Consistency of the Born approximation for the spin-1/2 Aharonov-Bohm scattering

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The relativistic scattering of a spin-1/2 particle from an infinitely long solenoid is considered in the framework of covariant perturbation theory. The first order term agrees with the corresponding term in the series expansion of the exact amplitude, and second order term vanishes, thus proving that the Born approximation is consistent.

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I. INTRODUCTION

It is by now well known that when particles of conventional statistics are coupled to a pure Chern-Simons (CS) gauge field, this field creates an Aharonov-Bohm (AB) like interaction which converts the particles to charge-flux-tube composites [1]. Somewhat later, it was shown, in the context of a Galilean field theory of scalar fields minimally coupled to a pure CS field, that one can approach the problem of calculating an arbitrary scattering process by restricting consideration to an *N*-body sector, allowing one to derive a Schrödinger equation for the *N*-body problem with each pair interacting as zero radius flux tubes. This has led to the claim that field theory, sector by sector, is formally equivalent to a conventional Schrödinger equation [2]. Specifically, in the two particle sector of this equivalent field theory, one gets a Schrödinger equation similar to the AB equation [3].

These developments brought back the long-standing issue of failure of the quantum mechanical perturbation theory for the Aharonov-Bohm (AB) scattering amplitude [4]. The failure of the Born approximation, for instance, is known to be due to the fact that the lowest partial wave amplitude satisfies an integral equation whose interaction term is quadratic in terms of the flux parameter. As the exact lowest partial wave contribution to the scattering amplitude is known to be linear in this parameter, then, it is absent in the first order Born approximation.

There have been several attempts to solve this problem for the spinless case in the context of direct AB scattering [5], anyon physics [6], and scalar Galilean Chern-Simons (CS) gauge field theory [7–9]. For instance, in Ref. [7], it was shown, through a perturbative calculation of the two-particle scattering amplitude, up to one-loop order, that this amplitude is nonrenormalizable, unless a contact interaction is introduced, which however for a given strength of the interaction (critical value corresponding to the self-dual limit) reduces to the same order term of the series expansion of the exact quantum mechanical amplitude. The same procedure is generalized to the non-Abelian case with similar conclusions in the second work of Ref. [7]. One should note that before the introduction of the contact interaction, the failure of the naive perturbation expansion of the Galilean CS field theory (namely the first Born term for *s* wave is wrong, while the second Born term is infinite) is very similar to that of the Born series in direct AB scattering. As the exact AB amplitude posseses scale invariance, it is natural that the agreement is obtained only after the introduction of contact interaction whose coupling strength has the critical value for which scale invariance is restored. This scale invariance at the critical coupling is explicitly checked up to three loops in Ref. [8], and up to all orders in Ref. [9].

The Born approximation problem for the spinless case was adressed from a more general point of view in Ref. [10] and Ref. [11] questioning whether the exact (nonperturbative) quantum mechanical AB amplitude can be reproduced order-by-order perturbatively in the framework of scalar Galilean CS field theory. Reference [10] concludes that the full agreement is obtained if the renormalized strength of the contact interaction is chosen to be related to the self-adjoint extension parameter, for fixed renormalization scale. However, the conclusion of more recent work [11] is not in full agreement with that of Ref. [10]. They show that the full agreement can be obtained only in some special regimes. Thus, we see that the general problem in the context of Galilean scalar field theory is not satisfactorily settled yet.

In Ref. [12], it was shown that if one starts from the relativistic scalar gauge field theory of the CS interaction, one finds a renormalizable one-loop scattering amplitude which remains so in the nonrelativistic limit as well, thus reproducing the correct series expansion of the exact quantum mechanical expression without the need to introduce a contact interaction term. It is not clear yet whether the issue raised in Ref. [10] and Ref. [11] would be relevant for the relativistic field theories. Obviously there are some fundamental differences between the nonrelativistic and the relativistic cases. For instance, in the nonrelativistic case, the necessity of a cutoff is not a relic of some unknown ultraviolet physics, but rather an artifact of the perturbative methods used. This is in contrast with the conventional wisdom on renormalization, whose natural habitat is the relativistic field theories.

AB scattering of spin-1/2 particles from an infinitely long solenoid was considered in the context of Dirac equation formalism in Ref. [13], and using covariant perturbation theory in Ref. [14]. In these works, it was shown that Born approximation works, that is, it agrees with the corresponding term in the series expansion of the exact amplitude. The agreement obtained in the framework of Dirac equation is not surprising at all. Because the Dirac Hamiltonian is linear in momenta, and the corresponding integral equation determining the lowest partial wave amplitude involves a term linear in flux parameter.

The spin-1/2 AB problem was also considered in the framework of equivalent Galilean CS gauge field theory in Ref. [15] and Ref. [16] from different perspectives. In these works the consistency of the perturbative treatment was checked up to one-loop order. As the exact amplitude is proportional to $\sin\pi\alpha$ (with $\alpha=-e\phi/2\pi$, and ϕ is the magnetic flux carried by the solenoid), the series expansion of this term contains terms of order $O(\alpha)$, $O(\alpha^3)$, . . . ; that is $O(\alpha^2)$ is missing. Thus a complete check of the consistency of perturbative approach, not only should get agreement on $O(\alpha)$ terms, but also should show the vanishing of $O(\alpha^2)$ terms (one-loop terms in the language of the field theory).

In Ref. [15], it was shown that the two-particle sector of the Galilean field theory again leads to an AB-like equation. Then, the two-particle scattering amplitude is computed up to one-loop order. The tree contribution $[O(\alpha)]$ agrees with the corresponding term in the series expansion of the exact amplitude; the one-loop contribution $[O(\alpha^2)]$ is finite and vanishes. This completes the check of consistency of the Born approximation to lowest order, in the (sector by sector) equivalent field theory framework.

Encouraged by the results of Ref. [15] and Ref. [16] in the Galilean field theory framework, it is aimed in this work to carry out the second order analysis in direct version of the problem, namely the relativistic scattering of spin-1/2 particles from an infinitely long solenoid, and check the consistency of the Born approximation fully, by demonstrating that $O(\alpha^2)$ contribution to the scattering amplitude in the framework of covariant perturbation theory vanishes. We will show that this is indeed what happens.

We would like to note that the subtleties pointed out in Ref. [10] and Ref. [11] for the spinless case are natually overcome in the relativistic case considered in this work. This does not create any difficulty in establishing parallelism with the results obtained in Ref. [15] in the context of Galilean CS field theory. Because it was already shown in Ref. [15] that, in contrast with the crucial role played by the contact interaction in the scalar case, the contribution of the Pauli term formally corresponding to the contact interaction (produced in the nonrelativistic limit of the fermionic CS gauge field theory with given coupling strength) to one-loop diagrams are finite and null, thanks to the statistics.

This paper is organized as follows: In Sec. II, we briefly review the results of Ref. [14] for the general discussion of the Helicity conservation. In Sec. III, we review the covariant perturbation theory approach to lowest order for the problem under consideration. In Sec. IV, the $O(\alpha^2)$ contribution to the scattering amplitude is computed; and it is shown that this contribution vanishes. Sec. V is devoted to the discussion of the results.

II. HELICITY CONSERVATION AND THE EXACT SCATTERING AMPLITUDE

The basic starting point of Ref. [14] is the well known observation that the helicity of a spin-1/2 particle is unchanged by a time-independent magnetic field [17].

Defining the Helicity eigenstates in the initial and final states as $|\pm\rangle_{i,f}$, the first observation is that $|\pm\rangle_i\to|\pm\rangle_f$ transitions proceed with unit probability in the Helicity space. Denoting the scattering matrix by S this reads as

$$\begin{aligned}|{}_f\langle\pm|S|\pm\rangle_i|^2&=1,\\ |{}_f\langle\pm|S|\mp\rangle_i|^2&=0.\end{aligned}\tag{1}$$

Thus, the differential cross-section for $|\pm\rangle_i\to|\pm\rangle_f$ per unit length is determined by the phase space only, and thus equal to the unpolarized cross section.

We next consider the scattering from an initial state polarized along the direction of an arbitrary unit vector $\hat{n}$ to a final state moving along θ, in which the beam is polarized again in the same $\hat{n}$ direction. Denoting the spherical angles of $\hat{n}$ with respect to the initial beam axis (chosen as x axis) by (θ',φ') these states are given as

$$\begin{aligned}|i(\vec{p}_i,\hat{n})\rangle&=\cos\frac{\theta'}{2}e^{-i\varphi'/2}|+\rangle_i+\sin\frac{\theta'}{2}e^{i\varphi'/2}|-\rangle_i,\\
|f(\vec{p}_f,\hat{n})\rangle&=\left(\cos\frac{\theta}{2}\cos\frac{\theta'}{2}e^{-i\varphi'/2}+\sin\frac{\theta}{2}\sin\frac{\theta'}{2}e^{i\varphi'/2}\right)|+\rangle_f\\
&\quad+\left(\cos\frac{\theta}{2}\sin\frac{\theta'}{2}e^{i\varphi'/2}-\sin\frac{\theta}{2}\cos\frac{\theta'}{2}e^{-i\varphi'/2}\right)|-\rangle_f.\end{aligned}\tag{2}$$

Using Eq. (2), one readily gets

$$\langle f|S|i\rangle=\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\sin\theta'\sin\varphi'.\tag{3}$$

Thus the polarized cross section per unit length of the solenoid is obtained as

$$\frac{d\sigma}{d\theta}=\left(1-(\hat{n}\times\hat{z})^2\sin^2\frac{\theta}{2}\right)\left(\frac{d\sigma}{d\theta}\right)_{unpol}.\tag{4}$$

Here $\hat{z}$ is the unit vector in the direction of the solenoid. Thus the cross section differs from the unpolarized case (or the spinless case) when the spin of the particle has components in the scattering plane(chosen here as x-y plane).

III. COVARIANT PERTURBATION THEORY: FIRST ORDER (BORN APPROXIMATION)

The purpose of this section is to show that Born approximation reproduces the correct result, that is it agrees with the corresponding terms in the series expansion of the exact amplitude.

The S-matrix element for a spin-1/2 particle scattering from an external electromagnetic field to lowest order is given by

$$S_{fi}^{(1)}=\int d^4z\,\overline{\psi}_f(z)(ie\gamma_\mu A^\mu(z))\psi_i(z)\tag{5}$$

where in the Bjorken-Drell convention

$$\psi_i(z)=\sqrt{\frac{m}{E_i V}}u(p_i,s_i)e^{-ip_{i\mu}z^\mu},$$

and

$$\psi_f(z)=\sqrt{\frac{m}{E_f V}}u(p_f,s_f)e^{-ip_{f\mu}z^\mu}. \tag{6}$$

The vector potential of the solenoid, taken along the third axis, in the Coulomb gauge $\vec{\nabla}\cdot\vec{A}=0$ is given as

$$A_1(z)=-\frac{\phi}{2\pi}\frac{z_2}{z_1^2+z_2^2},$$

$$A_2(z)=-\frac{\phi}{2\pi}\frac{z_1}{z_1^2+z_2^2}, \tag{7}$$

$$A_3(z)=A_0(z)=0$$

where ϕ is the magnetic flux carried by the solenoid. Denoting $\vec{q}=\vec{p}_f-\vec{p}_i$, and carrying out the z integrals, we find

$$S_{fi}^{(1)}=\frac{4\pi^2}{V}(me\phi)\,\delta(E_f-E_i)\,\delta(p_{f3}-p_{i3})\times\frac{\overline{u(f)}(\gamma^2q_1-\gamma^1q_2)u(i)}{\sqrt{E_fE_i}(q_1^2+q_2^2)}. \tag{8}$$

As the initial beam is in the first direction ($p_{i3}=0$), denoting $t=\overline{u(f)}(\gamma^2q_1-\gamma^1q_2)u(i)$, the differential cross section per unit solenoid length, to this order, is given as

$$\left(\frac{d\sigma}{d\theta}\right)^{Born}=\frac{m^2e^2\phi^2}{2\pi|\vec{p}_i|(q_1^2+q_2^2)^2}|t|^2 \tag{9}$$

with $|\vec{p}_i|=|\vec{p}_f|=k$ and $E_i=E_f$, as imposed by the δ functions. We can proceed in two ways: (a) we can sum over final polarizations, and average over the initial ones to get the unpolarized cross section by direct use of Dirac matrix algebra

$$\left(\frac{d\sigma}{d\theta}\right)^{Born}=\frac{e^2\phi^2}{8\pi k\sin^2\theta/2} \tag{10}$$

where $\vec{p}_i=k\hat{x}$. (b) we can compute the polarized amplitude, using the explicit expressions of the Dirac spinors for the polarized initial and final electrons:

$$u(i)=\cos\frac{\theta'}{2}e^{-i\varphi'/2}u_+(i)+\sin\frac{\theta'}{2}e^{i\varphi'/2}u_-(i)$$

$$u(f)=\left(\cos\frac{\theta}{2}\cos\frac{\theta'}{2}e^{-i\varphi'/2}+\sin\frac{\theta}{2}\sin\frac{\theta'}{2}e^{i\varphi'/2}\right)u_+(f)+\left(\cos\frac{\theta}{2}\sin\frac{\theta'}{2}e^{i\varphi'/2}-\sin\frac{\theta}{2}\cos\frac{\theta'}{2}e^{-i\varphi'/2}\right)\times u_-(f) \tag{11}$$

where

$$u_+(i)=N_i\begin{pmatrix}1\\0\\\mu_i\\0\end{pmatrix},\quad u_-(i)=N_i\begin{pmatrix}0\\1\\0\\-\mu_i\end{pmatrix},$$

$$N_i=\sqrt{\frac{E_i+m}{2m}},\quad \mu_i=\frac{|\vec{p}_i|}{E_i+m},$$

$$u_+(f)=N_f\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\\ \mu_f\cos\frac{\theta}{2}\\ \mu_f\sin\frac{\theta}{2}\end{pmatrix}, \tag{12}$$

$$u_-(f)=N_f\begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\\ \mu_f\sin\frac{\theta}{2}\\ -\mu_f\cos\frac{\theta}{2}\end{pmatrix},$$

$$N_f=\sqrt{\frac{E_f+m}{2m}},\quad \mu_f=\frac{|\vec{p}_f|}{E_f+m}.$$

Using Eqs. (11) and (12), we can compute t, and find

$$t=-\frac{2k^2}{m}\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\sin\theta'\sin\varphi'\right). \tag{13}$$

Substituting Eq. (13) in Eq. (9), we get

$$\left(\frac{d\sigma}{d\theta}\right)^{Born}_{pol}=\left(\frac{d\sigma}{d\theta}\right)^{Born}_{unpol}\left(1-(\hat{n}\times\hat{z})^2\sin^2\frac{\theta}{2}\right). \tag{14}$$

Thus, Born approximation indeed works in the polarized case. The scattering amplitude (and thus the cross section) is effected by the same expression in the Born approximation as in case of the exact amplitude. However this does not constitute a complete check of the consistency of the Born approximation in the relativistic spin-1/2 AB effect yet. As the exact amplitude is proportional to $\sin\pi\alpha$, a full consistency would require that the $O(\alpha^2)$ contribution to the scattering amplitude should vanish; and this is what we will check next.

IV. COVARIANT PERTURBATION THEORY: SECOND ORDER

The S matrix in the second order is given as

$$S^{(2)}_{fi}=\int\int d^4x d^4y\,\overline{\psi}_f(x)[-ie\gamma^\mu A_\mu(x)]iS_F(x-y)\times[-ie\gamma^\nu A_\nu(y)]\psi_i(y) \tag{15}$$

where

$$S_F(x-y)=\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}\frac{\gamma^\mu p_\mu+m}{p^2-m^2+i\varepsilon}.$$

Carrying out the spatial integrals we get

$$S^{(2)}_{fi}=\frac{i}{V}(e^2\phi^2)\frac{m}{\sqrt{E_iE_f}}\delta(E_f-E_i)\,\delta(p_{f3}-p_{i3})I,$$
$$I=\int d^2p_\perp\frac{N}{(\vec{p}^{\,2}_{f_\perp}-\vec{p}^{\,2}_\perp)(\overrightarrow{p_f-p})^2_\perp(\overrightarrow{p_i-p})^2_\perp}, \tag{16}$$

where N is obtained as

$$N=(p_i-p)_2(p_f-p)_1\overline{u}_f\gamma^1P\gamma^3u_i+(p_i-p)_1(p_f-p)_2\times\overline{u}_f\gamma^3P\gamma^1u_i-(p_i-p)_1(p_f-p)_1\overline{u}_f\gamma^1P\gamma^1u_i-(p_i-p)_2(p_f-p)_2\overline{u}_f\gamma^3P\gamma^3u_i \tag{17}$$

with

$$P=\gamma^0E_f-\gamma^3p_1-\gamma^1p_2+m. \tag{18}$$

Denoting the polar angle in the $p_\perp$ plane by φ, and making use of the energy conservation mandated by $\delta(E_f-E_i)$, $\vec{p}_i^{\,2}=\vec{p}_f^{\,2}\equiv k^2$ with $\vec{p}_i=k\hat{x}$, then N can be written as

$$N=\alpha+\beta\cos\varphi+\gamma\sin\varphi,$$
$$\alpha=k^3\left(\frac{E_i}{k}\{A\sin\theta-B\cos\theta\}-u^2\left\{\frac{E_i}{k}B+D\sin\theta+C(1+\cos\theta)\right\}+\frac{m}{k}\{A'\sin\theta+B'\cos\theta+B'u^2\}\right),$$
$$\beta=k^3\left[Cu^3+u\left(D\sin\theta+C\cos\theta+\frac{E_i}{k}\{(1+\cos\theta)B-A\sin\theta\}\right)-\frac{mu}{k}\{A'\sin\theta+B'(1+\cos\theta)\}\right],$$
$$\gamma=k^3\left[Du^3+u\left(C\sin\theta-D\cos\theta-\frac{E_i}{k}\{A(1-\cos\theta)-B\sin\theta\}\right)-\frac{mu}{k}\{A'(1-\cos\theta)+B'\sin\theta\}\right], \tag{19}$$

with $u\equiv p/k$ and

$$A=i\overline{u}_f\gamma^0\Sigma_2u_i,\quad A'=-i\overline{u}_f\Sigma_2u_i,\quad \text{with }\Sigma_2=\begin{pmatrix}\sigma_2&0\\0&\sigma_2\end{pmatrix},$$
$$B=\overline{u}_f\gamma^0u_i,\quad B'=\overline{u}_fu_i,\quad C=\overline{u}_f\gamma^3u_i,\quad D=\overline{u}_f\gamma^1u_i. \tag{20}$$

The φ integration can be carried out using the complex integration techniques. That is we define $z=e^{i\varphi}$, and the φ integration is converted into a contour integration over the unit circle $|z|=1$. Thus

$$I=\frac{e^{i\theta}}{2ik}\int_0^\infty\frac{du}{u(1-u^2)}\oint_{|z|=1}dz\frac{\mathcal{F}(z,\overline{z})}{(z^2+1-2az)(z^2+e^{2i\theta}-2aze^{i\theta})} \tag{21}$$

with $a=(u^2+1)/2u$ and

$$\mathcal{F}(z,\overline{z})=c_0+c_1z+c_2z^2 \tag{22}$$

where

$$c_0=\{C+iD\}u^3+u\left((C-iD)e^{i\theta}+\frac{E_i}{k}[B-iA+(B+iA)e^{i\theta}]-\frac{m}{k}[B'+iA'+(B'-iA')e^{i\theta}]\right),$$

$$c_1=\frac{2E_i}{k}\{A\sin\theta-B\cos\theta\}-2u^2\left(\frac{E_i}{k}B-\frac{m}{k}B'+D\sin\theta+C(1+\cos\theta)\right)+2\frac{m}{k}\{A'\sin\theta+B'\cos\theta\},\tag{23}$$

$$c_2=\{C-iD\}u^3+u\left((C+iD)e^{-i\theta}+\frac{E_i}{k}[B+iA+(B-iA)e^{-i\theta}]-\frac{m}{k}[B'-iA'+(B'+iA')e^{-i\theta}]\right).$$

The z integral now can be carried out, using the Cauchy theorem, and we get

$$J=-2\pi ie^{-i\theta}\frac{2u^2}{k}\ \frac{\{(E_iB-mB')u^2+E_i(A\sin\theta-B\cos\theta)+m(A'\sin\theta+B'\cos\theta)\}}{(u^2-e^{i\theta})(u^2-e^{-i\theta})}\varepsilon(u-1).\tag{24}$$

Substituting Eq. (24) in Eq. (21), we get

$$I=\frac{2\pi}{k^2}\int_0^\infty udu\varepsilon(u-1)\frac{\{(E_iB-mB')u^2+E_i(A\sin\theta-B\cos\theta)+m(A'\sin\theta+B'\cos\theta)\}}{(u^2-1)(u^2-e^{i\theta})(u^2-e^{-i\theta})}.\tag{25}$$

Changing variables, $u^2=v$, Eq. (25) could be rewritten as

$$I=\frac{\pi}{k^2}\int_0^\infty dv\,\varepsilon(v-1)\left(\frac{E_iB-mB'}{(v-e^{i\theta})(v-e^{-i\theta})}+\frac{(E_iA+mA')\sin\theta+(E_iB-mB')(1-\cos\theta)}{(v-1)(v-e^{i\theta})(v-e^{-i\theta})}\right).\tag{26}$$

The first integral in Eq. (26) can easily be shown to vanish with the help of a variable change $v=1/w$ in the $(1,\infty)$ interval. Thus, we finally end up with

$$I=\frac{\pi T}{k^2}\int_0^\infty dv\,\frac{\varepsilon(v-1)}{(v-1)(v-e^{i\theta})(v-e^{-i\theta})}\tag{27}$$

where

$$\begin{aligned}T&=(E_iA+mA')\sin\theta+(E_iB-mB')(1-\cos\theta)\\&=\overline{u}_f(E_i\gamma^0-m)(1-\cos\theta+i\sin\theta\Sigma_2)u_i.\end{aligned}\tag{28}$$

Using the definition in Eq. (20), the profactor T can be shown to vanish.

V. CONCLUSIONS AND DISCUSSION

In Ref. [14] it was claimed that the Born approximation for relativistic spin-1/2 AB scattering works, by demonstrating that this amplitude agrees with the corresponding terms in the series expansion of the exact amplitude. As the exact amplitude is proportional to $\sin\pi\alpha$, the demonstration of the full consistency of the Born approximation however requires a further step, namely the vanishing of the $O(\alpha^2)$ contributions. This was already done in the context of the Galilean invariant field theory whose two-particle sector is known to be equivalent to the AB Schrödinger equation. Encouraged by the success of these works, we have addressed the same issue directly, namely by considering the $O(\alpha^2)$ contribution for the relativistic scattering of spin-1/2 particles from an infinitely long solenoid in the context of covariant perturbation theory, and shown that it indeed vanishes, thus completing the consistency check of the Born approximation for the relativistic spin-1/2 problem.

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