

## Massive neutrinos and the Higgs boson mass window

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In the standard model, if there is no new physics up to a certain scale, there is a window for the Higgs boson mass, defined by the upper (perturbativity) and the lower (stability) bounds. If the new physics is in the form of right-handed neutrinos, responsible for the small neutrino masses through a seesaw mechanism, then the window narrows, and even disappears for  $M$  (the right-handed Majorana mass) ranging from  $10^{13}$  GeV for three quasidegenerate neutrinos with  $m_\nu \approx 2$  eV, to  $5 \times 10^{14}$  GeV for just one relevant generation with  $m_\nu \approx 0.1$  eV. A slightly weaker upper bound on  $M$ , coming from the requirement that the neutrino Yukawa couplings do not develop a Landau pole, is also discussed.

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Observations of the flux of atmospheric neutrinos by SuperKamiokande [1] provide strong evidence for neutrino oscillations, which in turn imply that (at least two species of) neutrinos must be massive. Additional support to this hypothesis is given by the need of neutrino oscillations to explain the solar neutrino flux deficit and the possible essential role of the neutrinos in the large scale structure of the universe [2]. Much work has been devoted in the last months in order to guess and to explain the structure and the origin of the neutrino mass matrices capable to account for the different observations [3].

In this paper, we would like to point out the consequences that massive neutrinos have for the standard model (SM), in particular for its still unprobed sector, namely the Higgs sector. As is known, if there is no new physics beyond the SM up to a certain scale,  $\Lambda$ , the Higgs boson mass is bounded from above from the requirement of validity of perturbation theory, and from below by the requirement of stability of the Higgs potential [4,5]. In particular, if we fix  $\Lambda = M_{\text{Pl}}$ , then the allowed window for the Higgs boson mass  $M_H$  is [137,175] GeV. Of course, the lower  $\Lambda$ , the wider the allowed window. On the other hand, if one assumes that new physics is present at  $M_{\text{GUT}}$ , then one should set  $\Lambda = M_{\text{GUT}} \approx 10^{16}$  GeV, in which case the window moves to [136,183] GeV. We will show that if neutrino masses are produced by a seesaw mechanism, the previous Higgs boson mass windows are substantially narrowed.

The simplest extension of the SM Lagrangian, capable to account for small neutrino masses, is the seesaw one

$$\mathcal{L}_\nu = -\bar{\nu}_R \mathcal{M}_D \nu_L + \frac{1}{2} \bar{\nu}_R \mathcal{M}_M \bar{\nu}_R^T + \text{H.c.}, \quad (1)$$

where  $\mathcal{M}_D$  is the Dirac mass matrix [ $\mathcal{M}_D = (1/\sqrt{2})\langle\phi\rangle\mathbf{Y}_\nu$ , where  $\phi$  is the neutral component of the Higgs field and  $\mathbf{Y}_\nu$  is the matrix of the neutrino Yukawa couplings] and  $\mathcal{M}_M$  is the Majorana mass matrix for the right-handed neutrinos. This scenario is so economical that it seems fair to examine its ultimate implications, in particular its impact on the Higgs sector. For that purpose, it is convenient to start with the case where there is a hierarchy of left-handed neutrino masses,  $m_{\nu_1}^2 \ll m_{\nu_2}^2 \ll m_{\nu_3}^2$ , presumably inherited by a similar hierarchy

in the Dirac-Yukawa couplings. In this case, there is only one relevant generation of neutrinos for our purposes (the most massive one), so  $\mathcal{M}_D$  and  $\mathcal{M}_M$  become the single parameters  $m_D$  and  $M$ . As we will see the extension of the results to the general case is straightforward. Now, the two associated neutrino eigenvalues arising from Eq. (1) are

$$m_{\nu_{1,2}} = \frac{1}{2}(M \mp \sqrt{M^2 + 4m_D^2}). \quad (2)$$

For  $M \gg \langle\phi\rangle$ , we get  $m_{\nu_1} \approx m_D^2/M$ ,  $m_{\nu_2} \approx M$ ;  $\nu_1, \nu_2$  correspond essentially to the left- and right-handed neutrinos respectively. For energy scales below  $M$  we can integrate out the  $\nu_2$  neutrino, so  $\mathcal{L}_\nu$  effectively becomes

$$\mathcal{L}_\nu^{\text{eff}} = -\frac{1}{2} \nu_L^T m_{\nu_1} \nu_L + \text{H.c.} \equiv -\frac{1}{4} \kappa \nu_L^T \langle\phi\rangle^2 \nu_L + \text{H.c.} \quad (3)$$

where  $v = \langle\phi\rangle$  at the physical vacuum (i.e.  $v = 246$  GeV) and for convenience we have introduced the effective coupling  $\kappa$  ( $\kappa v^2 = 2m_D^2/M$  at the  $M$ -scale). For scales below  $M$ ,  $\kappa$  is a running parameter, whose beta function is essentially given by [6]

$$\beta_\kappa = \frac{1}{16\pi^2} (-3g_2^2 + 2\lambda + 6Y_t^2) \kappa, \quad (4)$$

where  $g_2, \lambda, Y_t$  are the  $SU(2)$  gauge coupling, the Higgs quartic coupling and the top Yukawa coupling respectively. Thus, for a given physical mass of  $\nu_1$  (to be identified with the low-energy value of  $\kappa v^2/2$ ) and for a given value of the Majorana mass,  $M$ , the Dirac mass  $m_D$  (and thus  $Y_\nu$ ) is unambiguously fixed.

The one-loop effective potential  $V(\phi)$  has the form

$$V = V_{SM} + \Delta V_\nu \quad (5)$$

where  $V_{SM}$  is the usual one-loop SM potential, consisting of the tree level part  $V_{tree} = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4 + \Omega$ , plus the ordinary radiative corrections dominated by the top Yukawa coupling,  $Y_t$ , and

$$\Delta V_\nu = -\frac{1}{32\pi^2} \left[ m_{\nu_1}^4 \log \frac{m_{\nu_1}^2}{\mu^2} + \theta_\nu m_{\nu_2}^4 \log \frac{m_{\nu_2}^2}{\mu^2} \right], \quad (6)$$

where  $m_{\nu_{1,2}}$  are given by Eq. (2) (note that they are functions of  $\phi$ ),  $\mu$  is the renormalization scale<sup>1</sup> and  $\theta_\nu \equiv \theta(\mu - M)$  [7,8] is a step  $\theta$ -function accounting for the threshold at the  $M$ -scale.

Above  $M$ ,  $Y_\nu$  runs with the scale, while the various SM parameters get contributions from  $Y_\nu$  to their beta-coefficients. The relevant ones are [9]

$$\begin{aligned} \beta_{Y_\nu} &= \frac{1}{16\pi^2} \theta_\nu \left[ 3Y_\nu^2 - \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 \right) + \frac{5}{2}Y_\nu^2 \right] Y_\nu \\ \beta_\lambda^{(\nu)} &= \frac{1}{16\pi^2} [4\theta_\nu(\lambda Y_\nu^2 - Y_\nu^4)], \quad \gamma_\phi^{(\nu)} = \frac{1}{16\pi^2} [\theta_\nu Y_\nu^2] \\ \beta_\Omega^{(\nu)} &= -\frac{1}{16\pi^2} [\theta_\nu M^4], \quad \beta_{m^2}^{(\nu)} = \frac{1}{16\pi^2} [4\theta_\nu M^2 Y_\nu^2] \\ \beta_{Y_i}^{(\nu)} &= \frac{1}{16\pi^2} [\theta_\nu Y_\nu^2 Y_i], \end{aligned} \quad (7)$$

where  $\beta_i^{(\nu)}$  is the contribution of  $Y_\nu$  to the  $\beta_i$  coefficient. The matching of the complete and the effective theory at  $M$  requires to introduce threshold corrections. In particular, the matching of  $V$  requires to introduce a threshold contribution below  $M$ :  $\Delta_{th}V = -(2/64\pi^2)[m_{\nu_2}^4 \log(m_{\nu_2}^2/M^2)]$ , whose expansion gives the threshold corrections to the  $m^2$  and  $\lambda$  parameters, namely  $\Delta_{th}m^2 = (1/16\pi^2)Y_\nu^2 M^2$ ,<sup>2</sup>  $\Delta_{th}\lambda = -(5/16\pi^2)Y_\nu^4$ .

Now we are prepared to compute the modification of the perturbativity and stability bounds on the SM Higgs boson mass. The perturbativity bound arises from the requirement that  $\lambda$  does not enter the non-perturbative regime below a certain scale  $\Lambda$ . If the SM is to be valid until  $M_{pl}$  or some high scale  $M_X$ ,  $\Lambda$  should be identified with those scales. Since for  $\mu < M$  the running of  $\lambda$  is as in the SM, the perturbativity bound for  $\Lambda < M$  remains the same. However, for  $\mu > M$  the contribution of the neutrinos to the  $\lambda$  running [which for large  $\lambda$  is dominated by the positive  $\lambda Y_\nu^2$  term in Eq. (7)] speeds up the increasing of  $\lambda$ , thus leading to a more stringent upper bound on  $M_H$ .

<sup>1</sup>More precisely,  $\mu^2 = e^{3/2}\bar{\mu}^2$ , where  $\bar{\mu}$  is the usual modified minimal subtraction ( $\overline{\text{MS}}$ ) renormalization scale.

<sup>2</sup>Note that the threshold correction for  $m^2$  is very sizeable, exhibiting an explicit implementation of the SM gauge hierarchy problem. Working within the SM framework, it is necessary to fine-tune the  $m^2$  at high energy in order to reproduce the usual low-energy physics. However, this conceptual shortcoming does not play any role in our analysis and the corresponding results, since the  $m^2\phi^2$  contribution to the effective potential is always negligible compared to the  $\lambda\phi^4$  one.

On the other hand, the stability bound on the Higgs boson mass arises from demanding that the potential does not develop an instability for large values of  $\phi$ . The instability arises from the fact that  $\lambda$  can be driven to negative values at large enough scales. Notice that to evaluate the potential for large values of  $\phi$  one has to plug a renormalization scale  $\mu \sim \phi$  in order to avoid large logarithms. Hence, if the  $\lambda\phi^4$  term, which is dominant for large  $\phi$ , becomes negative, so does the potential. If we demand this not to happen below a scale  $\Lambda$ , i.e. we require  $\lambda(\Lambda) \geq 0$ , this translates into a lower bound on  $M_H$ . Again, for  $\Lambda < M$  the values of  $\lambda$  are as in the SM, and thus the corresponding lower bound on  $M_H$ . However, for  $\Lambda > M$  the values of  $\lambda$  get modified by the neutrino contribution to  $\beta_\lambda$ . For small values of  $\lambda$  this contribution is dominated by the negative  $Y_\nu^4$  term in (7). Hence,  $\lambda$  is driven more rapidly to negative values and the lower bound becomes more stringent too.

For a more careful analysis of the stability bound, one has to consider not just the tree-level potential (which is in fact dominated by the  $\lambda\phi^4$  term), but also the radiative corrections. A practical way to include them [5] is to take  $\mu = \alpha\phi$  ( $\alpha \approx 1$  is always a correct choice) and then to extract the  $\sim \phi^4$  contributions from  $\Delta V^{1-loop}$ , neglecting (for  $\phi \gg M$ ) the  $M/\phi$  factors. These  $\phi^4$  contributions can be incorporated into an effective quartic parameter:

$$\begin{aligned} \lambda_{eff}(\mu < M) &= \lambda - \frac{1}{16\pi^2} \left[ 6Y_i^4 \log \frac{Y_i^2}{2} \right] \\ \lambda_{eff}(\mu > M) &= \lambda - \frac{1}{16\pi^2} \left[ 6Y_i^4 \log \frac{Y_i^2}{2} + 2Y_\nu^4 \log \frac{Y_\nu^2}{2} \right]. \end{aligned} \quad (8)$$

Notice that for  $\mu < M$  the  $\nu_1$  contribution (the only neutrino which is present) is negligible, as  $m_{\nu_1} \ll m_t$ . However, for  $\mu > M$  both neutrinos contribute in the same (non-negligible) amount. The stability bound reads, in this more accurate form, as  $\lambda_{eff}(\Lambda) \geq 0$ .

To summarize, the presence of the neutrinos strengthens both the upper and the lower bound on the SM Higgs boson mass, thus narrowing the Higgs window. The quantitative effect depends just on the value of the ‘‘left-handed’’ neutrino mass,  $m_{\nu_1}$ , and on the value of the Majorana mass,  $M$  (the Yukawa coupling,  $Y_\nu$ , can be extracted from those two in the way explained above).

Before presenting numerical results, let us extend the analysis to the case where all the neutrinos have similar masses, and therefore all of them are equally relevant. Then,  $\kappa$ ,  $\mathcal{M}_M$ ,  $\mathcal{M}_D$  and  $\mathbf{Y}_\nu$  are flavor matrices. The corresponding effective mass matrix for the left-handed neutrinos (i.e. the analogue to  $m_{\nu_1} \simeq m_D^2/M$  in the single neutrino case) is given by

$$\mathcal{M}_\nu = \mathcal{M}_D^T \mathcal{M}_M^{-1} \mathcal{M}_D. \quad (9)$$

The beta functions given in Eqs. (7) get slightly modified, depending now on the whole matrix  $\mathbf{Y}_\nu$  (the extension is quite trivial and can be found in [9]). Consequently, the precise results will depend in principle on the textures of the

$\mathcal{M}_D$  (or  $\mathbf{Y}_\nu$ ) and  $\mathcal{M}_M$  matrices. However, as we will see, in practice the form of the textures is not important in our analysis.

One can always choose a basis of  $\nu_{L_i}, \nu_{R_i}$  in which  $m_D$ , and thus  $\mathbf{Y}_\nu$ , is diagonal (in this basis the matrix of Yukawa couplings of the charged leptons is in general non-diagonal, but these couplings are negligible in our analysis). Then, the relevant formulas, Eqs. (6)–(8) become trivially extended to the 3-generation case [9]. If the physical neutrinos are quasi-degenerated (which, according to the observations must occur for  $m_{\nu_i} \gtrsim 0.1$  eV), it is hard to believe that this fact can naturally come from a diagonal  $\mathcal{M}_D$  with non-degenerated entries, compensated by a non-trivial structure of the  $\mathcal{M}_M$  matrix, see Eq. (9). One can arrange things in this way, but it is extremely artificial. (In this case, the largest Yukawa coupling would dominate all the equations, and we simply would be back to the case of a single relevant neutrino.) Therefore, one expects that both  $\mathcal{M}_D$  and  $\mathcal{M}_M$  have essentially degenerated eigenvalues,  $m_D$  and  $M$ , so that at the end of the day all the neutrinos have masses  $m_{\nu_i} \approx m_D^2/M$ .

On the other hand, it has been claimed in the literature [10] that if neutrino masses are to play an essential cosmological role, beside explaining the atmospheric and solar neutrino fluxes, the texture of the effective left-handed neutrino mass must be of the bimaximal mixing type

$$\mathcal{M}_\nu = m_\nu \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} \quad (10)$$

where  $m_\nu$  is the common neutrino mass. This texture produces three degenerate left-handed neutrinos with maximal mixing angles  $\theta_{12}$  and  $\theta_{23}$ . This is the texture we have used for our numerical analysis, although, as explained above, other textures will produce similar results. Notice that for our analysis we need not just the form of  $\mathcal{M}_\nu$ , but also the form of the matrix of the Yukawa couplings,  $\mathbf{Y}_\nu$ , and thus the form of  $\mathcal{M}_M$  [see Eq. (9)]. Our choice has been to take  $\mathbf{Y}_\nu$  proportional to the identity, i.e.  $\mathbf{Y}_\nu = Y_\nu \text{diag}(1,1,1)$ . As discussed above, this is a perfectly reasonable choice in this case. Then the form of the right-handed Majorana matrix is forced to be

$$\mathcal{M}_M^{-1} = \frac{1}{M m_\nu} \mathcal{M}_\nu. \quad (11)$$

It is straightforward to check that with these ansatz, the mass-squared eigenvalues of the three ‘‘left-handed’’ and the three ‘‘right-handed’’ neutrinos, say  $(m_{\nu_{1,2}}^{(i)})^2$ , with  $i = 1,2,3$ , are as in Eq. (2). Thus, we see that in this particularly well-motivated case the texture plays no significant role indeed.

Let us now present the results. Figure 1 shows the evolution of the Higgs window as a function of the scale  $\Lambda$  until which the theory is assumed to be valid. In order to illustrate the effect of the neutrinos we have chosen a typical case, namely  $m_\nu = 0.1$  eV (for one or the three generations of neu-

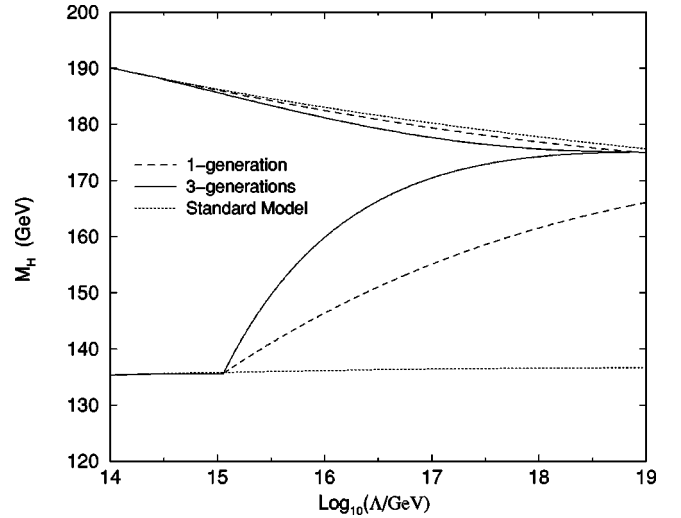


FIG. 1. The Higgs boson window as a function of the scale  $\Lambda$  for  $m_\nu = 0.1$  eV and  $M = 2.8 \times 10^{14}$  GeV.

trinos) and  $M \approx 3 \times 10^{14}$  GeV. The pure SM window is also shown to facilitate the comparison. We note that the strengthening of the stability (lower) bound is the main responsible of the substantial narrowing of the allowed window. Obviously, the effect is more important for larger  $\Lambda$ .

Figure 2 shows the variation of the Higgs window with the Majorana mass,  $M$  for just one relevant generation of massive neutrinos with  $m_\nu = 0.1$  eV (the other generations may be massive but hierarchically smaller). This case corresponds to the most conservative scenario concerning the effect of neutrinos on the Higgs window. We have represented the cases  $\Lambda = M_{\text{pl}}$  (continuous lines) and  $\Lambda = 10^{16}$  GeV (dashed lines). The last one basically gives the conservative bounds if new physics [in particular grand unified theory (GUT) physics] appears at  $\Lambda \approx 10^{16}$  GeV. In all the cases, the values of the bounds for ‘‘low’’ Majorana masses ( $M \lesssim 10^{13}$  GeV) coincide with those of the pure SM without

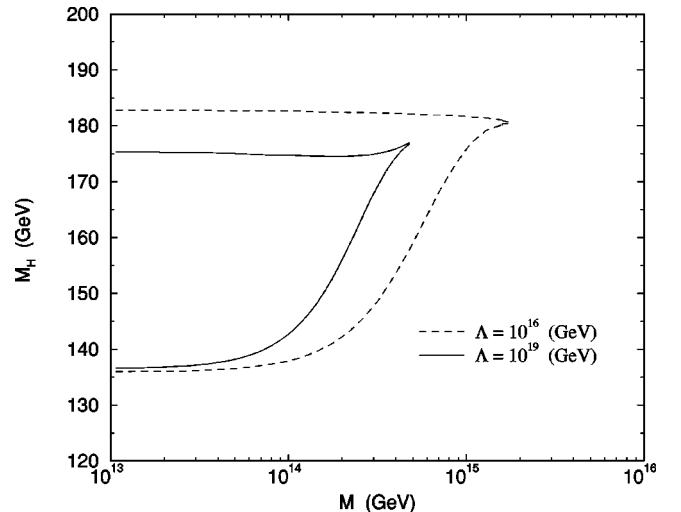


FIG. 2. The Higgs boson window as a function of the Majorana mass,  $M$  for  $\Lambda = M_{\text{pl}}$  (continuous lines) and  $\Lambda = 10^{16}$  GeV (dashed lines) for the case of just one massive neutrino with  $m_\nu = 0.1$  eV.

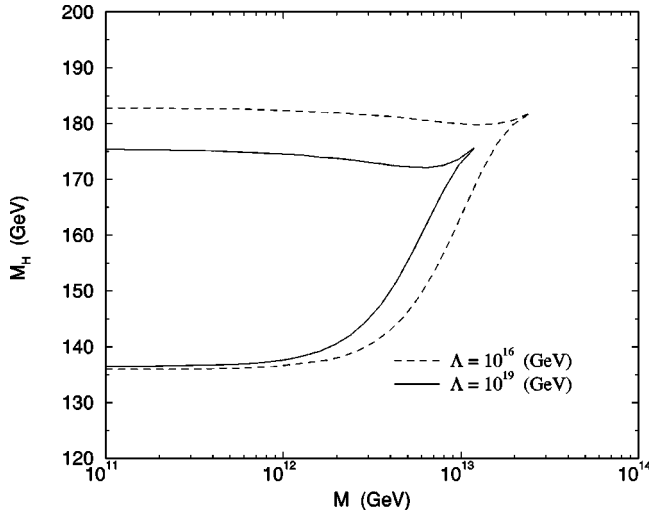


FIG. 3. The same as Fig. 1, but with three generations of massive neutrinos with  $m_\nu = 2$  eV.

neutrinos. Figure 3 is analogous to Fig. 2, but for three generations of neutrinos, with degenerate masses  $m_\nu = 2$  eV. This case corresponds to the scenario where the effect of the neutrinos is maximized. It is apparent from Figs. 2 and 3 that above a certain value of the Majorana mass the Higgs window closes up, disappearing, which puts an upper bound on  $M$ . For quasi-degenerate neutrinos this upper bound is well below  $M_{\text{GUT}}$ , even for  $\Lambda \approx M_{\text{GUT}}$ , which shows that even in the presence of GUT physics the bounds are relevant for model building. This complete closing of the window could seem paradoxical, since one expects that for some fine-tuned initial value of  $\lambda$  (and thus of  $M_H$ ) the evolution of  $\lambda(\mu)$  for large  $\mu$  will be just between the two bounds, say  $0 \leq \lambda(\mu) \leq 4\pi$ . So one would expect, for large values of  $M$ , an extremely narrow window, but a window after all. The fact that actually forbids that window is that for large values of  $M$  the neutrino Yukawa couplings themselves develop a Landau pole below  $\Lambda$ . Alternatively, we can set  $Y_\nu$  at the Landau pole at  $M_{\text{Pl}}$  [i.e.  $Y_\nu(M_{\text{Pl}}) \gg 1$ ] and evaluate the corresponding low energy value of  $m_\nu$ , through the renormalization group equations (RGE) of  $Y_\nu$  and  $\kappa$ , for a certain value of the Majorana mass  $M$ . This ‘‘infrared fixed point’’ value, say  $m_\nu^{\text{IR}}$ , represents an upper bound for the neutrino mass. The dependence of  $m_\nu^{\text{IR}}$  on  $M$  is illustrated in Fig. 4 for one and three generations of massive neutrinos. Due to the dependence of the  $\kappa$  renormalization group equation (RGE) on  $\lambda$ , the value of  $m_\nu^{\text{IR}}$  presents a slight dependence on the value of  $M_H$ , as is shown in the figure.

In conclusion, we have shown that if neutrino masses are produced by a see-saw mechanism, the SM prediction for the Higgs boson mass window (defined by upper (perturbativity)

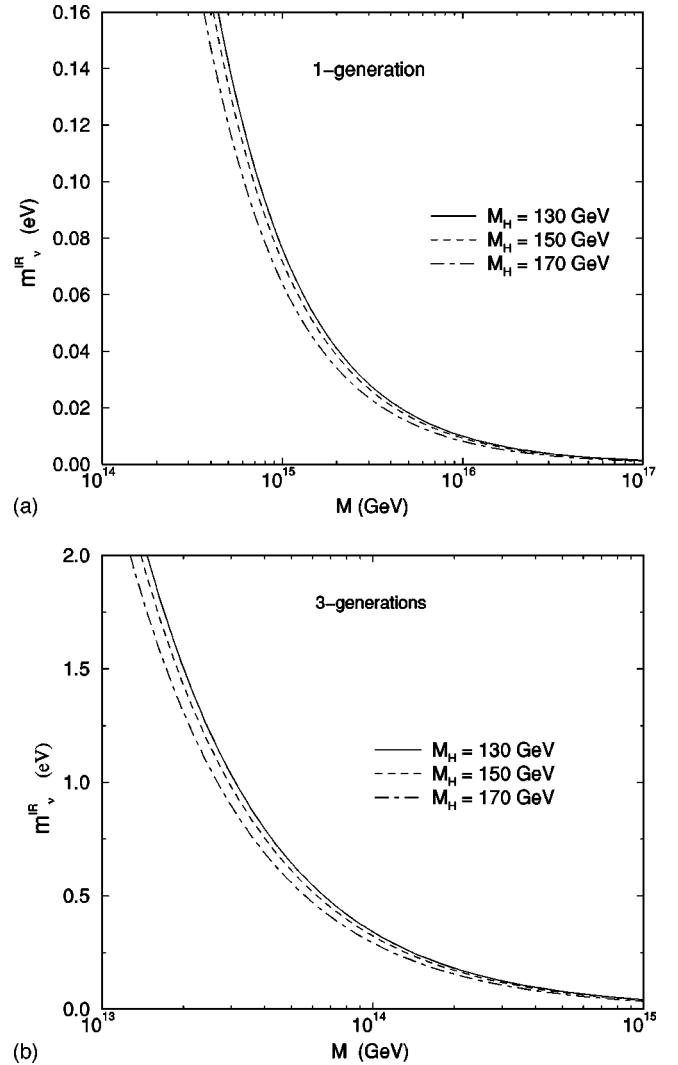


FIG. 4. Upper bound on the neutrino mass,  $m_\nu^{\text{IR}}$ , vs the Majorana mass  $M$  for one and three generations of massive neutrinos (left and right panels respectively).

and lower (stability) bounds) is substantially affected in an amount that depends on the value of the Majorana mass for the right-handed neutrinos,  $M$ . Actually, for values of  $M$  above a certain value, the Higgs window closes, setting an upper bound on  $M$ . This varies from  $10^{13}$  GeV for three massive neutrinos with  $m_\nu \approx 2$  eV to  $5 \times 10^{14}$  GeV for just one relevant generation with  $m_\nu \approx 0.1$  eV. We have also discussed a second (slightly weaker) upper bound on  $M$ , coming from the requirement that the neutrino Yukawa couplings do not develop a Landau pole. The whole analysis and results are practically independent of the details of the model (i.e. the particular structure of the neutrino mass matrices).

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