PHYSICAL REVIEW D, VOLUME 62, 051501(R)

Infrared behavior of the gluon propagator on a large volume lattice

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(Received 22 February 2000; published 14 July 2000)

The first calculation of the gluon propagator using an $\mathcal{O}(a^2)$ improved action with the corresponding $\mathcal{O}(a^2)$ improved Landau gauge fixing condition is presented. The gluon propagator obtained from the improved action and improved Landau gauge condition is compared with earlier unimproved results on similar physical lattice volumes of $3.2^3\times 6.4$ fm^4. We find agreement between the improved propagator calculated on a coarse lattice with lattice spacing $a=0.35$ fm and the unimproved propagator calculated on a fine lattice with spacing $a=0.10$ fm. This motivates us to calculate the gluon propagator on a coarse large-volume lattice $5.6^3\times 11.2$ fm^4. The infrared behavior of previous studies is confirmed in this work. The gluon propagator is enhanced at intermediate momenta and suppressed at infrared momenta. Therefore the observed infrared suppression of the Landau gauge gluon propagator is not a finite volume effect.

PACS number(s): 12.38.Gc, 11.15.Ha, 12.38.Aw, 14.70.Dj

I. INTRODUCTION

There has been considerable interest in the infrared behavior of the gluon propagator, as a probe into the mechanism of confinement and as input for other calculations. Lattice gauge theory is an excellent means to study such nonperturbative behavior. See, for example, Ref. [1] for a recent survey.

The infrared part of any lattice calculation may be affected by the finite volume of the lattice. Larger volumes mean either more lattice points (with increased computational cost) or coarser lattices (with corresponding discretization errors). The desire for larger physical volumes thus provides strong motivation for using improved actions. Improved actions have been shown to reduce discretization effects [2], although some concerns have been expressed that coarse lattices may miss important instanton physics.

In this study, no change is seen in the infrared gluon propagator, even with a lattice spacing as coarse as 0.35 fm. We find the gluon propagator to be less singular than $1/q^2$ in the infrared. Our results suggest that the gluon propagator is infrared finite, although more data is needed in the far infrared to be conclusive. This behavior is similar to that observed in three-dimensional SU(2) studies [3].

II. $\mathcal{O}(a^2)$ IMPROVEMENT

The $\mathcal{O}(a^2)$ tadpole-improved action is defined as

$$S_G=\frac{5\beta}{3}\sum_{\rm pl}\ \mathrm{Re\,Tr}[1-U_{\rm pl}(x)]-\frac{\beta}{12u_0^2}\times\sum_{\rm rect}\mathrm{Re\,Tr}[1-U_{\rm rect}(x)], \tag{2.1}$$

where the operators $U_{\rm pl}(x)$ and $U_{\rm rect}(x)$ are defined

$$U_{\rm pl}(x)=U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x) \tag{2.2}$$

and

$$U_{\rm rect}(x)=U_\mu(x)U_\nu(x+\hat{\mu})U_\nu(x+\hat{\nu}+\hat{\mu})U_\mu^\dagger(x+2\hat{\nu})\times U_\nu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)+U_\mu(x)U_\mu(x+\hat{\mu})\times U_\nu(x+2\hat{\mu})U_\mu^\dagger(x+\hat{\mu}+\hat{\nu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x). \tag{2.3}$$

The link product $U_{\rm rect}(x)$ denotes the rectangular 1×2 and 2×1 plaquettes. For the tadpole (mean-field) improvement parameter [4] we use the plaquette measure

$$u_0=(\tfrac{1}{3}\mathrm{Re\,Tr}\langle U_{\rm pl}\rangle)^{1/4}. \tag{2.4}$$

Equation (2.1) reproduces the continuum action as $a\to 0$, provided that β takes the standard value of $6/g^2$. Note that our $\beta=6/g^2$ differs from that used in [2,5,6]. Multiplication of our β in Eq. (2.1) by a factor of 5/3 reproduces their definition. $\mathcal{O}(g^2a^2)$ corrections to this action are estimated to be of the order of two to three percent [2].

Gauge configurations are generated using the Cabibbo-Marinari [7] pseudo-heat-bath algorithm with three diagonal SU(2) subgroups. The mean link, u_0, is averaged every 10 sweeps and updated during thermalization. Representative gauge fields are selected after 5000 thermalization sweeps.

Gauge fixing on the lattice is achieved by maximizing a functional, the extremum of which implies the gauge fixing condition. The usual Landau gauge fixing functional [8] is

$$\mathcal{F}_1^G[\{U\}]=\sum_{\mu,x}\frac{1}{2u_0}\mathrm{Tr}\{U_\mu^G(x)+U_\mu^G(x)^\dagger\}, \tag{2.5}$$

where

$$U_\mu^G(x)=G(x)U_\mu(x)G(x+\hat{\mu})^\dagger \tag{2.6}$$

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and

$$G(x)=\exp\left\{-i\sum_a \omega^a(x)T^a\right\}. \tag{2.7}$$

A maximum of Eq. (2.5) implies that $\Sigma_\mu\partial_\mu A_\mu=0$ up to $\mathcal{O}(a^2)$. To ensure that gauge dependent quantities are also $\mathcal{O}(a^2)$ improved, we implement the analogous $\mathcal{O}(a^2)$ improved gauge fixing functional

$$\mathcal{F}^G_{\rm Imp}=\sum_{\mu,x}\frac{1}{2u_0}\mathrm{Tr}\left\{\frac{4}{3}U^G_\mu(x)-\frac{1}{12u_0}U^G_\mu(x)\times U^G_\mu(x+\hat{\mu})+\mathrm{H.c.}\right\} \tag{2.8}$$

as described in [9]. We employ a conjugate gradient, Fourier accelerated gauge fixing algorithm [10] optimally designed for parallel machines.

III. THE LANDAU GAUGE GLUON PROPAGATOR

The gauge links $U_\mu(x)$ are expressed in terms of the continuum gluon fields as

$$U_\mu(x)=\mathcal{P}\exp\left(ig_0\int_x^{x+\hat{\mu}}A_\mu(z)dz\right), \tag{3.1}$$

where $\mathcal{P}$ denotes path ordering. From this, the dimensionless lattice gluon field $A_\mu(x)$ may be obtained via

$$A_\mu(x+\hat{\mu}/2)=\frac{1}{2ig_0}[U_\mu(x)-U^\dagger_\mu(x)]-\frac{1}{6ig_0}\times\mathrm{Tr}[U_\mu(x)-U^\dagger_\mu(x)], \tag{3.2}$$

accurate to $\mathcal{O}(a^2)$. $\mathcal{O}(a^2)$ improved gluon field operators have also been investigated. While the infrared behavior is unaffected by the improvement, the ultraviolet behavior suffers due to the extended nature of an improved operator. These results will be discussed in detail elsewhere [11].

We calculate the gluon propagator in coordinate space

$$D^{ab}_{\mu\nu}(x,y)\equiv\langle A^a_\mu(x)\,A^b_\nu(y)\rangle, \tag{3.3}$$

using Eq. (3.2). To improve statistics, we use translational invariance and calculate

$$D^{ab}_{\mu\nu}(y)=\frac{1}{V}\left\langle\sum_x A^a_\mu(x)A^b_\nu(x+y)\right\rangle. \tag{3.4}$$

In this paper we focus on the scalar part of the propagator,

$$D(y)=\frac{1}{N_d-1}\sum_\mu\frac{1}{N_c^2-1}\sum_a D^{aa}_{\mu\mu}(y), \tag{3.5}$$

where $N_d=4$ and $N_c=3$ are the number of dimensions and colors. This is then Fourier transformed into momentum space

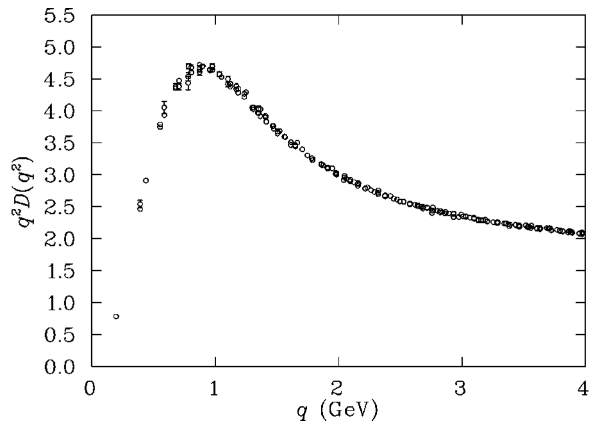

FIG. 1. Gluon propagator from 75 standard, Wilson configurations, on a $32^3\times64$ lattice with spacing $a=0.10$ fm.

$$D(q)=\sum_y e^{i\hat{q}\cdot y}D(y), \tag{3.6}$$

where the available momentum values, $\hat{q}$, are given by

$$\hat{q}_\mu=\frac{2\pi n_\mu}{aL_\mu},\quad n_\mu\in\left(-\frac{L_\mu}{2},\frac{L_\mu}{2}\right], \tag{3.7}$$

and L_μ is the length of the box in the μ direction. In the continuum, the scalar function $D(q^2)$ is related to the Landau gauge gluon propagator via

$$D^{ab}_{\mu\nu}(q)=\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\delta^{ab}D(q^2). \tag{3.8}$$

The bare, dimensionless lattice gluon propagator, $D(qa)$, is related to the renormalized continuum propagator, $D_R(q;\mu)$ via

$$a^2D(qa)=Z_3(\mu,a)D_R(q;\mu), \tag{3.9}$$

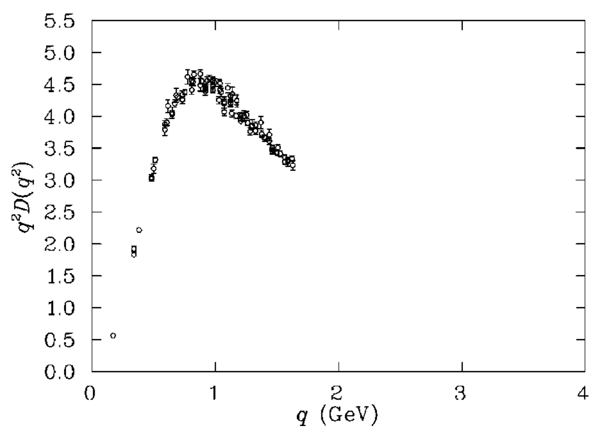

FIG. 2. Gluon propagator from 75 tree-level improved configurations on a $10^3\times20$ lattice with spacing $a=0.35$ fm, and a physical volume of $3.5^3\times7$ fm^4.

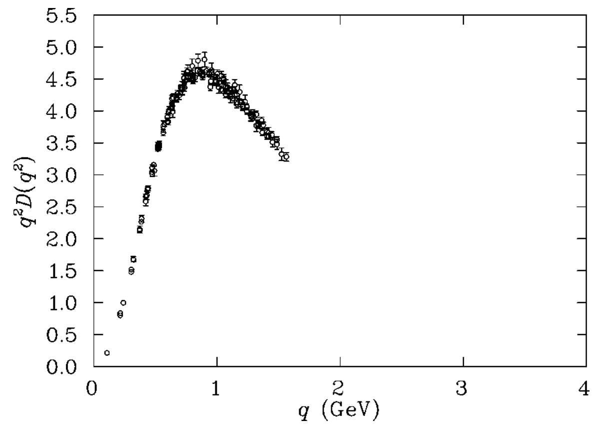

FIG. 3. Gluon propagator from 75 tree-level improved configurations, on a $16^3\times32$ lattice with spacing $a=0.35$ fm, and a physical volume of $5.6^3\times11.2$ fm^4.

where the renormalization constant, $Z_3(\mu,a)$ is determined by imposing a renormalization condition at some chosen renormalization scale, μ, e.g.,

$$D_R(q)|_{q^2=\mu^2}=\frac{1}{\mu^2}. \tag{3.10}$$

This means that there is an undetermined multiplicative renormalization factor, $Z_3(a)$, on each of our propagators. Since our purpose is to compare our two improved, coarse lattices with the unimproved, finer one, it is sufficient to consider only their relative renormalizations. We have slightly rescaled the improved propagators so as to provide a reasonable match with the old one. The relevant quantity is

$$Z_3(0.10)/Z_3(0.35)=1.02. \tag{3.11}$$

IV. ANALYSIS OF LATTICE ARTIFACTS

To emphasize the nonperturbative behavior of the gluon propagator, we divide the propagator by the tree-level result of lattice perturbation theory. Hence Figs. 1, 2, and 3 are plotted with $q^2D(q^2)$ on the y axis, which is expected to approach a constant up to logarithmic corrections as $q^2\to\infty$. Here

$$q_\mu\equiv\frac{2}{a}\sin\frac{\hat{q}_\mu a}{2}. \tag{4.1}$$

To reduce ultraviolet noise resulting from the lattice discretization, the available momenta are cut half way into the Brillouin zone, that is

$$q_{\max}=\frac{\pi}{2a}. \tag{4.2}$$

All figures have a cylinder cut imposed upon them, i.e., all momenta must lie within a cylinder of radius two spatial momentum units centered about the lattice diagonal. The propagators are plotted in physical units, which we obtain from the string tension [12] with $\sqrt{\sigma}=440$ MeV.

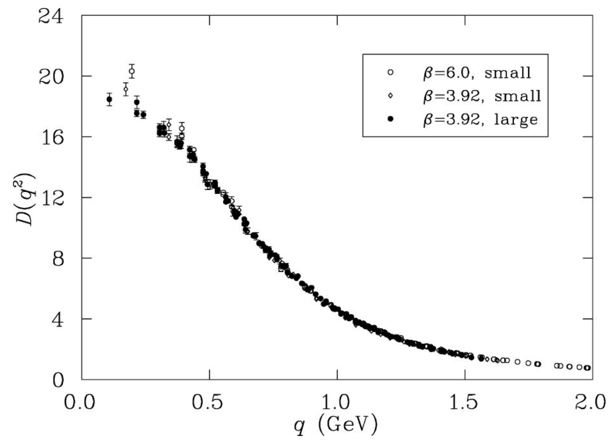

FIG. 4. Comparison of the gluon propagator on the three different lattices. The volumes are $3.2^3\times6.4$ fm^4, $3.5^3\times7.0$ fm^4, and $5.6^3\times11.2$ fm^4.

Figure 1 is reproduced from Ref. [13], where the standard Wilson action is used. The propagator is calculated on a $32^3\times64$ lattice at $\beta=6.0$, which corresponds to a lattice spacing of 0.1 fm. This propagator produces the correct asymptotic behavior and a detailed analysis shows that the anisotropic finite volume errors are small. However, it was impossible to rule out isotropic finite volume artifacts.

We use the improved action described above to calculate the gluon propagator on a small ($10^3\times20$), coarse ($a=0.35$ fm) lattice, which is shown in Fig. 2. Despite the coarse lattice spacing we see that it reproduces the infrared behavior of Fig. 1.

Finally, we calculate the propagator on a $16^3\times32$ lattice, at the same β providing $a=0.35$ fm. This corresponds to a very large physical volume of $5.6^3\times11.2$ fm^4. Figure 3 illustrates the results. These results largely agree with the previous two calculations of the propagators. The behavior of the gluon propagator is not changed by changing the volume.

In Fig. 4 we have superimposed the gluon propagators for all three lattices. Here we plot $D(q^2)$ on the y axis to allow an alternative examination of the most infrared momenta. The points at $\gtrsim$350 MeV are very robust with respect to volume. Only the very lowest momenta points show signs of finite volume effects. With more volumes it should be possible to extrapolate to the infinite volume limit. We see that the propagator, at the very lowest momentum points, decreases as the volume increases. This strongly suggests an infrared finite propagator.

V. CONCLUSIONS

The gluon propagator has been calculated on a coarse lattice with an $\mathcal{O}(a^2)$ improved action, in the $\mathcal{O}(a^2)$ improved Landau gauge. The infrared behavior of this propagator is consistent with that of a previous study on a finer lattice with an unimproved action, but comparable volume. The propagator was then calculated on another improved lattice with the same spacing, but larger volume. The increase in volume left the propagator largely unchanged. In particular, it has been shown that the turnover observed in [13] is not a finite volume effect.

With more lattices it may be possible to extrapolate to infinite volume, but from this study we can only make tentative conclusions. We have ruled out the q^{-4} behavior popular in Dyson-Schwinger studies, and any infrared singularity appears to be unlikely. An infrared finite propagator is most plausible. The gluon propagator would need to drop rapidly for momenta below ~350 MeV in order to vanish as suggested by Zwanziger [14] and others. Even larger volume lattices will be needed to study this possibility. The possible effects of lattice Gribov copies remains a very interesting question and we plan to study this in the near future.

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council and by grants of supercomputer time on the CM-5 made available through the South Australian Center for Parallel Computing. The authors also wish to thank J-I. Skullerud for his useful comments.

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