Photons, neutrinos, and large compact space dimensions

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(Received 27 April 2000; published 8 August 2000)

We compute the contribution of Kaluza-Klein graviton exchange to the cross section for photon-neutrino scattering. Unlike the usual situation where the virtual graviton exchange represents a small correction to a leading order electroweak or strong amplitude, in this case the graviton contribution is of the same order as the electroweak amplitude, or somewhat larger. Inclusion of the graviton contribution is not sufficient to allow high energy neutrinos to scatter from relic neutrinos in processes such as $\nu\bar{\nu}\to\gamma\gamma$, but the photon-neutrino decoupling temperature is substantially reduced.

PACS number(s): 13.15.+g, 04.50.+h, 14.60.Lm, 14.70.Bh

I. INTRODUCTION

The $2\to 2$ processes $\gamma\nu\to\gamma\nu$, $\gamma\gamma\to\nu\bar{\nu}$ and $\nu\bar{\nu}\to\gamma\gamma$ are of potential interest in astrophysics. However, because of the vector-axial-vector nature of the weak coupling, the leading term in these cross sections for these processes with massless neutrinos, nominally of order $G_F^2\alpha^2\omega^2$, vanishes due to Yang's theorem [1,2]. In the limit that the photon energy $\omega < m_e$, where m_e is the electron mass, these cross sections can be shown to be of order $G_F^2\alpha^2\omega^2(\omega/m_W)^4$ [3–5], and, in the annihilation channels at least, this ω^6 behavior persists to center of mass energies $\sqrt{s}\sim 2m_W$, where m_W is the mass of the W boson [6].

The ω^6 behavior of the photon-neutrino cross sections can be understood in terms of an effective Lagrangian of the form

$$\mathcal{L}_{\rm eff}^{\rm SM}=\frac{1}{32\pi}\frac{g^2\alpha}{m_W^4}A[\bar{\psi}\gamma_\nu(1+\gamma_5)(\partial_\mu\psi)-(\partial_\mu\bar{\psi})\gamma_\nu(1+\gamma_5)\psi]F_{\mu\lambda}F_{\nu\lambda}, \tag{1}$$

where g is the electroweak gauge coupling, ψ is the neutrino field and $F_{\mu\nu}$ is the electromagnetic field tensor. In Eq. (1), A is

$$A=\left[\frac{4}{3}\ln\left(\frac{m_W^2}{m_e^2}\right)+1\right] \tag{2}$$

in the low energy limit $\omega<m_e$, and A is obtained by fitting the numerical calculation of the cross section for $\omega>m_e$ [6]. Since $\mathcal{L}_{\rm eff}^{\rm SM}$ is a dimension 8 operator, it follows that the center of mass cross sections will behave as ω^6. Another property of Eq. (1) is that the scattered photons in the channel $\gamma\nu\to\gamma\nu$ are circularly polarized *in leading order* [5] due to the parity violating terms in $\mathcal{L}_{\rm eff}$. There is no linear polarization in this channel.

Because the scale of $\mathcal{L}_{\rm eff}^{\rm SM}$ is m_W, a typical photon-neutrino cross section is quite small, so small that a high energy neutrino beam is not attenuated by interactions with the present density of relic neutrinos via the process $\nu\bar{\nu}\to\gamma\gamma$ [6]. In the early universe, the photons and neutrinos decouple at a temperature $T\sim 1.6$ GeV, or about one microsecond after the Big Bang. If this temperature were a factor of 10 lower, i.e., $T\leqslant\Lambda_{\rm QCD}$, one might be justified in speculating that some remnant of the circular polarization mentioned above could be retained in the cosmic microwave background radiation and at this would provide evidence for the relic neutrino background.

Lowering the decoupling temperature necessitates increasing the cross section, $\sigma(\nu\bar{\nu}\to\gamma\gamma)$, or changing the dependence of the age of the universe, t, on the temperature T. The latter seems unlikely, since the $t\sim T^{-2}$ radiation dominated behavior of the early universe is insensitive changes such as including a non-vanishing cosmological constant. On the other hand, new interactions could increase $\sigma(\nu\bar{\nu}\to\gamma\gamma)$, provided they involve the exchange of particles with spin $\neq 1$. A new interaction of this type is provided by the recent proposal that the compact dimensions of string theory are sufficiently large to make the effective gravitational scale Λ of order a TeV rather than the usual $M_P=1.2\times 10^{19}$ GeV Planck scale [7–11]. That this new gravitational interaction will make a significant correction to the standard model photon-neutrino cross sections can be seen by rewriting Eq. (1) in the form

$$\mathcal{L}_{\rm eff}^{\rm SM}=\frac{1}{8\pi}\frac{g^2\alpha}{m_W^4}AT_{\alpha\beta}^{\nu}T_{\alpha\beta}^{\gamma}, \tag{3}$$

where $T_{\alpha\beta}^{\nu}$ and $T_{\alpha\beta}^{\gamma}$ are the symmetrical energy-momentum tensors of the neutrinos and the photons. Explicitly, we have

$$T^{\nu}_{\alpha\beta}=\frac{1}{8}[\overline{\psi}\gamma_{\alpha}(1+\gamma_5)(\partial_{\beta}\psi)+\overline{\psi}\gamma_{\beta}(1+\gamma_5)(\partial_{\alpha}\psi)$$
$$-(\partial_{\beta}\overline{\psi})\gamma_{\alpha}(1+\gamma_5)\psi-(\partial_{\alpha}\overline{\psi})\gamma_{\beta}(1+\gamma_5)\psi], \quad (4)$$

$$T^{\gamma}_{\alpha\beta}=F_{\alpha\lambda}F_{\beta\lambda}-\frac{1}{4}\delta_{\alpha\beta}F_{\lambda\rho}F_{\lambda\rho}. \quad (5)$$

In the next section, we show that an effective interaction which is the product of energy-momentum tensors also arises when the spin 2 graviton is exchanged between photons and neutrinos. This is followed by the calculation of $\sigma(\nu\overline{\nu}\to\gamma\gamma)$ and a discussion of the resulting astrophysical implications.

II. GRAVITON EXCHANGE BETWEEN PHOTONS AND NEUTRINOS

According to models in which only the graviton ($\mathcal{G}$) propagates in the additional n compact dimensions of a $D=4+n$ dimensional manifold, the compact spatial dimension R is related to Λ and M_P as [7]

$$\Lambda^{n+2}R^{n}\sim M_p^2/4\pi. \quad (6)$$

The graviton's propagation in all D dimensions implies the existence of a tower of spin 2 particles in ordinary space-time, whose masses are given by $m_{\vec{n}}^2=\vec{n}^2/R^2$, where $\vec{n}=(n_1,n_2,\ldots,n_n)$ and the n_i are integers. Furthermore, in these models the interaction between the spin 2 graviton, $\mathcal{G}_{\mu\nu}$, and any standard model field has the universal form

$$\mathcal{L}^{\mathcal{G}}_{\rm eff}=-\frac{\kappa}{2}T_{\mu\nu}\mathcal{G}_{\mu\nu}, \quad (7)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the standard model field.

Given the effective coupling, Eq. (7), it is a simple matter to calculate the $2\to2$ amplitudes [12,13]. In the annihilation channels ($\gamma\gamma\to\nu\overline{\nu}$ and $\nu\overline{\nu}\to\gamma\gamma$), the amplitude for a particular $m_{\vec{n}}$ has the form

$$\mathcal{A}_{\vec{n}}=\frac{\kappa^2}{4}T^{\nu}_{\alpha\beta}(p_1,p_2)\frac{\mathcal{P}_{\alpha\beta\lambda\rho}(k_1+k_2)}{m_{\vec{n}}^2-s-i\varepsilon}T^{\gamma}_{\lambda\rho}(k_1,k_2), \quad (8)$$

with $s=-(k_1+k_2)^2$. Here $\mathcal{P}_{\alpha\beta\lambda\rho}$ is the spin 2 projection operator

$$\mathcal{P}_{\alpha\beta\lambda\rho}(k)=\frac{1}{2}\bigg(d_{\alpha\lambda}(k)d_{\beta\rho}(k)+d_{\alpha\rho}(k)d_{\beta\lambda}(k)$$
$$-\frac{2}{3}d_{\alpha\beta}(k)d_{\lambda\rho}(k)\bigg), \quad (9)$$

with

$$d_{\alpha\beta}(k)=\delta_{\alpha\beta}+\frac{1}{m_{\vec{n}}^2}k_{\alpha}k_{\beta}. \quad (10)$$

Since the energy-momentum tensors are conserved, symmetrical and, in this case, traceless, Eq. (8) reduces to

$$\mathcal{A}_{\vec{n}}=\frac{\kappa^2}{4}T^{\nu}_{\alpha\beta}(p_1,p_2)\frac{1}{m_{\vec{n}}^2-s-i\varepsilon}T^{\gamma}_{\alpha\beta}(k_1,k_2). \quad (11)$$

To complete the calculation of the amplitude, it is necessary to sum $\mathcal{A}_{\vec{n}}$ over the values of $m_{\vec{n}}^2$. This is done by replacing the sum with an integral over the number density $d\mathcal{N}$ given by [9,11]

$$d\mathcal{N}=\frac{1}{2}\Omega_nR^n(m^2)^{(n-2)/2}dm^2, \quad (12)$$

where Ω_n is the surface area of an n-dimensional sphere. If the integral over dm^2 is cut off at Λ^2, we find as leading terms [11]

$$\int_0^{\Lambda^2}\frac{d\mathcal{N}}{m^2-s-i\varepsilon}=\frac{1}{2}\Omega_nR^n\Lambda^{n-2}\begin{cases}\ln\left(\frac{\Lambda^2}{s}\right) & \text{if} \quad n=2\\ \frac{2}{(n-2)} & \text{if} \quad n>2\end{cases}$$
$$=\frac{1}{2}\Omega_nR^n\Lambda^{n-2}I_n(\Lambda,s). \quad (13)$$

Apart from a $\ln(\Lambda^2/s)$ term when $n=2$, all values of n have the same Λ^{n-2} dependence. If we then take the specific realization of Eq. (6) for the scale parameter Λ to be

$$\Omega_n\Lambda^{n+2}R^n=M_P^2, \quad (14)$$

the summed version of Eq. (11) is

$$\mathcal{A}_{\mathcal{G}}=\frac{4\pi}{\Lambda^4}I_n(\Lambda,s)T^{\nu}_{\alpha\beta}(p_1,p_2)T^{\gamma}_{\alpha\beta}(k_1,k_2), \quad (15)$$

where we have used $\kappa^2=32\pi/M_P^2$ [9].

III. $\nu\overline{\nu}\to\gamma\gamma$ CROSS SECTION INCLUDING GRAVITON EXCHANGE

Using Eq. (3), the standard model amplitude for the annihilation processes is

$$\mathcal{A}^{\rm SM}=\frac{1}{8\pi}\frac{g^2\alpha}{m_W^4}AT^{\nu}_{\alpha\beta}(p_1,p_2)T^{\gamma}_{\alpha\beta}(k_1,k_2), \quad (16)$$

which, in view of Eq. (15), leads to the total amplitude

$$\mathcal{A}=\left(\frac{1}{8\pi}\frac{g^2\alpha}{m_W^4}A+\frac{4\pi}{\Lambda^4}I_n(\Lambda,s)\right)T^{\nu}_{\alpha\beta}(p_1,p_2)T^{\gamma}_{\alpha\beta}(k_1,k_2). \quad (17)$$

If the photon helicities are denoted by λ_1 and λ_2, the product of energy-momentum tensors in Eq. (17) is given by

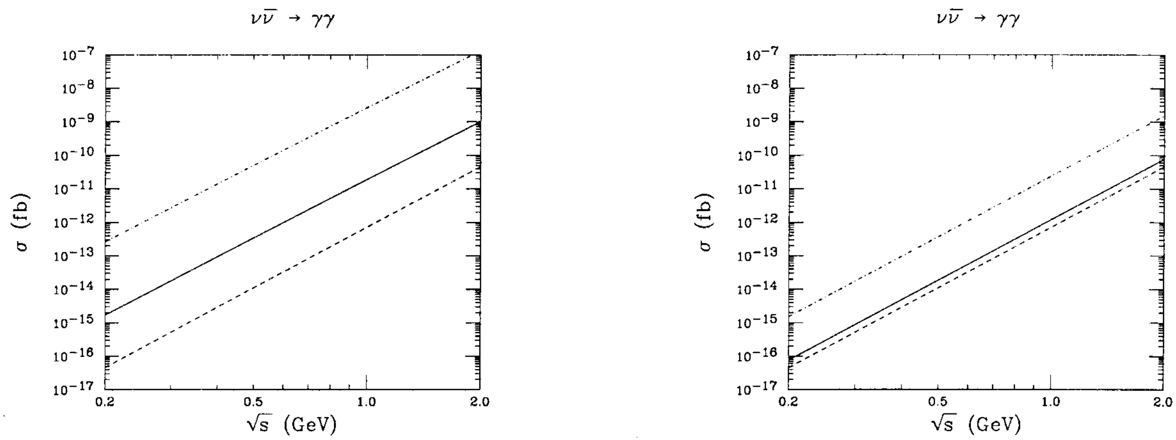

FIG. 1. The total cross section $\sigma(\nu\overline{\nu}\rightarrow\gamma\gamma)$ is shown. The left panel is the $n=2$ result and the right panel is the $n=4$ result. In each panel, the solid line corresponds to $\Lambda=1$ TeV, the dot-dash line to $\Lambda=.5$ TeV and the dashed line to $\Lambda=10$ TeV. The $\Lambda=10$ TeV curve is identical to the standard model result.

$$T^{\nu}_{\alpha\beta}(p_1,p_2)T^{\gamma}_{\alpha\beta}(k_1,k_2)$$

$$=\frac{1}{4}\sin\theta\left[st(1-\lambda_1\lambda_2)+\frac{1}{2}s^2(1-\lambda_1)(1+\lambda_2)\right] \qquad (18)$$

$$=\frac{1}{4}\mathcal{M}_{\lambda_1\lambda_2}, \qquad (19)$$

where $t=-(p_1-k_1)^2$, and θ is the scattering angle in the center of mass. The differential cross section for $\nu\overline{\nu}\rightarrow\gamma\gamma$ can then be calculated using

$$\frac{d\sigma}{dz}=\frac{1}{32\pi s}\sum_{\lambda_1\lambda_2}|\mathcal{A}_{\lambda_1\lambda_2}|^2, \qquad (20)$$

with $z=\cos\theta$, and

$$\mathcal{A}_{\lambda_1\lambda_2}=\left(\frac{1}{32\pi}\frac{g^2\alpha}{m_W^4}A+\frac{\pi}{\Lambda^4}I_n(\Lambda,s)\right)\mathcal{M}_{\lambda_1\lambda_2}. \qquad (21)$$

Summing over the helicities gives

$$\frac{d\sigma}{dz}=\frac{1}{16\pi}\left(\frac{1}{32\pi}\frac{g^2\alpha}{m_W^4}A+\frac{\pi}{\Lambda^4}I_n(\Lambda,s)\right)^2 s^3(1-z^4), \qquad (22)$$

which leads to the total cross section

$$\sigma(\nu\overline{\nu}\rightarrow\gamma\gamma)=\frac{1}{2!}\int_{-1}^{1}dz\,\frac{d\sigma}{dz} \qquad (23)$$

$$=\frac{s^3}{20\pi}\left(\frac{1}{32\pi}\frac{g^2\alpha}{m_W^4}A+\frac{\pi}{\Lambda^4}I_n(\Lambda,s)\right)^2. \qquad (24)$$

Using the value $A=14.4$ [6] and expressing Λ in TeV, we find

$$\sigma(\nu\overline{\nu}\rightarrow\gamma\gamma)=\frac{1}{8}\frac{s^3}{m_e^6}\left(1+.3\frac{I_n(\Lambda,s)}{\Lambda^4}\right)^2\times10^{-31}\ \text{fb}. \qquad (25)$$

The cross section is shown in Fig. 1 for the cases $n=2$ and $n=4$ with $\Lambda=.5$, 1 and 10 TeV. Although the logarithmic variation in the $n=2$ case is scarcely detectable in the range $.2\ \text{GeV}\leqslant\sqrt{s}\leqslant2$ GeV, its presence in the coefficient of Λ^{-4} makes the effect of the extra dimensions largest for this case.

IV. DISCUSSION AND CONCLUSIONS

The possibility of a high energy neutrino scattering from the current relic neutrino background is not materially enhanced by the inclusion of the effects of gravitons propagating in compact dimensions. Neglecting the electroweak contribution to the $\nu\overline{\nu}\rightarrow\gamma\gamma$ cross section, which is known to be too small [6] to produce any scattering, the condition $\sigma_{\nu\overline{\nu}\rightarrow\gamma\gamma}n_\nu ct_0=1$ for at least one scattering, gives

$$\frac{\pi}{20}\frac{s^3}{\Lambda^8}\ln^2\left(\frac{\Lambda^2}{s}\right)n_\nu ct_0=1 \qquad (26)$$

or, using the relic neutrino density $n_\nu=56\ \text{cm}^{-3}$, and the age of the universe $t_0=15\times10^9$ years, the condition can be written

$$x^6\ln^2(x^2)=0.0207\tilde{\Lambda}^2, \qquad (27)$$

with $x=\sqrt{s}/\Lambda$ and $\tilde{\Lambda}$ in GeV. For a 1 TeV scale, the solution to Eq. (27) is $x=3.78$, giving $\sqrt{s}=3.78$ TeV, which is beyond the range of validity of the effective theory.

The additional contribution to the cross section from gravition exchange will affect the decoupling temperature. The temperature at which the reaction $\nu\overline{\nu}\rightarrow\gamma\gamma$ ceases to occur can be determined from the reaction rate per unit volume

$$\rho = \frac{1}{(2\pi)^6}\int \frac{d^3p_1}{e^{E_1/T}+1}\int \frac{d^3p_2}{e^{E_2/T}+1}\sigma|\vec{v}|, \tag{28}$$

where $\vec{p}_1$ and $\vec{p}_2$ are the neutrino and antineutrino momenta, E_1 and E_2 their energies, $|\vec{v}|$ is the flux and T the temperature. Using the invariance of $\sigma E_1 E_2|\vec{v}|$, the relationship between $\sigma|\vec{v}|$ in the center of mass frame and any other frame is

$$\sigma|\vec{v}| = \sigma_{CM}\frac{2E_{\rm CM}^2}{E_1E_2}, \tag{29}$$

which gives

$$\sigma|\vec{v}| = \frac{s^4}{16E_1E_2m_e^6}\left(1+\frac{.3}{\Lambda^4}\ln\left(\frac{\Lambda^2}{s}\right)\right)^2\times 10^{-70}\ \mathrm{cm}^2, \tag{30}$$

for $n=2$ and Λ in TeV. Taking $s=4E_1E_2\sin^2(\theta_{12}/2)$, where θ_{12} is the angle between the incoming neutrinos, the angular integrations in Eq. (28) result in the integrand

$$\int d\Omega_1 d\Omega_{12} = \frac{(4\pi)^2}{5}\left[\left(1+\frac{.3}{\Lambda^4}\ln\left(\frac{\Lambda^2}{4E_1E_2}\right)\right)^2 + \frac{.12}{\Lambda^4}\left(1+\frac{.3}{\Lambda^4}\ln\left(\frac{\Lambda^2}{4E_1E_2}\right)\right)+.08\left(\frac{.3}{\Lambda^4}\right)^2\right]. \tag{31}$$

To estimate the decoupling temperature, the last two terms on the right in Eq. (31), which are suppressed relative to the first by small numerical factors, can be neglected. The reaction rate per unit volume is then given by

$$\rho = \frac{6.4\times 10^{-69}\ \mathrm{cm}^2}{5(2\pi)^4}\frac{T^{12}}{m_e^6}\int_0^\infty dx\frac{x^5}{e^x+1}\int_0^\infty dy\frac{y^5}{e^y+1} \times\left[1+\frac{.3}{\Lambda^4}\left(\ln\left(\frac{\Lambda^2}{4T^2}\right)-\ln x-\ln y\right)\right]^2. \tag{32}$$

The $\ln x$ and $\ln y$ terms in Eq. (32) result in contributions which are small relative to the remaining terms. Omitting these terms gives

$$\rho = \frac{6.4\times 10^{-69}\ \mathrm{cm}^2}{5(2\pi)^4}\frac{T^{12}}{m_e^6}\left[1+\frac{.3}{\Lambda^4}\ln\left(\frac{\Lambda^2}{4T^2}\right)\right]^2 \times\left[\frac{31}{32}\Gamma(6)\zeta(6)\right]^2, \tag{33}$$

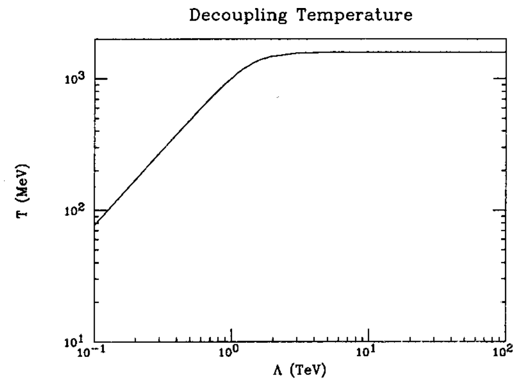

FIG. 2. The decoupling temperature is shown as a function of the effective gravitational scale Λ.

where $\zeta(z)$ is the Riemann Zeta function. The interaction rate R is obtained by dividing Eq. (33) by the neutrino density $n_\nu = 3\zeta(3)T^3/4\pi^2$, giving

$$R = 7.3\times 10^{-24}T_{10}^9\left[1+\frac{.3}{\Lambda^4}\ln\left(\frac{10^{12}\Lambda^2}{3T_{10}^2}\right)\right]^2 \sec^{-1}, \tag{34}$$

with $T_{10}=T/10^{10}$ K. Multiplying R by the age of the universe, $t=2T_{10}^{-2}$ sec, the condition for a single interaction to occur is

$$\frac{T_{10}}{1828}\left[1+\frac{.3}{\Lambda^4}\ln\left(\frac{10^{12}\Lambda^2}{3T_{10}^2}\right)\right]^{2/7} = 1. \tag{35}$$

The solution to this equation is shown in Fig. 2. While there is a substantial correction to the standard model decoupling temperature, a decoupling temperature of a few hundred MeV is only possible for $\Lambda\sim 250$–300 GeV, which is unrealistically low. The decoupling temperature for a 1 TeV scale is 1 GeV, down from the standard model result of 1.6 GeV. Thus, a mechanism for lowering the photon-neutrino decoupling temperature below $\Lambda_{\rm QCD}$ remains elusive.

ACKNOWLEDGMENTS

One of us (K.K.) wishes to thank her fellow participants in the Michigan State University High School Honors Science Program for numerous helpful conversations. This research was supported in part by the National Science Foundation under grant PHY-9802439 and by the Department of Energy under Contract No. DE-FG13-93ER40757.

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