Self-duality of various chiral boson actions

Yan-Gang Miao*
Department of Physics, University of Kaiserslautern, P.O. Box 3049, D-67653 Kaiserslautern, Germany and Department of Physics, Xiamen University, Xiamen 361005, People's Republic of China†

H. J. W. Müller-Kirsten‡
Department of Physics, University of Kaiserslautern, P.O. Box 3049, D-67653, Kaiserslautern, Germany

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The duality symmetries of various chiral boson actions are investigated using $D=2$ and $D=6$ space-time dimensions as examples. These actions involve the Siegel, Floreanini-Jackiw, Srivastava and Pasti-Sorokin-Tonin formulations. We discover that the Siegel, Floreanini-Jackiw and Pasti-Sorokin-Tonin actions have self-duality with respect to a common anti-dualization of chiral boson fields in $D=2$ and $D=6$ dimensions, respectively, while the Srivastava action is self-dual with respect to a generalized dualization of chiral boson fields. Moreover, the action of the Floreanini-Jackiw chiral bosons interacting with gauge fields in $D=2$ dimensions also has self-duality but with respect to a generalized anti-dualization of chiral boson fields.

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I. INTRODUCTION

Chiral p-forms, sometimes called chiral bosons, are described by an antisymmetric pth order tensor $A^{(p)}$ in the $D=2(p+1)$ dimensional space-time, whose external differential $F^{(p+1)}(A)=dA^{(p)}$ satisfies the self-duality condition

$$\mathcal{F}^{(p+1)}\equiv F^{(p+1)}(A)-{}^*F^{(p+1)}(A)=0, \qquad (1)$$

where ${}^*F^{(p+1)}(A)$ is defined as the dual partner of $F^{(p+1)}(A)$. In the space with the Lorentzian metric signature, the self-duality requires $A^{(p)}$ to be real if p is even, or complex if p is odd. In the latter case the theory can equivalently be described by a pair of real antisymmetric tensor fields related by a duality condition.

Chiral bosons have attracted much attention because they play an important role in many theoretical models. In $D=2$ dimensional space-time, they occur as basic ingredients and elements in the formulation of heterotic strings [1] and in a number of statistical systems [2]. In $D>2$ dimensional space-time, they form an integral part in $D=6$ and type IIB $D=10$ supergravity and M-theory five-branes [3–6]. Since the equation of motion of a chiral boson, i.e., the self-duality condition, is first order with respect to the derivatives of space and time, it is a key problem to construct the corresponding action and then to quantize the theory consistently. To this end, various formulations of actions have been proposed [7–12]. These actions can be classified by manifestly Lorentz covariant versions [7–10] and non-manifestly Lorentz covariant versions [11,12] when one emphasizes their formalism under the Lorentz transformation, or by polynomial versions [7–9] and non-polynomial version [10] when one focuses on auxiliary fields introduced in the actions. Incidentally, there are no auxiliary fields introduced in the non-manifestly Lorentz covariant actions [11,12].

Many proposals have been suggested to construct chiral boson actions, among which are four typical ones [7,11,8,10] we are interested in here. The first scheme, proposed by Siegel [7], is to impose the square of the self-duality condition upon a pth order antisymmetric tensor field through the introduction of an auxiliary tensor field as a Lagrange multiplier. The problem is that the Siegel action suffers from an anomaly of gauge symmetries. However, it is possible [7] to cancel the anomaly either by introducing a Liouville term or by taking a system of 26 chiral bosons. The second proposal, by Floreanini and Jackiw [11] only in $D=2$ dimensions, is to offer a unitary and Poincaré invariant formulation by means of a first order Lagrangian in the following three ways: (i) a nonlocal Lagrangian in terms of a local field, (ii) a local Lagrangian in terms of a nonlocal field, and (iii) a local Lagrangian in terms of a local field which is of fermionic character. The equivalence between item (ii), known as the Floreanini-Jackiw formulation, and the Siegel formulation in $D=2$ dimensions has been shown by Bernstein and Sonnenschein [13], and the intrinsic relation between items (i) and (iii) has also been uncovered by Girotti *et al.* [14] from the point of view of chiral bosonization. In addition, the Floreanini-Jackiw formulation has been generalized to $D=2(p+1)$ dimensional space-time by Henneaux *et al.* [12]. The third proposal, suggested by Srivastava [8] by following Siegel's idea but adding the self-duality condition itself, gives rise to the so-called linear formulation of chiral bosons in $D=2$ dimensions. Although it has some defects as pointed out by Harada [15] and Girotti *et al.* [16], the linear formulation strictly describes a chiral boson from the point of view of equations of motion at both the classical and quantum levels. Moreover, it is quite straightforward to generalize this formulation to $D=2(p+1)$ dimensional space-time (cf. Sec. IV B). The fourth scheme, recently proposed by Pasti, Sorokin and Tonin [10], is to construct a Lorentz covariant formulation of chiral p-forms in $D=2(p+1)$ dimensions that contains a finite number of auxiliary fields in a non-polynomial way. The simplest case is that only one auxiliary scalar field is introduced. This formulation reduces to the non-manifestly covariant Floreanini-Jackiw formulation [11]

*Email address: miao@physik.uni-kl.de
†Permanent address
‡Email address: mueller1@physik.uni-kl.de

provided appropriate gauge fixing conditions are chosen. On the other hand, it has a close relationship with the Lorentz covariant McClain-Wu-Yu formulation [9] that contains infinitely many auxiliary fields in the usual polynomial way. That is to say, the Pasti-Sorokin-Tonin formulation turns into the McClain-Wu-Yu formulation if one gets rid of the nonpolynomiality and eliminates the scalar auxiliary field at the price of introducing auxiliary $(p+1)$-forms, or, vice versa, if one consistently truncates the McClain-Wu-Yu infinite tail and puts on its end the auxiliary scalar field.

Because various types of strings are related by dualities, the duality symmetries of the Pasti-Sorokin-Tonin formulation have been studied and some interesting results have been obtained [10]. The chiral boson action in $D=2$ dimensions is self-dual with respect to both the dualization of the chiral boson field and the dualization of the auxiliary scalar field. In the $D=4$ case, the action is still self-dual under the dualization of the two real chiral 1-forms, but turns out to be a new covariant duality-symmetric Maxwell action that contains an auxiliary 2-form field under a duality transform of the auxiliary scalar field. The Pasti-Sorokin-Tonin action in $D=6$ dimensional space-time gives rise to such a dual version that includes an auxiliary 4-form field and has a different symmetry structure from that of its initial action when one performs a duality transform of the auxiliary scalar field. Incidentally, the self-duality of the action with respect to the dualization of the chiral 2-form field in the $D=6$ case was not explicitly verified in Ref. [10].

In this paper we investigate the duality properties of the four typical chiral p-form actions mentioned above by using $D=2$ and $D=6$ dimensions as examples. We pay our main attention to these actions' dual versions under duality transforms of chiral p-form fields since we expect to extract some common property from the four actions that have such big differences in formulation. As to the duality under transforms of auxiliary fields for the first three chiral p-form actions, it is a trivial problem because of the linearity of auxiliary fields in the Siegel and Srivastava actions [7,8] and of the non-existence of auxiliary fields in the Floreanini-Jackiw action [11,12]. As a result, we discover that the Siegel, Floreanini-Jackiw and Pasti-Sorokin-Tonin actions are self-dual under a common anti-dual transform of 1-form ''field strengths'' in $D=2$ dimensional space-time and of 3-form field strengths in the $D=6$ case, while the Srivastava action is self-dual under a generalized dual transform of 1-form ''field strength'' in $D=2$ dimensions and of 3-form field strength in $D=6$ dimensions. We also find that the self-duality conditions of the four actions in the $D=2$ and $D=6$ cases, respectively, have the same transformation although the transforms of the field strengths are quite different from one another. Moreover, we extend the self-duality of actions from free chiral bosons to interacting cases and choose, as an example, the action of the Floreanini-Jackiw chiral bosons interacting with gauge fields proposed by Harada [17]. We find that this action is also self-dual but with respect to a generalized anti-dualization of the chiral boson field, and that the transformation of the difference between the 1-form ''field strength'' and its dual partner is very different from that of the free cases because of interactions.

The paper is arranged as follows. In Secs. II–V, we discuss the duality symmetries of the four chiral p-form actions one by one in the Siegel, Floreanini-Jackiw, Srivastava, and Pasti-Sorokin-Tonin formulations. Each section is divided into two subsections for the $D=2$ and $D=6$ cases. Then we turn to the interacting theory of the Floreanini-Jackiw chiral bosons and gauge fields in Sec. VI, and finally make a conclusion in Sec. VII.

The metric notation we use throughout this paper is

$$g_{00}=-g_{11}=\cdots=-g_{D-1,D-1}=1,\qquad \epsilon^{012\cdots D-1}=1. \tag{2}$$

Greek letters stand for space-time indices $(\mu,\nu,\sigma,\ldots=0,1,\ldots,D-1)$ and Latin letters are spacial indices running from 1 to $D-1$.

II. SELF-DUALITY OF THE SIEGEL ACTION

A. The D=2 case

We begin with the Siegel action [7] in $D=2$ dimensional space-time

$$S=\int d^2x\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{1}{2}\lambda_{\mu\nu}(\partial^\mu\phi-\epsilon^{\mu\sigma}\partial_\sigma\phi)(\partial^\nu\phi-\epsilon^{\nu\rho}\partial_\rho\phi)\right], \tag{3}$$

where ϕ is a scalar field, and $\lambda_{\mu\nu}$ a symmetric and traceless auxiliary tensor field.

We investigate the duality property of Eq. (3) with respect to the dualization of the field $\phi(x)$ along the line of Ref. [10]. The first step is to introduce two independent vector fields, F_μ and G_μ, and replace Eq. (3) by the action

$$S=\int d^2x\left[\frac{1}{2}F_\mu F^\mu+\frac{1}{2}\lambda_{\mu\nu}\mathcal{F}^\mu\mathcal{F}^\nu+G^\mu(F_\mu-\partial_\mu\phi)\right], \tag{4}$$

where $\mathcal{F}^\mu$ is defined as the difference between F^μ and its dual partner $\epsilon^{\mu\nu}F_\nu$

$$\mathcal{F}^\mu=F^\mu-\epsilon^{\mu\nu}F_\nu. \tag{5}$$

Then, varying Eq. (4) with respect to G^μ gives the expression for the field F_μ in terms of ϕ

$$F_\mu=\partial_\mu\phi, \tag{6}$$

together with which Eq. (4) turns back to the original Siegel action Eq. (3). This shows the classical equivalence between actions Eqs. (3) and (4). The third step is to vary Eq. (4) with respect to F_μ, which yields the expression of G^μ in terms of F^μ

$$G^\mu=-F^\mu-(g^{\mu\sigma}+\epsilon^{\mu\sigma})\lambda_{\sigma\nu}\mathcal{F}^\nu. \tag{7}$$

Similar to Eq. (5), $\mathcal{G}^\mu$ is defined as

$$\mathcal{G}^{\mu}=G^{\mu}-\epsilon^{\mu\nu}G_{\nu}, \tag{8}$$

which, when Eq. (7) is substituted, gives a relationship between $\mathcal{F}^{\mu}$ and $\mathcal{G}^{\mu}$

$$\mathcal{F}^{\mu}=-\mathcal{G}^{\mu}. \tag{9}$$

It is necessary to point out in advance that Eq. (9) is generally satisfied for all the four chiral boson actions discussed in this paper (cf. Secs. III A, IV A, and V A) although the relations between F^{μ} and G^{μ} for these actions are very different from one another. With Eq. (9), it is easy to invert Eq. (7) and obtain F^{μ} in terms of G^{μ}

$$F^{\mu}=-G^{\mu}+(g^{\mu\sigma}+\epsilon^{\mu\sigma})\lambda_{\sigma\nu}\mathcal{G}^{\nu}. \tag{10}$$

We can check from Eq. (7) that when the self-duality condition is satisfied, i.e., $\mathcal{F}^{\mu}=0$, which is called ''on the mass shell'' in Ref. [10], F^{μ} and G^{μ} relate with an anti-duality $G^{\mu}=-\epsilon^{\mu\nu}F_{\nu}$. Note that in Ref. [10] they relate with a dual relation because of the distinct metric notation. We will see that this type of anti-duality also appears in the Floreanini-Jackiw and Pasti-Sorokin-Tonin actions in the D=2 case although Eqs. (7), (27) and (51) are quite different from one another (cf. Secs. III A and V A), but does not in the Srivastava action (cf. Sec. IV A). Substituting Eq. (10) into Eq. (4), we get the dual version of the Siegel action

$$S_{dual}=\int d^2x\left[-\frac{1}{2}G_{\mu}G^{\mu}+\frac{1}{2}\lambda_{\mu\nu}\mathcal{G}^{\mu}\mathcal{G}^{\nu}+\phi\partial_{\mu}G^{\mu}\right]. \tag{11}$$

Variation of Eq. (11) with respect to ϕ gives $\partial_{\mu}G^{\mu}=0$, whose solution should be

$$G^{\mu}(\psi)=-\epsilon^{\mu\nu}\partial_{\nu}\psi\equiv-\epsilon^{\mu\nu}F_{\nu}(\psi), \tag{12}$$

where ψ is an arbitrary scalar field. When Eq. (12) is substituted into Eq. (11), we obtain the dual action that is exactly the same as the Siegel action Eq. (3) only with the replacement of ϕ by ψ. As analyzed above, ϕ and ψ coincide with each other up to a constant when the self-duality condition is imposed. Therefore, the Siegel action is self-dual with respect to the $\phi(x)-\psi(x)$ anti-dualization expressed by Eqs. (6) and (12).

B. The D=6 case

The Siegel action in $D=6$ space-time dimensions takes the form [7]

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}(A)F^{\mu\nu\sigma}(A)+\frac{1}{2}\lambda_{\mu\nu}\mathcal{F}^{\mu\rho\sigma}(A)\mathcal{F}^{\nu}{}_{\rho\sigma}(A)\right], \tag{13}$$

where $F_{\mu\nu\sigma}(A)$ is the 3-form field strength of the real antisymmetric tensor field $A_{\mu\nu}(\mu,\nu=0,1,\ldots,5)$

$$F_{\mu\nu\sigma}(A)=\partial_{\mu}A_{\nu\sigma}+\partial_{\nu}A_{\sigma\mu}+\partial_{\sigma}A_{\mu\nu}\equiv\partial_{[\mu}A_{\nu\sigma]}, \tag{14}$$

and $\mathcal{F}_{\mu\nu\sigma}(A)$ is defined as

$$\mathcal{F}_{\mu\nu\sigma}(A)=F_{\mu\nu\sigma}(A)-\frac{1}{3!}\epsilon_{\mu\nu\sigma\rho\eta\delta}F^{\rho\eta\delta}(A). \tag{15}$$

In order to discuss the duality of the Siegel action, we introduce two 3-form fields $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$, and replace Eq. (13) by the following action:

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}F^{\mu\nu\sigma}+\frac{1}{2}\lambda_{\mu\nu}\mathcal{F}^{\mu\rho\sigma}\mathcal{F}^{\nu}{}_{\rho\sigma}+\frac{1}{3}G^{\mu\nu\sigma}(F_{\mu\nu\sigma}-\partial_{[\mu}A_{\nu\sigma]})\right], \tag{16}$$

where $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$ act, at present, as independent auxiliary fields. To vary Eq. (16) with respect to $G^{\mu\nu\sigma}$ gives

$$F_{\mu\nu\sigma}=\partial_{[\mu}A_{\nu\sigma]}, \tag{17}$$

which, when substituted into Eq. (16), yields the equivalence between actions Eqs. (13) and (16). On the other hand, variation of Eq. (16) with respect to $F_{\mu\nu\sigma}$ leads to the expression of $G^{\mu\nu\sigma}$ in terms of $F^{\mu\nu\sigma}$

$$G^{\mu\nu\sigma}=-F^{\mu\nu\sigma}-\lambda^{\rho[\mu}\mathcal{F}_{\rho}{}^{\nu\sigma]}-\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\lambda_{\theta[\rho}\mathcal{F}^{\theta}{}_{\eta\delta]}. \tag{18}$$

Like Eq. (15), we define $\mathcal{G}^{\mu\nu\sigma}$ to be

$$\mathcal{G}^{\mu\nu\sigma}=G^{\mu\nu\sigma}-\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}G_{\rho\eta\delta}, \tag{19}$$

and obtain, when Eq. (18) is substituted into Eq. (19), the relation

$$\mathcal{F}^{\mu\nu\sigma}=-\mathcal{G}^{\mu\nu\sigma}. \tag{20}$$

Note that this is generally satisfied for all the four actions in the $D=6$ case although relations of $F^{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$ in these actions are quite different from one another (cf. Secs. III B, IV B, and V B). With Eq. (20), we can invert Eq. (18) quite easily and solve $F^{\mu\nu\sigma}$ in terms of $G^{\mu\nu\sigma}$

$$F^{\mu\nu\sigma}=-G^{\mu\nu\sigma}+\lambda^{\rho[\mu}\mathcal{G}_{\rho}{}^{\nu\sigma]}+\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\lambda_{\theta[\rho}\mathcal{G}^{\theta}{}_{\eta\delta]}. \tag{21}$$

We can verify from Eq. (18) that when the self-duality condition is satisfied, i.e., $\mathcal{F}^{\mu\nu\sigma}=0$, $F^{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$ relate with an anti-duality $G^{\mu\nu\sigma}=-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}$. This relation also appears in the Floreanini-Jackiw and Pasti-Sorokin-Tonin actions in the $D=6$ case, but does not in the Srivastava action. Now substituting Eq. (21) into the action Eq. (16), we obtain the dual Siegel action in the $D=6$ case

$$S_{dual}=\int d^6x\left[-\frac{1}{6}G_{\mu\nu\sigma}G^{\mu\nu\sigma}+\frac{1}{2}\lambda_{\mu\nu}\mathcal{G}^{\mu\rho\sigma}\mathcal{G}^{\nu}{}_{\rho\sigma}+A_{\nu\sigma}\partial_{\mu}G^{\mu\nu\sigma}\right]. \tag{22}$$

Variation of Eq. (22) with respect to $A_{\nu\sigma}$ gives

$$\partial_{\mu}G^{\mu\nu\sigma}=0, \tag{23}$$

whose solution should be

$$G^{\mu\nu\sigma}(B)=-\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\partial_{[\rho}B_{\eta\delta]}\equiv-\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}(B), \tag{24}$$

where $B_{\mu\nu}$ is an arbitrary 2-form field. When Eq. (24) is substituted into the dual action Eq. (22), we get the result that the dual action is the same as the Siegel action Eq. (13) only with the replacement of $A_{\mu\nu}$ by $B_{\mu\nu}$. Consequently, the Siegel action is self-dual in $D=6$ dimensional space-time with respect to the $A_{\mu\nu}-B_{\mu\nu}$ anti-dualization given by Eqs. (17) and (24).

III. SELF-DUALITY OF THE FLOREANINI-JACKIW ACTION

A. The D=2 case

The Floreanini-Jackiw action in $D=2$ dimensions has the form [11]

$$S=\int d^2x[\partial_0\phi\partial_1\phi-(\partial_1\phi)^2], \tag{25}$$

in which no auxiliary fields are introduced. It is a non-manifestly Lorentz covariant action, but has Poincaré invariance from the point of view of Hamiltonian analyses.

As in Sec. II A, we introduce two independent auxiliary vector fields F^{μ} and G^{μ}, and replace Eq. (25) by the action

$$S=\int d^2x[F_0F_1-(F_1)^2+G^{\mu}(F_{\mu}-\partial_{\mu}\phi)]. \tag{26}$$

Variation of this action with respect to the Lagrange multiplier G^{μ} gives rise to the same result as Eq. (6), which leads to the equivalence between Eqs. (25) and (26). On the other hand, variation of Eq. (26) with respect to F_{μ} gives the expression of G^{μ} in terms of F_{μ}

$$G^0=-F_1,$$
$$G^1=-F_0+2F_1, \tag{27}$$

whose inversion is

$$F_0=-2G^0-G^1,$$
$$F_1=-G^0. \tag{28}$$

If we define $\mathcal{F}^{\mu}$ and $\mathcal{G}^{\mu}$ as in Eqs. (5) and (8), respectively, we discover that they still satisfy the relation Eq. (9) as pointed out in Sec. II A. Moreover, F^{μ} and G^{μ} have an anti-dual relation $G^{\mu}=-\epsilon^{\mu\nu}F_{\nu}$ if the self-duality condition in the $D=2$ case $\mathcal{F}^{\mu}=0$ is imposed into Eq. (27). Substituting Eq. (28) into Eq. (26), we obtain the dual Floreanini-Jackiw action

$$S_{dual}=\int d^2x[-(G_0)^2+G_0G_1+\phi\partial_{\mu}G^{\mu}]. \tag{29}$$

The remaining procedure is the same as that in Sec. II A. As a result, the Floreanini-Jackiw action in $D=2$ dimensional space-time is self-dual with respect to the $\phi(x)-\psi(x)$ anti-duality as shown in Eqs. (6) and (12).

B. The D=6 case

The non-manifestly Lorentz covariant formulation of Floreanini and Jackiw was generalized to chiral p-forms in Ref. [12]. The action for a chiral 2-form in $D=6$ dimensions is

$$S=\int d^6x\left[\frac{1}{2}\left(F_{0ij}(A)-\frac{1}{3!}\epsilon_{0ijklm}F^{klm}(A)\right)\times\frac{1}{3!}\epsilon_{0ijnpq}F^{npq}(A)\right], \tag{30}$$

where $F_{\mu\nu\sigma}(A)$ is the field strength of $A_{\mu\nu}$, as stated in Eq. (14), and Latin letters stand for spatial indices $(i,j,\ldots=1,\ldots,5)$. Note that no auxiliary fields appear in Eq. (30). In the following, we utilize the simplier form of Eq. (30)

$$S=\int d^6x\left[\frac{1}{12}\epsilon^{0ijklm}F_{0ij}(A)F_{klm}(A)-\frac{1}{6}F_{klm}(A)F_{klm}(A)\right]. \tag{31}$$

We begin with the duality property of the action Eq. (31) under the dualization of the antisymmetric tensor field $A_{\mu\nu}$. Introducing two auxiliary 3-forms $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$, we construct a new action to replace Eq. (31)

$$S=\int d^6x\left[\frac{1}{12}\epsilon^{0ijklm}F_{0ij}F_{klm}-\frac{1}{6}F_{klm}F_{klm}+\frac{1}{6}G^{\mu\nu\sigma}(F_{\mu\nu\sigma}-\partial_{[\mu}A_{\nu\sigma]})\right], \tag{32}$$

where $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$ are treated as independent fields. For the sake of convenience in the calculation, we rewrite Eq. (32) to be

$$S=\int d^6x\left[\frac{1}{12}\epsilon^{0ijklm}F_{0ij}F_{klm}-\frac{1}{6}F_{klm}F_{klm}+\frac{1}{2}G^{0ij}(F_{0ij}-\partial_{[0}A_{ij]})+\frac{1}{6}G^{klm}(F_{klm}-\partial_{[k}A_{lm]})\right]. \tag{33}$$

Variation of Eq. (33) with respect to the Lagrange multiplier $G^{\mu\nu\sigma}$ gives Eq. (17), which shows the equivalence between Eqs. (31) and (33). Moreover, variation of Eq. (33) with respect to $F_{\mu\nu\sigma}$ gives the expression of $G^{\mu\nu\sigma}$ in terms of $F_{\mu\nu\sigma}$

$$G^{0ij}=-\frac{1}{6}\epsilon^{0ijnpq}F_{npq},$$

$$G^{klm}=-\frac{1}{2}\epsilon^{0npklm}F_{0np}-2F^{klm}, \tag{34}$$

from which $F_{\mu\nu\sigma}$ can be calculated

$$F^{0ij}=-2G^{0ij}+\frac{1}{6}\epsilon^{0ijnpq}G_{npq},$$

$$F^{klm}=\frac{1}{2}\epsilon^{0npklm}G_{0np}. \tag{35}$$

If we define $\mathcal{F}^{\mu\nu\sigma}$ and $\mathcal{G}^{\mu\nu\sigma}$ as in Eqs. (15) and (19), respectively, we find that they still satisfy Eq. (20) although Eqs. (18) and (34), i.e., the relations of $F^{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$ for the Siegel and Floreanini-Jackiw formulations of chiral 2-forms, are quite different. In addition, when imposing the self-duality condition in the $D=6$ case, i.e., $\mathcal{F}^{\mu\nu\sigma}=0$, into Eq. (34), we still derive the anti-duality between $F^{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$, $G^{\mu\nu\sigma}=-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}$. Substituting Eq. (35) into Eq. (33), we obtain the dual formulation of the Floreanini-Jackiw chiral 2-form in the $D=6$ case

$$S_{dual}=\int d^6x\left[-\frac{1}{2}G^{0ij}G_{0ij}+\frac{1}{12}\epsilon_{0ijklm}G^{0ij}G^{klm}+\frac{1}{2}A_{\nu\sigma}\partial_\mu G^{\mu\nu\sigma}\right]. \tag{36}$$

The following steps are straightforward. Variation of Eq. (36) with respect to $A_{\nu\sigma}$ gives $\partial_\mu G^{\mu\nu\sigma}=0$, whose solution is Eq. (24) in which an antisymmetric tensor field $B_{\mu\nu}$ is introduced. With Eq. (24), the dual action Eq. (36) is the same as the action Eq. (31), only with the replacement of $A_{\mu\nu}$ by $B_{\mu\nu}$. Therefore, we verify that the Floreanini-Jackiw action for a chiral 2-form in $D=6$ dimensions is self-dual under the $A_{\mu\nu}-B_{\mu\nu}$ anti-duality transform of Eqs. (17) and (24).

IV. SELF-DUALITY OF THE SRIVASTAVA ACTION

A. The D=2 case

We write the linear formulation of chiral bosons suggested by Srivastava [8]

$$S=\int d^2x\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\lambda_\mu(\partial^\mu\phi-\epsilon^{\mu\nu}\partial_\nu\phi)\right], \tag{37}$$

where ϕ is a scalar field and λ_μ an auxiliary vector field. This action has some defects as pointed out by others [15,16], but it ‘‘synthesizes’’ the manifest Lorentz covariance and self-duality constraint.

Let us introduce two auxiliary vector fields F_μ and G^μ, and construct a new action to replace Eq. (37)

$$S=\int d^2x\left[\frac{1}{2}F^\mu F_\mu+\lambda_\mu(F^\mu-\epsilon^{\mu\nu}F_\nu)+G^\mu(F_\mu-\partial_\mu\phi)\right], \tag{38}$$

where F_μ and G^μ are independent of the other fields. When varying Eq. (38) with respect to G^μ, we have $F_\mu=\partial_\mu\phi$, i.e., Eq. (6), and with it we can prove that the new action Eq. (38) is equivalent to the original one Eq. (37). On the other hand, when varying Eq. (38) with respect to F_μ, we get G^μ in terms of F_μ

$$G^\mu=-F^\mu-(\lambda^\mu+\epsilon^{\mu\nu}\lambda_\nu), \tag{39}$$

or, vice versa, F_μ in terms of G^μ

$$F^\mu=-G^\mu-(\lambda^\mu+\epsilon^{\mu\nu}\lambda_\nu). \tag{40}$$

If $\mathcal{F}^\mu$ and $\mathcal{G}^\mu$ are defined as in Eqs. (5) and (8), respectively, they again satisfy the relation Eq. (9) although Eq. (39) is quite different from Eq. (7) and Eq. (27). However, F^μ and G^μ no longer relate with any anti-duality when the self-duality condition $\mathcal{F}^\mu=0$ is imposed into Eq. (39). This happens because the self-duality condition with a Lagrange multiplier is introduced linearly in the action Eq. (37). We may say that this anti-duality between F^μ and G^μ is not necessary when one considers the duality property of actions because the self-duality condition can not be directly imposed into actions. Substituting Eq. (40) into Eq. (38), we obtain the dual version of the Srivastava action

$$S_{dual}=\int d^2x\left[-\frac{1}{2}G^\mu G_\mu-\lambda_\mu(G^\mu-\epsilon^{\mu\nu}G_\nu)+\phi\partial_\mu G^\mu\right]. \tag{41}$$

When varying Eq. (41) with respect to ϕ, we get $\partial_\mu G^\mu=0$ and then solve

$$G^\mu(\psi)=\epsilon^{\mu\nu}\partial_\nu\psi\equiv\epsilon^{\mu\nu}F_\nu(\psi), \tag{42}$$

where ψ is an arbitrary scalar field. When Eq. (42) is substituted into Eq. (41), we find that the dual action is the same as the original one Eq. (37) only with the replacement of ϕ by ψ. Consequently, the Srivastava action in the $D=2$ case is self-dual under the generalized dualization Eq. (42). Here the word ‘‘generalized’’ means that $\phi(x)$ does not coincide with $\psi(x)$ even if the self-duality condition is considered.

B. The D=6 case

We can easily generalize the $D=2$ Srivastava action to the $D=6$ case

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}(A)F^{\mu\nu\sigma}(A)+\frac{1}{3}\lambda_{\mu\nu\sigma}\mathcal{F}^{\mu\nu\sigma}(A)\right], \tag{43}$$

where $F_{\mu\nu\sigma}(A)$ and $\mathcal{F}^{\mu\nu\sigma}(A)$ are defined as Eqs. (14) and (15), respectively, and $\lambda_{\mu\nu\sigma}$ is an auxiliary antisymmetric tensor field. Variation of this action with respect to $\lambda_{\mu\nu\sigma}$ gives the self-duality condition $\mathcal{F}^{\mu\nu\sigma}(A)=0$ that is in fact the equation of motion of $A_{\mu\nu}$. Therefore, Eq. (43) indeed describes a chiral 2-form field in $D=6$ dimensional spacetime. As to its canonical Hamiltonian analysis, it can be achieved straightforwardly by following the procedure shown in Ref. [8]. Here we omit it.

We introduce two auxiliary 3-form fields $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$, and construct a new action to replace Eq. (43)

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}F^{\mu\nu\sigma}+\frac{1}{3}\lambda_{\mu\nu\sigma}\mathcal{F}^{\mu\nu\sigma}+\frac{1}{3}G^{\mu\nu\sigma}(F_{\mu\nu\sigma}-\partial_{[\mu}A_{\nu\sigma]})\right], \qquad (44)$$

where $F_{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$ are treated as independent fields, and $\mathcal{F}^{\mu\nu\sigma}\equiv F^{\mu\nu\sigma}-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}$. By varying Eq. (44) with respect to $G^{\mu\nu\sigma}$, we get $F_{\mu\nu\sigma}=\partial_{[\mu}A_{\nu\sigma]}$ and then verify the equivalence between Eqs. (43) and (44). On the other hand, by varying Eq. (44) with respect to $F_{\mu\nu\sigma}$, we have the expression of $G^{\mu\nu\sigma}$ in terms of $F_{\mu\nu\sigma}$

$$G^{\mu\nu\sigma}=-F^{\mu\nu\sigma}-\left(\lambda^{\mu\nu\sigma}+\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\lambda_{\rho\eta\delta}\right), \qquad (45)$$

or, vice versa, that of $F_{\mu\nu\sigma}$ in terms of $G_{\mu\nu\sigma}$

$$F^{\mu\nu\sigma}=-G^{\mu\nu\sigma}-\left(\lambda^{\mu\nu\sigma}+\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\lambda_{\rho\eta\delta}\right). \qquad (46)$$

As usual, we define $\mathcal{G}^{\mu\nu\sigma}=G^{\mu\nu\sigma}-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}G_{\rho\eta\delta}$ and obtain, using Eq. (45), $\mathcal{F}^{\mu\nu\sigma}=-\mathcal{G}^{\mu\nu\sigma}$. This relation is generally correct for all the four formulations of chiral 2-forms although Eqs. (18), (34), (45) and (56) are quite different from one another. But, similar to the $D=2$ case, $F^{\mu\nu\sigma}$ and $G^{\mu\nu\sigma}$ do not relate with any anti-duality in the Srivastava action even if the self-duality condition $\mathcal{F}^{\mu\nu\sigma}=0$ is imposed to Eq. (45). The reason remains the linearity of the self-duality condition in the action Eq. (43). This situation does not occur in the Siegel, Floreanini-Jackiw and Pasti-Sorokin-Tonin actions. Substituting Eq. (46) into Eq. (44), we get the dual action

$$S_{dual}=\int d^6x\left[-\frac{1}{6}G_{\mu\nu\sigma}G^{\mu\nu\sigma}-\frac{1}{3}\lambda_{\mu\nu\sigma}\mathcal{G}^{\mu\nu\sigma}+A_{\nu\sigma}\partial_\mu G^{\mu\nu\sigma}\right]. \qquad (47)$$

Variation of Eq. (47) with respect to $A_{\nu\sigma}$ gives $\partial_\mu G^{\mu\nu\sigma}=0$, and the solution should be

$$G^{\mu\nu\sigma}(B)=\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\partial_{[\rho}B_{\eta\delta]}\equiv\frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}(B), \qquad (48)$$

where $B_{\mu\nu}$ is an arbitrary 2-form field. Substituting Eq. (48) into Eq. (47), we recover the Srivastava formulation with $B_{\mu\nu}$ as the argument. This shows the self-duality of the Srivastava action in the $D=6$ case with respect to the generalized duality transform Eq. (48). Here we add the word ''generalized'' because $A_{\mu\nu}$ no longer coincides with $B_{\mu\nu}$ on the mass shell.

V. SELF-DUALITY OF THE PASTI-SOROKIN-TONIN ACTION

A. The D=2 case

The self-duality of the Pasti-Sorokin-Tonin action in the $D=2$ case has been explicitly shown in Ref. [10]. In order to make our paper complete, we briefly repeat the main procedure by means of our metric notation that is different from that used in Ref. [10].

The non-polynomial formulation of chiral bosons proposed by Pasti, Sorokin and Tonin [10] takes the form

$$S=\int d^2x\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{1}{2(\partial_\nu a)(\partial^\nu a)}[\partial^\mu a(\partial_\mu\phi-\epsilon_{\mu\sigma}\partial^\sigma\phi)]^2\right\}, \qquad (49)$$

where $\phi(x)$ is a scalar field, and $a(x)$ an auxiliary scalar field introduced in a non-polynomial way. Note that we have adopted our metric notation in the action Eq. (49).

By introducing two auxiliary vector fields F_μ and G^μ, we construct a new action to replace Eq. (49):

$$S=\int d^2x\left[\frac{1}{2}F_\mu F^\mu+\frac{1}{2(\partial_\nu a)(\partial^\nu a)}(\partial^\mu a\mathcal{F}_\mu)^2+G^\mu(F_\mu-\partial_\mu\phi)\right], \qquad (50)$$

where F_μ and G^μ are dealt with as independent fields, and $\mathcal{F}_\mu\equiv F_\mu-\epsilon_{\mu\nu}F^\nu$. Variation of Eq. (50) with respect to the Lagrange multiplier G^μ gives $F_\mu=\partial_\mu\phi$, i.e., Eq. (6), which yields the equivalence between the Pasti-Sorokin-Tonin action and the new action Eq. (50). Moreover, variation of Eq. (50) with respect to F_μ leads to the expression of G^μ in terms of F_μ:

$$G^\mu=-F^\mu-\frac{\partial^\mu a+\epsilon^{\mu\rho}\partial_\rho a}{(\partial_\sigma a)(\partial^\sigma a)}(\partial^\nu a\mathcal{F}_\nu). \qquad (51)$$

In order to easily solve F_μ in terms of G^μ from the above equation, we define, like Eq. (8), $\mathcal{G}^\mu=G^\mu-\epsilon^{\mu\nu}G_\nu$. When Eq. (51) is substituted into $\mathcal{G}^\mu$, we get the relation $\mathcal{F}^\mu=-\mathcal{G}^\mu$, which also exists in the first three formulations of chiral bosons discussed in Secs. II A, III A, and IV A. By using $\mathcal{F}^\mu=-\mathcal{G}^\mu$, we therefore solve F^μ from Eq. (51):

$$F^\mu=-G^\mu+\frac{\partial^\mu a+\epsilon^{\mu\rho}\partial_\rho a}{(\partial_\sigma a)(\partial^\sigma a)}(\partial^\nu a\mathcal{G}_\nu). \qquad (52)$$

We can see that F^μ and G^μ satisfy an anti-duality $G^\mu=-\epsilon^{\mu\nu}F_\nu$ on the mass shell. Note that in Ref. [10] their relation is dual because of the distinct metric notation. We have known that this type of anti-duality also appears in the Siegel and Floreanini-Jackiw actions in the $D=2$ case although Eqs. (7), (27) and (51) are quite different from one another, but does not in the Srivastava action. Now substituting Eq. (52) into Eq. (50), we obtain the dual action

$$S_{dual}=\int d^2x\left[-\frac{1}{2}G_\mu G^\mu + \frac{1}{2(\partial_\nu a)(\partial^\nu a)}(\partial^\mu a\mathcal{G}_\mu)^2+\phi\partial_\mu G^\mu\right]. \quad (53)$$

Exactly following the discussions below Eq. (11), we can conclude that the Pasti-Sorokin-Tonin action in $D=2$ dimensional space-time is self-dual with respect to the $\phi(x)-\psi(x)$ anti-dualization given by Eqs. (6) and (12).

B. The D=6 case

Since the self-duality of the Pasti-Sorokin-Tonin action with respect to the dualization of chiral 2-form fields in $D=6$ dimensional space-time was not explicitly verified in Ref. [10], we add the verification here in terms of our metric notation.

First we write the Pasti-Sorokin-Tonin action for a chiral 2-form field $A_{\mu\nu}$:

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}(A)F^{\mu\nu\sigma}(A) + \frac{1}{2(\partial_\lambda a)(\partial^\lambda a)}\partial^\mu a\mathcal{F}_{\mu\nu\sigma}(A)\mathcal{F}^{\nu\sigma\rho}(A)\partial_\rho a\right], \quad (54)$$

where $F_{\mu\nu\sigma}(A)$ and $\mathcal{F}_{\mu\nu\sigma}(A)$ are defined as in Eqs. (14) and (15), respectively, and $a(x)$ is an auxiliary scalar field introduced in a non-polynomial way.

By introducing two auxiliary 3-form fields $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$, we construct a new action to replace Eq. (54):

$$S=\int d^6x\left[\frac{1}{6}F_{\mu\nu\sigma}F^{\mu\nu\sigma}+\frac{1}{2(\partial_\lambda a)(\partial^\lambda a)}\partial^\mu a\mathcal{F}_{\mu\nu\sigma}\mathcal{F}^{\nu\sigma\rho}\partial_\rho a + \frac{1}{3}G^{\mu\nu\sigma}(F_{\mu\nu\sigma}-\partial_{[\mu}A_{\nu\sigma]})\right], \quad (55)$$

where $F_{\mu\nu\sigma}$ and $G_{\mu\nu\sigma}$ are dealt with as independent fields, and $\mathcal{F}^{\mu\nu\sigma}\equiv F^{\mu\nu\sigma}-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}$. Variation of Eq. (55) with respect to the Lagrange multiplier $G^{\mu\nu\sigma}$ gives $F_{\mu\nu\sigma}=\partial_{[\mu}A_{\nu\sigma]}$, i.e., Eq. (17), which yields the equivalence between Eqs. (54) and (55). On the other hand, variation of Eq. (55) with respect to $F_{\mu\nu\sigma}$ leads to the expression of $G_{\mu\nu\sigma}$ in terms of $F_{\mu\nu\sigma}$

$$G^{\mu\nu\sigma}=-F^{\mu\nu\sigma}-\frac{1}{(\partial_\lambda a)(\partial^\lambda a)}\left[\partial^{[\mu}a\mathcal{F}^{\nu\sigma]\rho}\partial_\rho a + \frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\partial_{[\rho}a\mathcal{F}_{\eta\delta]\theta}\partial^\theta a\right]. \quad (56)$$

When we define $\mathcal{G}^{\mu\nu\sigma}=G^{\mu\nu\sigma}-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}G_{\rho\eta\delta}$, we obtain $\mathcal{F}^{\mu\nu\sigma}=-\mathcal{G}^{\mu\nu\sigma}$ once again. As we have pointed out in Sec. V A, this relation is generally correct for all the four chiral 2-form actions in $D=6$ dimensions although Eqs. (18), (34), (45) and (56) are quite different from one another. Considering the general relation, we can solve from Eq. (56) $F^{\mu\nu\sigma}$ in terms of $G^{\mu\nu\sigma}$

$$F^{\mu\nu\sigma}=-G^{\mu\nu\sigma}+\frac{1}{(\partial_\lambda a)(\partial^\lambda a)}\left[\partial^{[\mu}a\mathcal{G}^{\nu\sigma]\rho}\partial_\rho a + \frac{1}{3!}\epsilon^{\mu\nu\sigma\rho\eta\delta}\partial_{[\rho}a\mathcal{G}_{\eta\delta]\theta}\partial^\theta a\right]. \quad (57)$$

As discussed in Secs. II B and III B, we can prove that $G^{\mu\nu\sigma}$ relates to $F^{\mu\nu\sigma}$ by an anti-duality $G^{\mu\nu\sigma}=-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}$ on the mass shell, that is, under the condition $\mathcal{F}^{\mu\nu\sigma}=0$. The anti-dual relation is satisfied in the Siegel, Floreanini-Jackiw and Pasti-Sorokin-Tonin actions, but not in the Srivastava action. Now substituting Eq. (57) into Eq. (55), we get the dual action in terms of $G^{\mu\nu\sigma}$

$$S_{dual}=\int d^6x\left[-\frac{1}{6}G_{\mu\nu\sigma}G^{\mu\nu\sigma} + \frac{1}{2(\partial_\lambda a)(\partial^\lambda a)}\partial^\mu a\mathcal{G}_{\mu\nu\sigma}\mathcal{G}^{\nu\sigma\rho}\partial_\rho a + A_{\nu\sigma}\partial_\mu G^{\mu\nu\sigma}\right]. \quad (58)$$

We do not repeat the subsequent steps which are equally the same as below Eq. (22). As a result, the Pasti-Sorokin-Tonin action has self-duality under the $A_{\mu\nu}-B_{\mu\nu}$ anti-dual transform Eqs. (17) and (24).

VI. SELF-DUALITY OF THE GAUGED FLOREANINI-JACKIW CHIRAL BOSON ACTION

We extend the discussion of self-duality of chiral p-form actions from free theories to interacting cases, and choose the action of Floreanini-Jackiw chiral bosons interacting with gauge fields [17] as our example.

We first write the action of this interacting theory

$$S=\int d^2x\left[\partial_0\phi\partial_1\phi-(\partial_1\phi)^2+2e\partial_1\phi(A_0-A_1) - \frac{1}{2}e^2(A_0-A_1)^2+\frac{1}{2}e^2aA_\mu A^\mu-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \quad (59)$$

where ϕ is a scalar field, A_μ a gauge field and $F_{\mu\nu}$ its field strength; e is the electric charge and a a real parameter caused by ambiguity in bosonization. It is a non-manifestly Lorentz covariant action but indeed has Lorentz invariance [17]. In the following discussion, the interacting term, i.e., the third term in Eq. (59), is important, while the last three terms that relate only to gauge fields have nothing to do with the duality property of the action.

By introducing two auxiliary vector fields F_μ and G^μ, we construct a new action to replace Eq. (59)

$$S=\int d^2x\bigg[F_0F_1-(F_1)^2+2eF_1(A_0-A_1)-\frac{1}{2}e^2(A_0-A_1)^2+\frac{1}{2}e^2aA_\mu A^\mu-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+G^\mu(F_\mu-\partial_\mu\phi)\bigg], \tag{60}$$

where F_μ and G^μ are treated as independent fields. Variation of Eq. (60) with respect to the Lagrange multiplier G^μ gives $F_\mu=\partial_\mu\phi$, which yields the equivalence between the two actions Eqs. (59) and (60). Furthermore, variation of Eq. (60) with respect to F_μ leads to the expression of G^μ in terms of F_μ

$$G^0=-F_1,$$

$$G^1=-F_0+2F_1-2e(A_0-A_1). \tag{61}$$

It is easy to solve for F_μ from the above equation

$$F_0=-2G^0-G^1-2e(A_0-A_1),$$

$$F_1=-G^0. \tag{62}$$

If we define $\mathcal{F}_\mu=F_\mu-\epsilon_{\mu\nu}F^\nu$ and $\mathcal{G}_\mu=G_\mu-\epsilon_{\mu\nu}G^\nu$, we find that they satisfy the relation

$$\mathcal{F}_\mu=-\mathcal{G}_\mu-2e(g_{\mu\nu}-\epsilon_{\mu\nu})A^\nu, \tag{63}$$

which is different from that of the free Floreanini-Jackiw case because of interactions. In other words, if the interaction did not exist, i.e., $e=0$, Eq. (63) would reduce to the free theory case $\mathcal{F}_\mu=-\mathcal{G}_\mu$. Substituting Eq. (62) into Eq. (60), we obtain the dual action in terms of G^μ

$$S_{dual}=\int d^2x\bigg[-(G^0)^2-G^0G^1-2eG^0(A_0-A_1)-\frac{1}{2}e^2(A_0-A_1)^2+\frac{1}{2}e^2aA_\mu A^\mu-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\phi\partial_\mu G^\mu\bigg]. \tag{64}$$

Variation of Eq. (64) with respect to ϕ gives $\partial_\mu G^\mu=0$, whose solution should be

$$G^\mu(\psi)=-\epsilon^{\mu\nu}\partial_\nu\psi\equiv-\epsilon^{\mu\nu}F_\nu(\psi), \tag{65}$$

where $\psi(x)$ is an arbitrary scalar field. Substituting Eq. (65) into Eq. (64), we get the dual action in terms of ψ

$$S_{dual}=\int d^2x\bigg[\partial_0\psi\partial_1\psi-(\partial_1\psi)^2+2e\partial_1\psi(A_0-A_1)-\frac{1}{2}e^2(A_0-A_1)^2+\frac{1}{2}e^2aA_\mu A^\mu-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\bigg]. \tag{66}$$

It has the same formulation as the original action Eq. (59) only with the replacement of ϕ by ψ. Note that because of interactions, $\phi(x)$ no longer coincides with $\psi(x)$ up to a constant on the mass shell, which is different from that of the free theory case. This means that Eq. (65) shows a generalized anti-dualization of F_μ and G_μ. Therefore, we prove that the action of gauged Floreanini-Jackiw chiral bosons has self-duality with respect to the generalized anti-dualization of ''field strength'' expressed by Eq. (65). Incidentally, if we chose the solution $G^\mu(\psi)=\epsilon^{\mu\nu}\partial_\nu\psi$ instead of Eq. (65), the dual action would have a minus sign in the third term. That is to say, the dual action derived in this way would be different from the action Eq. (59) in formulation. However, the physical spectrum is the same whether the third term of Eq. (66) is positive or negative.

VII. CONCLUSION

By following the procedure of duality analyses illustrated by Pasti, Sorokin and Tonin [10], we have proved that the Siegel, Floreanini-Jackiw and Pasti-Sorokin-Tonin actions are self-dual with respect to a common anti-dualization of 1-form ''field strengths'' given by Eq. (12) in $D=2$ dimensional space-time, and that they are self-dual with respect to another common anti-dualization of 3-form field strengths given by Eq. (24) in $D=6$ dimensional space-time. For the Srivastava action, we have verified that it has self-duality under a generalized dual transform of 1-form ''field strength'' expressed by Eq. (42) in the $D=2$ case, and that it has self-duality under another generalized dual transform of 3-form field strength expressed by Eq. (48) in the $D=6$ case. Here the word ''generalized'' means that $G^\mu(\psi)$ and $F^\mu(\phi)$ do not relate with an anti-duality $G^\mu(\psi)=-\epsilon^{\mu\nu}F_\nu(\phi)$ on the mass shell $\mathcal{F}^\mu(\phi)=0$ in $D=2$ dimensions, and that $G^{\mu\nu\sigma}(B)$ and $F^{\mu\nu\sigma}(A)$ do not relate with another anti-duality $G^{\mu\nu\sigma}(B)=-(1/3!)\epsilon^{\mu\nu\sigma\rho\eta\delta}F_{\rho\eta\delta}(A)$ on the mass shell $\mathcal{F}^{\mu\nu\sigma}(A)=0$ in $D=6$ dimensions. The reason is the linearity of the self-duality condition introduced with an auxiliary field in the Srivastava action. We emphasize that this type of anti-duality is not necessary for self-duality of actions because the self-duality condition, i.e., the mass shell condition, cannot directly be imposed on actions. Moreover, we have found a generally satisfied relation for all the four actions discussed in this paper, that is, Eq. (9) for the $D=2$ case and Eq. (20) for the $D=6$ case. This relation means that the self-duality condition remains unchanged although the transforms of field strengths are quite different from one action to another. Incidentally, we do not mention in our paper the duality property of actions under transforms of auxiliary fields because on one hand it is a trivial problem for the first three chiral p-form actions, and on the other hand it has been studied in detail for the Pasti-Sorokin-Tonin action [10]. The triviality is caused by the linearity of auxiliary fields in the Siegel and Srivastava actions [7,8] and by the non-existence of auxiliary fields in the Floreanini-Jackiw action [11,12].

We have tried to extend the self-duality of actions from free theories to interacting ones and chosen, as our example, the action of the Floreanini-Jackiw chiral bosons interacting with gauge fields. By utilizing the concept of the generalized dualization extracted from the self-duality of the Srivastava action, we obtain that the action of the interacting theory is self-dual with respect to a generalized anti-dualization of the 1-form ''field strength'' of chiral scalars.

As stated in Ref. [10] that the self-duality of the Pasti-Sorokin-Tonin action remains in $D=2(p+1)$ dimensions, we can conclude that the Siegel, Floreanini-Jackiw and Srivastava actions are also self-dual in $D=2(p+1)$ dimensional space-time. Finally, we point out that the self-duality also exists in a wider context of theoretical models that relate to chiral p-forms, such as the generalized chiral Schwinger model (GCSM) [18], whose self-duality corresponds to the vector and axial vector current duality. This work is arranged in a separate paper [19].

Note added. The Kavalov-Mkrtchyan formulation [20] can be proved to be self-dual with respect to an anti-dualization of chiral 2-form fields along the line of this paper. We thank Dr. R. Manvelyan for pointing this out.

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