# Model of quark and lepton masses: The neutrino sector

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If neutrinos have masses, why are they so tiny? Are these masses of the Dirac type or of the Majorana type? We are already familiar with the mechanism of how to obtain a tiny Majorana neutrino mass by the famous seesaw mechanism. The question is: Can one build a model in which a tiny *Dirac* neutrino mass arises in a more or less ''natural'' way? What would be the *phenomenological consequences* of such a scenario, other than just merely reproducing the neutrino mass patterns for the oscillation data? In this article, a systematic and detailed analysis of a model is presented, with, as key components, the introduction of a family symmetry as well as a new SU(2) symmetry for the right-handed neutrinos. In particular, in addition to the calculations of light neutrino Dirac masses, interesting phenomenological implications of the model will be presented.

## I. INTRODUCTION

There are strong indications—the latest of which came from the SuperKamiokande Collaboration [1]—that neutrinos do have a mass, albeit a very tiny one, and, as a result, ''oscillate.'' The exact nature of the masses as well as the oscillation angles is an important subject which is under intense investigation [2]. Consequently, there exists many interesting models which, in one way or another, try to accommodate most of the known data. It is perhaps prudent to think that the subject of neutrino masses and oscillation is still a very open one.

It is fair to say that the extreme smallness of neutrino masses suggests something very peculiar about these particles. This peculiarity could come from the way the neutrinos obtain their masses and/or from the very special nature of the neutrinos themselves which distinguish them from all other particles. For example, do right-handed neutrinos (present in most models of neutrino masses) carry quantum numbers which are absent in some or all other (left- or right-handed) fermions? After all, right-handed neutrinos, if present, would be singlets under $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ anyway.

Most efforts on the problem of neutrino masses, at least on the model-building front, are concentrated on the construction of lepton mass matrices based on various *Ansätze*. There is one common assumption present in many of such models: light neutrino masses arise from a seesaw mechanism [3]. The smallness of the neutrino masses would come from an expression that goes as $m_D^2/\mathcal{M}$, where $m_D$ is the Dirac mass and $\mathcal{M}$ is the Majorana mass which typically is very much larger than $m_D$. In these models, the scale of new physics $\mathcal{M}$, as suggested by the lightness of neutrino masses, would be some kind of grand unified scale or even the breaking scale of left-right symmetry models [4]. (Lepton number is not a conserved quantity in this class of models.) The seesaw mechanism is a very elegant approach which is widely embraced.

However, one could not help but wonder if there might be some other mechanism for obtaining tiny neutrino masses, and if so, how it would fare compared with the see-saw mechanism. Would this new mechanism shed light on other important issues? What would be its scale of new physics? Can one find an experimental distinction between the two mechanisms? This was the topic discussed in Ref. [5].

At the present time, it is not clear that, if neutrinos do have a mass, it would be of the Majorana or Dirac type. As we have mentioned above, with Majorana neutrinos and the seesaw mechanism, one could ''easily'' obtain small neutrino masses. Now if the mass were to be of the Dirac type, one can straightforwardly write down a gauge-invariant Yukawa coupling in the SM itself (endowed with right-handed neutrinos, of course). But to obtain a small neutrino mass, one has to put in *by hand* a Yukawa coupling which is incredibly small, of the order of $10^{-11}$. Such a fine tuning is highly unnatural and that might be the reason why little attention is given to the construction of models based on Dirac neutrino masses. Did we leave something out by ignoring it? What if the mass is truly of the Dirac type? Until this question is settled, it is worthwhile to investigate possible alternatives to the seesaw mechanism. This paper and a previous one [5] propose one of such alternatives by constructing a model of *Dirac* neutrino masses where the smallness of their values arises dynamically. One of the criteria used in building such a model is the wish to go beyond the mere presentation of a neutrino mass matrix. In particular, we would like to see if there might be other *phenomenological consequences* which could be testable: New particles, new physics signals, etc. This is the aim we had in mind in building our model.

The construction of the model presented in Ref. [5] was based on the following questions. If neutrino masses were so small compared to all other known masses, would there be an appearance of a special symmetry when one lets the mass go to zero? Could this special symmetry, if it exists, be a peculiar feature of the right-handed neutral leptons alone? Could there be additional purposes for its existence other than providing a small mass for the neutrinos? In other words, can one learn something more from it? It was found in Ref. [5] that there is indeed an interesting symmetry which acts only on the right-handed neutrinos and which, in addi-

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tion to providing a reason for the smallness of the neutrino masses, also constrains the nature (even or odd) of the number of generations. Furthermore, the way in which neutrino masses are constructed can be used to build a model for charged lepton and quark masses. In addition, this particular way of constructing masses might even have some bearing on the strong $CP$ problem. Last but not least, are there additional tests of various neutrino models other than neutrino oscillations? For the seesaw mechanism with Majorana neutrinos, one already sees that one of such additional signals is, for example, the phenomenon of neutrinoless double beta decay. As it will be presented below, the addtional signals of the model presented here will involve a number of very concrete predictions: the absence of neutrinoless double beta decay, the possible presence of ''low mass'' (a couple of hundreds of GeV, e.g.) vectorlike fermions, among other things. In particular, the detection of these vectorlike fermions do not in any way involve neutrinos.

One particularly important feature of our model is the following predictions for neutrino oscillations, assuming only the validity of the atmostpheric and solar neutrino data: (1) The three light neutrinos are nearly degenerate; (2) If the light neutrinos have a mass large enough to form a component of the hot dark matter (HDM) [6] then *only* the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino oscillation is favored; (3) If the vacuum solution to the solar neutrino problem turns out to be the correct one, our model will only be able to accomodate tiny neutrino masses, around $10^{-3}$ eV or less, ruling out near-degenerate neutrinos as components of HDM. As a result, in our model, one cannot have both vacuum solution and HDM. We will show below the correlation between the masses and the differences of mass squared, $\Delta m^2$, which enter the neutrino oscillation phenomena.

Assuming the existence of the aforementioned symmetry, how can one construct Dirac neutrino masses to be dynamically small? By ''dynamically,'' it is meant that the mass is zero at tree level and that any nonzero value would have to arise at the one-loop (or more) level. Now, the peculiar (and toughest) thing about neutrinos is the fact that their mass is so small—at least eleven orders of magnitude smaller than the electroweak scale. In constructing our model for Dirac neutrino masses, it is then reasonable to ask under what conditions would the dynamical Dirac mass of the neutrinos obtained at the one loop level be ''naturally'' small, i.e., devoid of excessive fine-tuning. In this paper, we present the following interesting results. In the four-generation model, it is found that the fourth neutrino can be naturally heavy while the other three obtain their masses at one loop, with the result that these masses can be tiny provided some ratios of masses of particles which participate in the loop integration are ''large,'' *regardless* of their actual values. This is interesting because, as we shall see below, some of the particles which participate in the loop integration, in particular the lightest ones, can have masses as low as a few hundred GeVs and which could provide a direct test of this model. We will also see that, in order to obtain very small neutrino masses, at least one of the particles needs to be much heavier than the lightest one—a result which is somewhat reminescent of the

seesaw mechanism. We will also see that the mass of the light neutrinos is intrinsically tied to the extra global symmetry present in the scalar sector of the model. In fact, the extra Nambu-Goldstone (NG) bosons which are not absorbed by the [family and $SU(2)_{\nu_R}$] gauge bosons acquire a mass due to the presence of the gauge-invariant ''cross-coupling'' terms in the potential which explicitly break the extra global symmetry.

The above brief statement will be made clearer in the discussion of neutrino masses. Notice, in particular, that the result given for light neutrino masses in Ref. [5] is only a very special case of the present discussion.

The plan of the paper is as follows. First, the model is presented with a description of the gauge structure along with its particle content. It is shown how a new symmetry prevents neutrinos from obtaining a mass unless it is broken. Next, the special properties of this extra symmetry associated with the right-handed neutrinos are discussed. In particular, if that symmetry is a chiral SU(2) as is the case in this paper, nontrivial constraints coming from the nonperturbative Witten anomaly [7] can be applied to the nature of the number of families. This is the extra bonus mentioned above. The paper then proceeds to discuss the generation of light neutrino masses, principally by radiative corrections of the type mentioned above. It is then followed by a discussion of the neutrino mass matrix. In particular, we will present the correlation between the values of the neutrino masses and $\Delta m^2$. Most importantly, we will show how $\Delta m^2$ increases or decreases with the masses themselves, with two resulting implications: either one has HDM *and* MSW or vacuum solution and *no* HDM. Either of these solutions will have an important cosmological implication. We end the paper with a brief discussion of the charged lepton mass matrix, the primary purpose of which being the wish to complete the discussion by presenting some examples of what the oscillation angles might look like. A follow-up paper will deal seperately with the charged lepton sector and, as a consequence, with a full discussion of the angles.

We would like to emphasize for the purpose of clarity that the charged lepton sector (which will be dealt with in a separate paper) is different in structure from the neutrino sector, as we shall see below, and does not have the same hierarchical structure. The fact that, in this model, the three light neutrinos are nearly degenerate does not imply that it would be the same in the charged lepton sector. In fact, it is not as we will show in a subsequent paper.

Finally, a section will be devoted to various other phenomenological implications of the model. We shall assume throughout this paper the existence of right-handed neutrinos.

Since this manuscript is meant to be comprehensive, and hence lengthy, one could skip the three subsections of the next section, after first reading its introduction. (Its reading is nevertheless recommended because the physics motivations are discussed there.)

## II. A MODEL

It is well known that all that is needed to give neutrinos a mass is to simply add extra right-handed neutrinos to the

standard model. One can then construct a (Dirac) mass term with an arbitrary Yukawa coupling $g_\nu \bar{l}_L \phi \nu_R + $H.c., which can be made to be as small as one wishes. This, of course, is unsatisfactory because, if neutrinos have masses in the eV range or less, this would require the Yukawa coupling $g_\nu$ to be of $O(10^{-11})$ or less. Fine-tuning to such a precision is normally considered to be unnatural. At this point, one might be tempted to try to explain this fact by simply invoking a fourth generation with a democratic mass matrix, at least for the neutrinos, as has been done by Ref. [8]. The diagonalization of the neutrino mass matrix would then give one heavy eigenstate and three massless states. By adding some arbitrary phases to the mass matrix, one can ''provide'' a small mass (depending on the values of those phases) to the three neutrinos. This purely phenomenological *Ansatz* (Ref. [8]) appears to ''fit'' the recent data on neutrino oscillations with the appropriate *choices* of the phases. However, the fourth generation lepton masses came out to be extremely heavy and split, which practically seems to be ruled out by analyses of precision experiments [9].

In Ref. [5], a model of Dirac neutrino masses was constructed and based on a four generation scenario that was very different from the democratic *Ansatz* made in Ref. [8]. One of the reasons for using such a scenario is the fact that, as of the present time, a fourth generation is *not* ruled out by experiment and, as a consequence, it is interesting to explore its possible implications. A recent review [11] gave a comprehensive discussion of various topics concerning quarks and leptons beyond the third generation, including the present experimental status and future searches.

If a fourth generation were to be used in the investigation of neutrino masses, one should keep in mind various phenomenological constraints concerning not only leptons but also quarks. For instance, constraints on the $\rho$ parameter limit the mass splitting within each doublet of extra quarks and leptons: the up and down members of a fourth generation should be very close in mass. They should be long-lived enough to escape present detection. This, in turns, tells us something about the mixing between the fourth generation and the other three. All of these issues have been described in Ref. [11]. In the construction of the model presented in Ref. [5], these phenomenological constraints were kept in mind.

As mentioned briefly in the Introduction, our approach, as described in Ref. [5], is based on a dynamical justification for the small value of the neutrino Yukawa couplings. The question that was asked was: Could there be a scenario in which a symmetry appears as one lets the Yukawa coupling go to zero? The tiny Yukawa coupling which would give the neutrino a very small mass would then arise dynamically when that symmetry is broken. These Yukawa couplings then appear as effective couplings which could be small for dynamical reasons and are not fundamental parameters that are put in by hand and which are needed to be fine-tuned. What is the nature of that symmetry and how a dynamical Yukawa coupling appears will be the subject of this section and the following two.

It is obvious that an extension of the standard model (SM) is needed in addressing the above issues. One simply cannot stay solely within the SM if one wishes to deal with the mass of the neutrinos. What it is that one needs when one goes beyond the SM is a matter of taste, modulo a very obvious requirement: predictability of new phenomena or particles which can be tested.

We first describe the model, presenting its gauge structure and representations. Next, explanations are provided for the reasons behind the choices of the extended gauge group and its particle content. The crucial assumption here is the existence of two new symmetries, one of which will be particular to right-handed neutrinos, as alluded to earlier, and the other one is a family gauge symmetry. As we shall see below, it is the breaking of these new symmetries that will give a mass to the neutrinos.

In this work, the SM is extended in the following way. Generically, it takes the form $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ $\otimes$(family symmetry)$\otimes$(right-handed neutrino special symmetry). Why a ''family symmetry''? This is so for two reasons: (a) We wish to investigate the family replication problem and the mixing among different generations and (b) the special symmetry endowed by the right-handed neutrinos might have some bearing on the family symmetry itself. After all, if one would like to investigate the family problem, some kind of family symmetry—be it discrete or continuous, global, or gauge—is needed. Why a special symmetry for the right-handed neutrinos? The reasons were already expounded above: To provide a framework for an understanding of the smallness of neutrino masses. Our next task is then to determine what this special symmetry might be and what form the family symmetry might take.

Our model is described by

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SO(N_f) \otimes SU(2)_{\nu_R}, \quad (1)$$

where $SO(N_f)$ and $SU(2)_{\nu_R}$ are the family gauge group and the special gauge group for the right-handed neutrinos, respectively. The particle content of the model is listed in Table I. Notice that we have denoted the right-handed neutrinos by $\eta_R = (\nu_R^\alpha, \tilde{\nu}_R^\alpha)$ because they are assumed to transform as doublets under $SU(2)_{\nu_R}$. The two options listed for the right-handed neutrinos as well as the meaning of the nonstandard particles will be discussed below. We would first like to explain the choices of the extra gauge groups. Here, the extra symmetries are chosen to be *gauge symmetries* because, as it is well known, powerful constraints can be obtained from models built on the gauge assumption.

### A. Why $SU(2)_{\nu_R}$?

Let us first look at Table I. In this model, all standard (left-handed and right-handed) particles are singlets under $SU(2)_{\nu_R}$. Hence the subscript $\nu_R$. In this respect, $SU(2)_{\nu_R}$ is very different from $SU(2)_R$ of the popular left-right model [4]. In that model, right-handed quarks and leptons form doublets under $SU(2)_R$, for every family. Because of our assignment, all weak interactions among standard particles are pure *V-A*, in contrast with the left-right model. What is the motivation behind our choice that makes it so different from the left-right model? To answer that question, let us

TABLE I. Particle content and quantum numbers of $SU(3)_c$ $\otimes SU(2)_L \otimes U(1)_Y \otimes SO(N_f) \otimes SU(2)_{\nu_R}$.

| | |
|---|---|
| Standard fermions | $q_L = (3,2,1/6,N_f,1)$ |
| | $l_L = (1,2,-1/2,N_f,1)$ |
| | $u_R = (3,1,2/3,N_f,1)$ |
| | $d_R = (3,1,-1/3,N_f,1)$ |
| | $e_R = (1,1,-1,N_f,1)$ |
| Right-handed $\nu$'s | Option 1: $\eta_R = (1,1,0,N_f,2)$ |
| | Option 2: $\eta_R = (1,1,0,N_f,2)$; |
| | $\eta'_R = (1,1,0,1,2)$ |
| Vectorlike fermions | $F_{L,R} = (1,2,-1/2,1,1)$ |
| | $\mathcal{M}_{1L,R} = (1,1,-1,1,1)$ |
| | $\mathcal{M}_{2L,R} = (1,1,0,1,1)$ |
| Scalars | $\Omega^\alpha = (1,1,0,N_f,1)$ |
| | $\rho_i^\alpha = (1,1,0,N_f,2)$ |
| | $\phi = (1,2,1/2,1,1)$ |

recall an interesting feature of chiral $SU(2)$: the presence or absence of the so-called Witten global anomaly.

If chiral fermions transform as doublets under $SU(2)$, there exists a nonperturbative anomaly—the so-called Witten anomaly [7]—associated with an *odd* number of doublets. Briefly speaking, this is so because the fermionic determinant $\sqrt{\det i \overline{\nabla}(A_\mu)}$ changes sign under a ''large'' gauge transformation $A_\mu^U = U^{-1}A_\mu U - i U^{-1}\partial_\mu U$ if the number of chiral doublets is odd. This would make the partition function $Z$ vanish and the theory would be ill defined. This nonperturbative anomaly would then require the number of Weyl doublets to be *even* in order for the theory to be consistent. (This ambiguity in sign stems from the fact that the fourth homotopy group $\Pi_4[SU(2)] = Z_2$.) Other groups that also have similar nontrivial constraints are $Sp(N)$ for any $N$ and $O(N)$ for $N \leq 5$.

It is amusing to recall a well-known but forgotten fact about the SM. There the chiral gauge group is $SU(2)_L$. Each family contains one lepton and three quark doublets and, as such, is free from the global Witten anomaly. (Let us recall that the cancellation of the *perturbative* triangle anomaly in the SM only relates the lepton charge to that of the quark.) If, instead of three, the number of colors $N_c$ were arbitrary, the freedom from such an anomaly would require $1 + N_c$ to be even, and hence, $N_c$ to be odd, namely, $N_c = 3,5,\ldots$. Why nature chooses $N_c = 3$ instead of some other odd number is a question which can only be answered in the context of some deeper theory such as, e.g., $SU(5)$. Although the Witten anomaly does not fix the size of $N_c$, it is nevertheless a powerful constraint in the sense that, *once a fermion content is known* [e.g., one color singlet (leptons) and one fundamental representation (quarks) in the SM], $N_c$ is constrained (e.g., odd in the case of the SM).

The above simple lesson taught us something about the powerful constraint that a chiral $SU(2)$ exerts on the number of chiral doublets. This is the reason why it is chosen to be

the special symmetry of the right-handed neutrinos. Let us contrast the constraint coming from $SU(2)_{\nu_R}$ with that coming from $SU(2)_R$ (left-right model). For our model, with $SU(2)_{\nu_R}$, *only* $\eta_R$ transforms as doublets. Absence from the Witten anomaly then requires the number of such doublets to be *even*. If $\eta_R$ carries, in addition, family indices then the anomaly requirement restricts the number of generations to be *even* such as in option 1 as indicated in Table I. If there exists an $\eta_R$ which is a family singlet (denoted by $\eta'_R$), the number of generations would be *odd* such as in option 2 of Table I. With the left-right model, *each family* contains four doublets of $SU(2)_R$: $(\nu_R, e_R)$ and $(u_R, d_R)_i$ with $i = 1,\ldots,3$. Therefore, the Witten anomaly requirement is automatically satisfied *per family*. This is one of the few differences between our model and the left-right model.

A final word of caution is in order here. Although the Witten anomaly constraint allows us to make a statement on the evenness or oddness of the number of generations—a subject to which we shall come back in the next subsection—it *does not* determine that number. This should come from a deeper and as-yet-unknown theory. Our goal is much more modest: Given a fermion content (option 1 or 2 below), we can say whether or not the number of generations is odd or even, and that is all. We shall, however, try to constraint that number from a different route which is more phenomenological in nature, and point out the differences between options 1 and 2.

### B. Why $SO(N_f)$?

In the construction of any model, there is a time-honored requirement: the absence of the perturbative triangle anomaly. Even if the Witten anomaly were absent, this requirement is a must for any gauge theory. (It just happens that, in the SM, both requirements are simultaneously satisfied.) In our case, if a family index is assigned to all standard fermions *and* to $\eta_R$, the family gauge group that is chosen cannot be a vectorlike theory, which is anomaly-free, because $\eta_R$ possesses an additional quantum number, that of $SU(2)_{\nu_R}$. This is unlike QCD or even the left-right model if left and right-handed fermions carry similar family quantum numbers. A safe group and representations have to be chosen.

The choice made in this paper is $SO(N_f)$ for the family gauge group, with chiral (left- and right-handed) fermions transforming as (real) vector representations with $N_f$ components each. As such, the model is also free of the perturbative triangle anomaly. Our model based on $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SO(N_f) \otimes SU(2)_{\nu_R}$ with an *even* number of $SU(2)_{\nu_R}$ doublets and chiral fermions transforming as *vector representations* of $SO(N_f)$ is free from both nonperturbative and perturbative anomalies.

### C. Constraints on $N_f$

As shown in Table I, there are two options for $\eta_R$, each of which should contain an even number of $SU(2)_{\nu_R}$ doublets.

(a) Option 1: $\eta_R^\alpha$ carries the family index $\alpha = 1 \ldots N_f$ where $N_f = 2, 4, 6, 8, \ldots$.

(b) Option 2: Here we have $\eta_R'$ (a family singlet) and $\eta_R^\alpha$. The constraint is now $1 + N_f =$ even, which means that $N_f = 3, 5, 7, \ldots$ (excluding the trivial case of 1 family).

Unlike the SM where one knows the fermion content for each family, i.e., quarks and leptons, and hence the nature of $N_c$—it is *odd*—our scenario involves incomplete experimental informations, and as such, the nature (odd or even) of $N_f$ cannot be completely fixed. Each choice, however, represents a distinct particle content (no family singlets for the even option and one family singlet for the odd option) which implies a possible distinct route for a yet-unknown unification.

Recognizing the fact that there are deep differences between the even and odd options—a point to be discussed below—and in the absence of a deeper theory, one might wonder what can be done to narrow down the choices, not between odd or even, but within each option itself. Below we present an argument that could help in finding a way to further restrict $N_f$. This argument is only suggestive, being a combination of ''theoretical prejudice'' and phenomenological constraint.

One might require that gauge couplings are free from Landau singularities below the Planck scale in such a way that unification of the SM gauge couplings, if it exists, occurs in the perturbative regime [10]. With this criterion, one can see that the even option can only accomodate $N_f = 2, 4, 6$, while the odd option can only accomodate $N_f = 3, 5$. This is because for $N_f \geq 7$, one or more gauge couplings will ''blow up'' before the Planck scale. There are no reasons, in the absence of a deeper theory, to rule out any of the above choices. This will require other yet-unknown conditions. The only thing one can say, in the context of our model, is that electroweak precision experiments appear to rule out $N_f \geq 5$ [9] and and that existential facts tell us that $N_f$ is at least three. This leaves us with the choice $N_f = 4$ for the even option and $N_f = 3$ for the odd option.

If $N_f \leq 4$ comes from the above argument, what then is the role of the Witten anomaly in all of this? It tells us about the particle content of the right-handed neutrinos. For $N_f = 4$, the right-handed neutrinos are simply $\eta_R = (1, 1, 0, 4, 2)$ while for $N_f = 3$, one has $\eta_R = (1, 1, 0, 3, 2)$ plus a family singlet $\eta_R' = (1, 1, 0, 1, 2)$. What observed differences can there be between these two options? The former predicts the existence of a fourth generation whose consequences have been recently discussed in Refs. [10] and [11,12]. The latter predicts the existence of a neutral family-singlet $\eta_R'$ [doublet under $SU(2)_{\nu_R}$] which could have cosmological consequences of a yet-unknown nature. In addition, as we point out below, it appears that the even option prefers three almost degenerate light neutrinos while the odd option prefers a hierarchical structure for the light neutrinos. If a fourth generation is discovered, which alone does not necessarily imply the even option presented here, and if the light neutrino masses are convincingly ''proven'' to be nearly degenerate (instead of a hierarchical structure), the even option might be viable. Furthermore, as we shall see below, another
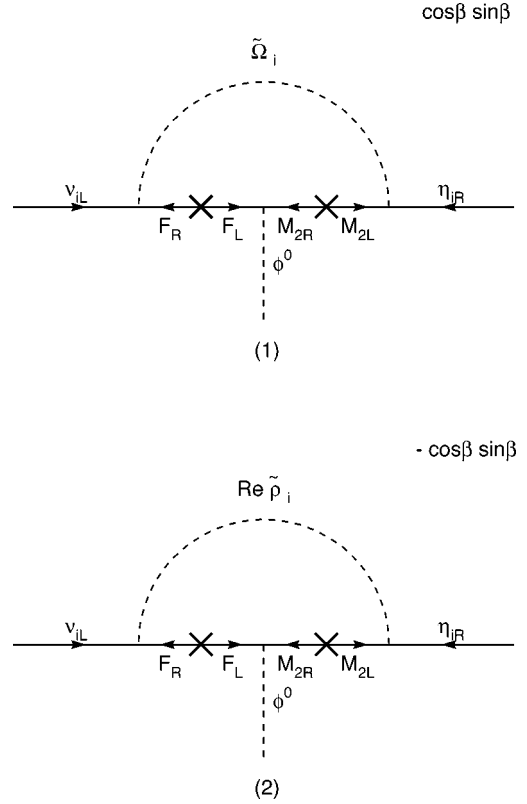


FIG. 1. Feynman graph showing the computation of $\tilde{G}_\nu$, where $m_\nu = \tilde{G}_\nu (v / \sqrt{2})$.

possibility for testing this model is to look for signals of some of the lightest particles—the vectorlike fermions—which participate in the loop diagram of Fig. 1. As discussed below, the light neutrino masses depend only on the ratios of these masses and not on their magnitudes and these vectorlike fermions can be as light as a few hundred GeVs.

## III. NEUTRINO MASSES

This section will be devoted to the discussion of how neutrino masses can be generated in our model for option 1. We shall comment on option 2 at the end of the manuscript. We shall concentrate only on the lepton sector and, in particular, on the neutrino one, leaving the full discussion of the charged lepton and quark sectors for a subsequent publication.

Since we will be dealing only with *Dirac* neutrino masses, we shall require that *all* fermions be endowed with a global *B-L* symmetry. Since we are concerned only with leptons in this section, a global *L* symmetry is sufficient for the present purpose. This global *L* symmetry would *prevent* a Majorana mass term of the type $\eta_R^{i\,\alpha} \eta_{i\,\alpha R}$, where $i = 1, 2$ and $\alpha = 1, \ldots, 4$. Only Dirac masses will be allowed.

There might be other suggestive reasons as to why Dirac masses for the neutrinos might be attractive. For example, a combined fit of massive neutrinos as components of hot dark matter (HDM) and atmospheric neutrino oscillations seems to prefer a scenario in which two or three light neutrinos are nearly degenerate and have mass in the $O(\text{eV})$ range. Recent

data on neutrinoless double beta decay (or absence thereof) [13] appear to rule out Majorana neutrinos heavier than 0.2 eV, at least in the simplest versions. Here it will be shown how, in our scenario, one can obtain three near-degenerate neutrinos whose mass can be of the order of a few eV's and is of the Dirac type. Consequently, in our model, there will be *no* neutrinoless double beta decay, and hence no contraint on the Dirac neutrino masses from such a search.

As we have discussed above, option 1 contains no family-singlet fermion field and freedom from the Witten anomaly dictates that the number of families should be even. Furthermore, we have argued that this even number should be four. As a result, the gauge group for this option is

$$\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \otimes \text{SO}(4) \otimes \text{SU}(2)_{\nu_R} \quad (2)$$

The reader is referred to Table I for a list of particles that participate in this model.

## A. Computation of the diagonal elements of the 4×4 neutrino mass matrix

Without the extra vector-like fermions, $F$, $M_1$, and $M_2$, the only gauge-invariant Yukawa coupling involving leptons would be $\mathcal{L}_Y = g_E \bar{l}_L^\alpha \phi e_{\alpha R} + \text{H.c.}$ (where $\alpha = 1, \ldots, 4$ is the family index), giving rise to equal masses for the charged leptons. Unbroken $\text{SU}(2)_{\nu R}$ forbids a similar term for the neutrinos and they remain massless at this level. (Notice that, since we are only interested in Dirac neutrino masses, a gauge-invariant Majorana mass term of the type $\eta_R^{i \alpha} \eta_{i \alpha R}$ is forbidden by $L$ symmetry.) We *know* that the charged leptons are not degenerate in mass. We also *know* that the width of the $Z$ boson [14] constrains the mass of the fourth neutrino to be larger than half the $Z$ mass. This is where the vectorlike fermions listed in Table I come in. Because of their vector-like nature, they can have *arbitrary* gauge-invariant bare masses. We shall assume that the $F$ fermions which transform as a doublet under $\text{SU}(2)_L$ are degenerate in mass. It is seen below that some of these masses can be as low as a few hundred GeV and are thus accessible to future experimental searches. As for the S parameter constraint, it is well-known that, to leading order, vectorlike fermions which carry electroweak $\text{SU}(2)_L$ quatum numbers *do not* contribute to S if one has a *degenerate* $\text{SU}(2)$ doublet. The reason for this being so is because the right-handed contribution cancels exactly the left-handed contribution. Therefore, to leading order, there is no constraint from the S parameter on the mass of the $F$ fermions. This point and other issues concerning quarks and leptons beyond the third generation are discussed in Ref. [12].

The Yukawa part of the Lagrangian involving leptons can be written as

$$\mathcal{L}_{\text{lepton}}^Y = g_E \bar{l}_L^\alpha \phi e_{\alpha R} + G_1 \bar{l}_L^\alpha \Omega_\alpha F_R + G_{M_1} \bar{F}_L \phi \mathcal{M}_{1R}$$
$$+ G_{M_2} \bar{F}_L \tilde{\phi} \mathcal{M}_{2R} + G_2 \bar{\mathcal{M}}_{1L} \Omega_\alpha e_R^\alpha + G_3 \bar{\mathcal{M}}_{2L} \rho_m^\alpha \eta_{\alpha R}^m$$
$$+ M_F \bar{F}_L F_R + M_1 \bar{\mathcal{M}}_{1L} \mathcal{M}_{1R} + M_2 \bar{\mathcal{M}}_{2L} \mathcal{M}_{2R} + \text{H.c.} \quad (3)$$

The assumption of an unbroken $L$ symmetry forbids the presence of Majorana mass terms as mentioned above.

Notice that the values of $M_{F,1,2}$ are arbitrary. What they might be will be the subject of the discussion presented below. After integrating out the $F$, $\mathcal{M}_1$, and $\mathcal{M}_2$ fields, the relevant part of the effective Lagrangian below $M_{F,1,2}$ reads

$$\mathcal{L}_{\text{lepton}}^{Y,\text{eff}} = g_E \bar{l}_L^\alpha \phi e_{\alpha R} + G_E \bar{l}_L^\alpha (\Omega_\alpha \phi \Omega^\beta) e_{\beta R}$$
$$+ G_N \bar{l}_L^\alpha (\Omega_\alpha \tilde{\phi} \rho_i^\beta) \eta_{\beta R}^i + \text{H.c.}, \quad (4)$$

where

$$G_E = \frac{G_1 G_{M_1} G_2}{M_F M_1}; \quad G_N = \frac{G_1 G_{M_2} G_3}{M_F M_2}. \quad (5)$$

This is a tree-level effective Lagrangian whose consequences are now presented.

Let us discuss the implication of each term on the right-hand side of Eq. (4). As stated in the preceding paragraph, the first term gives rise to equal masses for the charged leptons. The second term would lift the degeneracy of the charged lepton sector once $\Omega$ acquires a vacuum expectation value (VEV). The third term gives rise to a neutrino mass once *both* $\Omega$ and $\rho$ acquire a VEV. It is clear that, in our model, neutrino masses can appear only when *both* SO(4) and $\text{SU}(2)_{\nu_R}$ are spontaneously broken while the charged lepton masses are nonzero (but equal) even if SO(4) is unbroken. Only when SO(4) is broken will the charged lepton mass degeneracy be lifted.

Let us assume $\langle \Omega \rangle = (0,0,0,V)$ and $\langle \rho \rangle = (0,0,0,V' \otimes s_1)$, where $s_1 = \binom{1}{0}$. Notice that each component [under SO(4)] of $\rho$ transforms as a doublet under $\text{SU}(2)_{\nu_R}$. If we denote the fourth element of $\eta_R$ by $(N_R, \tilde{N}_R)$, one can use the above two VEV's along with $\langle \phi \rangle = (0, v/\sqrt{2})$ ($v \approx 246$ GeV) in Eq. (4) to write down a Dirac mass term for the fourth generation neutrino, namely,

$$\tilde{G}_N \frac{v}{\sqrt{2}} \bar{N}_L N_R + \text{H.c.}; \quad \tilde{G}_N = G_1 G_{M_2} G_3 \frac{V V'}{M_F M_2}, \quad (6)$$

giving

$$m_N = \tilde{G}_N \frac{v}{\sqrt{2}}. \quad (7)$$

At *tree level*, all other neutrinos are massless. Their masses arise at the one-loop level as shown below. The *Dirac* mass of the fourth neutrino could be rather *heavy*. In fact, it is not unreasonable to expect $G_1$, $G_{M_2}$, and $G_3$ to be of the order of unity. In consequence, as long as

$$V V' / M_F M_2 \sim O(1), \quad (8)$$

one might expect the fourth neutrino to be even as heavy as 175 GeV. Certainly, the CERN $e^+ e^-$ collider LEP bound of $M_Z/2$ can easily be satisfied.

Why are the other three neutrinos massless at tree level? First, it is so because, from Eq. (3) and Eq. (4), one can see that, after integrating out the heavy vectorlike fermions, there is no (tree-level, dimension 6) operator which contains, as a factor, a term such as $\bar{l}_L^m \tilde{\phi} \eta_{iR}^m$, where $m=1,2,3$ is a family index, which would give rise to a mass term for the three light neutrinos. An effective (dimension 6) operator which contains the aforementioned term would necessarily come from a loop integration such as the one shown in Fig. 1. Just like the various terms which appear in Eq. (4), this effective operator would also contain the scalar fields $\Omega$ and $\rho$. It would appear as

$$\bar{l}_L^m \tilde{\phi} \eta_{iR}^m (\Omega_\alpha \rho^{\alpha\, i}). \qquad (9)$$

As pointed out in the Appendix, a term such as $(\Omega^\alpha \rho_{\alpha\, i})$ appears as part of a quartic term in the potential which explicitly breaks the extra global symmetry that the scalar sector posesses. As a result, the extra NG bosons are, in fact, pseudo-NG bosons and acquire a mass which is proportional to the coupling $\lambda_4$ as shown in Eq. (A8) of the Appendix.

In order to compute the one-loop contributions to neutrino masses, let us recall, in this section, the results obtained in the Appendix concerning the relevant mass eigenstates in the scalar sector. We have

$$H_4 = \cos \alpha \tilde{H}_4 - \sin \alpha \tilde{h}_4, \qquad (10a)$$

$$h_4 = \sin \alpha \tilde{H}_4 + \cos \alpha \tilde{h}_4, \qquad (10b)$$

$$\Omega_i = \cos \beta \tilde{\Omega}_i - \sin \beta \, \mathrm{Re}\, \tilde{\rho}_i, \qquad (10c)$$

$$\mathrm{Re}\, \rho_i = \sin \beta \tilde{\Omega}_i + \cos \beta \, \mathrm{Re}\, \tilde{\rho}_i, \qquad (10d)$$

where $i=1,2,3$ and where the states with the tildes sign are mass eigenstates. The Yukawa couplings which will be involved in the computation of neutrino masses can now be written in terms of the mass eigenstates. For example, $G_1 \bar{l}_L^\alpha \Omega_\alpha F_R$ can be written as

$$G_1 \bar{l}_L^4 \Omega_4 F_R = G_1 \bar{l}_L^4 (\cos \alpha \tilde{H}_4 - \sin \alpha \tilde{h}_4) F_R, \qquad (11)$$

$$G_1 \bar{l}_L^i \Omega_i F_R = G_1 \bar{l}_L^i (\cos \beta \tilde{\Omega}_i - \sin \beta \, Re \tilde{\rho}_i) F_R, \qquad (12)$$

where $i=1,2,3$. Also, $G_3 \bar{M}_{2L} \rho_m^\alpha \eta_{\alpha R}^m$ ($m=1,2$) can now be written as

$$G_3 \bar{M}_{2L} \rho_1^4 \eta_{4R}^1 = G_3 \bar{M}_{2L} (\sin \alpha \tilde{H}_4 + \cos \alpha \tilde{h}_4$$
$$+ i\, \mathrm{Im}\, \rho_4)_1 \eta_{4R}^1, \qquad (13)$$

$$G_3 \bar{M}_{2L} \rho_1^i \eta_{iR}^1 = G_3 \bar{M}_{2L} (\sin \beta \tilde{\Omega}_i + \cos \beta \, \mathrm{Re}\, \tilde{\rho}_i$$
$$+ i\, \mathrm{Im}\, \rho_i)_1 \eta_R^{1,i}. \qquad (14)$$

The above equations, in addition to $G_{M_2} \bar{F}_L \tilde{\phi} M_{2R}$, form the basis for constructing the one-loop diagrams as shown in Fig. 1. As one can immediately see, the only scalars that participate in the loop integration are $\tilde{H}_4$, $\tilde{h}_4$, $\tilde{\Omega}_i$, and $\tilde{\rho}_i$. The contributions to the light neutrino masses will contain a factor $\cos \beta \sin \beta = \sin(2\beta)/2$ for $\tilde{\Omega}_i$ and $-\cos \beta \sin \beta$ for $\mathrm{Re}\, \tilde{\rho}_i$.

The masses of the physical Higgs scalars $H_4$ and $h_4$, and those of the pseudo-NG bosons $\mathrm{Re}\, \tilde{\rho}_i$ ($i=1,2,3$), are given by Eqs. (A4),(A8) in the Appendix. Since the one-loop contributions to the fourth neutrino mass are expected to be small compared with its tree-level value, we shall concentrate in this section on the light neutrino masses. There we shall be concerned only with $\tilde{\Omega}_i$ (NG bosons) and $\mathrm{Re}\, \tilde{\rho}_i$ (pseudo-NG bosons) ($i=1,2,3$). In the 't Hooft-Feynman gauge, the NG bosons will have a propagator with a mass which is that of the family gauge bosons. We shall denote it by $M_G$. We shall call the mass of the pseudo-NG bosons, $M_P$.

The result obtained from the diagrams as shown in Fig. 1 for the three light neutrinos is

$$m_\nu = \tilde{G}_\nu \frac{v}{\sqrt{2}}, \qquad (15)$$

where

$$\tilde{G}_\nu = G_1 G_{M2} G_3 \frac{\sin(2\beta)}{32\, \pi^2} [I(\tilde{\Omega}) - I(\mathrm{Re}\, \tilde{\rho})], \qquad (16)$$

and where

$$I(\tilde{\Omega}) - I(\mathrm{Re}\, \tilde{\rho}) = \frac{1}{M_F - M_2} \left\{ \frac{M_F\{M_F^2[M_G^2 \ln(M_G^2/M_F^2) - M_P^2 \ln(M_P^2/M_F^2)] + M_G^2 M_P^2 \ln(M_P^2/M_G^2)\}}{(M_G^2 - M_F^2)(M_P^2 - M_F^2)} - (M_F \leftrightarrow M_2) \right\}. \qquad (17)$$

For notational convenience, we shall define

$$\Delta I(G,P) \equiv I(\tilde{\Omega}) - I(\mathrm{Re}\, \tilde{\rho}), \qquad (18)$$

It is convenient to express the mass of the light neutrinos by the following ratio:

$$\frac{m_\nu}{m_N} = \frac{M_F M_2}{VV'} \frac{\sin(2\beta)}{32\, \pi^2} \Delta I(G,P), \qquad (19)$$

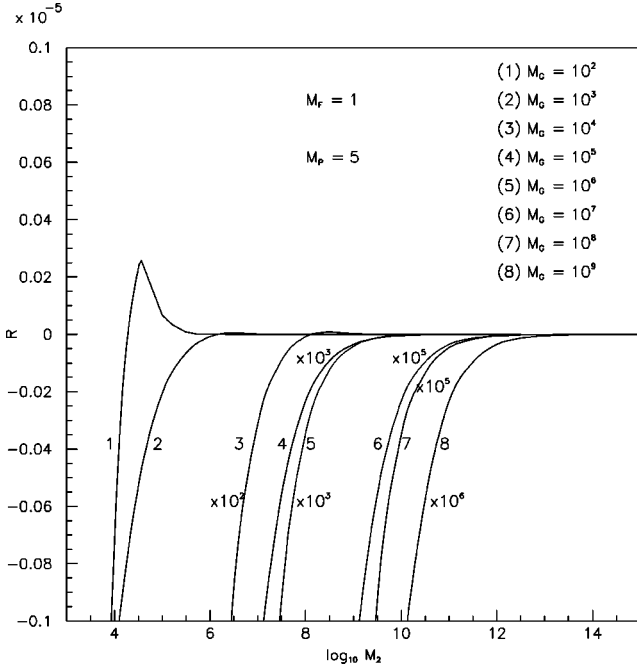where $m_N$ is defined by Eq. (7).

FIG. 2. The ratio $R \equiv m_\nu/m_N$ [Eq. (23)] as a function of $M_2$ (in units of $M_F$, and hence the notation $M_F=1$), for $M_P=5$ and for various values of $M_G$. For visibility purpose, a few curves have been inflated by factors $\times 10^{2,3,5,6}$.

One should mention for completeness the tiny one-loop contribution to the fourth neutrino mass. If we denote by this contribution by $\delta m_4$, it is straightforward to see that it is given precisely by the same formula for the light neutrino mass, Eq. (15), with the following replacements: $\beta \to \alpha$, $M_G \to M_{H_4}$, $M_P \to M_{h_4}$, namely,

$$\delta m_4 = \widetilde{G}_4 \frac{v}{\sqrt{2}}, \tag{20a}$$

$$\widetilde{G}_4 = G_1 G_{M2} G_3 \frac{\sin(2\alpha)}{32\pi^2} \Delta I(G,P), \tag{20b}$$

where the form of $I(\widetilde{H}_4)-I(\widetilde{h}_4)$ is identical to Eq. (17) with the replacements as mentioned above. This contribution will play an insignificant role in the mass matrix, but it has to be mentioned for completeness.

The above results were obtained at one loop. One wonders if higher loop contributions might be significant. It turns out that, because of the nature of the interactions, the next correction occurs at the three loop level. It means that the correction to the one-loop light neutrino mass is at the two-loop order. Considering that already the one-loop result is $O(<10^{-10})$, a two-loop correction to that result would most likely be insignificant, even for the mass splitting to be discussed below. Above all, the experimental results are far from being precise enough to even contemplate such a tiny correction. From here on, we shall assume that these three-loop corrections are insignificant in the computation of the mass splittings.

At this stage, the three light neutrinos are degenerate. A discussion of the lifting of the degeneracy will follow a more general discussion of the implications of Eq. (19). It is clear that the ''light family'' symmetry would have to be broken in order for the ''light'' fermions to mix. It is also clear that the neutrino masses (one heavy and three light) derived so far represent only the diagonal elements of a $4 \times 4$ neutrino mass matrix. If the discussion presented in this section on light neutrino masses is to be at all interesting, it is imperative to assume that the bulk of at least one, if not all, of the light neutrino masses comes from Eq. (15).

At this point, an important remark is in order here. As we have stressed above, the near degeneracy of the light neutrinos in no way implies that a similar situation will occur in the charged lepton sector. In fact, we will show in a separate paper that this will not be the case.

Under what conditions will $\widetilde{G}_\nu$ be of the order of $10^{-11}$ or less? First of all, as we have seen from Eq. (7), in order to have a ''heavy'' fourth neutrino, one should have $G_1 G_{M2} G_3 (V V'/M_F M_2) \approx O(1)$. This puts a condition on the angle $\beta$ itself, namely, $(\tan\beta \equiv V'/V)$

$$\tan\beta \approx \frac{1}{G_1 G_{M2} G_3} \frac{M_F M_2}{V^2}. \tag{21}$$

As we have stated earlier, it is not unreasonable to assume that $G_1$, $G_{M_2}$, and $G_3$ to be of the order of unity. With $M_G^2 \sim g^2 V^2$ [where $g$ is the SO(4) gauge coupling], Eq. (21) becomes

$$\tan\beta \approx g^2 \frac{M_F}{M_G} \frac{M_2}{M_G}. \tag{22}$$

The above estimate for the constraint on the angle $\beta$ will be used in our computation of the light neutrino masses. With this in mind, we can now proceed to make an estimate of the ratio $m_\nu/m_N$, where now $V V'/M_F M_2 \sim O(1)$ and Eq. (19) becomes

$$\frac{m_\nu}{m_N} = \frac{\sin(2\beta)}{32\pi^2} [I(\widetilde{\Omega})-I(\mathrm{Re}\,\widetilde{\rho})]. \tag{23}$$

As we have seen above, the result (23) depends only on ratios of masses of the particles in the loop integral and not on their absolute values. Because of that fact, the results will be shown in units of $M_F$ which can be as small or as large as one wishes.

Before moving on to discuss the implications of Eqs. (19) and (23), one remark is in order here. From Eq. (17), one can see that the light neutrino mass vanishes when $M_G=M_P$. Since there is no reason (as far as the present construction of the model is concerned) for this equality to be valid, we shall dismiss this possibility. We shall concentrate instead on the criteria for having small $m_\nu$ for arbitrary $M_G$ and $M_P$ (and $M_F$ and $M_2$ as well).

The results are shown in Figs. 2–5. A few comments are in order here. First of all, as we have mentioned above, our results depend on ratios of the four masses which enter the
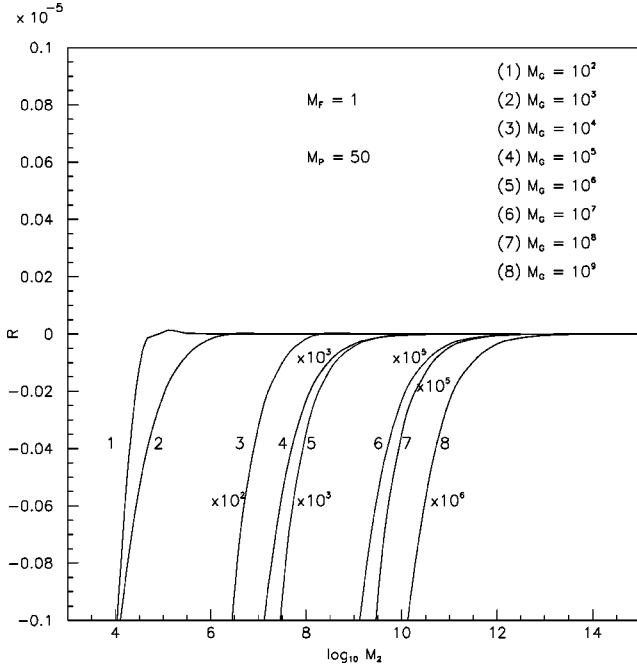
FIG. 3. The ratio $R \equiv m_\nu/m_N$ [Eq. (23)] as a function of $M_2$ (in units of $M_F$, and hence the notation $M_F = 1$), for $M_P = 50$ and for various values of $M_G$. For visibility purpose, a few curves have been inflated by factors $\times 10^{2,3,5,6}$.



FIG. 5. The ratio $R \equiv m_\nu/m_N$ [Eq. (23)] as a function of $M_2$ (in units of $M_F$, and hence the notation $M_F = 1$), for $M_P = 5000$ and for various values of $M_G$. For visibility purpose, a few curves have been inflated by factors $\times 10^{2,3,5,6}$.

loop integral $M_F$, $M_P$, $M_G$, and $M_2$. One can symbolically denote one of the masses as $M = 1$, and the other three will be multiples of that chosen one. Which one should be chosen is a matter of convenience and phenomenological interest. In
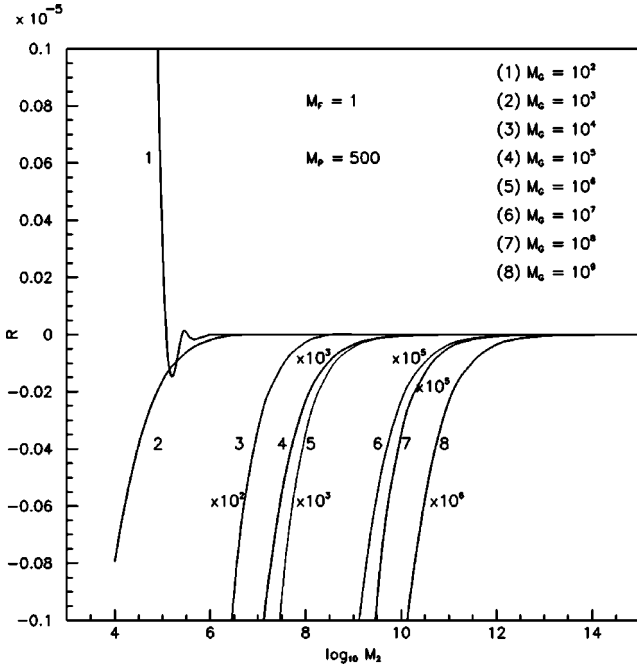


FIG. 4. The ratio $R \equiv m_\nu/m_N$ [Eq. (23)] as a function of $M_2$ (in units of $M_F$, and hence the notation $M_F = 1$), for $M_P = 500$ and for various values of $M_G$. For visibility purpose, a few curves have been inflated by factors $\times 10^{2,3,5,6}$.
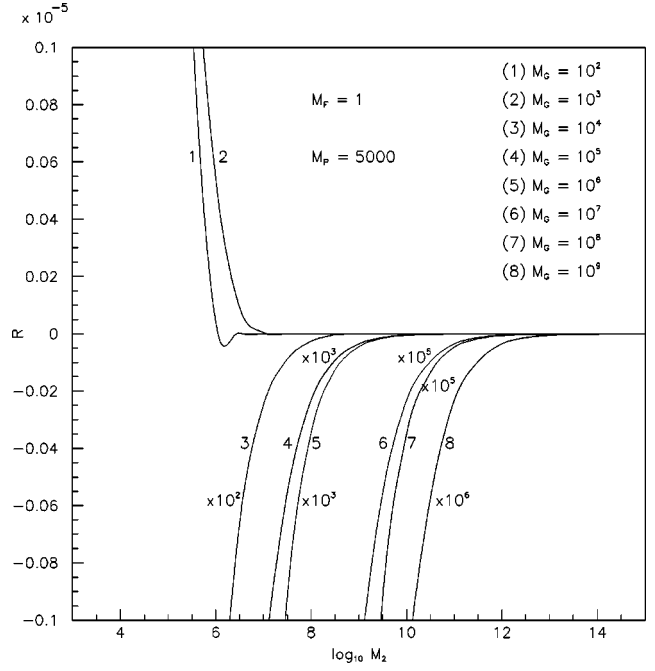
particular, we choose $M_F = 1$ because there is a possibility that the vectorlike fermions $F$ could be detected if their masses are low enough.

A glance at Figs. 2–5 reveal that it is relatively easy to obtain a very small ratio $R \equiv m_\nu/m_N$. In particular, one can see that large values of $M_2$, the mass of the singlet fermion field $\mathcal{M}_2$, are sufficient to obtain small values for $R \equiv m_\nu/m_N$. For instance, one can see that, roughly speaking, $R \equiv m_\nu/m_N \lesssim 10^{-11}$ when $M_2 \gtrsim 10^6$ (in units of $M_F$). Although conceptually quite different, the above fact is very reminiscent of the seesaw mechanism in that there is one large scale: Majorana for seesaw, $M_2$ for this scenario, and one "small" scale: Dirac mass $m_D$ for seesaw, $M_F$ for this scenario. The important point that we wish to make is the fact that the general result obtained here, namely the smallness of light neutrino masses, does not depend on one particular combination of masses which would imply fine tuning, a point which was not made quite clear in Ref. [5], but only on "large" ratio of masses whatever they might be. In this sense, the smallness of neutrino masses in our scenario is no less natural than the ones obtained from the seesaw mechanism.

In Figs. 2–5, we show the results for the case $M_2 > M_G$. There is, of course, absolutely no reason for this ordering. It is a matter of presentation. We obtain exactly the same results with the roles of $M_2$ and $M_G$ reversed. As can be inferred from the figures, for a given value of $M_P$ ($M_F = 1$), $R \equiv m_\nu/m_N \lesssim 10^{-11}$ if the ratio $M_G/M_2$ is below a certain value. For example, for $M_G \lesssim 10^5$, one has $M_G/M_2 \lesssim 10^{-3}$, while for $M_G \gtrsim 10^7$, one has $M_G/M_2 \approx 10^{-2} - 10^{-1}$. What this says is that the larger the mass is (e.g., $M_G$), the less mass splitting is needed in order to have a small $R$.

At this stage, we can only say that $m_\nu$ can be very small. What we cannot say is exactly what its value should be. This should come from some deeper theory. Instead, we shall use present constraints to restrict the range of values for $M_{G,P,2}$.

Having seen how one can obtain *very* small $m_\nu$, the next question would be: How small can one allow $m_\nu$ to be if one takes into account the neutrino oscillation data? First of all, atmospheric neutrino oscillation data gives a difference of mass squared $\Delta m^2 \approx 10^{-3}$ eV$^2$ while solar neutrino oscillation data gave $\Delta m^2 \approx 10^{-5}$ eV$^2$ (MSW) or $10^{-10}$ eV$^2$ (vacuum). In anticipation of new data, the Liquid Scintillation Neutrino Detector (LSND) results are not taken into account in our rough estimation of various mass scales. Without any need for a specific model, one can say that the atmospheric data implies that at least one of the three neutrinos should have a mass of *at least* $3 \times 10^{-2}$ eV, while the solar data implies that at least one of the remaining two should have a mass of *at least* $3 \times 10^{-3}$ eV (MSW) or $10^{-5}$ eV (vacuum). As we have seen above, the fourth neutrino can be quite heavy. For the sake of argument, let us assume here that its mass is approximately 100 GeV. Since our three light neutrinos are practically degenerate—a lifting of which will be discussed below, the atmospheric data alone constrains $R$ to be greater than approximately $10^{-14}$. This in turn constrains $M_2 \lesssim 10^{12}$ (in units of $M_F$) for the case $M_2 > M_G$, or $M_G \lesssim 10^{12}$ for the reverse case. Notice that this rough estimate is only for illustration purpose.

There is, however, one interesting piece of information which could be quite interesting, phenomenologically speaking: the presence of vectorlike fermions which carry weak quatum numbers and which could be relatively "light." These are the fermions $F$ with mass $M_F$ as indicated above. Let us recall from the above discussions that $M_{G,P,2}$ are all expressed in units of $M_F$ which itself could take on any value, even a few hundreds of GeV. The sole restriction will be from experimental constraints, a subject to which we shall come back below. Furthermore, we can see from the results that the mass of the pseudo Nambu-Goldstone (NG) bosons can also be "low" as well (Fig. 1) which could provide a further experimental clue.

We now turn to an important issue: the lifting of the mass degeneracy of the light neutrinos. The analysis presented below will reveal quite interesting implications such as the correlation between the actual values of the masses and $\Delta m^2$, which can have a profound cosmological consequence. For neutrino masses which are large enough to provide part of HDM, the MSW solution of the solar neutrino problem is preferred. If the vacuum solution turns out to be the correct one, the neutrino masses will be much too light in our scenario to play a role in HDM.

We shall divide the discussion presented below into two parts. First we analyze the case when there is no mixing between the fourth neutrino and the lighter three. It will be seen that an interesting feature emerges $\Delta m_{23}^2 \approx \Delta m_{21}^2$—a quasisymmetric splitting. ($\Delta m_{31}^2$ is of the same order.) This phenomenon could be called a mass splitting quasidegeneracy. Of course, solar and atmospheric neutrino data suggest otherwise. Next, we will show how this mass splitting quasidegeneracy can be lifted, suggesting—at least in our

scenario—the presence of a fourth neutrino.

In what follows, we will neglect any possible $CP$ phase in the neutrino mass matrix since we will be concerned only with $\Delta m^2$ and present data on neutrino oscillations are not sensitive to the presence of such a phase. In addition, we shall concentrate in the next two subsections only on $\Delta m^2$. A full comparison with the data will necessitate the inclusion of the leptonic "Cabibbo-Kobayashi-Maskawa" (CKM) angles coming from $V_L = U_l^\dagger U_\nu$. In the two subsections presented below, we shall see what $U_\nu$ might look like. To complete the discussion, we shall use a model for $U_l$ in order to make some statements about the size of the mixing angles. The subject of the charged lepton mass matrix itself will be dealt with in a subsequent publication.

### B. Neutrino mass matrix I: What if there is no mixing between the fourth and the lighter three neutrinos?

The $4 \times 4$ neutrino mass matrix obtained at this point is purely *diagonal*. We would like to examine how mass mixing might arise. In particular, we would like to lift the degeneracy of the three light neutrinos. In this section we will concentrate on the scenario where there is mass mixing only among the three light neutrinos. We will show that, in this scenario, $\Delta m_{23}^2 \approx \Delta m_{21}^2$. If this were experimentally the case, it would be hard to detect the influence of the fourth neutrino since it does not mix with the other three. Since the atmostpheric and solar data appear to point to $\Delta m_{23}^2 \gg \Delta m_{21}^2$, we will present in the next section what can be done in order to be in agreement with the data. It turns out that this can be accomplished if one introduces a mixing with the fourth neutrino. This implies that, at least in our model, $\Delta m_{23}^2 \gg \Delta m_{21}^2$ implies the existence of a fourth neutrino, and hence a fourth generation.

The degeneracy of the three light neutrinos at this level comes from the fact that there is a remaining global SO(3) symmetry which manifests itself through the equality of the masses of the family gauge bosons ($M_G$) as well as those of the pseudo-NG bosons ($M_P$). It is then clear that one needs to break that remaining global symmetry in order to remove the degeneracy of the light neutrino masses. We would want to do this in such a way as to preserve the quasidegeneracy of the light neutrinos. There are probably several ways to achieve this, and we will present one of them here.

Since we have seen how the diagonal elements of the neutrino mass matrix for the three light neutrinos are obtained at the one loop level, it is natural to envision a scenario in which the mixings themselves are obtained at *one loop*. A look at Figs. 1 reveals that the most "straightforward" way to induce mixings at one loop is for $\tilde{\Omega}_i$ and/or Re $\tilde{\rho}_i$ to have mixed couplings, i.e., to both $\nu_{Li}$ and $\nu_{Lj}$ as well as to both $\eta_{Ri}$ and $\eta_{Rj}$. This could come from mixings among $\tilde{\Omega}_i$ with different family indices and/or the mixings among Re $\tilde{\rho}_i$. Before getting into the details of what kinds of interactions are needed to break the remaining global SO(3) symmetry and hence inducing the mixings, it is instructional to assume that such a mixing among the boson masses occurs and to write down the Yukawa couplings (13),(14) in

terms of the new boson mass eigenstates.

Let us first look at the states $\tilde{\Omega}_i$. As we have discussed earlier, these are the NG bosons which are absorbed by the corresponding family gauge bosons. When these NG bosons get mixed, there will be mass mixings among the corresponding family gauge bosons. Let us denote the orthogonal matrix which diagonalizes these family gauge bosons by $A_\Omega$. We shall choose the following representation for $A_\Omega$:

$$A_\Omega = \begin{pmatrix} c_2 c_3 & -s_1 s_2 c_3 + c_1 s_3 & c_1 s_2 c_3 + s_1 s_3 \\ -c_2 s_3 & c_1 c_3 + s_1 s_2 s_3 & -c_1 s_2 s_3 + s_1 c_2 \\ -s_2 & -s_1 c_2 & c_1 c_2 \end{pmatrix},$$

$$(24)$$

where $c$ and $s$ represent the cosine and sine. If we denote by $\tilde{\Omega}'_i$ the longitudinal components of the gauge boson mass eigenstates, its relationship with $\tilde{\Omega}_i$ in the unmixed case is given by

$$\begin{pmatrix} \tilde{\Omega}_1 \\ \tilde{\Omega}_2 \\ \tilde{\Omega}_3 \end{pmatrix} = A_\Omega^T \begin{pmatrix} \tilde{\Omega}'_1 \\ \tilde{\Omega}'_2 \\ \tilde{\Omega}'_3 \end{pmatrix},$$

$$(25)$$

where $A_\Omega^T$ is given by

$$A_\Omega^T = \begin{pmatrix} c_2 c_3 & -c_2 s_3 & -s_2 \\ -s_1 s_2 c_3 + c_1 s_3 & c_1 c_3 + s_1 s_2 s_3 & -s_1 c_2 \\ c_1 s_2 c_3 + s_1 s_3 & -c_1 s_2 s_3 + s_1 c_2 & c_1 c_2 \end{pmatrix}.$$

$$(26)$$

The masses of the corresponding gauge bosons are now denoted by

$$M_{G_1}^2 = M_G^2 + \delta_1; \quad M_{G_2}^2 = M_G^2 + \delta_2; \quad M_{G_3}^2 = M_G^2,$$

$$(27)$$

where $\delta_{1,2}$ can be positive or negative. Notice that $\delta_{1,2}$ and the mixing angles shown above are *related*, i.e., they are all derived from the same boson mass matrix. We will show an example of such fact below.

We can now replace the unprimed states in Eqs.(12),(14) by the primed states using Eq. (25). We can then compute the one-loop contributions to the elements of the neutrino mass matrix $\mathcal{M}_\nu$. Let us first look at the contributions to the light neutrino masses and mixings coming from the $\tilde{\Omega}_i$ states. The two terms which are crucial for this computation are

$$G_1 \bar{l}_L^i \cos\beta \tilde{\Omega}_i F_R = G_1 \bar{l}_L^i \cos\beta A_{\Omega,i}^{T,j} \tilde{\Omega}'_j F_R \qquad (28)$$

and

$$G_3 \bar{M}_{2L} \sin\beta \tilde{\Omega}_i \eta_R^{1,i} = G_3 \bar{M}_{2L} \sin\beta \tilde{\Omega}'_j A_{\Omega,i}^j \eta_R^{1,i}. \qquad (29)$$

In the loop integrations, one will encounter the following propagators:

$$\frac{1}{k^2 - M_{G3}^2} = \frac{1}{k^2 - M_G^2}, \qquad (30\text{a})$$

$$\frac{1}{k^2 - M_{G1}^2} = \frac{1}{k^2 - M_G^2} + \frac{\delta_1}{(k^2 - M_{G1}^2)(k^2 - M_G^2)}, \qquad (30\text{b})$$

$$\frac{1}{k^2 - M_{G2}^2} = \frac{1}{k^2 - M_G^2} + \frac{\delta_2}{(k^2 - M_{G2}^2)(k^2 - M_G^2)}. \qquad (30\text{c})$$

With the above remarks in mind, let us proceed to calculate the contributions of $\tilde{\Omega}'$ to the neutrino mass matrix. We shall concentrate first on the $3 \times 3$ submatrix of the light neutrino sector. As a prelude to the computation of the full submatrix, let us show how two elements are calculated: $\mathcal{M}_\nu^{11}$ and $\mathcal{M}_\nu^{12}$. In these computations. we shall use, as an example, the explicit form for $A_\Omega$ shown in Eq. (24). For the complete calculations of the matrix elements, we shall use the notations $A_{ij}$ for $A_\Omega$.

(a) In the calculation of the contribution of $\tilde{\Omega}'$ to $\mathcal{M}_\nu^{11}$, one combines Eq. (25) with Eq. (26) to get the following combination of $\tilde{\Omega}'$:

$$(c_2 c_3 \tilde{\Omega}'_1 - c_2 s_3 \tilde{\Omega}'_2 - s_2 \tilde{\Omega}'_3)^2, \qquad (31)$$

which gives the combination of propagators

$$c_2^2 c_3^2 \langle \tilde{\Omega}'_1 \tilde{\Omega}'_1 \rangle + c_2^2 s_3^2 \langle \tilde{\Omega}'_2 \tilde{\Omega}'_2 \rangle + s_2^2 \langle \tilde{\Omega}'_3 \tilde{\Omega}'_3 \rangle. \qquad (32)$$

Upon using the propagators listed in Eqs. (30) in the one-loop integral (Fig. 1), one obtains the following replacement [the reader is referred to Eq. (16) for a comparison]:

$$\frac{\sin(2\beta)}{32\pi^2} I(\tilde{\Omega}) \rightarrow \frac{\sin(2\beta)}{32\pi^2} [I(\tilde{\Omega}) + c_2^2 c_3^2 \delta_1 I(M_G, M_{G1})$$

$$+ c_2^2 s_3^2 \delta_2 I(M_G, M_{G2})], \qquad (33)$$

where $(i = 1,2)$

$$\delta_i I(M_G, M_{Gi}) = \frac{1}{M_F - M_2} \left\{ \frac{M_F \{M_F^2 [M_G^2 \ln(M_G^2/M_F^2) - M_{Gi}^2 \ln(M_{Gi}^2/M_F^2)] + M_G^2 M_{Gi}^2 \ln(M_{Gi}^2/M_G^2)\}}{(M_G^2 - M_F^2)(M_{Gi}^2 - M_F^2)} - (M_F \leftrightarrow M_2) \right\}.$$

$$(34)$$

One can see that, in the symmetry limit where $\delta_i \to 0$ ($M_{Gi} \to M_G$), $\delta_i I(M_G, M_{Gi})$ vanishes identically.

One interesting remark worth mentioning is the following: In Eq. (33), the first term $I(\tilde{\Omega})$ contains no mixing angles. In fact, the coefficient in front of $I(\tilde{\Omega})$ is $c_2^2 c_3^2 + c_2^2 s_3^2 + s_2^2 = 1$, which is the result of $A_\Omega$ being an orthogonal matrix.

We do not give the explicit form for $I(\tilde{\Omega})$ because, after taking into account the contribution of Re $\tilde{\rho}_i$, one obtains the combination $I(\tilde{\Omega}) - I(\text{Re } \tilde{\rho})$ which is already given by Eq. (17). When the boson mass differences, represented by $\delta_i$, are small compared with $M_G^2$, another useful form which could be used is given by ($i = 1, 2$)

$$\delta_i I(M_G, M_{Gi}) = -x_i I(M_G, x_i), \qquad (35)$$

where

$I(M_G, x_i)$

$$= \frac{M_G^2}{M_F - M_2}$$

$$\times \left\{ \frac{M_F\{-M_F^2[1 + x_i + \ln(M_G^2/M_F^2)] + M_G^2(1 + x_i)\}}{(M_G^2 - M_F^2)^2 \{1 + x_i[M_G^2/(M_G^2 - M_F^2)]\}} \right.$$

$$\left. - (M_F \leftrightarrow M_2) \right\}, \qquad (36)$$

and where

$$x_i = \frac{\delta_i}{M_G^2}, \qquad (37)$$

so that

$$M_{G3}^2 = M_G^2; \quad M_{G1}^2 = M_G^2(1 + x_1); \quad M_{G2}^2 = M_G^2(1 + x_2). \qquad (38)$$

Here one could explicitely see the vanishing of $\delta_i I(M_G, M_{Gi})$ in the symmetry limit because of the explicit appearance of $\delta_i$ on the right-hand side of the equation.

The other diagonal elements of the neutrino mass matrix can be analogously calculated. One just needs to replace the combination of angles in Eq. (32) with the appropriate ones.

(b) For the 1-2 element, the appropriate combination of propagators is given by

$$c_2 c_3(-s_1 s_2 c_3 + c_1 s_3)\langle \tilde{\Omega}_1' \tilde{\Omega}_1' \rangle - c_2 s_3(c_1 c_3 + s_1 s_2 s_3)$$

$$\times \langle \tilde{\Omega}_2' \tilde{\Omega}_2' \rangle + s_1 s_2 c_2 \langle \tilde{\Omega}_3' \tilde{\Omega}_3' \rangle. \qquad (39)$$

It is now straigthforward to compute $\mathcal{M}_\nu^{12}$. It is given by

$$\mathcal{M}_\nu^{12}(\tilde{\Omega}) = \frac{\sin(2\beta)}{32 \pi^2} [c_2 c_3(-s_1 s_2 c_3 + c_1 s_3) \delta_1 I(M_G, M_{G1})$$

$$- c_2 s_3(c_1 c_3 + s_1 s_2 s_3) \delta_2 I(M_G, M_{G2})], \qquad (40)$$

where we have the appearance of the same $\delta_i I(M_G, M_{Gi})$. Notice that $\mathcal{M}_\nu^{12}(\tilde{\Omega})$ denotes the contribution coming from $\tilde{\Omega}$ only. The full element will also include the contribution coming from Re $\tilde{\rho}$.

Notice that the term $I(\tilde{\Omega})$ is not present in Eq. (40). Again this is due to the orthogonality of $A_\Omega$. The coefficient appearing in front of $I(\tilde{\Omega})$ is $c_2 c_3(-s_1 s_2 c_3 + c_1 s_3) - c_2 s_3(c_1 c_3 + s_1 s_2 s_3) + s_1 s_2 c_2 = 0$. The orthogonality of $A_\Omega$ implies that the product of any two columns is equal to zero. As a result we can see that, in the symmetry limit, $\mathcal{M}_\nu^{12}$ vanishes identically. This applies to all the other off-diagonal elements.

In order to complete the computation of the matrix elements (including the 1-1 and 1-2 elements), one has to say something about the contributions coming from the pseudo-NG bosons themselves. One might imagine that the same mechanism which breaks the global SO(3) symmetry also induces mixing among the degenerate pseudo-NG bosons. We will *assume* that the same matrix $A_\Omega$ diagonalizes the pseudo-NG boson sector so that, instead of the combination of $\tilde{\Omega}_i$ and Re $\tilde{\rho}_i$ used in Eqs. (12) for the NG and pseudo-NG bosons, we shall use $A_\Omega \tilde{\Omega}$ and $A_\Omega$ Re $\tilde{\rho}$, where $\tilde{\Omega}$ and Re $\tilde{\rho}$ are now column vectors. With these definitions, one simply gets $\tilde{\Omega}^\dagger$ Re $\tilde{\rho} = \tilde{\Omega}^\dagger A_\Omega^{-1} A_\Omega$ Re $\tilde{\rho}$. This simple assumption is used for two purposes: (1) To reduce the number of arbitrary parameters and (2) to see how far one can go with it before one needs to modify it. With this assumption, the mass splitting among the pseudo-NG bosons are given as in Eq. (38), namely,

$$M_{P3}^2 = M_P^2; \quad M_{P1}^2 = M_P^2(1 + x_1); \quad M_{P2}^2 = M_P^2(1 + x_2), \qquad (41)$$

with the same $x_i$ as for the gauge boson masses. Furthermore, the mixing angles are the same as above. The contributions of the "rotated" pseudo-NG bosons to the neutrino mass matrix elements will therefore be accompanied by a factor $-\sin(2\beta)/32 \pi^2$, just as in Eq. (33).

As mentioned above, in the full computation of the matrix elements, we shall use, for convenience, $A_{ij}$ to denote the matrix elements of $A_\Omega$ instead of the representation of Eq. (24). One should then recall that, because $A_\Omega$ is an orthogonal matrix, one has $\Sigma_j A_{ij}^2 = 1$ and $\Sigma_k A_{ki} A_{kj} = 0$. The form of the neutrino mass matrix elements will make use of these properties, just as we have done above.

With the above remarks in mind, the full $4 \times 4$ neutrino mass matrix is now given by

$$\mathcal{M}_\nu/m_N = \begin{pmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{12} & m_{22} & m_{23} & 0 \\ m_{13} & m_{23} & m_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (42)$$

where $m_N$ is the mass of the fourth generation neutrino shown in Eq. (7). In Eq. (42), we have ignored the tiny one-loop contribution to $m_{44} \equiv 1$, in particular when there is no mixing between the fourth neutrino and the lighter three. As we shall see later on, it can be ignored even if there is mixing, the reason being the fact that $m_{ij}$, $i,j = 1,2,3$, are so much smaller than $m_{44} \equiv 1$. A change of $m_{44}$ to a value slightly less than or greater than one will not significantly affect the eigenvalues, as we shall see in the numerical examples below.

With

$$\Delta I(G,P,x_i) \equiv I(M_G,x_i) - I(M_P,x_i), \qquad (43)$$

where $I(M_P,x_i)$ is given by Eq. (36) with the substitution $M_G \rightarrow M_P$, one obtains for $m_{ij}$

$$m_{11} = \frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P) - A_{11}^2 x_1 \Delta I(G,P,x_1)$$

$$- A_{12}^2 x_2 \Delta I(G,P,x_2)\}, \qquad (44a)$$

$$m_{22} = \frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P) - A_{21}^2 x_1 \Delta I(G,P,x_1)$$

$$- A_{22}^2 x_2 \Delta I(G,P,x_2)\}, \qquad (44b)$$

$$m_{33} = \frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P) - A_{31}^2 x_1 \Delta I(G,P,x_1)$$

$$- A_{32}^2 x_2 \Delta I(G,P,x_2)\}, \qquad (44c)$$

$$m_{12} = -\frac{\sin(2\beta)}{32\,\pi^2}\{A_{11}A_{21}x_1 \Delta I(G,P,x_1)$$

$$+ A_{12}A_{22}x_2 \Delta I(G,P,x_2)\}, \qquad (44d)$$

$$m_{13} = -\frac{\sin(2\beta)}{32\,\pi^2}\{A_{11}A_{31}x_1 \Delta I(G,P,x_1)$$

$$+ A_{11}A_{32}x_2 \Delta I(G,P,x_2)\}, \qquad (44e)$$

$$m_{23} = -\frac{\sin(2\beta)}{32\,\pi^2}\{A_{21}A_{31}x_1 \Delta I(G,P,x_1)$$

$$+ A_{22}A_{32}x_2 \Delta I(G,P,x_2)\}, \qquad (44f)$$

where $A_{ij}$ denote the matrix elements of $A_\Omega$, as mentioned above, and where $\Delta I(G,P)$ was already defined in Eq. (18).

A few remarks are in order here. First, one can see that, in the limit $x_i \rightarrow 0$, $\mathcal{M}_\nu$ reduces to a diagonal matrix with three equal diagonal elements $[\sin(2\beta)/32\,\pi^2]\Delta I(G,P)$. Secondly,
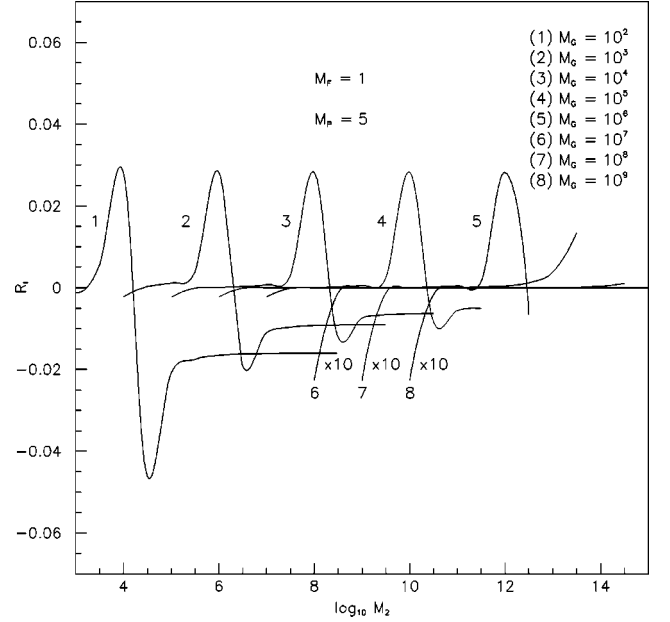


FIG. 6. The ratio $R_I \equiv \Delta I(G,P,x_i)/\Delta I(G,P)$ for $b = 0.035$.

apart from various mixing angles, the off-diagonal elements depend on results of loop integrals, $\Delta I(G,P,x_i)$ which, in turns, depend on the same parameters as the ones that enter the loop integrals of the diagonal elements in the unbroken case $\Delta I(G,P)$. The ratio $R_I \equiv \Delta I(G,P,x_i)/\Delta I(G,P)$ is plotted in Figs. 6,7, for two values of the parameter $x$, as a function of $M_2$ in the similar manner to Figs. 2–5. (The two values of $x$ were chosen for the purpose of illustration and to coincide with the two examples given below.) It can be seen that the ratio $R_I$ is at most of $O(10^{-2})$, even for $x$ as large as 0.5. Therefore, in our model, a small mass splitting in the scalar and gauge sectors results in a scenario with almost degenerate light neutrinos. The difference of the mass squared, $\Delta m^2$, depends, however, on the size of the off-diagonal elements. To see how it actually works, a simple model of mixings will be presented below along with some numerical examples.

We starts out with a very simplistic model of mixing and try to see how far one can go. It is

$$\mathcal{M}_{G,P}^2 = M_{G,P}^2 \begin{pmatrix} 1 & b & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (45)$$

where $b$ is a small parameter less than unity. This simple model has the merit of elucidating the points that we have made above. (An extension of this model, showing similar results, will be discussed below.) The above mass mixing (45) could come, for example, from a term in the Lagrangian of the form: $\lambda_5[(\Omega^\alpha \rho'_\alpha)(\Omega^\beta \rho''_\beta) + (\rho^\alpha \rho'_\alpha)(\rho^\beta \rho''_\beta)]$. Assuming $\langle \rho' \rangle = (v',0,0,0)$, $\langle \rho'' \rangle = (0,v'',0,0)$, with $v',v'' \ll V,V'$, one can obtain the above mass mixing matrix.

It is easy to see that the eigenvalues of Eq. (45) are

$$M^2_{G1,P1}=M^2_{G,P}(1+b), \quad M^2_{G2,P2}=M^2_{G,P}(1-b),$$

$$M^2_{G3,P3}=M^2_{G,P}. \tag{46}$$

$A_\Omega$ as discussed above is now given by

$$A_\Omega=\begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\[2ex] -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\[2ex] 0 & 0 & 1 \end{pmatrix}. \tag{47}$$

Now we can make the following identifications: $x_1 \equiv b$, $x_2 \equiv -b$. The various angles are given in $A_\Omega$. The matrix elements of the neutrino mass matrix are now fairly simple:

$$m_{11}=\frac{\sin(2\beta)}{32\,\pi^2}\left\{\Delta I(G,P)-\frac{1}{2}b[\Delta I(G,P,b)\right.$$
$$\left.-\Delta I(G,P,-b)]\right\}, \tag{48a}$$

$$m_{22}=\frac{\sin(2\beta)}{32\,\pi^2}\left\{\Delta I(G,P)-\frac{1}{2}b[\Delta I(G,P,b)\right.$$
$$\left.-\Delta I(G,P,-b)]\right\}, \tag{48b}$$

$$m_{33}=\frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P)\}, \tag{48c}$$

$$m_{12}=-\frac{\sin(2\beta)}{32\,\pi^2}\left\{\frac{1}{2}b[\Delta I(G,P,b)+\Delta I(G,P,-b)]\right\}, \tag{48d}$$

$$m_{13}=0, \tag{48e}$$

$$m_{23}=0. \tag{48f}$$

The above matrix elements are surprisingly easy to handle. When they are substituted into Eq. (42), one obtains straightforwardly the following mass eigenvalues:

$$m_1=m_N\frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P)-b\Delta I(G,P,b)\}, \tag{49a}$$

$$m_2=m_N\frac{\sin(2\beta)}{32\,\pi^2}\Delta I(G,P), \tag{49b}$$

$$m_3=m_N\frac{\sin(2\beta)}{32\,\pi^2}\{\Delta I(G,P)+b\Delta I(G,P,-b)\}, \tag{49c}$$

$$m_4=m_N. \tag{49d}$$

The matrix which diagonalizes the above neutrino mass matrix is simply

$$U_\nu=\begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 & 0 \\[2ex] 0 & 0 & 1 & 0 \\[2ex] -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 & 0 \\[2ex] 0 & 0 & 0 & 1 \end{pmatrix}. \tag{50}$$

One obtains the following mass splittings:

$$m_3^2-m_2^2=\left(m_N\frac{\sin(2\beta)}{32\,\pi^2}\right)^2\{2b\Delta I(G,P)\Delta I(G,P,-b)$$
$$+[b\Delta I(G,P,-b)]^2\}, \tag{51a}$$

$$m_2^2-m_1^2=\left(m_N\frac{\sin(2\beta)}{32\,\pi^2}\right)^2\{2b\Delta I(G,P)\Delta I(G,P,b)$$
$$+[b\Delta I(G,P,b)]^2\}. \tag{51b}$$

In general, $\Delta I(G,P,x_i)\ll\Delta I(G,P)$, and combined with the fact that $b<1$, one has $[b\Delta I(G,P,-b$ or $b)]^2 \ll 2b\Delta I(G,P)\Delta I(G,P,-b$ or $b)$. One can then neglect the last terms in Eq. (51). Numerically, one has $\Delta I(G,P,b) \approx \Delta I(G,P,-b)$. This implies that $m_3^2-m_2^2\approx m_2^2-m_1^2$, a quasidegenerate mass splitting. This holds for any value of $b$. Solar and atmospheric data suggest otherwise. This necessitates the lifting of this quasidegeneracy of the mass splitting. To do this, we need to invoke some kind of mixing between the fourth neutrino and the lighter three. In an indirect way, the disparity between $\Delta m_{23}^2$ and $\Delta m_{21}^2$ indicates—in our model—the influence of a fourth generation. Before discussing this issue which will be presented in the next section, let us illustrate numerically a few examples of the quasidegenerate case.

First, a few useful points are in order here. Since $m_2 = m_N[\sin(2\beta)/32\,\pi^2]\Delta I(G,P)$, one can rewrite the above equations (51) as (neglecting the last terms on the right-hand side)

$$m_3^2-m_2^2=m_2\left(m_N2b\frac{\sin(2\beta)}{32\,\pi^2}\Delta I(G,P,-b)\right), \tag{52a}$$

$$m_2^2-m_1^2=m_2\left(m_N2b\frac{\sin(2\beta)}{32\,\pi^2}\Delta I(G,P,b)\right). \tag{52b}$$

For a fixed value of $m_2$, the size of the mass splitting, $\Delta m^2$, depends on the size of the factor $m_N(2b)[\sin(2\beta)/32\,\pi^2]\Delta I(G,P,-b$ or $b)$. At first glance, it appears that one can obtain $\Delta m^2$ to be as small as one wants with the appropriate choice of $b$. Although it is true that it can be so, we will show that, $\Delta m^2$ can also be very small ($<10^{-10}$ eV$^2$), even when $b\approx1$. This depends on how large the masses of
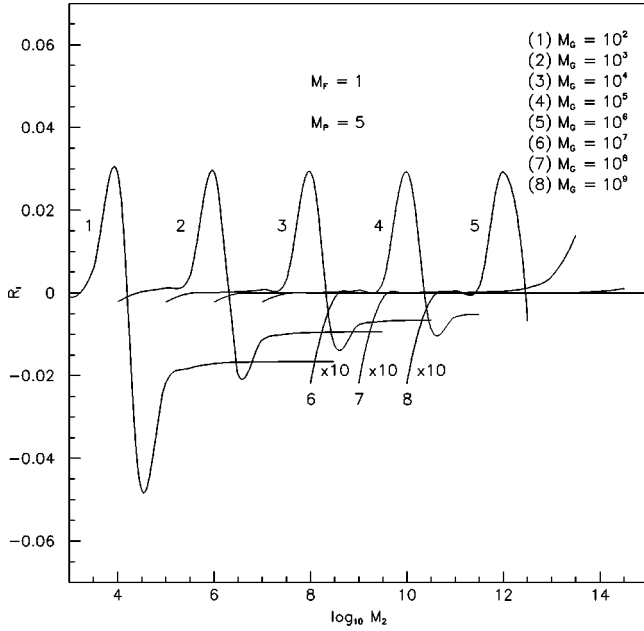
FIG. 7. The ratio $R_I \equiv \Delta I(G,P,x_i)/\Delta I(G,P)$ for $b = 0.000095$.

some of the particles participating in the loop diagrams are. As a result, by limiting $\Delta m^2 \geqslant 10^{-10}$ eV$^2$, one puts a constraint on those masses.

In Fig. 8, we present the ''median'' mass $m_2$ as a function of $M_2$ and $M_G$ for a given $M_P$ (as presented in Figs. 2–5). The mass is given in units of $(m_N/100 \text{ GeV})$. Similarly, we present in Figs. 9,10 $m_3^2 - m_2^2$ and $m_2^2 - m_1^2$ as a function of the same masses, but also for a given value of $b$. The results are expressed in units of $(m_N/100 \text{ GeV})^2$. For a more stream-
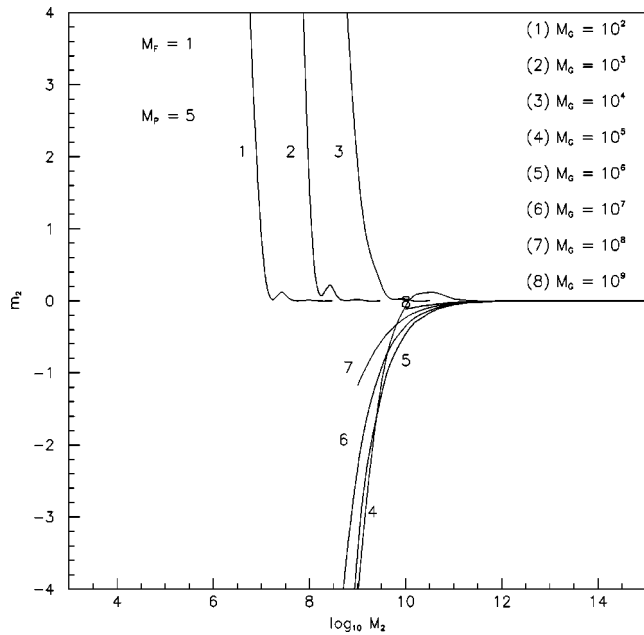


FIG. 8. The median mass $m_2$ as defined by Eq. (49b). Notice the correlation with $m_2^2 - m_1^2$ and $m_3^2 - m_2^2$ shown in the next two figures.

lined presentation of the results, we shall limit ourselves to the case $m_2 \lesssim 1.67$ eV, coming from the suggestion that the sum of neutrino masses lies between 4 and 5 eV in order to form a component of HDM. Similarly, we shall restrict $\Delta m^2 < 1$ eV$^2$. In our model, for a given value of $b$, $m_2$ and $\Delta m^2$ are correlated as one can see from Figs. 8,9,10.

Three major remarks are in order here. (1) One can see from Figs. 9,10 the quasidegeneracy of the mass splitting in this particular scenario. (In the next section, we shall see how one can lift that degeneracy.) (2) One can also see from Figs. 8–10 that, were the vacuum solution to the solar neutrino problem favored, i.e., $\Delta m^2 \approx 10^{-10}$ eV$^2$, the median value $m_2$ will always be less than 0.1 eV. (The lifting of the mass splitting degeneracy to satisfy the atmospheric neutrino data will not change this conclusion.) This simply means that, at least in this model, the solar vacuum solution is incompatible with the light neutrinos being significant components of HDM. (3) Also from Figs. 8–10 it can be seen that the MSW solution $\Delta m^2 \approx 10^{-5}$ eV$^2$ can correspond to values of $m_2$ larger than 1 eV. (Again, the lifting of the mass splitting degeneracy to satisfy the atmospheric neutrino data will not change this conclusion.) So, in our scenario, the MSW solution is compatible with the light neutrinos being significant components of HDM while the vacuum solution is not. This is a very specific prediction of this model.

The above discussion leaves out the question of the size of the mixing angles. As mentioned above, we have already fixed the neutrino mixing matrix $U_\nu$, as given by Eq. (50). To complete the task, one has to model the charged lepton mixing matrix $U_l$. This is something that we shall do in the last section. We wish however to reemphasize the main result of this section: the values of $\Delta m^2$ are *independent* of $U_l$. As one can see from Figs. 9,10, $\Delta m^2$ depends only on the various masses and on the parameter $b$, regardless of $U_l$. As a consequence, the large angle or small angle solutions as deduced from the data basically constrains, in our scenario, the matrix $U_l$ ($U_\nu$ being already fixed).

To finish the discussion of this section, we wish to present another form for the boson mass matrix, namely,

$$\mathcal{M}_{G,P}^2 = M_{G,P}^2 \begin{pmatrix} 1 & b & 0 \\ b & 1 & b \\ 0 & b & 1 \end{pmatrix}. \quad (53)$$

The mass eigenvalues are

$$M_{G1,P1}^2 = M_{G,P}^2(1 + \sqrt{2}b), \quad M_{G2,P2}^2 = M_{G,P}^2(1 - \sqrt{2}b),$$

$$M_{G3,P3}^2 = M_{G,P}^2. \quad (54)$$

$A_\Omega$ is now given by

$$A_\Omega = \begin{pmatrix} \dfrac{1}{2} & \dfrac{1}{\sqrt{2}} & \dfrac{1}{2} \\[2mm] \dfrac{1}{2} & -\dfrac{1}{\sqrt{2}} & \dfrac{1}{2} \\[2mm] -\dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \end{pmatrix}. \quad (55)$$

It is now straightforward to see that the neutrino mass matrix elements are

$$m_{11} = m_{33} = \frac{\sin(2\beta)}{32\,\pi^2} \left\{ \Delta I(G,P) - \frac{b}{2\sqrt{2}} \right.$$

$$\left. \times [\Delta I(G,P,b) - \Delta I(G,P,-b)] \right\}, \qquad (56)$$

$$m_{22} = \frac{\sin(2\beta)}{32\,\pi^2} \left\{ \Delta I(G,P) - \frac{b}{\sqrt{2}} [\Delta I(G,P,b) \right.$$

$$\left. - \Delta I(G,P,-b)] \right\}, \qquad (57)$$

$$m_{12} = m_{23} = -\frac{\sin(2\beta)}{32\,\pi^2} \left\{ \frac{1}{2} b [\Delta I(G,P,b) \right.$$

$$\left. + \Delta I(G,P,-b)] \right\}, \qquad (58)$$

$$m_{13} = -\frac{\sin(2\beta)}{32\,\pi^2} \left\{ \frac{b}{2\sqrt{2}} [\Delta I(G,P,b) \right.$$

$$\left. - \Delta I(G,P,-b)] \right\}. \qquad (59)$$

The eigenvalues are now simply given by

$$m_1 = m_N \frac{\sin(2\beta)}{32\,\pi^2} \{ \Delta I(G,P) - \sqrt{2} b \Delta I(G,P,b) \},$$
$$(60a)$$

$$m_2 = m_N \frac{\sin(2\beta)}{32\,\pi^2} \Delta I(G,P), \qquad (60b)$$

$$m_3 = m_N \frac{\sin(2\beta)}{32\,\pi^2} \{ \Delta I(G,P) + \sqrt{2} b \Delta I(G,P,-b) \},$$
$$(60c)$$

$$m_4 = m_N. \qquad (60d)$$

These masses have exactly the same form as those of Eq. (49), except for the factor of $\sqrt{2} b$ instead of $b$. The matrix

$U_\nu$ which diagonalizes the above matrix is exactly the same as in Eq. (50). Furthermore, $m_3^2 - m_2^2$ and $m_2^2 - m_1^2$ are of the same form as Eqs. (51), with the following replacement in Eqs. (51): $b \to b' = \sqrt{2} b$. The analysis which follows is exactly the same as the one presented above.

One can envision various scenarios for the boson mass matrices, but it is certainly beyond the scope of this paper. To make things more complicated than the simple assumption (45) does not appear to add much to the discussion. Although it might be possible that a more involved *Ansatz* than Eq. (45) could lead to the lifting of the mass splitting "quasidegeneracy," we have not succeeded in finding it. For this reason, we now turn our attention to the more appealing scenario, at least within our model: the mixing between the fourth neutrino and the rest.
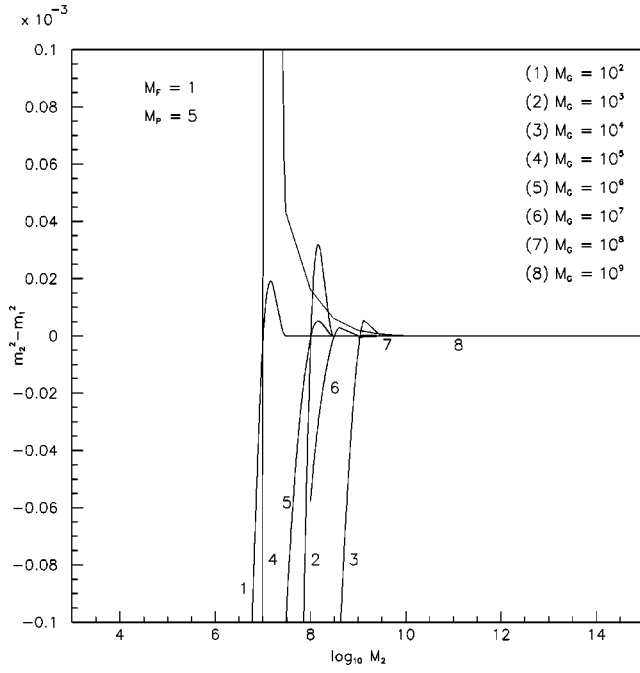
### C. Neutrino mass matrix II: Mixing between the fourth and the lighter three neutrinos

We have seen above that the simple ansatz for the boson mass matrices (45) leads to a situation in which the mass splittings are quasidegenerate. This, of course, is in contradiction with the data. In this model, in order to lift that quasidegeneracy, one needs a mixing between the fourth neutrino and at least one of the lighter three. To get a feel for what might be needed, we shall first present a few numerical examples. Based on these examples, we shall attempt to give a theoretical basis for these numerical examples.

As an example, we shall choose a specific value for the parameter $b$ and for the masses $M_2$, $M_G$, $M_P$, and $M_F$ which enter the loop integrals for the neutrino masses. This will fix a definite value for the matrix elements of the neutrino mass matrix. As we have already discussed earlier, the integrals depend only on the ratio of the above masses. We will present two examples for the purpose of comparison. We shall see the reasons why we wish to do so below.

(1) First example. We shall set (in units of $M_F$): $M_F = 1$, $M_P = 5$, $M_G = 10^6$, $M_2 = 2.5 \times 10^9$. For $b$, we shall choose $b = 0.035$. (A smaller value of $b$ will give a smaller mass splitting.) The reason for this choice (other choices are *equally valid*) is the fact that it will give a typical mass of approximately 1.5 eV and a desired mass splitting. Putting these values into the expressions for the integrals as given by Eq. (36), we obtain the following neutrino mass matrix, where $m_N$ is assumed to be 100 GeV for convenience:

$$\mathcal{M}_\nu = (100 \text{ GeV}) \begin{pmatrix} -1.579332216 \times 10^{-11} & 0.8697647852 \times 10^{-17} & 0 & 0 \\ 0.8697647852 \times 10^{-17} & -1.579332216 \times 10^{-11} & 0 & 0 \\ 0 & 0 & -1.579332184 \times 10^{-11} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (61)$$

FIG. 9. $m_3^2 - m_2^2$ as defined by Eq. (52a) for $b = 0.000095$.



FIG. 10. $m_2^2 - m_1^2$ as defined by Eq. (52b) for $b = 0.000095$.

Notice that the above matrix has no mixing between the fourth neutrino and the lighter three. The eigenvalues are just

$$|m_1| = 1.579331346 \ \text{eV}, \quad |m_2| = 1.579332184 \ \text{eV},$$

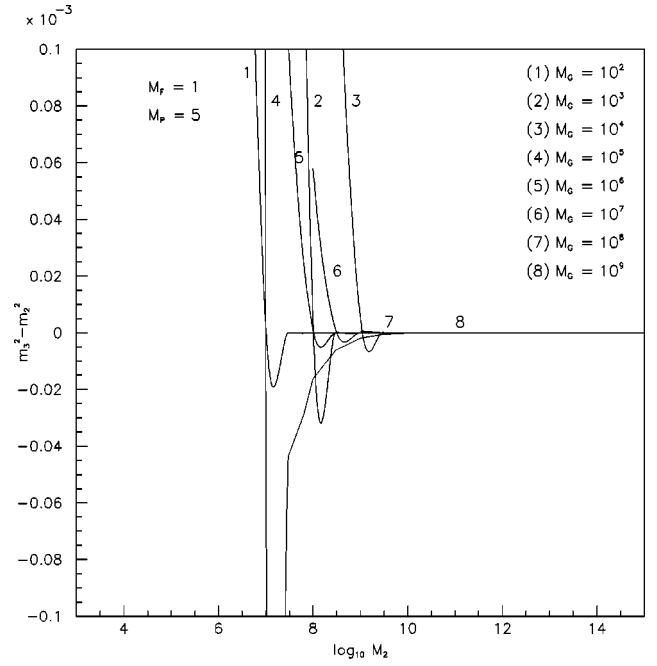$$|m_3| = 1.579333086 \ \text{eV}, \quad |m_4| = 100 \ \text{GeV}. \tag{62}$$

As we have discussed in the previous section, this gives a quasidegenerate mass splitting, namely,

$$\Delta m_{32}^2 = 1.601195367 \times 10^{-6} \ \text{eV}^2, \tag{63}$$

$$\Delta m_{21}^2 = 1.535757079 \times 10^{-6} \ \text{eV}^2, \tag{64}$$

where $\Delta m_{ji}^2 = m_j^2 - m_i^2$.

Let us now assume that the mixing with the fourth neutrino is nonzero. We start out with the *simplest assumption*, namely, one in which only the third neutrino mixes with the fourth one. This means that $m_{34}$ and $m_{43}$ are both nonvanishing. If we wish to have $m_3^2 - m_2^2 \approx 10^{-3}$ eV$^2$ as suggested by the atmospheric neutrino data, it turns out that $m_{34}$ and

$m_{43}$ cannot be too small nor too large, being of order $10^{-7} m_N$. Notice that $m_{34}$ and $m_{43}$ do not have to be equal. We shall see how it might be possible to obtain such a number. Let us first see how it works from a numerical viewpoint.

To guide our understanding of how things work, let us notice that, by adding $m_{34}$ and $m_{43}$ to $\mathcal{M}_\nu$ above, one changes only one of the three light mass eigenvalues, leaving the other two the same. Now the two unchanged eigenvalues will be the ones that fix one of the two mass splittings $\Delta m^2$. For convenience, we shall choose the $\Delta m^2$ corresponding to the unmodified mass eigenvalues as the one which corresponds to the solar neutrino problem. As we have learned from the above analysis in Sec. III B, if one chooses the MSW solution, then one can find masses which are large enough for HDM, while, if the vacuum solution is chosen, the masses will be too small to form any significant component of HDM. For the numerical example given here, we shall choose the MSW solution as shown above. For $m_{34}$ and $m_{43}$, we shall first choose a symmetric case (there is no particular reason for this being so) as an example. We have

$$\mathcal{M}_\nu = (100 \ \text{GeV}) \begin{pmatrix} -1.579332216 \times 10^{-11} & 0.8697647852 \times 10^{-17} & 0 & 0 \\ 0.8697647852 \times 10^{-17} & -1.579332216 \times 10^{-11} & 0 & 0 \\ 0 & 0 & -1.579332184 \times 10^{-11} & 0.8 \times 10^{-7} \\ 0 & 0 & 0.8 \times 10^{-7} & 1 \end{pmatrix}. \tag{65}$$

The eigenvalues are

$$|m_1| = 1.579331346 \ \text{eV}, \quad |m_2| = 1.579333086 \ \text{eV}, \quad |m_3| = 1.579972184 \ \text{eV}, \quad m_4 = 100 \ \text{GeV}. \tag{66}$$

We then get

$$\Delta m_{32}^2 = 2.02 \times 10^{-3} \ \text{eV}^2, \tag{67}$$

$$\Delta m_{21}^2 = 5.497 \times 10^{-6} \ \text{eV}^2. \tag{68}$$

The matrix which diagonalizes the mass matrix is

$$U_\nu = \begin{pmatrix} -\dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\[2mm] \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\[2mm] 0 & 0 & -1 & 0.8 \times 10^{-7} \\[2mm] 0 & 0 & 0.8 \times 10^{-7} & 1 \end{pmatrix}. \tag{69}$$

Two remarks are in order here. First, from the values of the light neutrino masses, one obtains $\Sigma_{i=1}^3 |m_i| \approx 4.7$ eV, which is in the range of mass for HDM. Secondly, Eq. (67) corresponds to the best fit for the atmospheric neutrino data, while Eq. (68) corresponds to the best fit for the (small angle) MSW solution to the solar neutrino data. One word of caution: this is not a prediction because we chose the masses ($M_G$, $M_2$, etc.) in such a way as to ''reproduce'' the experimental results. It nevertheless shows a *dynamical* basis for these numbers. Also, for nothing more than a numerical example, the values of $m_{34,43}$ were chosen arbitrarily in order to have the desired mass splitting. How to justify these values is the subject to be discussed below.

The next numerical example deals with the case when $m_{34} \neq m_{43}$. In doing the analysis, we notice that it does not matter whether $m_{34}$ is greater than $m_{43}$ or the other way around. One obtains the same result either way. We shall require that $\Delta m_{32}^2 (\text{eV}^2) = 10^{-3} - 10^{-2}$. It turns out that $m_{34}$ and $m_{43}$ can range (in units of $m_N$) only between approximately $0.4 \times 10^{-6}$ and $0.8 \times 10^{-8}$. To be explicit, one has

$$\mathcal{M}_\nu = (100 \ \text{GeV}) \begin{pmatrix} -1.579332216 \times 10^{-11} & 0.8697647852 \times 10^{-17} & 0 & 0 \\ 0.8697647852 \times 10^{-17} & -1.579332216 \times 10^{-11} & 0 & 0 \\ 0 & 0 & -1.579332184 \times 10^{-11} & 0.4 \times 10^{-6} \\ 0 & 0 & 0.8 \times 10^{-7} & 1 \end{pmatrix} \tag{70}$$

gives $\Delta m_{32}^2 (\text{eV}^2) \approx 10^{-2}$, while

$$\mathcal{M}_\nu = (100 \ \text{GeV}) \begin{pmatrix} -1.579332216 \times 10^{-11} & 0.8697647852 \times 10^{-17} & 0 & 0 \\ 0.8697647852 \times 10^{-17} & -1.579332216 \times 10^{-11} & 0 & 0 \\ 0 & 0 & -1.579332184 \times 10^{-11} & 0.4 \times 10^{-6} \\ 0 & 0 & 0.8 \times 10^{-8} & 1 \end{pmatrix} \tag{71}$$

gives $\Delta m_{32}^2 (\text{eV}^2) \approx 10^{-3}$. Notice that $\Delta m_{21}^2$ stays the same. The above numerical results show that $m_{34}$ can differ from $m_{43}$ by a large factor (50 in this case) while keeping $\Delta m_{32}$ within the desired range.

(2) Second example. In this example, we choose (in units of $M_F$): $M_F = 1$, $M_P = 5$, $M_G = 10^4$, $M_2 = 1.2 \times 10^9$. For $b$, we shall choose: $b = 0.000095$. For simplicity, we shall assume, as we have already done above, the following values for $m_{34,43}$, namely, $m_{34} = m_{43} = 0.8 \times 10^{-7} (100 \ \text{GeV})$. The mass matrix is now

$$\mathcal{M}_\nu = (100 \ \text{GeV}) \begin{pmatrix} 1.382258467 \times 10^{-11} & 0.981382953 \times 10^{-17} & 0 & 0 \\ 0.981382953 \times 10^{-17} & 1.382258467 \times 10^{-11} & 0 & 0 \\ 0 & 0 & 1.382258467 \times 10^{-11} & 0.8 \times 10^{-7} \\ 0 & 0 & 0.8 \times 10^{-7} & 1 \end{pmatrix}. \tag{72}$$

The eigenvalues are

$$m_1 = 1.382259448 \ \text{eV}, \quad m_2 = 1.382257486 \ \text{eV}, \quad m_3 = 1.381618467 \ \text{eV}, \quad m_4 = 100 \ \text{GeV}, \tag{73}$$

with the corresponding diagonalization matrix given by

$$U_\nu = \begin{pmatrix} -\dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\[2mm] \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\[2mm] 0 & 0 & -1 & 0.8\times 10^{-7} \\[2mm] 0 & 0 & 0.8\times 10^{-7} & 1 \end{pmatrix}.$$

(74)

The mass splittings are

$$|\Delta m_{32}^2| = 1.77\times 10^{-3}\ \text{eV}^2,$$

(75)

$$|\Delta m_{21}^2| = 5.42\times 10^{-6}\ \text{eV}^2.$$

(76)

The above two examples are chosen solely for illustration. Other values of $\Delta m^2$ are possible with different choices of various masses ($M_G$, $M_2$, etc.) and/or the parameter $b$.

Before turning to the discussion on the possible origins of $m_{34,43}$, let us briefly discuss the ''tiny'' one-loop contribution to $m_{44}$, namely, $\delta m_4$ as given by Eq. (20). One might wonder how it would affect the light mass eigenvalues. It turns out, however, that, as long as $\delta m_4 \ll 1$ (which is the case in this paper), it does not matter what value it takes. It is easy to see how. A $2\times 2$ matrix of the form $(a,c;c,b)$, where $a,c \ll b$, has as eigenvalues $b + c^2/b + (1/4)a^2/b + O(c^4,a^4)$ and $a - c^2/b - (1/4)a^2/b + O(c^4,a^4)$. One can see that, for the smaller eigenvalue, a small change in $b$ affects very little its value. As an example, we put 0.99 instead of 1 in Eq. (71). We obtain $\Delta m_{32}^2(\text{eV}^2) \approx 1.02\times 10^{-3}$ instead of $1.06\times 10^{-3}$ (for 1). If we put 1.1 instead of 1, we obtain $\Delta m_{32}^2(\text{eV}^2) \approx 0.923\times 10^{-3}$. Considering the kind of accuracy that one has at the present time, this is completely irrelevant.

There are probably several scenarios for calculating $m_{34,43}$. However, considering the fact that the present experimental status is not accurate enough for a detailed model, we will present below a more or less ''generic'' scenario which will show how one can obtain $m_{34,43}$ of the right order of magnitude.

What might be the origin of $m_{34,43}$? It might be obvious up until now that the vacuum expectation values of $\Omega$ and $\rho$ shown in Sec. III A cannot generate such a mixing. One needs at least one additional scalar with a nonvanishing vacuum expectation value along the third direction. Let that field be $\Omega'$ and let us assume that $\langle \Omega' \rangle = (0,0,\tilde{v},0)$. Let us also assume that there are couplings of the type

$$\lambda_{34}\Omega^\alpha \Omega'_\alpha \rho^\beta \rho_\beta; \quad \lambda_{43}\Omega^\alpha \rho_\alpha \Omega',_\beta \rho_\beta,$$

(77)

where, for convenience, we have omitted the $SU(2)_{\nu R}$ index in $\rho$. With the above couplings, one can construct diagrams for $m_{34}$ and $m_{43}$ as shown in Fig. 11.

We shall denote the masses of $\tilde{H}_4$ and $\tilde{h}_4$ by $M_{H_4}$ and $M_{h_4}$, respectively. Let us define the following quantities:

$$\Delta M^2(G,\tilde{H}_4) = M_G^2 - M_{H_4}^2,$$

(78a)

$$\Delta M^2(G,\tilde{h}_4) = M_G^2 - M_{h_4}^2,$$

(78b)

$$\Delta M^2(P,\tilde{H}_4) = M_P^2 - M_{H_4}^2,$$

(78c)

$$\Delta M^2(P,\tilde{h}_4) = M_P^2 - M_{h_4}^2.$$

(78d)

From Fig. 8, we obtain

$$m_{34}/m_N = \left(\frac{\lambda_{34}}{16\pi^2}\right)\left(\frac{\tilde{v}M_F M_2}{V}\right)\left(c_\beta^2 s_\alpha^2 \frac{\Delta I(G,\tilde{H}_4)}{\Delta M^2(G,\tilde{H}_4)}\right.$$
$$+ c_\beta^2 c_\alpha^2 \frac{\Delta I(G,\tilde{h}_4)}{\Delta M^2(G,\tilde{h}_4)} + s_\beta^2 s_\alpha^2 \frac{\Delta I(P,\tilde{H}_4)}{\Delta M^2(P,\tilde{H}_4)}$$
$$\left. + s_\beta^2 c_\alpha^2 \frac{\Delta I(G,\tilde{h}_4)}{\Delta M^2(G,\tilde{h}_4)}\right),$$

(79a)

$$m_{43}/m_N = \left(\frac{\lambda_{34}}{16\pi^2}\right)\left(\frac{\tilde{v}M_F M_2}{V}\right)\left(s_\beta^2 c_\alpha^2 \frac{\Delta I(G,\tilde{H}_4)}{\Delta M^2(G,\tilde{H}_4)}\right.$$
$$+ s_\beta^2 s_\alpha^2 \frac{\Delta I(G,\tilde{h}_4)}{\Delta M^2(G,\tilde{h}_4)} + c_\beta^2 c_\alpha^2 \frac{\Delta I(P,\tilde{H}_4)}{\Delta M^2(P,\tilde{H}_4)}$$
$$\left. + c_\beta^2 s_\alpha^2 \frac{\Delta I(G,\tilde{h}_4)}{\Delta M^2(G,\tilde{h}_4)}\right),$$

(79b)

where $c$ and $s$ stand for cos and sin, and $\Delta I(G,\tilde{H}_4)$ and the other similar quantities in Eq. (79) are given by Eq. (18), with the substitution of the appropriate masses taken into account.

As one can see from the above equations, the expressions appear rather complicated at first look. However, one can make an estimate as to which term in $m_{34}$ and $m_{43}$ is the most important. Each term in Eqs. (79) is of the form $\lambda(\tilde{v}/V)(M_F/M_2)(M_2^2/\Delta M^2)\Delta I$ (mixing angles), where $\lambda$ stands for $\lambda_{34,43}$. First, we have seen from the above numerical analysis that, if we wish to have a mass of $O(1-2\ \text{eV})$, then $M_F/M_2 \approx 10^{-9}$. It is reasonable to assume that
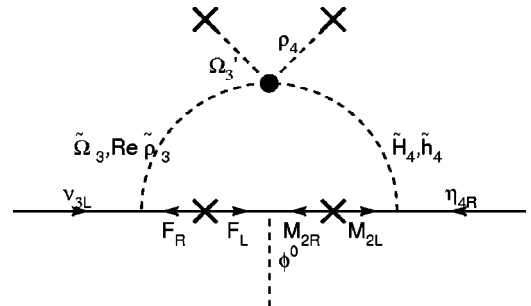


FIG. 11. Diagram for $m_{34,43}$.

$\lambda(\tilde{v}/V) \times$ (mixing angles) $\lesssim 1$. If one of the terms in Eq. (79) were to be the dominant one and that $m_{34,43} \approx 10^{-7}$, then one should have $(M_2^2/\Delta M^2)\Delta I \gtrsim 10^2$. Let us first look at the $(G;\tilde{H},\tilde{h})$ contribution. Assuming that $M_{H_4,h_4} < M_G$ so that $(M_2^2/\Delta M^2) \approx M_2^2/M_G^2$, it turns out numerically that $(M_2^2/M_G^2)\Delta I$ is always less than $\sim 10$. For $M_{H_4,h_4} > M_G$, $\Delta I$ is larger in value than the previous case, but then with $(M_2^2/\Delta M^2) \approx M_2^2/M_{H_4,h_4}^2$, one will again have $(M_2^2/M_{H_4,h_4}^2)\Delta I$ less than $10^2$. Taking into account the actual calculation of $m_{34,43}$ which includes mixing angles and various factors, the $(G;\tilde{H},\tilde{h})$ would be too small to actually affect the mass splittings. This leaves us with the contribution coming from $(P;\tilde{H},\tilde{h})$. Here, as we have done above, we will set $M_P = 5$ in units of $M_F$. There are several possibilities that one can explore. We will present here one of such possibilities. The main purpose will be to show that, under reasonable assumptions, one can obtain the desired order of magnitude for $m_{34,43}$. In addition, one would like to see phenomenological implications coming from such a scenario—something extra other than just a mass matrix.

Let us assume that, by an appropriate choice of parameters in the Higgs potential, one has $M_{H_4}$ to be of $O(M_G)$, and that $M_{h_4} \ll M_2$. Furthermore, let us assume that one also has $\beta \approx \alpha$. Although it is not really necessary, let us further assume that $\lambda_{34} \sim \lambda_{43}$. Now numerically, $(M_2^2/\Delta M^2)\Delta I < 10^2$ when one of the masses in $\Delta M^2$ is much larger than the other one and not too much different from $M_2$. This is just the case for $M_{H_4} = O(M_G) \gg M_P$. Under these assumptions, we are left with the $(P;h)$ contribution. In this case, one has $m_{34} \approx m_{43}$. So we get

$$|m_{34}| \approx |m_{43}| \approx m_N \lambda_{34} \frac{\tilde{v}}{V} \frac{M_F}{M_2} \left| \frac{M_2^2}{M_P^2 - M_{h_4}^2} \right| \Delta M^2(G,\tilde{h}_4) s_\beta^2 c_\alpha^2.$$
(80)

Typically, $\Delta M^2(G,\tilde{h}_4) = O(10^{-7} - 10^{-11})$. In most of our examples, $M_F/M_2 \sim 10^{-9}$. So one would expect $(M_F/M_2)\Delta M^2(G,\tilde{h}_4) \sim 10^{-16} - 10^{-20}$. If we wish $m_{34} \approx m_{43} \approx m_N 0.8 \times 10^{-7}$, for example, the other factors have to be sufficiently large. First, the ratio $|M_2^2/(M_P^2 - M_{h_4}^2)|$ can be rather large if $M_{h_4}$ is small compared with $M_2$. Secondly, even if the previous ratio can be large, it can still be offset by $s_\beta^2 c_\alpha^2$. Let us recall from Eq. (21) that $\tan\beta \approx g^2(M_F/M_2)(M_2^2/M_G^2) \gtrsim g^2 10^{-9}(M_2^2/M_G^2)$. Therefore the angle can be very small if $M_G$ is too "close" in mass to $M_2$. A numerical investigation reveals that, if one wants to have a mass of $O(1 \text{ eV})$ and, at the same time, a large enough angle, $M_G$ can be relatively "low" ($\sim 10^4$ in units of $M_F$). (This would imply that the scale of family symmetry could be a few thousands of TeV if $M_F$ is a few hundred GeV.) We now give a couple of numerical estimates. We shall take the second example as a prototype. There one can calculate the

factor $s_\beta^2 c_\alpha^2$ to be $\approx 0.134$. (1) For $M_{h_4} = 100$ with all other masses being the same as those of the second example, we obtain

$$m_{34} \approx m_{43} \approx m_N \lambda_{34} \frac{\tilde{v}}{V} \times 4.7 \times 10^{-7}.$$
(81)

If we wish $m_{34} \approx m_{43} \approx m_N 0.8 \times 10^{-7}$, then $\lambda_{34}(\tilde{v}/V) \approx 0.17$. So one could either have $\lambda_{34} \approx 0.2$ and $\tilde{v} \approx V$, or some other combination. (2) For $M_{h_4} = 10$, we have

$$m_{34} \approx m_{43} \approx m_N \lambda_{34} \frac{\tilde{v}}{V} \times 1.4 \times 10^{-5},$$
(82)

which would imply $\lambda_{34}(\tilde{v}/V) \approx 0.006$—a reasonable constraint.

It turns out that the cases with $M_{h_4} \gtrsim 1000$ (in units of $M_F$) do not work because then the mass ratios are not large enough to compensate for the smallness of the integrals. It is interesting that one can have scenarios where $\tilde{h}_4$ is light enough (i.e. not too much heavier than $F$)—a feature which could have interesting phenomenological implications.

### D. Oscillation angles

To discuss the neutrino oscillation angles, one needs to give the leptonic "CKM" matrix, namely, $V_L = U_l^\dagger U_\nu$. It is beyond the scope of this paper to discuss the charged lepton sector, and hence $U_l$. This will be the subject of the following publication. However, we can give an example of $U_l$ by adopting, at least for this paper, a simple model of charged lepton masses of Ref. [15], which is a phenomenological model based on a generalization to the lepton sector of the "democratic mass" *Ansatz* of the quark sector. The reason why we use, as an example, Ref. [15] is because the matrix which diagonalizes the neutrino mass matrix, $U_\nu$, is *identical* to the $3 \times 3$ submatrix of our Eq. (50) (apart from a difference in the overall sign), namely,

$$U_\nu^{(3)} = \begin{pmatrix} -\dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 \\[2mm] \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 \\[2mm] 0 & 0 & -1 \end{pmatrix}.$$
(83)

Although Ref. [15] discussed an *Ansatz* for three generations, we will use it here because the mixing with the fourth generation is not relevant for the oscillation angles we are interested in. (It was relevant for the mass splitting.) So, basically, we will be using *only* the phenomenological ansatz for the *charged lepton* mass matrix of Ref. [15]. In fact, we will *only* use the matrix which diagonalizes that mass matrix.

The $3 \times 3$ leptonic "CKM" matrix written down by Ref. [15] is

$$V_l = (AB_l)^\dagger U_\nu$$

$$\approx \begin{pmatrix} 1 & -(1/\sqrt{3})\sqrt{m_e/m_\mu} & (2/\sqrt{6})\sqrt{m_e/m_\mu} \\ \sqrt{m_e/m_\mu} & 1/\sqrt{3} & -2/\sqrt{6} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix},$$

(84)

where $AB_l$ is the matrix which diagonalizes the charged lepton mass matrix, $U_\nu$ is given above, and $m_e$ and $m_\mu$ are the electron and muon masses, respectively. Now, the probability for $\nu_e \to \nu_\mu$ is

$$P(\nu_e \to \nu_\mu) \approx 2(V_{11}^2 V_{21}^2 + V_{12}^2 V_{22}^2 - V_{13}^2 V_{23}^2)$$
$$\times \sin^2(1.27 \Delta m_{12}^2 L/E), \qquad (85)$$

where the usual notation $\sin^2(2\theta_{12})$ is simply the coefficient of $\sin^2(1.27\Delta m_{12}^2 L/E)$. Similarly

$$P(\nu_\mu \to \nu_\tau) \approx 4 V_{23}^2 V_{33}^2 \sin^2(1.27\Delta m_{23}^2 L/E), \qquad (86)$$

with $\sin^2(2\theta_{23})$ being the coefficient of $\sin^2(1.27\Delta m_{23}^2 L/E)$. Putting in the values of $m_e$ and $m_\mu$ to evaluate the matrix elements of $V_l$, one readily obtains

$$\sin^2(2\theta_{12}) \approx 6.5 \times 10^{-3}; \quad \sin^2(2\theta_{23}) \approx 0.89. \qquad (87)$$

These results correspond to the *small angle* MSW solution, and to the *large angle* atmospheric solution, respectively. This is consistent with the best fit for the two neutrino oscillation problems.

The above results should be viewed with caution. The *small angle* MSW solution given above, as well as the *large angle* solution for the atmospheric oscillation, depends on the charged lepton sector—the neutrino sector diagonalizatin matrix being already fixed by Eq. (50). One can easily imagine how these angles can drastically change if the charged lepton mass matrix has a different texture. This will be the subject of a subsequent paper where we will examine the charged lepton mass matrix in the context of the present model—the basic interaction Lagrangian being already given by Eq. (3).

## IV. EPILOGUE

The above discussions focused entirely on the atmospheric and solar neutrino data. We have left out the LSND [16] result for two reasons. First, it is because it might be prudent to wait for future experiments, either to confirm or to refute these results. Secondly, it is because it is extremely hard to incorporate *all three* experiments simultaneously in a "natural" model. In general, one needs to invoke some kind of sterile neutrino that mixes with the lightest neutrino to explain the solar data. If this sterile neutrino were to arise from some kind of model, it is rather hard to invent, in a "natural" way, a scenario to explain why this sterile neutrino is so light and close in mass to one of the three active light neutrinos.

Let us suppose that the LSND result are verified by future experiments. What does the model presented in this paper have to say about a sterile neutrino? Let us remember that $\eta_R = (\nu_R^\alpha, \widetilde{\nu}_R^\alpha)$ is an electroweak *singlet*. Furthermore we have seen that it is $\nu_R^\alpha$ which mixes with $l_L^\alpha$ to give masses to the neutrinos. Its $\mathrm{SU}(2)_{\nu_R}$ partner, $(\widetilde{\nu}_R^\alpha)$, remains massless, at least within the framework of the preceding sections. Could these be the so-called sterile neutrinos? If so, how would they get a mass? How would they mix with the light neutrinos? These are the questions which are under investigation.

We have concentrated in this manuscript on the even option. One might wonder about the odd option and its implication on neutrino masses. It is beyond the scope of this paper to investigate this issue, however, a preliminary investigation of the odd option, with three families and one family singlet $\eta'$, appears to indicate that the preferred solution for the neutrino masses is that in which there is a hierarchy $m_1 \ll m_2 \ll m_3$.

There are numerous phenomenological consequences to be worked out in subsequent publications. One can, however, make one rather solid prediction: neutrinos, being of the Dirac nature, will not give rise to the phenomenon of neutrinoless double beta decay. Another interesting consequence is the possible existence of "light" (i.e., 200 GeV or so) vectorlike fermions: $F$, as well as TeV-scale pseudo-NG bosons which carry family and $\mathrm{SU}(2)_{\nu_R}$ quantum numbers. This will be dealt with in a separate paper.

Several other phenomenological issues remain to be investigated. For instance, what are the consequences of a broken $\mathrm{SU}(2)_{\nu R}$ and what might the cosmological implications of $\widetilde{\nu}_R$'s and $\eta_i'$ be? When $\mathrm{SU}(2)_{\nu R}$ is broken by $\rho_R^\alpha$, the gauge bosons are expected to acquire a mass of $O(V')$ and can be quite heavy. Since only *right-handed* neutral leptons participate in $\mathrm{SU}(2)_{\nu R}$ interactions, a place where the effects of those gauge bosons might show up is in the decays of neutrinos. Without going into detail, it is easy to see that the decay of the light (near-degenerate) neutrinos into each other is completely negligible for lack of phase space and for the fact that neutrino masses are *tiny* compared with $V'$ (even if the latter is in the TeV region). This leaves us with the decay of the (heavy) fourth-generation neutral lepton $N$ for which we have $N \to \widetilde{N} + \nu_i + \widetilde{\nu}_i$ (1) via the exchange of $\mathrm{SU}(2)_{\nu R}$ gauge bosons, and $N \to l_i^- + l_j^+ + \nu_j$ (2) if $m_N < m_W$ or $N \to l_i^- + W$ (3) if $m_N > m_W$. In addition, one could have $N \to E + l_j^+ + \nu_j$ when $m_N > m_E$, via the exchange of $W$. Whether or not $m_N$ is larger or smaller than $m_E$, the relevant decays to compare with each other are (1) and (3). To make an estimate, let us assume the the family gauge coupling is about the same size as the electroweak coupling ($g \sim 0.7$). The ratio of the decay widths for (1) and (3) is approximately

$$\Gamma(1)/\Gamma(3) \sim 7.5 \times 10^{-4}(M_W/M_{\widetilde{G}})^2(m_N/M_{\widetilde{G}})^2$$
$$\times [1 - (M_W/m_N)^4]^{-2} x^{-2},$$

where $M_{\widetilde{G}}$ represent the mass of the $\mathrm{SU}(2)_{\nu_R}$ gauge bosons and $x$ represents the mixing cofficient between the fourth neutrino and a light charged lepton. Now let us remember that the computation of the neutrino masses does not involve

$M_{\widetilde{G}}$ and as a result there appears to be no constraint there. However, $M_{\widetilde{G}} \sim g V'$ and $M_G \sim g V$, and as a result $\tan \beta \equiv V'/V \sim M_{\widetilde{G}}/M_G \sim g^2 10^{-9}(M_2^2/M_G^2)$. In the second example discussed in the previous example, $V' \sim V$ (with $M_G = 10^4 M_F$) which implies $M_{\widetilde{G}} \sim M_G$. Now $\Gamma(1)/\Gamma(3)$ can also be appreciable if $m_N$ is close to $m_W$. For example, if $M_F \sim 200$ GeV and $m_N \sim 82$ GeV, $\Gamma(1)/\Gamma(3) \sim 1$ provided $x \sim 10^{-9}$. If this were the case, the signal would be quite interesting: a long-lived massive neutral lepton whose electroweak decay width is not what it should be. It is certainly beyond the scope of this paper to explore numerous phenomenological consequences which might arise from our scenario.

Other phenomenological issues such as the magnitude of flavor-changing neutral currents, e.g., $\mu \rightarrow e \gamma$, will be discussed in an upcoming paper dealing with the charged lepton sector. However, a preliminary statement can be made. For example, in the case of $\mu \rightarrow e \gamma$, there are two kinds of contributions: One coming from the propagation of neutrinos with a nonzero mass inside the loop diagram for the process, and the other one coming from diagrams involving the new vectorlike fermions. It turns out that both contributions are negligible: (1) In the first case, it is because $m_\nu \ll M_W$, (2) in the second case, it is because of the cancellations of the type described in Sec. III B. It is beyond the scope of this paper to discuss these points in more detail. They will be part of the phenomenological consequences of the charged lepton sector.

As for the cosmological consequences of $\widetilde{\nu}_R$'s and $\eta'_R$, if they are massless, one should recall our earlier discussion: These particles *only* have family and $SU(2)_{\nu R}$ gauge interactions (both for $\widetilde{\nu}_R$'s and the latter only for $\eta'_R$). Therefore, they *cannot* influence big-bang nucleosynthesis. One can estimate their decoupling temperatures by comparing the interaction rate $\Gamma_{\text{int}} \sim G^2 T^5$, where $G^2 \sim 1/(64 V^{(')4})$, with the Hubble rate $H \sim T^2/m_{pl}$. Decoupling occurs when $\Gamma_{\text{int}} < H$ which gives a temperature of $O(10^6)$ GeV if $V^{(')} \sim 10^9$ GeV, for example. After this, their temperature would scale as $T \sim 1/R$. It is not clear what else they can do except to exist as almost noninteracting relativistic relics with an energy density negligible compared with normal matter. At this stage, it is also not clear if they really do need to have a mass. The cosmology of these objects is probably worth exploring further.

Another interesting cosmological subject to explore is the "heaviest" particle in our scenario: The vectorlike neutral fermion $\mathcal{M}_2$ which is singlet under all the listed gauge groups in Eq. (2). $\mathcal{M}$ couples to other fermions via 3. The decay modes obtained from (3) are $\mathcal{M}_{2R} \rightarrow \phi^\pm F_L^\mp$ (1) and $\mathcal{M}_{2L} \rightarrow \rho_\alpha \eta^\alpha$ (2). Notice that, in the examples given above for the calculations of the neutrino masses, the mass of this fermion is typically $M_2 \sim 10^9 M_F$. So, if $M_F \sim 200$ GeV (or a few hundred GeV), one would then expect the mass of $\mathcal{M}_2$ to be around a few times $10^{11}$ GeV. If $M_F \sim 1$ TeV, $\mathcal{M}_2$ would have a mass around $10^{12}$ GeV. The questions that we would like to investigate are (1) how many $\mathcal{M}_2$ are left in the present universe? and (2) could the decay of the relic $\mathcal{M}_2$'s

manifest itself as ultra high energy cosmic rays (UHECR) (with energy exceeding $10^{20}$ eV $= 10^{11}$ GeV) whose origins are still unknown? It does appear that the mass of $\mathcal{M}_2$ is in the right energy ballpark. This would be the case of a non-accelerated source of UHECR and is part of the "top-down" approach to UHECR [17]. For example, $\mathcal{M}_{2R}$ would decay into the longitudinal component of $W$ ($\phi^\pm$) and $F_L^\mp$. $\phi^\pm$ would in turn decay into extremely high-energy quarks and leptons. The quarks will hadronize into hadrons such as pions which will eventually convert into photons, neutrinos, etc.

Last but not least, in the subsequent series of papers, we shall deal with the charged lepton sector and with the quark sector. In particular, we shall see how the generalization of Eq. (3) to the quark sector might yield interesting results.

*Note added*. It has come recently to the attention of the author that there exists a class of models [18] based on the left-right symmetry model which made use of vectorlike fermions to achieve loop-suppressed Dirac masses for neutrinos. The model presented in this manuscript is very different from the aforementioned class of models.

## APPENDIX: HIGGS POTENTIAL

In this appendix, we shall discuss a simple form of the Higgs potential for the group $SO(4) \otimes SU(2)_{\nu_R}$. For simplicity, we shall assume that there is no cross coupling between $(\Omega, \rho)$ and the SM Higgs field $\phi$. (One might wonder about the fact that even if the cross coupling were vanishing, it might still be induced through radiative corrections. This, however, would be very small in our model.)

The potential containing $\Omega$ and $\rho$ reads

$$V(\Omega,\rho) = \lambda_1(\Omega^\alpha \Omega_\alpha - V^2)^2 + \lambda_2(\rho^{\dagger\,\alpha}\rho_\alpha - V'^{\,2})^2$$
$$+ \lambda_3[(\Omega^\alpha \Omega_\alpha - V^2) - (\rho^{\dagger\,\alpha}\rho_\alpha - V'2)]^2$$
$$+ \lambda_4[(\Omega^\alpha \Omega_\alpha)(\rho^{\dagger\,\beta}\rho_\beta) - (\Omega^\alpha \rho_\alpha^\dagger)(\Omega^\beta \rho_\beta)],$$
$$(A1)$$

where $\langle \Omega \rangle = (0,0,0,V)$ and $\langle \rho \rangle = (0,0,0,V' \otimes s_1)$, with $s_1 = \binom{1}{0}$. Here, we will assume that $\Omega$ is real and $\rho$ is complex. We will be particularly interested in the mass eigenstates resulting from Eq. (A1).

With $\Omega_4 = H_4 + V$ and

$$\rho_4 = \begin{pmatrix} h_4 + V' + i\phi_4 \\ \rho'_4 \end{pmatrix},$$

Eq. (A1) gives rise to the following mass matrix for $H_4$ and $h_4$:

$$8\begin{pmatrix} (\lambda_1+\lambda_3)V^2 & -\lambda_3 VV' \\ -\lambda_3 VV' & (\lambda_2+\lambda_3)V'^2 \end{pmatrix}. \qquad \text{(A2)}$$

The eigenvectors are

$$\tilde{H}_4 = \cos\alpha\, H_4 + \sin\alpha\, h_4, \qquad \text{(A3a)}$$

$$\tilde{h}_4 = -\sin\alpha\, H_4 + \cos\alpha\, h_4. \qquad \text{(A3b)}$$

The associated eigenvalues are

$$m_{H_4}^2 = 4(\lambda_2+\lambda_3)V'^2 m_1^2, \qquad \text{(A4a)}$$

$$m_{h_4}^2 = 4(\lambda_2+\lambda_3)V'^2 m_2^2, \qquad \text{(A4b)}$$

where

$$m_{1,2}^2 = \frac{1+a \pm \sqrt{(1-a)^2+4b^2}}{2}, \qquad \text{(A5a)}$$

$$a = \left(\frac{\lambda_1+\lambda_3}{\lambda_2+\lambda_3}\right)\tan^2\beta, \qquad \text{(A5b)}$$

$$b = \left(\frac{\lambda_3}{\lambda_2+\lambda_3}\right)\tan\beta, \qquad \text{(A5c)}$$

$$\tan\beta = \frac{V'}{V}, \qquad \text{(A5d)}$$

$$\cos\alpha = \frac{1}{\sqrt{1+[(1-m_1^2)/b]^2}}. \qquad \text{(A5e)}$$

The mass matrix for $\Omega_i$ and Re $\rho_i$ with $i=1,2,3$, is

$$2\lambda_4\begin{pmatrix} V'^2 & -VV' \\ -VV' & V^2 \end{pmatrix}. \qquad \text{(A6)}$$

The eigenvectors are

$$\tilde{\Omega}_i = \cos\beta\, \Omega_i + \sin\beta\, \text{Re}\,\rho_i, \qquad \text{(A7a)}$$

$$\text{Re}\,\tilde{\rho}_i = -\sin\beta\, \Omega_i + \cos\beta\, \text{Re}\,\rho_i, \qquad \text{(A7b)}$$

The associated eigenvalues are

$$m_{\tilde{\Omega}} = 0, \qquad \text{(A8a)}$$

$$m_{\text{Re}\,\tilde{\rho}} = 2\lambda_4(V^2+V'^2). \qquad \text{(A8b)}$$

Notice that $\tilde{\Omega}_i$ are NG Goldstone bosons which are absorbed by some of the SO(4) gauge bosons.

Since it is not of immediate relevance to the paper, we will simply quote the masses of the other scalars obtained from Eq. (A1). Scalars (pseudo NG bosons) which have a mass $2\lambda_4 V^2$: Im $\rho_i$, Re $\rho_i'$, Im $\rho_i'$. Goldstone bosons which are absorbed by some of the $SO(4)\otimes SU(2)_{\nu_R}$ gauge bosons: Re $\rho_4'$, Im $\rho_4'$. Notice that the pseudo-NG boson masses are all proportional to $\lambda_4$. As a result, their masses tend to zero as $\lambda_4 \to 0$.

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