Non-Abelian Aharonov-Bohm scattering of spin 1/2 particles

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We study the low energy regime of the scattering of two fermionic particles carrying isospin $\frac{1}{2}$ and interacting through a non-Abelian Chern-Simons field. We calculate the one-loop scattering amplitude for both the nonrelativistic and also for the relativistic theory. In the relativistic case we introduce an intermediate cutoff, separating the regions with low and high loop momenta integration. In this procedure purely relativistic field theory effects as the vacuum polarization and anomalous magnetic moment corrections are automatically incorporated.

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I. INTRODUCTION

Starting from different perspectives, a scalar non-Abelian Aharonov-Bohm (AB) effect has been discussed by several authors [1–4]. This subject has interesting implications to the physics of peculiar objects such as cosmic strings and black holes; it also has applications to some aspects of gravitation in $2+1$ dimensions [5–10]. Cosmic strings, for example, may have trapped non-Abelian magnetic flux tubes so that the scattering of charged particles by these strings is just a manifestation of the non-Abelian AB effect.

The study of the AB effect was started through the exact calculation of the scattering amplitude of scalar particles by a thin magnetic flux tube at the origin [11]. As is nowadays well known, in that situation the perturbative Born approximation fails to reproduce the expansion of the exact result [12] and, moreover, the second term of the Born series is divergent. This discrepancy is due to the fact that the perturbative wave function does not satisfy the same boundary condition as the exact one. Actually, in a perturbative treatment for a nonrelativistic field theory describing spinless Abelian particles scattered through a Chern-Simons field, it was shown that to eliminate the divergences, to recuperate the scale invariance, and to reproduce the result of the expansion of the exact solution, it is necessary to add a contact term $(\phi^*\phi)^2$ [13].

Recently, the perturbative treatment was applied to relativistic non-Abelian scalar particles [14] minimally coupled to a non-Abelian CS field. By considering the low momentum limit, it was shown that, up to leading order, the same results are obtained through the calculation of a non-Abelian nonrelativistic field theory [15]. In the next-to-leading approximation, new corrections appear which are absent in the direct nonrelativistic approach. These corrections also differ from the ones obtained in the Abelian theory [16].

By analyzing the Abelian AB effect, it has been verified that new features appear if spin is introduced [17–20]. For example, the Pauli's magnetic term plays the role of a contact interaction and no quartic self-interaction is needed. Besides that, as shown in Ref. [20] new effective low momentum interactions are induced if one starts from a fully relativistic theory.

Completing our study of the non-Abelian AB effect began in Ref. [14], in this work we analyze the AB scattering for non-Abelian spin $\frac{1}{2}$ particles. We start by calculating the AB scattering in a nonrelativistic setting. We then consider the AB scattering from a more basic standpoint, starting from a relativistic quantum field theory, and then taking the appropriate nonrelativistic limit of the scattering amplitudes. One of the advantages of such procedure is in the fact that it automatically incorporates quantum radiative corrections as the vacuum polarization and induced magnetic moment. To take the nonrelativistic limit most easily, we use an intermediate auxiliary cutoff separating the low and high loop momenta in the Feynman integrals. As it happened in our previous studies, it is also convenient to work in the Coulomb gauge, since in this gauge the Chern-Simons propagator depends only on the spatial part of the loop momentum variable.

II. NONRELATIVISTIC THEORY

We consider the non-Abelian Pauli-Schrödinger model for fermions minimally coupled to a non-Abelian Chern-Simons field specified by the Lagrangian

$$\mathcal{L}=-\Theta\varepsilon^{\alpha\beta\lambda}\mathrm{tr}\left(A_\alpha\partial_\beta A_\lambda+\frac{2g}{3}A_\alpha A_\beta A_\lambda\right)+\psi^\dagger\left[i\partial_t+\frac{(\nabla-g\mathbf{A})^2}{2m}+igA_0-i\frac{g}{2m}B\right]\psi-\frac{1}{\xi}\mathrm{tr}(\nabla\cdot\mathbf{A})^2-\mathbf{c}^{*a}(\delta_{\mathbf{ab}}\nabla^2+g\varepsilon_{\mathbf{abc}}\mathbf{A}^{\mathbf{c}}\cdot\nabla)\mathbf{c}^{\mathbf{b}}, \tag{1}$$

where ψ is a one-component anticommuting field, belonging

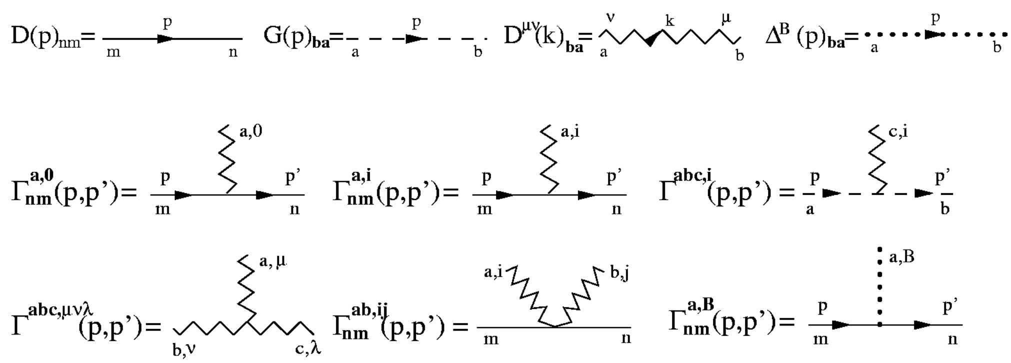

FIG. 1. Feynman rules—nonrelativistic theory.

to the fundamental representation of the SU(2) group, and $A_\mu = A_\mu^a T^a$, with T^a being the generator of the Lie Algebra of SU(2) satisfying

$$[T_a, T_b] = \varepsilon_{abc} T^c, \tag{2}$$

and normalized such that

$$T^a T^b = -\frac{\delta_{ab}}{4} I + \frac{1}{2} \varepsilon^{abc} T_c . \tag{3}$$

The term containing the ''magnetic'' field B, is the Pauli term and **c** is the ghost field needed to guarantee unitarity. For convenience, we will work in a strict Coulomb gauge obtained by letting $\xi \to 0$.

We will use a graphical notation where the CS field, the matter field, and the ghost field propagators are represented by wavy, continuous, and dashed lines respectively. The analytic expression for the A_μ free propagator is

$$D^{\mu\nu}(k)_{ba} = D^{\mu\nu}(k)\,\delta_{ba} = \frac{1}{\Theta} \varepsilon^{\mu\nu\lambda} \frac{\overline{k_\lambda}}{\mathbf{k}^2} \quad \delta_{ba}, \tag{4}$$

where $\overline{k_\lambda} \equiv (0, \mathbf{k})$. The matter field propagator is

$$S(p)_{nm} = S(p)\,\delta_{nm} = \frac{i}{p_0 - \mathbf{p}^2/2m + i\epsilon}\, \delta_{nm}, \tag{5}$$

whereas the ghost field propagator is

$$G(p)_{ba} = G(p)\,\delta_{ba} = \frac{-i}{\mathbf{p}^2} \quad \delta_{ba} . \tag{6}$$

Since the B field occurs in Eq. (1) it is convenient to have at hand

$$\Delta_B^{ba}(x) = \langle T B^b(x) A_0^a(0) \rangle = -\frac{i}{\Theta}\, \delta^{(3)}(x)\, \delta^{ba}, \tag{7}$$

which is the only nonvanishing propagator involving the B field; graphically it will be represented by a dotted line. Expression (7) shows that the Pauli term, i.e., the interaction $\psi^\dagger B \psi$, plays the same role as the quartic term $(\phi^\dagger \phi)^2$ in the scalar case.

The graphical representation for the vertices is given in Fig. 1 and the corresponding analytical expressions are

$$\Gamma_{nm}^{a,0}(p,p') = -g (T^a)_{nm}, \tag{8}$$

$$\Gamma_{nm}^{a,i}(p,p') = -\frac{g}{2m} (T^a)_{nm} (p^i + p'^i), \tag{9}$$

$$\Gamma_{nm}^{ab,ij}(p,p') = -i \frac{g^2}{2m} (T^a T^b + T^b T^a)_{nm} g^{ij}, \tag{10}$$

$$\Gamma_{nm}^{a,B}(p,p') = \frac{g}{2m} (T^a)_{nm}, \tag{11}$$

$$\Gamma^{abc,\mu\nu\lambda}(p,p') = i g \Theta \varepsilon^{abc} \varepsilon^{\mu\nu\lambda}, \tag{12}$$

$$\Gamma_{nm}^{abc,i}(p,p') = -g \varepsilon^{abc} p'^i \delta_{nm} . \tag{13}$$

In the tree approximation and in the center-of-mass frame the two-body scattering amplitude is given by

$$\mathcal{M}(\theta) = \frac{i g^2}{m\Theta} [T^a \otimes T_a] \left[1 + i \frac{\sin\theta}{(1-\cos\theta)} \right], \tag{14}$$

where θ is the scattering angle. Here and in what follows we employ a simplified notation where the isospin indices are omitted. If the incoming and outgoing particles have isospin (n,m) and (n',m') the total amplitude for the process is given by

$$\mathcal{M}_{n'm';nm} = \langle n',m' | \mathcal{M}(\theta) | n,m \rangle - \langle m',n' | \mathcal{M}(\theta+\pi) | n,m \rangle. \tag{15}$$

The one-loop contribution to AB scattering is depicted in Fig. 2. The incoming and outgoing fermions are assumed to have momenta $p_1 = (\mathbf{p}_1^2/2m, \mathbf{p}_1)$, $p_2 = (\mathbf{p}_2^2/2m, \mathbf{p}_2)$ and $p_3 = (\mathbf{p}_3^2/2m, \mathbf{p}_3)$, $p_4 = (\mathbf{p}_4^2/2m, \mathbf{p}_4)$, respectively. We work in the center-of-mass frame where $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$, $\mathbf{p}_3 = -\mathbf{p}_4 = \mathbf{p}'$, and $|\mathbf{p}| = |\mathbf{p}'|$. For the first graph, Fig. 2(a), we get

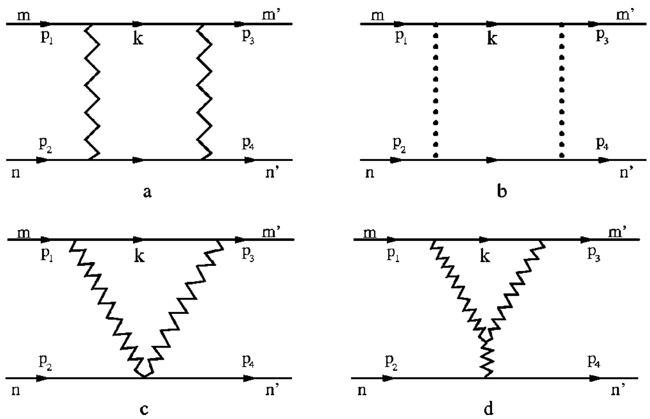

FIG. 2. One-loop scattering—nonrelativistic theory.

$$\mathcal{M}_{\rm a}(\theta)=\int\frac{d^3k}{(2\pi)^3}[\Gamma^{d,\alpha}(p_1+p_2-k,p_4)\times S(p_1+p_2-k)\Gamma^{c,\nu}(p_2,p_1+p_2-k)D^{ac}_{\mu\nu}(k-p_1)\times D^{db}_{\alpha\beta}(k-p_3)\Gamma^{b,\beta}(k,p_3)S(k)\Gamma^{a,\mu}(p_1,k)]. \quad (16)$$

After performing the k_0 integration, this gives

$$\mathcal{M}_{\rm a}(\theta)=-\frac{4ig^4}{m\Theta^2}[T^bT^a\otimes T_bT_a]\times\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{\mathbf{p}^2-\mathbf{k}^2+i\epsilon}\left[\frac{(\mathbf{p}_1\wedge\mathbf{k})(\mathbf{p}_3\wedge\mathbf{k})}{(\mathbf{k}-\mathbf{p}_1)^2(\mathbf{k}-\mathbf{p}_3)^2}\right]. \quad (17)$$

As a general rule, whenever dealing with divergent spatial integrals we will introduce a nonrelativistic cutoff Λ_{NR}. However, in Eq. (17) such regulator is not necessary as the integral is ultraviolet finite. The final result is

$$\mathcal{M}_{\rm a}(\theta)=-\frac{ig^4}{4\pi m\Theta^2}[T^bT^a\otimes T_bT_a]\left\{\log\left[\frac{\mathbf{q}^2}{\mathbf{p}^2}\right]+i\pi\right\}. \quad (18)$$

where $\mathbf{q}=\mathbf{p_3}-\mathbf{p_1}$ is the momentum transferred.

The same procedure can be used to calculate the other graphs in Fig. 2. Here the spatial integrals are logarithmically divergent and are done after the introduction of the aforementioned cutoff. Graph 2(b) gives

$$\mathcal{M}_{\rm b}(\theta)=-\frac{g^4}{m^2\Theta^2}[T^bT^a\otimes T_bT_a]\times\int\frac{d^3k}{(2\pi)^3}S(p_1+p_2-k)\,S(k), \quad (19)$$

from which we obtain

$$\mathcal{M}_{\rm b}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^bT^a\otimes T_bT_a]\left\{\log\left[\frac{\Lambda^2_{NR}}{\mathbf{p}^2}\right]+i\pi\right\}. \quad (20)$$

Similarly, graph 2(c) corresponds to

$$\mathcal{M}_{\rm c}(\theta)=2\int\frac{d^3k}{(2\pi)^3}\Gamma^{cd,ij}D^{ac}_{0i}(k)D^{db}_{j0}(k+q)\Gamma^{b,0}\Gamma^{a,0}. \quad (21)$$

The k^0 integration is straightforward and gives

$$\mathcal{M}_{\rm c}(\theta)=\frac{ig^4}{2m\Theta^2}[(T^aT^b+T^bT^a)\otimes T_bT_a]\times\int\frac{d^2k}{(2\pi)^2}\frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{q})}{\mathbf{k}^2(\mathbf{k}+\mathbf{q})^2}. \quad (22)$$

Effectuating the remaining integral produces

$$\mathcal{M}_{\rm c}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^bT^a\otimes T_bT_a+\tfrac{1}{2}\varepsilon^{cab}T_c\otimes T_bT_a]\times\left\{\log\left[\frac{\mathbf{q}^2}{\Lambda^2_{NR}}\right]\right\}. \quad (23)$$

The last diagram, graph 2(d) gives

$$\mathcal{M}_{\rm d}(\theta)=2\int\frac{d^3k}{(2\pi)^3}[\Gamma^{b,\nu}(p_2,p_4)D^{ab}_{\nu\mu}(q)\Gamma^{ac'd',\mu\rho\sigma}D^{d'd}_{\sigma\alpha}\times(k-p_3)\Gamma^{d,\alpha}(k,p_3)S(k)\Gamma^{c,\beta}(p_1,k)D^{cc'}_{\beta\rho}\times(k-p_1)] \quad (24)$$

so that, after the k^0 integration,

$$\mathcal{M}_{\rm d}(\theta)=\frac{ig^4}{m\Theta^2}[\varepsilon^{cab}T_c\otimes T_bT_a]\times\int\frac{d^2k}{(2\pi)^2}\frac{[\mathbf{q}\wedge\mathbf{k}-\mathbf{p}_1\wedge\mathbf{p}_3](\mathbf{q}\wedge\mathbf{k})}{\mathbf{q}^2(\mathbf{k}-\mathbf{p}_1)^2(\mathbf{k}-\mathbf{p}_3)^2} \quad (25)$$

leading to

$$\mathcal{M}_{\rm d}(\theta)=\frac{ig^4}{8\pi m\Theta^2}[\varepsilon^{cab}T_c\otimes T_bT_a]\left\{1-\log\left[\frac{\mathbf{q}^2}{\Lambda^2_{NR}}\right]\right\}. \quad (26)$$

Thus, the sum of the one-loop contribution is

$$\mathcal{M}_{\text{1-loop}}(\theta)=\frac{ig^4}{8\pi m\Theta^2}[\varepsilon^{cab}T_c\otimes T_bT_a]=-\frac{ig^4}{8\pi m\Theta^2}[T^a\otimes T_a]. \quad (27)$$

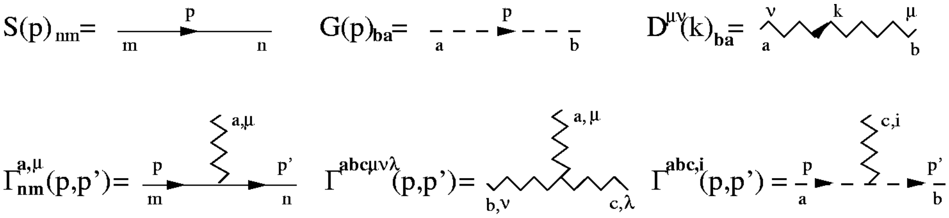

FIG. 3. Feynman rules—relativistic theory.

It happens that the nonvanishing result in the last equation is only due to the regularization used. Really, as the original expression was logarithmically divergent, different regularization schemes will produce results that for the finite part will differ at most by a constant. This remark holds even for the sum of the Feynman integrals which is only conditionally convergent and leads to different results depending on the way it is treated. In particular, had we used the dimensional regularization, as it was done in Ref. [13] for the scalar case, Eq. (27) would be zero. Our constant term in that result may be eliminated through a redefinition of the cutoff Λ_{NR} in Eq. (26) or by adding a counterterm of the form $(\psi^\dagger T^a \psi)^2$ to the original Lagrangian. In the relativistic theory the divergences are milder, the graphs are individually finite and no such counterterms are needed.

III. RELATIVISTIC THEORY

We will now consider the non-Abelian scattering within the full relativistic context. The Lagrangian describing the model is

$$\mathcal{L}=-\Theta\varepsilon^{\alpha\beta\lambda}\mathrm{tr}\left(A_\alpha\partial_\beta A_\lambda+\frac{2g}{3}A_\alpha A_\beta A_\lambda\right)+i\overline{\Psi}(\not{D}-m)\Psi$$
$$-\frac{1}{\xi}\mathrm{tr}(\nabla\cdot\mathbf{A})^2-\mathbf{c}^{*\mathbf{a}}(\nabla^2+g\varepsilon_{\mathbf{abc}}\mathbf{A}^{\mathbf{c}}\cdot\nabla)\mathbf{c}^b. \tag{28}$$

where $D_\mu=\partial_\mu+gA_\mu$ and Ψ is a two-component Dirac field belonging to the fundamental representation of the SU(2) gauge group. Ψ represents particles and antiparticles with the same spin and we take m to be positive. Our graphical notation is specified in Fig 3. The corresponding analytical expressions for the gauge and ghost field propagators are the same as in the previous section. The matter field propagator and the vertices, however, are now given by

$$S(p)_{nm}=S(p)\delta_{nm}=\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}\delta_{nm}, \tag{29}$$

$$\Gamma^{a,\mu}_{nm}(p,p')=-g(T^a)_{nm}(\gamma^\mu), \tag{30}$$

$$\Gamma^{abc,\mu\nu\lambda}(p,p')=ig\Theta\varepsilon^{abc}\varepsilon^{\mu\nu\lambda}, \tag{31}$$

$$\Gamma^{abc,i}_{nm}(p,p')=-g\varepsilon^{abc}p'^{i}\delta_{nm}. \tag{32}$$

The model is renormalizable. Actually, without the matter field it was found that there are no radiative corrections to the Green functions [21]. We can therefore restrict our study of one-loop renormalization to superficially divergent graphs arising from the coupling to the matter field, i.e., the one-loop correction to the self-energy, vacuum polarization and vertex corrections. The nonvanishing self-energy graph depicted in Fig. 4 is given by [22]

$$\Sigma(p)=-\int\frac{d^3k}{(2\pi)^3}[\Gamma^{a,\mu}(p+k,p)S(p+k)\Gamma^{b,\nu}$$
$$\times(p,p+k)D^{ab}_{\nu\mu}(k)]$$
$$=\frac{ig^2}{\Theta}[T^aT_a]\int\frac{d^3k}{(2\pi)^3}\frac{[\gamma^\mu(\not{p}+\not{k}+m)\gamma^\nu]\ \ \varepsilon_{\mu\nu\lambda}\overline{k^\lambda}}{[(p+k)^2-m^2+i\epsilon]\ \ \mathbf{k}^2}, \tag{33}$$

so that the inverse of the complete fermion propagator is written as $\mathcal{S}^{-1}(p)=\not{p}-m+i\Sigma$. Notice that the self-energy is diagonal in isospin space. After doing the k_0 integration we obtain

$$\Sigma(p)=-\frac{ig^2}{8\pi\Theta}[T^aT_a]\int_0^{\Lambda_0^2}d\mathbf{k}^2\frac{1}{w_k}$$
$$\times\left\{\frac{m}{\mathbf{p}^2}\gamma\cdot\mathbf{p}[1-\epsilon(\mathbf{k}^2-\mathbf{p}^2)]+[1+\epsilon(\mathbf{k}^2-\mathbf{p}^2)]\right\}, \tag{34}$$

where $\epsilon(x)$ is the signal function, $w_k=\sqrt{\mathbf{k}^2+m^2}$, and a cutoff Λ_0 was introduced to take care of the ultraviolet divergence of the integral. The integral is easily done and gives

$$\Sigma(p)=-\frac{ig^2}{2\pi\Theta}[T^aT_a]\left[\gamma\cdot\mathbf{p}\frac{m}{\mathbf{p}^2}(w_p-m)+\sqrt{\Lambda_0^2+m^2}-w_p\right] \tag{35}$$

and so, for $\Lambda_0\to\infty$,

$$\Sigma(p)=-\frac{ig^2}{2\pi\Theta}[T^aT_a]\left\{\frac{m\ \ \gamma\cdot\mathbf{p}-\mathbf{p}^2}{w_p+m}-m+\Lambda_0\right\}. \tag{36}$$

The linear ultraviolet divergence may be eliminated through the imposition of an adequate renormalization condition. Due to our use of the Coulomb gauge, a convenient condition is the one adopted in the work [23]; denoting the renor-

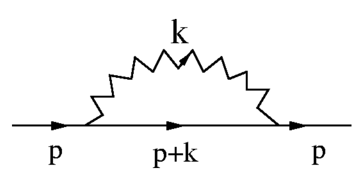

FIG. 4. Matter field self-energy.

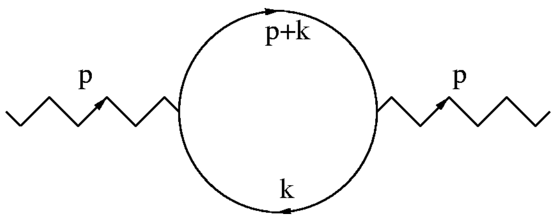

FIG. 5. Vacuum polarization.

malized propagator by $\mathcal{S}_R$, this condition reads $\mathcal{S}_R(p_0,\mathbf{p}=0)=S(p_0,\mathbf{p}=0)$. Proceeding in this way, we get for the renormalized propagator

$$\mathcal{S}_R(p)=i\frac{(\not p+m)+\alpha(m-w_p)[1+(m/\mathbf{p}^2)\gamma.\mathbf{p}]}{(p^2-m^2)}, \quad (37)$$

where $\alpha=-g^2[T^aT_a]/(2\pi\Theta)$.

Let us now turn our attention to the vacuum polarization correction. The only graph that contributes is the one drawn in Fig. 5. As this graph is gauge independent, the would be linear divergence may be eliminated if one employs a gauge invariant regularization scheme. Use of dimensional regularization gives

$$\Pi^{\mu\nu}(q)=\frac{ig^2}{4\pi}\mathrm{tr}[T^aT^b] \times\left[\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\Pi_1(q^2)+im\varepsilon^{\mu\nu\lambda}q_\lambda\Pi_2(q^2)\right], \quad (38)$$

with

$$\Pi_1(q^2)=\int_0^1 dx\frac{2q^2x(1-x)}{[m^2-q^2x(1-x)]^{1/2}}\approx\frac{q^2}{3|m|}, \quad (39)$$

$$\Pi_2(q^2)=\int_0^1 dx\frac{1}{[m^2-q^2x(1-x)]^{1/2}}\approx\frac{1}{|m|}, \quad (40)$$

where the expressions on the right of these equations are the leading approximations for low momenta q. From these results, we see that for low momentum a Yang-Mills term may be induced, as one would expect on general grounds.

The one-loop corrections to the CS matter field vertex are given by the graphs in Figs. 6(a) and 6(b). The on-shell analytic expression associated to graph 6(a) is

$$\overline{u}(p')\Gamma_{\mathrm{a}}^{a,\mu}u(p) =\frac{g^3}{\Theta}[T^bT^aT_b]\int\frac{d^3k}{(2\pi)^3} \times\frac{\varepsilon_{\rho\sigma\lambda}\overline{k^\lambda}\,\overline{u}(p')[\gamma^\sigma(\not p'-\not k+m)\gamma^\mu(\not p-\not k+m)\gamma^\rho]u(p)}{[(p-k)^2-m^2+i\epsilon][(p'-k)^2-m^2+i\epsilon][-\mathbf{k}^2]}. \quad (41)$$

Here on in, what follows the isospin indices (n,m) will be omitted. Up to the group factor $T^bT^aT_b$, this agrees with the vertex for the Abelian theory discussed in [18]. Using dimensional regularization, the result can be read from that reference but for general momenta it is not particularly illuminating. Nevertheless, for small momenta (i.e., for $|\mathbf{p}|\approx|\mathbf{p}'|\ll m$) a great simplification occurs and one finds ($\eta=|\mathbf{p}|/m$)

$$\overline{u}(p')\Gamma_{\mathrm{a}}^{a,0}u(p)=\mathcal{O}(\eta^2), \quad (42)$$

$$\overline{u}(p')\Gamma_{\mathrm{a}}^{a,i}u(p)=\frac{g^3}{4\pi\Theta}[T^bT^aT_b]\frac{1}{2m}[P^i-i\varepsilon^{ij}q_j] +\mathcal{O}(\eta^2), \quad (43)$$

where $P^i=p^i+p'^i$ and $q=p'^i-p^i$

Similarly, the graph 6(b) which corresponds to

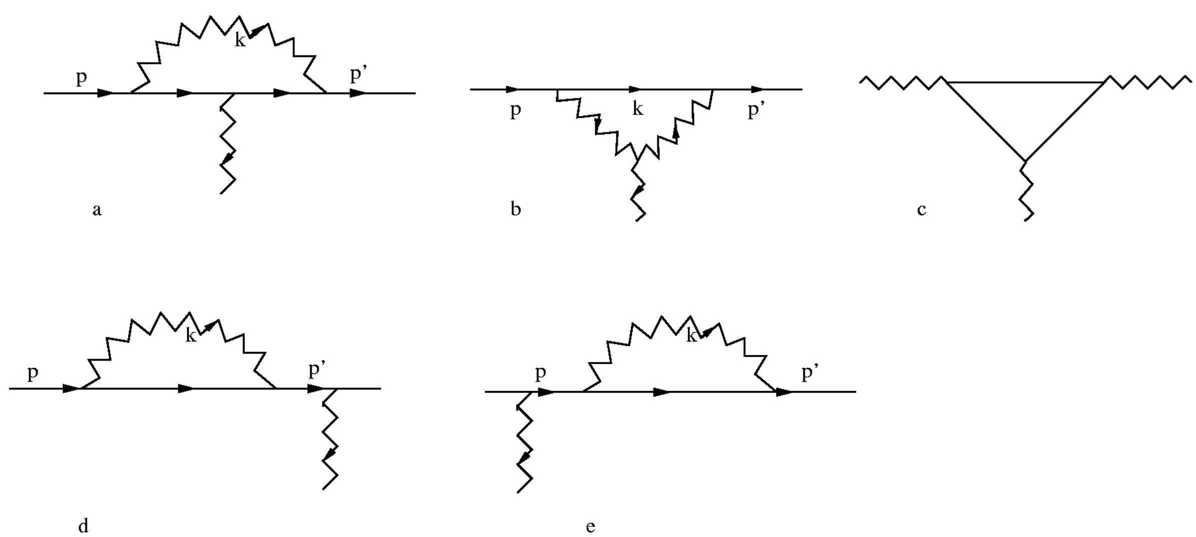

FIG. 6. Vertices correction.

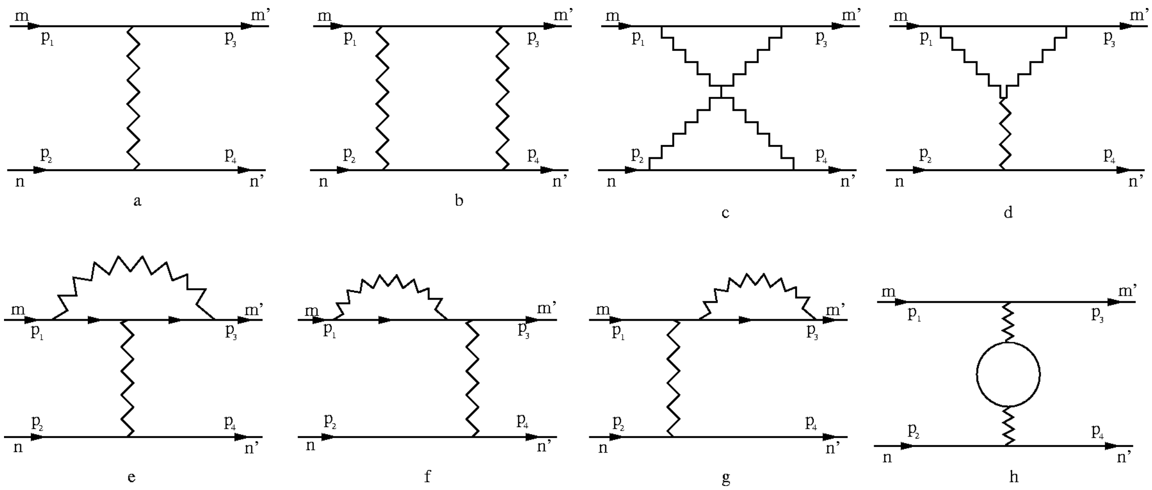

FIG. 7. Fermion-fermion scattering—relativistic theory. Similar graphs in which self-energies or vertex parts are inserted in the bottom lines have not been drawn, for convenience.

$$\overline{u}(p')\Gamma_{\rm b}^{a,\mu}u(p) = -\frac{g^3}{\Theta}[\varepsilon^{abc}T_bT_c]\int \frac{d^3k}{(2\pi)^3}\frac{\overline{u}(p')[\gamma^\sigma(\not k+m)\gamma^\beta]u(p)\varepsilon^{\mu\sigma\rho}\varepsilon_{\sigma\beta\lambda}\overline{(p-k)^\lambda}\varepsilon_{\alpha\rho\xi}\overline{(p'-k)^\xi}}{[k^2-m^2+i\epsilon](\mathbf{p}-\mathbf{k})^2(\mathbf{p}'-\mathbf{k})^2} \tag{44}$$

gives for small momenta the result

$$\overline{u}(p')\Gamma_{\rm b}^{a,0}u(p) = \mathcal{O}(\eta^2) \tag{45}$$

and

$$\overline{u}(p')\Gamma_{\rm b}^{a,i}u(p) = \frac{g^3}{8\pi m\Theta}[\varepsilon^{abc}T_cT_b] \times\left\{P^i + i\varepsilon^{ij}q_j\left[1+\log\left(\frac{4m^2}{\mathbf{q}^2}\right)\right]\right\}. \tag{46}$$

The renormalized vertex part is defined by $\Gamma_R^{a\mu} = Z_1\Gamma^{a,\mu}$ where $\Gamma^{a\mu} = -gT^a\gamma^\mu + \Lambda^{a\mu}$ is the unrenormalized one. Fixing the vertex renormalization constant Z_1 by the condition that for $\mathbf{p}=\mathbf{p}'=0$ and $p^\circ = p'^\circ = m$,

$$\overline{u}\Gamma_R^{a\mu}u = -gT^a g^{0\mu}, \tag{47}$$

we get $Z_1 = 1$, so that up to one-loop there is no coupling constant renormalization. This result is also in accord with the computation of the correction to the trilinear Chern-Simons (CS) vertex shown in Fig. 6(c); simple symmetry considerations shown that the result is finite and no counterterm is necessary. Actually, graph 6(c), plus the graphs with four external gauge lines, and the polarization tensor give an induced Yang-Mills term (and also a finite correction for the Chern-Simons term), as commented before. However, up to one-loop, graph 6(c) does not contribute to the scattering and for that reason it will not be considered any longer.

Summarizing, up to one-loop one needs just a mass renormalization counterterm to fix the fermion mass. There are neither vertex nor wave function renormalizations.

Although not 1PI, we have drawn in Fig. 6, graphs 6(d) and 6(e) which are needed to compute the anomalous magnetic moment of the fermions. At low momenta these graphs give the contributions

$$\overline{u}\Gamma_{\rm d}^{a,0}u = \overline{u}\Gamma_{\rm e}^{a,0}u = \mathcal{O}(\eta^2), \tag{48}$$

$$\overline{u}\Gamma_{\rm de}^{a,i}u = \overline{u}\Gamma_{\rm d}^{a,i}u + \overline{u}\Gamma_{\rm e}^{a,i}u = -\frac{g^3}{4\pi\Theta}[(T^bT_b)T^a]\left\{\frac{1}{2m}[P^i + i\varepsilon^{ij}q_j]\right\}. \tag{49}$$

In the Abelian situation the contribution in Eq. (46) is absent and, in the expressions corresponding to Eqs. (43) and (49) the P^i dependent part is exactly canceled. Here, due to the group factors, to get cancellation it is necessary to take into account the new contribution arising from Eq. (46). This can be easily verified using the identity $T^bT^aT_b = T^a(T^bT_b) + \varepsilon^{abc}T_cT_b$. The remaining local parts occurring in $\Gamma_{\rm a-e}^{a\mu}$ will contribute to the (matrix) magnetic moment and we get

$$\mu_{\text{1-loop}}^a = \frac{ig^3}{4\pi m\Theta}[T^a(T^bT_b)]. \tag{50}$$

This expression only differs from the corresponding result in the Abelian case by the group factor. In the ''Abelian limit'' ($g = e\sqrt{2}$ and $T = i/\sqrt{2}$) the result of [18] is recovered.

To complete our discussion of the one-loop properties of the model, one still has to calculate the fermion-fermion scattering. Figure 7 shows the contributing graphs. The only tree level graph, depicted in Fig. 7(a), furnishes

$$\mathcal{M}_{\rm a}(\theta)=[\overline{u(\mathbf{p_4})}\Gamma^{b,\nu}(p_2,p_4)u(\mathbf{p_2})]D^{ba}_{\nu\mu}(q)\times[\overline{u(\mathbf{p_3})}\Gamma^{a,\mu}(p_1,p_3)u(\mathbf{p_1})]. \tag{51}$$

To the leading order of $\mathbf{p}/m$, this gives

$$\mathcal{M}_{\rm a}(\theta)=\frac{ig^2}{m\Theta}[T^a\otimes T_a]\left\{1+i\frac{\sin\theta}{(1-\cos\theta)}\right\}, \tag{52}$$

which exactly agrees with that obtained previously for the nonrelativistic theory.

The one-loop graphs are represented in Figs. 7(b)–7(h). To facilitate our computation we will use an intermediate cutoff Λ, satisfying $|\mathbf{p}|\ll\Lambda\ll m$, which separates the loop integrals in two regions. In the *low* (L) region ($0\leqslant|\mathbf{k}|^2\leqslant\Lambda^2$) the integrand is expanded in power of $1/m$, and in *high* (H) region ($|\mathbf{k}|^2\geqslant\Lambda^2$) we make a Taylor series of the integrand around $|\mathbf{p}|\approx 0$. We will retain terms up to order $\eta=|\mathbf{p}|/m\approx(\Lambda/m)^2\approx(|\mathbf{p}|/\Lambda)^2$.

Using that

$$S(p)=i\left[\frac{u(\mathbf{p})\overline{u(\mathbf{p})}}{p^0-w_p+i\epsilon}+\frac{v(-\mathbf{p})\overline{v(-\mathbf{p})}}{p^0+w_p-i\epsilon}\right], \tag{53}$$

we may decompose the amplitude for the graph in Fig. 7(b)

$$\mathcal{M}_{\rm b}(\theta)=\int\frac{d^3k}{(2\pi)^3}[\overline{u(\mathbf{p}_4)}\Gamma^{c,\alpha}(t,p_4)S(t)\Gamma^{d,\beta}\times(p_2,t)u(\mathbf{p}_2)]D^{ca}_{\alpha\mu}(l')\times[\overline{u(\mathbf{p}_3)}\Gamma^{a,\mu}(r,p_3)S(r)\Gamma^{b,\nu}(p_1,r)u(\mathbf{p}_2)]D^{bd}_{\nu\beta}(l) \tag{54}$$

where $l=(k^0,\mathbf{k}-\mathbf{p}_1)$, $l'=(k^0,\mathbf{k}-\mathbf{p}_3)$, $r=(w_p+k^0,\mathbf{k})$ and $t=(w_p+k^0,-\mathbf{k})$, into a sum of terms

$$\mathcal{M}^{uu}_b+\mathcal{M}^{vv}_b \tag{55}$$

where $\mathcal{M}^{uu}_b$ and $\mathcal{M}^{vv}_b$ designate the contributions of the u and v fermion wave functions to the two internal lines of the graph. The mixed contributions in which one has u in one line and v in the other vanish. After integrating in k^0 we obtain

$$\mathcal{M}^{uu}_b=\frac{ig^4}{2}[T^aT^b\otimes T_aT_b]\int\frac{d^2k}{(2\pi)^2}\left[\frac{T(k,p_1)T^*(k,p_3)}{w_k-w_p}\right] \tag{56}$$

and

$$\mathcal{M}^{vv}_b=\frac{ig^4}{2}[T^aT^b\otimes T_aT_b]\int\frac{d^2k}{(2\pi)^2}\left[\frac{H(p_3,k)H^*(p,k)}{w_k+w_p}\right], \tag{57}$$

where

$$T(k,p)=[\overline{u(\mathbf{k})}\gamma^\nu u(\mathbf{p})]D_{\nu\beta}(k-p)[\overline{u(-\mathbf{k})}\gamma^\nu u(-\mathbf{p})], \tag{58}$$

$$H(p,k)=[\overline{u(\mathbf{p})}\gamma^\nu v(-\mathbf{k})]D_{\nu\beta}(k-p)[\overline{u(-\mathbf{p})}\gamma^\nu v(\mathbf{k})]. \tag{59}$$

Introducing the intermediate cutoff to separate the *low* and *high* parts we get

$$\mathcal{M}^{uu}_{{\rm b}_{low}}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^aT^b\otimes T_aT_b]\left\{\log\left(\frac{\Lambda^2}{\mathbf{q}^2}\right)+\mathcal{O}(\eta)\right\}, \tag{60}$$

$$\mathcal{M}^{uu}_{{\rm b}_{high}}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^aT^b\otimes T_aT_b]\times\left\{\log\left(\frac{2m^2}{\Lambda^2}\right)+\mathcal{O}(\eta)\right\}, \tag{61}$$

$$\mathcal{M}^{vv}_{{\rm b}_{low}}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^aT^b\otimes T_aT_b]\{\mathcal{O}(\eta)\}, \tag{62}$$

$$\mathcal{M}^{vv}_{{\rm b}_{high}}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^aT^b\otimes T_aT_b]\{\log(2)+\mathcal{O}(\eta)\}. \tag{63}$$

Putting these results together we arrive at

$$\mathcal{M}_{\rm b}(\theta)=\frac{ig^4}{4\pi m\Theta^2}[T^aT^b\otimes T_aT_b]\left\{\log\left(\frac{4m^2}{\mathbf{q}^2}\right)\right\} \tag{64}$$

as the leading contribution.

For the crisscross graph, Fig. 7(c), we proceed analogously and obtain (in this case what survives are the mixed uv and vu contributions)

$$\mathcal{M}_{{\rm c}_{low}}(\theta)=-\frac{ig^4}{2\pi m\Theta^2}[T^bT^a\otimes T_aT_b]\times\left\{\frac{1}{2}\log\left(\frac{\Lambda^2}{\mathbf{q}^2}\right)+\mathcal{O}(\eta)\right\}, \tag{65}$$

$$\mathcal{M}_{{\rm c}_{high}}(\theta)=\frac{ig^4}{2\pi m\Theta^2}[T^bT^a\otimes T_aT_b]\times\left\{1+\frac{1}{2}\log\left(\frac{\Lambda^2}{4m^2}\right)+\mathcal{O}(\eta)\right\}. \tag{66}$$

i.e.,

$$\mathcal{M}_{\rm c}(\theta)=\frac{ig^4}{2\pi m\Theta^2}[T^bT^a\otimes T_aT_b]\times\left\{1+\frac{1}{2}\log\left(\frac{\mathbf{q}^2}{4m^2}\right)+\mathcal{O}(\eta)\right\}. \tag{67}$$

The graph 7(d) does not exist in the Abelian theory but it is here essential to cancel the extra contribution coming through group factors in other graphs. It corresponds to

$$M_d = \int \frac{d^3k}{(2\pi)^3} \{[\overline{u}(p_4)\Gamma^{b,\nu}u(p_2)]D^{ba}_{\nu\mu}(q)\Gamma^{ac'd',\mu\sigma\rho} \times D^{dd'}_{\sigma\alpha}(k-p_3)D^{cc'}_{\beta\rho}(k-p_1) \times [\overline{u}(p_3)\,\Gamma^{d,\alpha}\,S(k)\,\Gamma^{c,\beta}u(p_1)]\}, \quad (68)$$

and has *low* and *high* momentum parts given by

$$M_{\mathrm{d}_{low}} = \frac{ig^4}{8\pi m\Theta^2}[\varepsilon^{acd}T_a \otimes T_d T_c] \times \left\{ 1 + i\frac{\sin\theta}{1-\cos\theta} + \left[\log\left(\frac{\Lambda^2}{\mathbf{q}^2}\right) - \frac{\Lambda^2}{2m^2} - (1+2\cos\theta)\frac{\mathbf{p}^2}{\Lambda^2} \right] \right\}, \quad (69)$$

and

$$M_{\mathrm{d}_{high}} = \frac{ig^4}{8\pi m\Theta^2}[\varepsilon^{acd}T_a \otimes T_d T_c] \times \left\{ -\log\left(\frac{\Lambda^2}{4m^2}\right) + \frac{\Lambda^2}{2m^2} + (1+2\cos\theta)\frac{\mathbf{p}^2}{\Lambda^2} \right\}, \quad (70)$$

respectively. Summing Eqs. (69) and (70) we get

$$M_{\mathrm{d}} = \frac{ig^4}{8\pi m\Theta^2}[\varepsilon^{acd}T_a \otimes T_d T_c] \times \left\{ 1 + i\frac{\sin\theta}{1-\cos\theta} + \log\left(\frac{4m^2}{\mathbf{q}^2}\right) \right\}. \quad (71)$$

Finally, incorporating the radiative correction, Figs. 7(e)–7(h), we obtain

$$\mathcal{M}_{\mathrm{e-g}}(\theta) = -\frac{ig^4}{4\pi m\Theta^2}[T^a \otimes (T^b T_b)T_a] - \frac{ig^4}{8\pi m\Theta^2} \times [\varepsilon^{abc}T_a \otimes T_c T_b]\left\{ 1 + i\frac{\sin\theta}{(1-\cos\theta)} \right\} \quad (72)$$

and

$$\mathcal{M}_{\mathrm{h}}(\theta) = \frac{ig^4}{24\pi m\Theta^2}[T^a \otimes T_a]. \quad (73)$$

Summing all these contributions and using the relation (3) to simplify the result, we get the total one-loop amplitude

$$\mathcal{M}_{1\text{-loop}}(\theta) = \frac{ig^4}{4\pi m\Theta^2}\{\tfrac{3}{8}[I\otimes I] + \tfrac{2}{3}[T^a \otimes T_a]\}. \quad (74)$$

IV. CONCLUSIONS

In this work we studied the scattering of isospin $\frac{1}{2}$ fermionic particles interacting through a non-Abelian Chern-Simons field. In the nonrelativistic formulation we found that, up to a finite constant term, there is no one-loop correction to the tree approximation to the scattering amplitude. This is similar to what happens in the scalar theory where the constant one-loop contribution may be eliminated by a finite quartic counterterm [14].

We have also considered the same problem starting from the fully relativistic theory. After discussing the one-loop renormalizability of the model and determining anomalous contributions to the matrix magnetic moment of the fermions, we considered the low momenta limit of the two-body scattering amplitude obtaining a nonvanishing one-loop contribution. This result, shown in Eq. (74), is a correction to the scattering which does not appear in the nonrelativistic theory. It is a leading order contribution and implies that the effective low momentum Lagrangian contains a four-fermion self-interaction with a coupling which can be read from Eq. (74). These terms cannot be eliminated by adding counterterms to the original Lagrangian (28) without destroying the renormalizability of the relativistic model. Furthermore, as also happens in the Abelian case, these new terms come from the *high* part of the original theory and could not be suspected in a direct nonrelativistic approach.

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$$u(\vec{p})=\left(\frac{w_p+m}{2\,w_p}\right)^{1/2}\begin{bmatrix}1\\ \dfrac{p^2-ip^1}{w_p+m}\end{bmatrix},\quad v(\vec{p})=\left(\frac{w_p+m}{2\,w_p}\right)^{1/2}\begin{bmatrix}\dfrac{p^2+ip^1}{w_p+m}\\ 1\end{bmatrix},$$

where $w_p=(m^2+\vec{p}^{\,2})^{1/2}$ and the normalizations were chosen so that $\bar{u}u=-\bar{v}v=m/w_p$.
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