

# $1/m_Q$ corrections to the heavy-to-light-vector transitions in HQET

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Within the heavy quark effective theory (HQET), the heavy-to-light-vector meson transitions are systematically analyzed to the order of  $1/m_Q$ . In addition to the four universal functions at leading order, there are 22 independent universal form factors on the order of  $1/m_Q$ . Both the semileptonic decay  $B \rightarrow \rho$  which is relevant to the  $|V_{ub}|$  extraction, and the penguin induced decay  $B \rightarrow K^*$ , which is important to new physics discovery, depend on these form factors. The phenomenological implications are discussed.

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There are two main reasons for studying the heavy-meson-to-light-meson weak transitions. To extract the Cabibbo–Kobayashi–Maskawa matrix element  $V_{ub}$  precisely, which has important implications for  $CP$  violation, the exclusive  $B \rightarrow \rho(\pi)l\nu$  decays are suitable channels.<sup>1</sup> Another reason is to investigate the rare  $B$  exclusive decays induced by penguin diagrams which are important for testing the standard model and for discovering new physics. They are the processes  $B \rightarrow K^*\gamma$  and  $B \rightarrow K^{(*)}l^+l^-$ , respectively. A large number of samples of the  $B$  to light meson processes produced in the current experiments [1] and to be produced in the near future  $B$  factories will make precise measurements available. Thus the main task is to reduce the theoretical uncertainties in the calculations of the hadronic matrix elements.

In this paper, we focus on the  $B$  to light vector decays. The matrix elements responsible for the decay  $H \rightarrow V l \nu$  can be parametrized in terms of four invariant form factors, which are conventionally defined as

$$\begin{aligned} \langle V(p, \epsilon) | \bar{q} \gamma_\mu Q | H(P) \rangle &= i g(q^2) \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} \\ &\quad \times (P+p)^\lambda (P-p)^\sigma, \\ \langle V(p, \epsilon) | \bar{q} \gamma_\mu \gamma_5 Q | H(P) \rangle &= f(q^2) \epsilon_\mu^* + a_+(q^2) (\epsilon^* \cdot P) \\ &\quad \times (P+p)_\mu + a_-(q^2) (\epsilon^* \cdot P) \\ &\quad \times (P-p)_\mu, \end{aligned} \quad (1)$$

where  $q^2 = (P-p)^2$ . The matrix elements for the decays  $B \rightarrow K^*\gamma$  and  $B \rightarrow K^*l^+l^-$  are parametrized by the following three invariant form factors:

$$\begin{aligned} \langle V(p, \epsilon) | \bar{q} \sigma_{\mu\nu} Q | H(P) \rangle &= g_+ \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (P+p)^\sigma + g_- \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (P-p)^\sigma \\ &\quad + h \varepsilon_{\mu\nu\lambda\sigma} (P+p)^\lambda (P-p)^\sigma (\epsilon^* \cdot P), \\ \langle V(p, \epsilon) | \bar{q} \sigma_{\mu\nu} \gamma_5 Q | H(P) \rangle &= i g_+ [\epsilon_\nu^* (P+p)_\mu - \epsilon_\mu^* (P+p)_\nu] \\ &\quad + i g_- [\epsilon_\nu^* (P-p)_\mu - \epsilon_\mu^* (P-p)_\nu] \\ &\quad + i h [(P+p)_\nu (P-p)_\mu - (P+p)_\mu (P-p)_\nu] (\epsilon^* \cdot P). \end{aligned} \quad (2)$$

Here the second relation is obtained from the first one using  $\sigma^{\mu\nu} = (i/2) \varepsilon^{\mu\nu\lambda\sigma} \sigma_{\lambda\sigma} \gamma_5$ .

We use the heavy quark effective theory (HQET) [2] to study these form factors to the order of  $1/m_Q$ . The HQET provides a clear physical description for the hadrons containing a single heavy quark. It has been successfully applied to the analysis of the  $B \rightarrow D^{(*)}$  decays. It has also been used for the heavy-to-light meson exclusive weak decays [3]. In this latter case, the form factors have no normalization point. The heavy quark symmetry (HQS) does not simplify the analysis significantly. Nevertheless, the relations between various heavy-to-light meson transitions can be found by the HQS. At the order of  $1/m_Q$  many form factors are introduced. However, they are universal for all the heavy to light transitions. The systematic nature of the heavy quark expansion means uncertainties are easier to identify and estimate. Furthermore the analysis based on HQET is model independent.

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<sup>1</sup>An alternative way for the  $|V_{ub}|$  extraction is from the inclusive  $B \rightarrow X_u$  decays.

It is therefore meaningful to consider the  $1/m_Q$  corrections to the heavy-to-light meson transitions, in addition to the leading order results. They are also practically important for the analysis of the  $D \rightarrow \rho$ ,  $K^*$  weak decays. For the heavy-to-light pseudoscalar weak decays, the  $1/m_Q$  corrections have been considered in Ref. [4]. We will calculate the  $1/m_Q$  corrections to the heavy-to-light vector meson transitions.

Let us make a brief review of the HQET. In the heavy quark limit, the velocity of the heavy quark  $Q$ ,  $v$ , is a well defined quantity and the heavy quark field can be represented by the velocity-dependent field

$$h_v(x) = \exp(im_Q v \cdot x) P_+ Q(x), \quad (3)$$

where  $P_+ = (1 + \not{v})/2$ . The effective Lagrangian is

$$\mathcal{L}_{\text{eff}}^0 = \bar{h}_v i v \cdot D h_v, \quad (4)$$

where the gauge-covariant derivative generates the residual momentum. To the  $1/m_Q$  order [2,5], the relation between  $Q$  and  $h_v$  is obtained by treating  $1/m_Q$  as a perturbation

$$Q(x) = \exp(-im_Q v \cdot x) \left( 1 + \frac{i \not{D}_\perp}{2m_Q} \right) h_v(x), \quad (5)$$

where  $D_\perp^\mu = D^\mu - v^\mu v \cdot D$ , and  $h_v$  satisfies exactly the equation of motion  $i v \cdot D h_v = 0$ . The effective Lagrangian becomes

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^0 + \frac{1}{2m_Q} [O_{\text{kin}} + O_{\text{mag}}] + \mathcal{O}(1/m_Q^2), \quad (6)$$

where

$$O_{\text{kin}} = \bar{h}_v (iD)^2 h_v, \quad O_{\text{mag}} = \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v. \quad (7)$$

$O_{\text{kin}}$  describes the kinetic energy of the heavy quark in the hadron, and  $O_{\text{mag}}$  the heavy quark chromomagnetic energy.

To study the hadronic matrix elements in the HQET, the form factors are considered as functions of the kinematic variable

$$v \cdot p = \frac{m_H^2 + m_V^2 - q^2}{2m_H}. \quad (8)$$

Accordingly, Eqs. (1) and (2) can be reexpressed as

$$\begin{aligned} \langle V(p, \epsilon) | \bar{q} \gamma_\mu h_v | H(v) \rangle &= 2i \tilde{g}(v \cdot p) \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} \hat{p}^\lambda v^\sigma, \\ \langle V(p, \epsilon) | \bar{q} \gamma_\mu \gamma_5 h_v | H(v) \rangle &= 2[\tilde{f}(v \cdot p) \epsilon_\mu^* + \tilde{a}_1(v \cdot p) \\ &\quad \times (\epsilon^* \cdot v) \hat{p}_\mu + \tilde{a}_2(v \cdot p) \\ &\quad \times (\epsilon^* \cdot v) v_\mu], \end{aligned} \quad (9)$$

and

$$\begin{aligned} \langle V(p, \epsilon) | \bar{q} \sigma_{\mu\nu} Q | H(v) \rangle &= 2[\tilde{g}_v \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} v^\sigma + \tilde{g}_p \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} \hat{p}^\sigma \\ &\quad + \tilde{h} \epsilon_{\mu\nu\lambda\sigma} v^\lambda \hat{p}^\sigma (\epsilon^* \cdot v)], \\ \langle V(p, \epsilon) | \bar{q} \sigma_{\mu\nu} \gamma_5 Q | H(v) \rangle &= 2\{i \tilde{g}'_v [\epsilon_\nu^* v_\mu - \epsilon_\mu^* v_\nu] + i \tilde{g}'_p [\epsilon_\nu^* \hat{p}_\mu - \epsilon_\mu^* \hat{p}_\nu] \\ &\quad + i \tilde{h}' [v_\nu \hat{p}_\mu - v_\mu \hat{p}_\nu] (\epsilon^* \cdot v)\}, \end{aligned} \quad (10)$$

where the dimensionless variable is

$$\hat{p}^\mu = \frac{p^\mu}{v \cdot p}, \quad v \cdot \hat{p} = 1, \quad (11)$$

so that all of the form factors have the same dimension. It is convenient to work in the matrix representation of the hadrons [6]. These wave functions are only dependent on the HQS and their Lorentz transformation properties. The ground-state pseudoscalar and vector heavy mesons are described by

$$\begin{aligned} \mathcal{M}(v) &= \frac{1 + \not{v}}{2} \\ &\times \begin{cases} -\gamma_5 & \text{pseudoscalar meson,} \\ \not{\epsilon} & \text{vector meson with polarization vector } \epsilon^\mu. \end{cases} \end{aligned} \quad (12)$$

Based on the symmetry and the Feynman rules of the HQET, one can express the hadronic matrix element by evaluating some trace over the above wave functions. At the leading order of  $1/m_Q$ , the matrix element of the relevant current  $\bar{q} \Gamma h_v$  can be written as

$$\langle V(p, \epsilon) | \bar{q} \Gamma h_v | H(v) \rangle = -\text{Tr}\{\Omega_L(v, p) \Gamma \mathcal{M}(v)\}, \quad (13)$$

where the matrix  $\Omega_L(v, p)$  transforms as a Lorentz scalar as functions of  $v \cdot p$ . And it has linear dependence on the polarization of the meson  $V$ . Considering  $\mathcal{M}(v) \not{v} = -\mathcal{M}(v)$ , the general form for  $\Omega_L$  is

$$\Omega_L = L_1 \not{\epsilon}^* + L_2 v \cdot \epsilon^* + [L_3 \not{\epsilon}^* + L_4 v \cdot \epsilon^*] \not{p}, \quad (14)$$

where the universal functions  $L_i$  ( $i=1-4$ ) depend on the kinematic variable  $v \cdot p$ , but not on the heavy quark mass  $m_Q$ .

The power corrections proportional to  $1/m_Q$  result from both the effective currents and the effective Lagrangian of the HQET. We first consider the corrections coming from the expansion of the currents. Weak current of the heavy-to-light transition in the effective theory can be expanded as

$$\bar{q} \Gamma Q = \bar{q} \Gamma \left( 1 + \frac{i \not{D}_\perp}{2m_Q} \right) h_v(x). \quad (15)$$

In the same manner as shown in leading order, one can find that the matrix elements of the operators containing a covariant derivative which acts on the heavy quark field have the formal structure

$$\langle V(p, \epsilon) | \bar{q} \Gamma i D^\mu h_v | M(v) \rangle = -\text{Tr} \{ \Omega_D^\mu(v, p) \Gamma \mathcal{M}(v) \}. \quad (16)$$

The matrix  $\Omega_D^\mu(v, p)$  also contains some universal functions depending only on the variable  $v \cdot p$ , and transforms as a vector. The generic structure of  $\Omega_D^\mu$  is

$$\begin{aligned} \Omega_D^\mu = & (D_1 v^\mu + D_2 \hat{p}^\mu + D_3 \gamma^\mu) \epsilon^{*\mu} + (D_4 v^\mu + D_5 \hat{p}^\mu + D_6 \gamma^\mu) (v \cdot \epsilon^*) + (D_7 v^\mu + D_8 \hat{p}^\mu + D_9 \gamma^\mu) \epsilon^{*\mu} \hat{p} \\ & + (D_{10} v^\mu + D_{11} \hat{p}^\mu + D_{12} \gamma^\mu) (v \cdot \epsilon^*) \hat{p} + (D_{13} + D_{14} \hat{p}) \epsilon^{*\mu}. \end{aligned} \quad (17)$$

The functions  $D_i$  are functions of  $v \cdot p$ . Not all of these 14 universal functions are independent. Using equation of motion of the heavy quark,  $iv \cdot Dh_v = 0$ , we can easily obtain

$$\begin{aligned} D_1 + D_2 - D_3 &= 0, \\ D_4 + D_5 - D_6 + D_{13} &= 0, \\ D_7 + D_8 - D_9 &= 0, \\ D_{10} + D_{11} - D_{12} + D_{14} &= 0. \end{aligned} \quad (18)$$

Furthermore, using the following relations:

$$i \partial^\mu (\bar{q} \Gamma h_v) = \bar{q} \Gamma (i D^\mu) h_v + i \bar{q} (\overline{D}^\mu) \Gamma h_v, \quad (19)$$

and

$$\begin{aligned} D_1 - 2D_3 + 2D_7 + \hat{p}^2 D_8 + D_{13} &= -\bar{\Lambda} L_1 - (2\bar{\Lambda} - v \cdot p \hat{p}^2) L_3, \\ 2D_2 - D_4 + 4D_6 + 2D_{10} + \hat{p}^2 D_{11} &= \bar{\Lambda} (L_2 - 2L_1) - (2\bar{\Lambda} - v \cdot p \hat{p}^2) L_4, \\ D_2 - D_7 - D_{14} &= \bar{\Lambda} L_3 + v \cdot p L_1, \\ D_5 - 2D_7 + D_{10} - 2D_{12} &= \bar{\Lambda} (2L_3 - L_4) + v \cdot p L_2. \end{aligned} \quad (22)$$

The relations, Eqs. (18) and (22), imply that only six of the 14 universal functions are independent. Note that the light quarks have been taken to be massless. This reduces the number of HQET operators appearing in the expansion of QCD currents.

The corrections to the effective states should be included. The  $1/m_Q$  terms in the effective lagrangian are treated as perturbation,  $h_v$  is still defined by Eq. (4) at the subleading order of the heavy quark expansion. Therefore the effective states of Eq. (12) are not the eigenstates of the operators  $O_{\text{kin}}$  and  $O_{\text{mag}}$ . The corrections to the effective states can be accounted for by including time-ordered products in which  $O_{\text{kin}}$  or  $O_{\text{mag}}$  is inserted into matrix elements of the leading-order currents. By using the Feynman rules in HQET, one can obtain

$$\begin{aligned} \langle V(p, \epsilon) | i \partial^\mu (\bar{q} \Gamma h_v) | H(v) \rangle &= (\bar{\Lambda} v^\mu - p^\mu) \\ &\times \langle V(p, \epsilon) | \bar{q} \Gamma h_v | H(v) \rangle, \end{aligned} \quad (20)$$

where  $\bar{\Lambda} = m_M - m_Q$  denotes the finite mass difference between a heavy meson and the heavy quark in the infinite quark mass limit, and equation of motion for the light quark field,  $i \not{D} q = 0$ , we can obtain

$$\begin{aligned} \text{Tr} \{ \Omega_D^\mu(v, p) \gamma_\mu \Gamma' \mathcal{M}(v) \} &= (\bar{\Lambda} v^\mu - p^\mu) \text{Tr} \{ \Omega_L(v, p) \\ &\times \gamma_\mu \Gamma' \mathcal{M}(v) \}, \end{aligned} \quad (21)$$

where we have substituted  $\Gamma$  by  $\gamma_\mu \Gamma'$ . It yields

$$\begin{aligned} \langle V(p, \epsilon) | i \int dy T \{ \bar{q} \Gamma h_v(0), O_{\text{kin}}(y) + O_{\text{mag}}(y) \} | H(v) \rangle \\ = -\text{Tr} \left\{ \Omega_K(v, p) \Gamma \mathcal{M}(v) \right. \\ \left. + \Omega_G^{\alpha\beta}(v, p) \Gamma \frac{1 + \not{v}}{2} \sigma_{\alpha\beta} \mathcal{M}(v) \right\}, \end{aligned} \quad (23)$$

where the properties of matrix  $\Omega_K(v, p)$  are very similar to matrix  $\Omega_L$ , and the matrix  $\Omega_G^{\alpha\beta}(v, p)$  also has similar properties except it must transform as a tensor. They can be described in terms of 16 additional universal functions  $S_i(v \cdot p)$  as follows:

$$\Omega_K = S_1 \hat{\epsilon}^* + S_2 (v \cdot \epsilon^*) + [S_3 \hat{\epsilon}^* + S_4 (v \cdot \epsilon^*)] \hat{p},$$

$$\begin{aligned} \Omega_G^{\alpha\beta} &= (iS_5 \hat{p}^\alpha \gamma^\beta + S_6 \sigma^{\alpha\beta}) \hat{\epsilon}^* + (iS_7 \hat{p}^\alpha \gamma^\beta + S_8 \sigma^{\alpha\beta}) \\ &\quad \times (v \cdot \epsilon^*) + (iS_9 \hat{p}^\alpha \gamma^\beta + S_{10} \sigma^{\alpha\beta}) \hat{\epsilon}^* \hat{p} \\ &\quad + (iS_{11} \hat{p}^\alpha \gamma^\beta + S_{12} \sigma^{\alpha\beta}) \hat{p} (v \cdot \epsilon^*) + (iS_{13} \gamma^\alpha \epsilon^{*\beta} \\ &\quad + iS_{14} \gamma^\alpha \epsilon^{*\beta} \hat{p}) + (iS_{15} \epsilon^{*\alpha} \hat{p}^\beta + iS_{16} \epsilon^{*\alpha} \hat{p}^\beta \hat{p}). \end{aligned} \quad (24)$$

To get the independent universal functions in  $\Omega_G^{\alpha\beta}$ , let us consider the following matrix element of the time-ordered operator products [7]:

$$\begin{aligned} \langle V(p, \epsilon) | i \int dy T \left\{ \bar{q} \Gamma h_v(0), \frac{i}{2} g_s \bar{h}_v \Gamma_1 G^{\alpha\beta} h_v \right\} | H(v) \rangle \\ = -\text{Tr} \left\{ \bar{\Omega}_G^{\alpha\beta}(v, p) \Gamma \frac{1+\not{v}}{2} \Gamma_1 \mathcal{M}(v) \right\}, \end{aligned} \quad (25)$$

where  $\Gamma_1$  is any fixed Dirac matrix, and

$$\begin{aligned} \bar{\Omega}_G^{\alpha\beta} &= \frac{1}{2} [(iS_5 \hat{p}^{[\alpha} \gamma^{\beta]} + 2S_6 \sigma^{\alpha\beta}) \hat{\epsilon}^* + (iS_7 \hat{p}^{[\alpha} \gamma^{\beta]} + 2S_8 \sigma^{\alpha\beta}) (v \cdot \epsilon^*) + (iS_9 \hat{p}^{[\alpha} \gamma^{\beta]} + 2S_{10} \sigma^{\alpha\beta}) \hat{\epsilon}^* \hat{p} + (iS_{11} \hat{p}^{[\alpha} \gamma^{\beta]} \\ &\quad + 2S_{12} \sigma^{\alpha\beta}) \hat{p} (v \cdot \epsilon^*) + (iS_{13} \gamma^{[\alpha} \epsilon^{*\beta]} + iS_{14} \gamma^{[\alpha} \epsilon^{*\beta]} \hat{p}) + (iS_{15} \epsilon^{*[\alpha} \hat{p}^{\beta]} + iS_{16} \epsilon^{*[\alpha} \hat{p}^{\beta]} \hat{p}) + (iS_{17} \epsilon^{*[\alpha} v^{\beta]} \\ &\quad + iS_{18} \epsilon^{*[\alpha} v^{\beta]} \hat{p}) + (iS_{19} \hat{p}^{[\alpha} v^{\beta]} \hat{\epsilon}^* + iS_{20} \hat{p}^{[\alpha} v^{\beta]} \hat{\epsilon}^* \hat{p}) + (iS_{21} \hat{p}^{[\alpha} v^{\beta]} + iS_{22} \hat{p}^{[\alpha} v^{\beta]} \hat{p}) (v \cdot \epsilon^*) + (iS_{23} \gamma^{[\alpha} v^{\beta]} \hat{\epsilon}^* \\ &\quad + iS_{24} \gamma^{[\alpha} v^{\beta]} \hat{\epsilon}^* \hat{p}) + (iS_{25} \gamma^{[\alpha} v^{\beta]} + iS_{26} \gamma^{[\alpha} v^{\beta]} \hat{p}) (v \cdot \epsilon^*)], \end{aligned} \quad (26)$$

with  $[\alpha, \beta]$  being the antisymmetric index. Note that  $i g_s G^{\alpha\beta} = [iD^\alpha, iD^\beta]$ , and

$$-\bar{h}_v \Gamma_1 D^\alpha D^\beta h_v = -\partial^\alpha (\bar{h}_v \Gamma_1 D^\beta h_v) + \bar{h}_v \bar{D}^\alpha \Gamma_1 D^\beta h_v. \quad (27)$$

After the integration over  $x$ , the total divergence term can be neglected. Consequently, using the equation of motion of heavy quark  $iv \cdot Dh_v = 0$ , one finds that

$$v_\beta \text{Tr} \left\{ \bar{\Omega}_G^{\alpha\beta}(v, p) \Gamma \frac{1+\not{v}}{2} \Gamma_1 \mathcal{M}(v) \right\} = 0. \quad (28)$$

This means  $v_\beta \bar{\Omega}_G^{\alpha\beta} = 0$ , which yields

$$\begin{aligned} S_5 &= S_{19}, \quad 2S_6 = S_{19} - S_{23}, \quad S_7 + S_{15} = S_{21}, \\ 2S_8 &= S_{17} + S_{21} - S_{25}, \quad S_9 = S_{20}, \quad 2S_{10} = -S_{24}, \\ S_{11} + S_{16} &= S_{22}, \quad 2S_{12} = S_{18} + S_{22} - S_{26}, \\ S_{13} + S_{15} &= -S_{17}, \quad S_{14} + S_{16} = -S_{18}. \end{aligned} \quad (29)$$

Therefore, all of the 12 universal functions  $S_i (i=5-16)$  are independent.

Until now all the independent universal functions for heavy meson to light vector meson transitions are obtained, four at leading order, 22 at next-to-leading order. Note that even to the order of  $1/m_Q$ , the form factors in  $B \rightarrow \rho$  semi-leptonic decay and that in  $B \rightarrow K^*$  rare decays are connected.

By evaluating the traces, we get the relevant form factors to the order of  $1/m_Q$  in terms of the universal functions

$$\begin{aligned} \tilde{g} &= -L_3 + \frac{1}{2m_Q} (D_2 - D_7 - 2D_9 - D_{14} - S_3 - 2S_5 + 2S_9 \\ &\quad - 6S_{10} - 2S_{14} + S_{15} - S_{16}), \\ \tilde{f} &= -(L_1 + L_3) + \frac{1}{2m_Q} (-D_1 - D_2 + 4D_3 - D_7 - \hat{p}^2 D_8 \\ &\quad - 2D_9 + D_{13} - D_{14} - S_1 - S_3 - 6S_6 + (1 - \hat{p}^2) S_9 - 6S_{10} \\ &\quad - 2S_{13} - 2S_{14} + (\hat{p}^2 - 1) S_{16}), \\ \tilde{a}_2 &= L_2 + \frac{1}{2m_Q} (D_4 + 2D_6 + \hat{p}^2 D_{11} + S_2 - 2S_7 + 6S_8 \\ &\quad - 2\hat{p}^2 S_{11} + 2S_{13} - \hat{p}^2 S_{16}), \\ \tilde{a}_1 &= (L_3 - L_4) + \frac{1}{2m_Q} (-D_2 + D_5 + D_7 + 2D_9 - D_{10} - 2D_{11} \\ &\quad + 4D_{12} - D_{14} + S_3 - S_4 + 2S_5 + 2S_7 - 2S_9 + 6S_{10} \\ &\quad + 2S_{11} - 6S_{12} + S_{16}), \end{aligned} \quad (30)$$

and

$$\begin{aligned} \tilde{g}_v &= L_1 + \frac{1}{2m_Q} (D_1 + 2D_2 - 3\hat{p}^2 D_8 + D_{13} + S_1 - S_5 + 6S_6 \\ &\quad + 5\hat{p}^2 S_9 + 2S_{13} + S_{14} - S_{15} + 3S_{16}), \\ \tilde{g}_p &= L_3 + \frac{1}{2m_Q} (D_2 + D_7 - 4D_9 - D_{14} + S_3 + 3S_5 + S_9 \\ &\quad + 6S_{10} + 3S_{14} + S_{15} + S_{16}), \end{aligned}$$

$$\begin{aligned}
 \tilde{h} &= L_4 + \frac{1}{2m_Q}(D_5 + D_{10} + 2D_{12} + S_4 - S_5 - 2S_7 - 2S_{11} + 6S_{12} + S_{14} + S_{15}), \\
 \tilde{g}'_v &= L_1 + \frac{1}{2m_Q}(D_1 + 2D_2 - \hat{p}^2 D_8 + D_{13} + S_1 - 2S_5 + 6S_6 + 2\hat{p}^2 S_9 + 3S_{13} + S_{15} - \hat{p}^2 S_{16}), \\
 \tilde{g}'_p &= L_3 + \frac{1}{2m_Q}(D_2 + D_7 + 2D_8 - 4D_9 - D_{14} + S_3 + 2S_5 - 2S_9 + 6S_{10} + 2S_{14} - S_{15} + S_{16}), \\
 \tilde{h}' &= L_4 + \frac{1}{2m_Q}(D_5 + D_{10} + 2D_{12} + S_4 - 2S_7 - 2S_{11} + 6S_{12} + 2S_{14} - S_{15}).
 \end{aligned} \tag{31}$$

In summary, within the HQET, we have systematically analyzed the heavy-to-light vector meson transitions to the order of 1/m<sub>Q</sub>. Besides the four universal functions at the leading order, there are 22 independent universal form factors at the order of 1/m<sub>Q</sub>. Both the semileptonic decay  $B \rightarrow \rho$  which is relevant to the  $|V_{ub}|$  extraction, and the penguin induced decay  $B \rightarrow K^*$  which is important to new physics discovering, depend on these form factors. Once they are given, we can use them to calculate all kinds of decays involving such transitions to a good precision.

Some model-independent observations can be made. Consider the decay  $B \rightarrow \rho l \nu$ ; the decay rate will be largely simplified if we work at the zero recoil point of  $\rho$  meson. Only  $|\tilde{f}|^2$  has nonvanishing contribution. So we have

$$\begin{aligned}
 \frac{d\Gamma(B \rightarrow \rho l \bar{\nu}_l)}{d(v \cdot p)} \Big|_{v \cdot p = m_\rho} &= \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \sqrt{(v \cdot p)^2 - m_\rho^2} \cdot \frac{Q(v \cdot p) - m_l^2}{Q(v \cdot p)} \\
 &\times \left\{ \left[ -(L_1 + L_3) + \frac{1}{2m_b} (2(D_1 - D_8 + D_{13}) + (\bar{\Lambda} + v \cdot p)L_1 \right. \right. \\
 &\quad \left. \left. + (3\bar{\Lambda} - v \cdot p \hat{p}^2)L_3 - S_1 - S_3 - 6S_6 + (1 - \hat{p}^2)S_9 - 6S_{10} - 2S_{13} - 2S_{14} + (\hat{p}^2 - 1)S_{16} \right]^2 \right. \\
 &\quad \left. \times \left[ \frac{m_B^2}{m_\rho^2} \left( (v \cdot p) - \frac{m_\rho^2}{m_B} \right)^2 \left( 1 - \frac{m_l^4}{Q^2(v \cdot p)} \right) - 2(Q(v \cdot p) - m_l^2) \right] \right\}, \tag{32}
 \end{aligned}$$

where  $Q(v \cdot p) = m_B^2 + m_\rho^2 - 2m_B(v \cdot p)$ . Only 13 universal functions are needed to determine the decay rate at zero recoil point. We may also include the flavor changing neutral current decays, such as  $B \rightarrow K^* \gamma$  and  $B \rightarrow K^* l \bar{l}$  to get some information of the unknown universal functions that appeared at the order of 1/m<sub>Q</sub>.

The 1/m<sub>Q</sub> corrections are more important for the decays  $D \rightarrow K^*$  and  $D \rightarrow \rho$ . The information of the 1/m<sub>Q</sub> corrections can be drawn through the following way. First, from the  $B \rightarrow \rho$  decay, neglecting the 1/m<sub>b</sub> effect, we can get a certain result for the leading order heavy quark expansion by comparing it with the experimental data. Then by inputting this knowledge to the  $D \rightarrow K^*(\rho)$  decays, while keeping the 1/m<sub>c</sub> corrections, the information on the 1/m<sub>Q</sub> correction to the decay can be obtained with an uncertainty subject to  $m_c/m_b \sim 30\%$ .

To obtain more detailed results of the decays, knowledge about the universal form factors themselves is needed. While the HQS simplifies the analysis, it does not predict the  $v \cdot p$  dependence of the universal functions. This dependence must be determined separately by using nonperturbative techniques, such as QCD sum rules or lattice simulation, which are the next important steps to obtain quantitative results.

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[1] CLEO Collaboration, B. H. Behrens *et al.*, Phys. Rev. D **61**, 052001 (2000); CLEO Collaboration, T. E. Coan *et al.*, Phys. Rev. Lett. **84**, 5283 (2000).  
 [2] For reviews, see N. Isgur and M. B. Wise, in *B Decays*, 2nd ed., edited by B. Stone (World Scientific, Singapore, 1994), p.

231; M. Neubert, Phys. Rep. **245**, 259 (1994); I. Bigi, M. Shifman, and N. Uraltsev, Annu. Rev. Nucl. Part. Sci. **47**, 591 (1997).  
 [3] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); Phys. Rev. D **42**, 2388 (1990); G. Burdman and J.

- F. Donoghue, Phys. Lett. B **270**, 55 (1991); G. Kramer, T. Mannel, and G. A. Schuler, Z. Phys. C **51**, 649 (1991); D. Du and C. Liu, Phys. Rev. D **50**, 4558 (1994); D. Du, C. Liu, and D.-X. Zhang, Phys. Lett. B **317**, 179 (1993); A. I. Sanda and A. Yamada, Phys. Rev. Lett. **75**, 2807 (1995); Z. Ligeti and M. B. Wise, Phys. Rev. D **53**, 4937 (1996); Z. Ligeti, I. W. Stewart, and M. B. Wise, Phys. Lett. B **420**, 359 (1998); Z. Ligeti and M. B. Wise, Phys. Rev. D **60**, 117506 (1999); E. M. Aitala *et al.*, Phys. Rev. Lett. **80**, 1393 (1998).
- [4] G. Burdman, Z. Ligeti, M. Neubert, and Y. Nir, Phys. Rev. D **49**, 2331 (1994).
- [5] M. E. Luke, Phys. Lett. B **252**, 447 (1990).
- [6] A. F. Falk *et al.*, Nucl. Phys. **B343**, 1 (1990); J. D. Bjorken, in *Gauge Bosons and Heavy Quarks*, Proceedings of the 18th SLAC Summer Institute on Particle Physics, Stanford, California, 1990, edited by J. F. Hawthorne (SLAC Summer Inst., 1990), p. 167.
- [7] Y.-B. Dai, X.-H. Guo, and C.-S. Huang, Nucl. Phys. **B412**, 277 (1994).