^-$	10.578
$(3D1s)3/2'^+$	10.430	$(1S2p)5/2^-$	10.580
$(3D1s)1/2^+$	10.463	$(1S2p)3/2'^-$	10.581
$(3D1s)3/2^+$	10.483	$(3S1s)1/2^+$	10.563

pression is given by a small value of phase space. Therefore, we have to expect the presence of narrow resonances in the mass spectra of pairs $\Xi_{bb}\pi$, as they are produced in the decays of quasistable states with $J^P=3/2^-,\ 1/2^-$. The experimental observation of such levels could straightforwardly confirm the existence of diquark excitations and provide the information on the character of dependency in $f(r_l)$, i.e., on the nonhomogeneous chromomagnetic field in the nonperturbative region.

Sure, the $3D1sJ^P=7/2^+,\ 5/2^+$ states are also quasistable, since in the framework of multipole expansion in QCD they transform into the ground state due to the quadrupole emission of gluon (the $E2$ transition with the hadronization $gq\to q'\pi$).

As for the higher excitations, the $3P1s$ states are close to the $1S2p$ levels with $J^P=3/2^-,\ 1/2^-$, so that the operators changing both the orbital momentum of diquark and its spin, can lead to the essential mixing with an amplitude $\delta V_{nn'}/\Delta E_{nn'}\sim 1$, despite suppression by the inverse heavy quark mass and small size of diquark. We are sure that the mixing slightly shifts the masses of states. The most important effect is a large admixture of $1S2p$ in $3P1s$. It makes the state to be unstable because of the transition into the ground $1S1s$ state with the emission of gluon (the $E1$ transition). This transition leads to decays with the emission of π mesons.[2]

The level $1S2pJ^P=5/2^-$ has the definite quantum numbers of diquark and light quark motion, because there are no levels with the same values of J^P in its vicinity. However, its width of transition into the ground state and π meson is not suppressed and seems to be large, $\Gamma\sim 100$ MeV.

The following transitions take place:

$$\frac{3}{2}^- \to \frac{3}{2}^+\pi \ \text{in } S \text{ wave},$$

$$\frac{3}{2}^- \to \frac{1}{2}^+\pi \ \text{in } D \text{ wave},$$

$$\frac{1}{2}^- \to \frac{3}{2}^+\pi \ \text{in } D \text{ wave},$$

$$\frac{1}{2}^- \to \frac{1}{2}^+\pi \ \text{in } S \text{ wave}.$$

The D-wave transitions are suppressed by the ratio of low recoil momentum to the mass of the baryon. The width of state $J^P=3/2^+$ is completely determined by the radiative electromagnetic $M1$ transition into the basic $J^P=1/2^+$ state.

B. Ξ_{cc} baryons

The calculation procedure described above leads to the results for the doubly charmed baryons as presented below. For $1S2p$, the splitting is equal to

$$\Delta^{(5/2)}=17.4 \text{ MeV}. \tag{32}$$

For $J=\frac{3}{2}$, the mixing is determined by the matrix

$$\begin{pmatrix} 4.3 & -1.7 \\ -1.7 & 7.8 \end{pmatrix} \text{ MeV}, \tag{33}$$

so that the eigenvectors

$$|1S2p(\tfrac{3}{2}')\rangle = 0.986|J_l=\tfrac{3}{2}\rangle + 0.164|J_l=\tfrac{1}{2}\rangle,$$

$$|1S2p(\tfrac{3}{2})\rangle = -0.164|J_l=\tfrac{3}{2}\rangle + 0.986|J_l=\tfrac{1}{2}\rangle, \tag{34}$$

have the eigenvalues

$$\Delta'^{(3/2)}=\lambda_1'=3.6 \text{ MeV},$$

$$\Delta^{(3/2)}=\lambda_1=8.5 \text{ MeV}. \tag{35}$$

For $J=\frac{1}{2}$, the mixing matrix equals

$$\begin{pmatrix} -3.6 & -55.0 \\ -55.0 & -73.0 \end{pmatrix} \text{ MeV}, \tag{36}$$

where the vectors

$$|1S2p(\tfrac{1}{2}')\rangle = 0.957|J_l=\tfrac{3}{2}\rangle - 0.291|J_l=\tfrac{1}{2}\rangle,$$

$$|1S2p(\tfrac{1}{2})\rangle = 0.291|J_l=\tfrac{3}{2}\rangle + 0.957|J_l=\tfrac{1}{2}\rangle, \tag{37}$$

have the eigenvalues

$$\Delta'^{(1/2)}=\lambda_2'=26.8 \text{ MeV},$$

$$\Delta^{(1/2)}=\lambda_2=-103.3 \text{ MeV}. \tag{38}$$

The splitting of the $3D$ diquark level is given by

$$\Delta^{(3)}=-3.02 \text{ MeV},$$

$$\Delta^{(2)}=2.19 \text{ MeV},$$

$$\Delta^{(1)}=3.39 \text{ MeV}. \tag{39}$$

Further, we have to take into account the hyperfine spin-spin corrections in the quark-diquark system.

For the $1S$ and $2S$ wave levels of diquark, the shifts of vector states are equal to

$$\Delta(1S)=6.3 \text{ MeV},$$

$$\Delta(2S)=4.6 \text{ MeV}.$$

The mass spectra of the Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons are presented in Fig. 3 and Table VIII.

C. Ξ_{bc} baryons

As we have already mentioned in the Introduction, the heavy diquark composed of the quarks of different flavors, turns out to be unstable under the emission of soft gluons.

[2]Remember, that the $\Xi_{QQ'}$ baryons are the isodoublets.

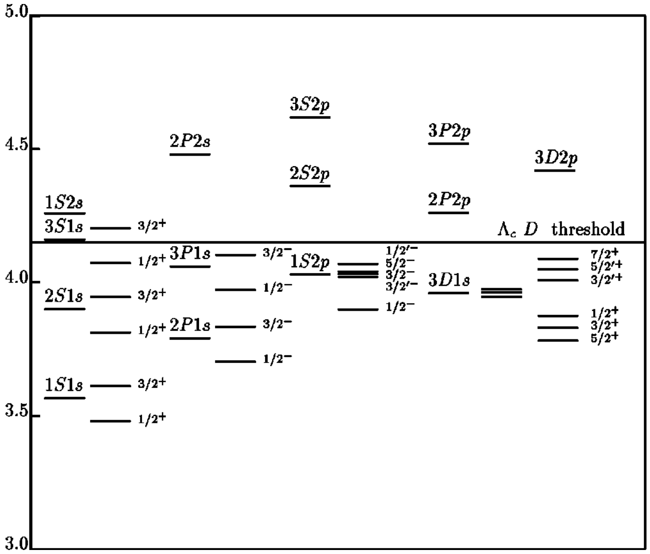

FIG. 3. The spectrum of Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons. The masses are given in GeV.

So, in the Fock state of doubly heavy baryon, there is a sizable nonperturbative admixture of configurations including the gluons and diquark with the various values of its spin S_d and orbital momentum L_d,

$$|B_{bcq}\rangle = O_B|bc^{S_d,L_d}_{\bar{3}_c},q\rangle + H_1|bc^{S_d\pm 1,L_d}_{\bar{3}_c},g,q\rangle + H_2|bc^{S_d,L_d\pm 1}_{\bar{3}_c},g,q\rangle + \cdots,$$

whereas the amplitudes of H_1, H_2 are not suppressed with respect to O_B. In heavy quarkonium, the analogous operators for the octet-color states are suppressed by the probability of emission by the nonrelativistic quarks inside a small volume determined by the size of singlet-color system of heavy quark and antiquark. In the baryonic system under consideration, a soft gluon is restricted only by the ordinary scale of confinement, and, hence, there is no suppression.

We suppose that the calculations of masses for the excited Ξ_{bc} baryons are not so justified in the given scheme. Therefore, we present only the result for the ground state with $J^P = 1/2^+$

TABLE VIII. The mass spectrum of Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons.

$(n_d L_d n_l L_l)$, J^P	Mass (GeV)	$(n_d L_d n_l L_l)$, J^P	Mass (GeV)
$(1S1s)1/2^+$	3.478	$(3P1s)1/2^-$	3.972
$(1S1s)3/2^+$	3.61	$(3D1s)3/2'^+$	4.007
$(2P1s)1/2^-$	3.702	$(1S2p)3/2'^-$	4.034
$(3D1s)5/2^+$	3.781	$(1S2p)3/2^-$	4.039
$(2S1s)1/2^+$	3.812	$(1S2p)5/2^-$	4.047
$(3D1s)3/2^+$	3.83	$(3D1s)5/2'^+$	4.05
$(2P1s)3/2^-$	3.834	$(1S2p)1/2'^-$	4.052
$(3D1s)1/2^+$	3.875	$(3S1s)1/2^+$	4.072
$(1S2p)1/2^-$	3.927	$(3D1s)7/2^+$	4.089
$(2S1s)3/2^+$	3.944	$(3P1s)3/2^-$	4.104

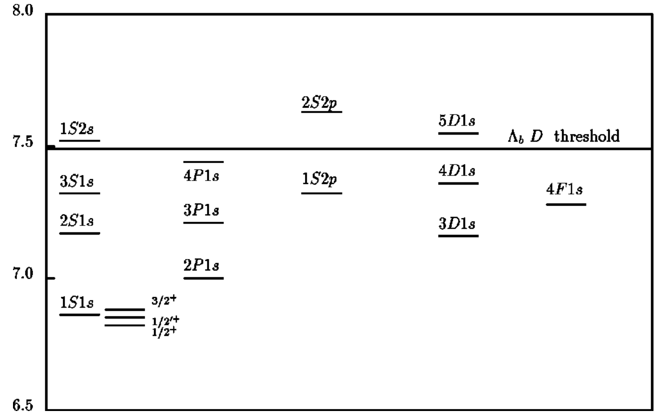

FIG. 4. The spectrum of Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons without the splittings of higher excitations. The masses are given in GeV.

$$M_{\Xi'_{bc}} = 6.85 \text{ GeV}, \quad M_{\Xi_{bc}} = 6.82 \text{ GeV},$$

whereas for the vector diquark we have assumed that the spin-dependent splitting due to the interaction with the light quark is determined by the standard contact coupling of magnetic moments for pointlike systems. The picture for the baryon levels is shown in Fig. 4 with no account for the spin-dependent perturbations suppressed by the heavy quark masses.

D. The doubly heavy baryons with the strangeness $\Omega_{QQ'}$

In the leading approximation, we suppose that the wave functions and the excitation energies of strange quark in the field of doubly heavy diquark repeat the characteristics for the analogous baryons containing the ordinary quarks u,d. Therefore, the level system of baryons $\Omega_{QQ'}$ reproduces that of $\Xi_{QQ'}$ up to an additive shift of the masses by the value of current mass of strange quark $m_s \approx M(D_s) - M(D) \approx M(B_s) - M(B) \approx 0.1$ GeV.

Furthermore, we suppose that the spin-spin splitting of $2P1s$ and $3D1s$ levels of $\Omega_{QQ'}$ is 20–30 % less than in $\Xi_{QQ'}$ (the factor of $m_{u,d}/m_s$). As for the $1S2p$ level, the procedure described above can be applied. So, for Ω_{bb}, the matrix of mixing for the states with the different values of total momentum J_l practically can be assigned to be diagonal. This fact means that the following term of perturbation is dominant:

$$\frac{1}{4}\left(\frac{2\mathbf{L}\cdot\mathbf{S}_l}{2m_l^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right).$$

Therefore, we can think that the splitting of $1S2p$ is determined by the factor of $m_{u,d}^2/m_s^2$ with respect to the splitting of corresponding Ξ_{bb}, i.e., it is 40% less than in Ξ_{bb}. Hence, the splitting is very small.

For the baryon Ω, the factor of m_s/m_c is not small. Hence, for $1S2p$, the mixing matrix is not diagonal, so that the arrangement of $1S2p$ states of Ω can be slightly different than that of Ξ.

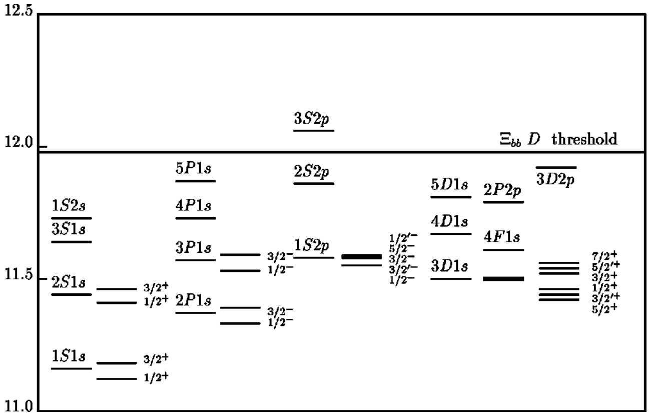

FIG. 5. The spectrum of Ω_{bbc}^0 baryons which account for the spin-dependent splittings for the low-lying excitations. Masses are given in GeV.

The following peculiarity of $\Omega_{QQ'}$ is of great interest. The low-lying S and P excitations of the diquark are stable. Indeed, even after taking into account the mixing of levels, a gluon emission makes a hadronization into the K meson (the transitions of $\Omega_{QQ'} \to \Xi_{QQ'} + K$), while a single emission of the π meson is forbidden because of the conservation of isospin and strangeness. The hadronic transitions with kaons are forbidden because of insufficient splitting between the masses of $\Omega_{QQ'}$ and $\Xi_{QQ'}$. The decays with the emission of pion pairs belonging to the isosinglet state, are suppressed by a small phase space or even forbidden. Thus, the radiative electromagnetic transitions into the ground state are the dominant modes of decays for the low-lying excitations of $\Omega_{QQ'}$.

E. Ω_{bbc} baryons

In the framework of quark-diquark picture, we can build the model for the baryons containing three heavy quarks bbc. However, as we estimate, the size of diquark turns out to be comparable with the average distance to the charmed quark. So, the model assumption on the compact heavy diquark cannot be quite accurate for the calculations of mass levels in this case. The spin-dependent forces are negligibly small inside the diquark, as we have already pointed out above. The spin-spin splitting of vector diquark interacting with the charmed quark, is given by

$$\Delta(1s) = 33 \text{ MeV}, \quad \Delta(2s) = 18 \text{ MeV}.$$

For $1S2p$, the level shifts are small. So, for the state $J^P = 1/2^-$ we have to add the correction of -33 MeV. For the $3D1s$ state the splitting is determined by the spin-spin interaction. The characteristics of excitations for the charmed quark in the model with the potential by Buchmüller and Tye have been presented above. Finally, we obtain the picture of Ω_{bbc} levels presented in Fig. 5 and Table IX.

Further, the excitations of ground Ω_{bbc}^0 state can strongly mix with large amplitudes because of small splittings between the levels, but they have small shifts of masses. This effect takes place for $3P1s - 1S2p$ with $J^P = 1/2^-, 3/2^-$,

TABLE IX. The mass spectrum of Ω_{bbc}^0 baryons.

$(n_d L_d n_l L_l)$, J^P	Mass (GeV)	$(n_d L_d n_l L_l)$, J^P	Mass (GeV)
$(1S1s)1/2^+$	11.12	$(3D1s)3/2'^+$	11.52
$(1S1s)3/2^+$	11.18	$(3D1s)5/2'^+$	11.54
$(2P1s)1/2^-$	11.33	$(1S2p)1/2^-$	11.55
$(2P1s)3/2^-$	11.39	$(3D1s)7/2^+$	11.56
$(2S1s)1/2^+$	11.40	$(1S2p)3/2'^-$	11.58
$(3D1s)5/2^+$	11.42	$(1S2p)3/2^-$	11.58
$(3D1s)3/2^+$	11.44	$(1S2p)1/2'^-$	11.59
$(3D1s)1/2^+$	11.46	$(1S2p)5/2^-$	11.59
$(2S1s)3/2^+$	11.46	$(3P1s)3/2^-$	11.59
$(3P1s)1/2^-$	11.52	$(3S1s)1/2^+$	11.62

and for $2S1s - 3D1s$ with $J^P = 1/2^+, 3/2^+$. We suppose the prediction to be quite reliable for the states of $1S1s$ with $J^P = 1/2^+, 3/2^+$, $1S2p$ with $J^P = 5/2^-$, and $3D1s$ with $J^P = 5/2^+, 7/2^+$. For these excitations, we might definitely predict the widths of their radiative electromagnetic transitions into the ground state in the framework of multipole expansion in QCD. The widths for the transitions will be essentially determined by the amplitudes of admixtures, which have a strong model dependence. Therefore, the experimental study of electromagnetic transitions in the family of Ω_{bbc}^0 baryons could provide a valuable information on the mechanism of mixing between the different levels in the baryonic systems. The electromagnetic transitions combined with the emission of pion pairs, if not forbidden by the phase space, saturate the total widths of excited Ω_{bbc}^0 levels. The characteristic value of total width is about $\Gamma \sim 10-100$ keV, in the order of magnitude. Thus, the system of Ω_{bbc}^0 can be characterized by a large number of narrow quasistable states.

IV. CONCLUSION

In this paper we have calculated the spectroscopic characteristics of baryons containing two heavy quarks, in the model with the quark-diquark factorization of wave functions. We have explored the nonrelativistic model of constituent quarks with the potential by Buchmüller and Tye. The region of applicability of such the approximations has been pointed out.

We have taken into account the spin-dependent relativistic corrections to the potential in the subsystems of the diquark and light quark diquark. Below the threshold of decay into the heavy baryon and heavy meson, we have found the system of excited bound states, which are quasistable under the hadronic transitions into the ground state. We have considered the physical reasons for the quasistability taking place for the baryons with two identical quarks. In accordance with the Pauli principle, the operators responsible for the hadronic decays and the mixing between the levels, are suppressed by the inverse heavy quark mass and the small size of diquark. This suppression is caused by the necessity of instantaneous change in both the spin and the orbital momentum of compact diquark. In the baryonic systems with two heavy quark and the strange quark, the quasistability of diquark excitations is provided by the absence of transitions with the emission of both a single kaon and a single pion. These transitions are forbidden because of small splitting between the levels and the conservation of the isospin and strangeness. The characteristics of wave functions can be used in calculations of cross sections for the doubly heavy baryons in the framework of the quark-diquark approximation.

ACKNOWLEDGMENTS

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