Canonical approach to 2D WZNW model, non-Abelian bosonization, and anomalies

B. Sazdović*
Institute of Physics, 11001 Belgrade, P.O. Box 57, Yugoslavia
(Received 20 December 1999; published 20 July 2000)

The gauged WZNW model has been derived as an effective action, whose Poisson brackets algebra of the constraints is isomorphic to the commutator algebra of operators in quantized fermionic theory. As a consequence, the Hamiltonian as well as the usual Lagrangian non-Abelian bosonization rules have been obtained for the chiral currents and chiral densities. The expression for the anomaly has been obtained as a function of the Schwinger term, using canonical methods.

PACS number(s): 11.10.Kk

I. INTRODUCTION

It is well known that in 1+1 dimensions there exists an equivalence between Fermi and Bose theories in the Abelian [1] and non-Abelian case [2]. In the latter Witten demonstrated that free field theory of N massless Majorana fermions is equivalent to the nonlinear σ model with a Wess-Zumino term at the infrared-stable fixed point, because both theories obey the same Kac-Moody (KM) algebras. The extension of this equivalence has been considered by several authors [3]. They introduced external chiral gauge fields and showed the identity of the effective actions which implies the identity of correlation functions.

In this paper, starting with non-Abelian fermionic theory coupled with background gauge fields, we are going to construct the equivalent bosonic theory for general gauge group. Our approach is different from the previous one and naturally works in the *Hamiltonian* formalism. We believe that it gives a simpler resolution of the problem.

The classical fermionic theory is invariant under local non-Abelian gauge transformations. Consequently, the first class constraints (FCC's) $j_{\pm a}$ are present in the theory and satisfy non-Abelian algebra as a Poisson brackets (PB) algebra. In the quantum theory the *central term* appears in the commutator algebra of the operators $\hat{j}_{\pm a}$, so that the constraints become second class (SCC's) which implies the existence of the anomaly [4]. These known results will be repeated in Sec. II for completeness of the paper and in order to fix our notation.

We define the effective bosonized theory, as a classical theory whose PB algebra of the constraints $J_{\pm a}$ is isomorphic to the commutator algebra of the operators $\hat{j}_{\pm a}$, in the quantized fermionic theory. This is how the bosonized theory at the classical level incorporates anomalies of the quantum fermionic theory.

In Sec. III we find the effective action W, for given algebra as its PB algebra. The similar problems has been considered before in the literature [5]. Using the method of coadjoint orbits, they showed that KM algebra yields the Wess-Zumino-Novikov-Witten (WZNW) model. Here we are going to present a new, canonical approach. We introduce the phase-space coordinate q^α, π_α and parametrize the constraints $J_{\pm a}$ by them. One of the main points of the paper is to find the expressions for the constraints $J_{\pm a}$ and for the canonical Hamiltonian $\mathcal{H}_c$ in terms of phase-space coordinate, satisfying a specific PB algebra. We then use the general canonical method [6,7] for constructing the effective action W with the known representation of the constraints. By eliminating the momentum variables on their equations of motion we obtain the Bose theory in the background fields, which is equivalent to the quantum fermi theory in the same background. This Bose theory is known as a *gauged WZNW action*.

In Sec. IV we derive *Hamiltonian* non-Abelian bosonization rules. It is easy to obtain the formulas for the currents, just differentiating the functional integral with respect to the background fields. We also derive the rules for $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$ terms, using the approach of this paper. Note that our Hamiltonian bosonization formulas for the currents depend on the momenta, while those for mass term depend only on the coordinates. Witten's non-Abelian bosonization rules can be obtained from the Hamiltonian ones, after eliminating the momenta.

In Sec. V we obtain the expression for the anomaly, using canonical method. We extend the general canonical formalism, from systems with FCC's to the systems with SCC's where the central term appears. We find the expressions for the left-right, as well as for the axial anomaly.

Section VI is devoted to concluding remarks. The derivation of the central term, using normal ordering prescription is presented in the Appendix.

II. CANONICAL ANALYSIS OF THE FERMIONIC THEORY

A. Classical theory

Let us consider the theory of two-dimensional (2D) massless Majorana fermions ψ^i $(i=1,\ldots,N)$, interacting with the external Yang-Mills fields A_μ and B_μ, with the action

$$S=\int d^2\xi\left[\bar{\psi}i\hat{\partial}\psi-i\bar{\psi}\hat{A}\frac{1+\gamma_5}{2}\psi-i\bar{\psi}\hat{B}\frac{1-\gamma_5}{2}\psi\right]. \quad (2.1)$$

We can rewrite it as

*Email address: sazdovic@phy.bg.ac.yu

$$S=\int d^2\xi[i\psi_-^*\dot{\psi}_- + i\psi_+^*\dot{\psi}_+ + i\psi_-^*\psi_-' - i\psi_+^*\psi_+' - i\sqrt{2}(A_+^a\psi_-^*t_a\psi_- + B_-^a\psi_+^*t_a\psi_+)]. \tag{2.2}$$

We chose anti-Hermitian matrices t_a as the generators of the gauge group G, introduce light-cone components $V_\pm = 1/\sqrt{2}(V_0 \pm V_1)$ for the vectors, and write the gauge potentials as $A_+ = A_+^a t_a$ and $B_- = B_-^a t_a$. We use the basis $\gamma^0 = \sigma_1$, $\gamma^1 = -i\sigma_2$, $\gamma_5 = \gamma^0\gamma^1 = \sigma_3$, and define the Weyl spinors by the conditions $\gamma_5\psi_\pm = \mp\psi_\pm$. For simplicity, we write $\psi_\pm^*\psi_\pm$ and $\psi_\pm^* t_a \psi_\pm$ instead of $\Sigma_i \psi_\pm^{*i}\psi_\pm^i$ and $\Sigma_{ij}\psi_\pm^{*i}t_{aij}\psi_\pm^j$.

The fermionic action (2.2) is already in the Hamiltonian form and we can conclude that there are two basic Lagrangian variables ψ_-^i and ψ_+^i appearing with time derivative, whose conjugate momenta are $\pi_\pm^i = i\psi_\pm^{*i}$. Variables without time derivatives, A_+^a and B_-^a, are Lagrange multipliers and the primary constraints corresponding to them are the currents

$$j_{\pm a} = i\psi_\pm^* t_a \psi_\pm = \pi_\pm t_a \psi_\pm . \tag{2.3}$$

The canonical Hamiltonian density can be expressed in terms of the chiral quantities $\theta_\pm$

$$\mathcal{H}_c = \theta_+ - \theta_- \quad (\theta_\pm = i\psi_\pm^*\psi_\pm' = \pi_\pm\psi_\pm'). \tag{2.4}$$

Starting with the basic PB

$$\{\psi_\pm^i(\sigma), \pi_\pm^j(\bar{\sigma})\} = \delta^{ij}\delta(\sigma-\bar{\sigma}), \tag{2.5}$$

we can find that PB of the currents satisfies two independent copies of KM algebras *without central charges*

$$\{j_{\pm a}, j_{\pm b}\} = f_{ab}{}^c j_{\pm c}\delta, \quad \{j_{+a}, j_{-b}\} = 0. \tag{2.6}$$

We also have the relations

$$\{\theta_\pm, j_{\pm a}\} = j_{\pm a}\delta', \quad \{\theta_\pm, j_{\mp a}\} = 0, \tag{2.7}$$

which imply

$$\{\mathcal{H}_c, j_{\pm a}\} = \pm j_{\pm a}\delta'. \tag{2.8}$$

The total Hamiltonian takes the form

$$H_T = \int d\sigma[\mathcal{H}_c + \sqrt{2}(A_+^a j_{-a} + B_-^a j_{+a})]. \tag{2.9}$$

The consistency conditions for the currents

$$\dot{j}_{+a} = \{j_{+a}, H_T\} = j_{+a}' + \sqrt{2} f_{ab}{}^c B_-^b j_{+c},$$
$$\dot{j}_{-a} = \{j_{-a}, H_T\} = -j_{-a}' + \sqrt{2} f_{ab}{}^c A_+^b j_{-c}, \tag{2.10}$$

do not lead to new constraints, because the right-hand sides of Eqs. (2.10) are weakly equal to zero. In fact, the last equation means that chiral currents are separately conserved

$$D_- j_{+a} \equiv \partial_- j_{+a} - f_{ab}{}^c B_-^b j_{+c} = 0,$$

$$D_+ j_{-a} \equiv \partial_+ j_{-a} - f_{ab}{}^c A_+^b j_{-c} = 0, \tag{2.11}$$

or that both vector and axial vector currents are conserved.

The currents j_{-a} and j_{+a} correspond to the arbitrary multipliers A_+^a and B_-^a, respectively in Eq. (2.9), and consequently they are FCC's. Equations (2.6) lead to the same conclusion.

Therefore, the classical theory has local non-Abelian gauge symmetries, whose generators $j_{\pm a}$ satisfies the corresponding PB algebra (2.6).

B. Quantum theory

In passing from the classical to the quantum domain, we introduce the operators $\hat{\psi}_\pm^i$ and $\hat{\pi}_\pm^j$ instead of the fields $\psi_\pm^i$ and $\pi_\pm^j$, replace the PB by the commutators, and define the composite operators using the normal ordered prescription

$$\hat{j}_{\pm a} = :\hat{\pi}_\pm t_a \hat{\psi}_\pm:, \quad \hat{\theta}_\pm = :\hat{\pi}_\pm \hat{\psi}_\pm':. \tag{2.12}$$

The gauge fields A_+^a and B_-^a will be considered as classical background fields.

Then, instead of the PB algebra (2.6) and (2.7) we obtain corresponding commutator algebra

$$[\hat{j}_{\pm a}, \hat{j}_{\pm b}] = i\hbar[f_{ab}{}^c \hat{j}_{\pm c}\delta \pm 2\kappa\delta_{ab}\delta'], \quad [\hat{j}_{+a}, \hat{j}_{-b}] = 0,$$
$$[\hat{\theta}_\pm, \hat{j}_{\pm a}] = i\hbar \hat{j}_{\pm a}\delta', \quad [\hat{\theta}_\pm, \hat{j}_{\mp a}] = 0, \tag{2.13}$$

with $\kappa = -\hbar/8\pi$. For details of the derivations see the Appendix.

In the quantum theory, as well as in the classical one, we also have a pair of commuting KM algebras but this time with a *central charge*, in this case known as the Schwinger term. Therefore, the constraints $j_{\pm a}$ which were FCC's in the classical theory, become SCC operators $\hat{j}_{\pm a}$ in the quantum theory. This means that the theory is anomalous, because the classical symmetry generated by FCC's $j_{\pm a}$ is destroyed at the quantum level. After quantization the theory obtains new degrees of freedom.

Note that under parity transformation $P: \hat{\psi}_\pm \to \hat{\psi}_\mp$, so that $P\hat{j}_{\pm a}(\tau,\sigma)P = \hat{j}_{\mp a}(\tau,-\sigma)$ and $P\hat{\theta}_\pm(\tau,\sigma)P = -\hat{\theta}_\mp(\tau,-\sigma)$. Consequently, relations (2.13) with plus and minus indices are connected by parity transformation. This means that our regularization scheme is left-right symmetric, because the normal order prescription takes the regularization role of the theory.

III. GAUGE WZNW MODEL AS AN EFFECTIVE ACTION

The PB algebra (2.6) is the symmetry generator algebra, because $j_{\pm a}$ are of the FCC. The commutator algebra (2.13) is the algebra of dynamical variables (except the zero modes, see [8]), because there is a constant central term on the right-hand side. Our intention is to find the *effective theory* for these variables, which means the quantum version of the action (2.1).

We introduce new variables $J_{\pm a}$ and $\Theta_\pm$ and postulate that their classical PB algebra

$$\{J_{\pm a},J_{\pm b}\}=f_{ab}{}^{c}J_{\pm c}\delta\pm 2\kappa\delta_{ab}\delta', \quad \{J_{+a},J_{-b}\}=0,$$
$$\{\Theta_{\pm},J_{\pm a}\}=J_{\pm a}\delta', \quad \{\Theta_{\pm},J_{\mp a}\}=0, \tag{3.1}$$

is isomorphic to the commutator algebra (2.13) of the operators $\hat{j}_{\pm a}$ and $\hat{\theta}_{\pm}$. We also define the canonical and the total Hamiltonian densities in analogy with Eqs. (2.4) and (2.9)

$$\mathcal{H}_c=\Theta_{+}-\Theta_{-}, \quad \mathcal{H}_T=\mathcal{H}_c+\sqrt{2}(A_{+}^{a}J_{-a}+B_{-}^{a}J_{+a}). \tag{3.2}$$

We should construct the canonical effective action W, for the theory with PB algebra (3.1) and with Hamiltonian density (3.2). In Sec. III A we are going to find the expressions for the currents and Hamiltonian density in terms of the phase-space variables, and then in Sec. III B we will apply general canonical method [6,7] to find the action W.

A. Bosonic representation for the PB algebra

Let us ''solve'' Eqs. (3.1), i.e., find the expressions for the currents $J_{\pm a}$ and for the energy-momentum tensor $\Theta_{\pm}$ in terms of the coordinate q^{α} and the corresponding momenta π_{α}, which satisfies

$$\{q^{\alpha},\pi_{\beta}\}=\delta^{\alpha}_{\beta}\delta. \tag{3.3}$$

We will start with the *ansatz* that the currents are linear in the momenta

$$J_{\pm a}=-E_{\pm a}{}^{\alpha}(\pi_{\alpha}+R_{\pm\alpha}), \tag{3.4}$$

where the coefficients $E_{\pm a}{}^{\alpha}$ and $R_{\pm\alpha}$ are the functions of q^{α} only, and do not depend on the π_{α}. We also suppose that the matrices $E_{\pm a}{}^{\alpha}$ have inverses, which we denote by $E_{\pm\alpha}{}^{a}$. The indices $\alpha,\beta,\ldots$ run over the same range as $a,b,\ldots$.

Substituting Eq. (3.4) into the first equation (3.1) we obtain an equation linear in π_{α}. The vanishing coefficient in front of momentum gives

$$E_{\pm b}{}^{\beta}\partial_{\beta}E_{\pm a}{}^{\alpha}-E_{\pm a}{}^{\beta}\partial_{\beta}E_{\pm b}{}^{\alpha}=-f_{ab}{}^{c}E_{\pm c}{}^{\alpha}, \tag{3.5}$$

or equivalently

$$\partial_{\beta}E_{\pm\alpha}{}^{c}-\partial_{\alpha}E_{\pm\beta}{}^{c}=f_{ab}{}^{c}E_{\pm\alpha}{}^{a}E_{\pm\beta}{}^{b}. \tag{3.6}$$

The second condition (term without π) yields

$$E_{\pm a}{}^{\alpha}E_{\pm b}{}^{\beta}[\{\pi_{\alpha},R_{\pm\beta}\}+\{R_{\pm\alpha},\pi_{\beta}\}]=\pm 2\kappa\delta_{ab}\delta'. \tag{3.7}$$

On the right side there is a derivative of the δ function, so there must also be a derivative on the left side, and we suppose that

$$R_{\pm\alpha}=P_{\pm\alpha\beta}(q)q^{\beta'}. \tag{3.8}$$

Using this in Eq. (3.7) we obtain two conditions,

$$E_{\pm a}{}^{\alpha}E_{\pm b}{}^{\beta}(P_{\pm\alpha\beta}+P_{\pm\beta\alpha})=\pm 2\kappa\delta_{ab}, \tag{3.9}$$

and

$$E_{\pm a}{}^{\alpha}\partial_{\gamma}E_{\pm b}{}^{\beta}P_{\pm\alpha\beta}+E_{\pm a}{}^{\alpha}\partial_{\gamma}(E_{\pm b}{}^{\beta}P_{\pm\beta\alpha})$$
$$+E_{\pm a}{}^{\alpha}E_{\pm b}{}^{\beta}(\partial_{\beta}P_{\pm\alpha\gamma}-\partial_{\alpha}P_{\pm\beta\gamma})=0, \tag{3.10}$$

because the coefficients in front of δ' and δ must vanish separately.

If we define the symmetric tensor

$$\gamma_{\pm\alpha\beta}=E_{\pm\alpha}{}^{a}E_{\pm\beta}{}^{b}\delta_{ab}, \tag{3.11}$$

we can rewrite Eq. (3.9) as $P_{\pm\alpha\beta}+P_{\pm\beta\alpha}=\pm 2\kappa\gamma_{\pm\alpha\beta}$ and find its general solution

$$P_{\pm\alpha\beta}=2\kappa(\tau_{\pm\alpha\beta}\pm\tfrac{1}{2}\gamma_{\pm\alpha\beta}), \tag{3.12}$$

where $\tau_{\pm\alpha\beta}=-\tau_{\pm\beta\alpha}$ is some antisymmetric tensor. The first term is a solution of the homogeneous part and the second one is a particular solution of the full equation.

With the help of Eqs. (3.6) and (3.12) we can obtain, from Eq. (3.10),

$$\partial_{\alpha}\tau_{\pm\beta\gamma}+\partial_{\beta}\tau_{\pm\gamma\alpha}+\partial_{\gamma}\tau_{\pm\alpha\beta}=\mp\tfrac{1}{2}f_{abc}E_{\pm\alpha}{}^{a}E_{\pm\beta}{}^{b}E_{\pm\gamma}{}^{c}. \tag{3.13}$$

Therefore, from the first Eq. (3.1) we got two relations, (3.6) and (3.13). The first one is a condition on the $E_{\pm\alpha}{}^{a}$'s and the second one defines $\tau_{\pm\alpha\beta}$'s in terms of $E_{\pm\alpha}{}^{a}$'s.

From the second Eq. (3.1) we also obtain three equations:

$$E_{+a}{}^{\alpha}\partial_{\alpha}E_{-b}{}^{\beta}-E_{-b}{}^{\alpha}\partial_{\alpha}E_{+a}{}^{\beta}=0, \tag{3.14}$$

$$[-E_{+a}{}^{\alpha}\partial_{\alpha}E_{-b}{}^{\beta}P_{-\beta\gamma}+E_{-b}{}^{\alpha}\partial_{\alpha}E_{+a}{}^{\beta}P_{+\beta\gamma}$$
$$+E_{+a}{}^{\alpha}\partial_{\gamma}E_{-b}{}^{\beta}(P_{-\beta\alpha}+P_{+\alpha\beta})$$
$$+E_{+a}{}^{\alpha}E_{-b}{}^{\beta}(-\partial_{\alpha}P_{-\beta\gamma}+\partial_{\beta}P_{+\alpha\gamma}+\partial_{\gamma}P_{-\beta\alpha})]q^{\gamma'}=0, \tag{3.15}$$

and

$$P_{+\alpha\beta}+P_{-\beta\alpha}=0, \tag{3.16}$$

as a coefficients in front of $\pi_{\beta}\delta$, δ and $E_{+a}{}^{\alpha}E_{-b}{}^{\beta}\delta'$ respectively. From Eqs. (3.12), (3.16), and the symmetry properties of τ and γ, follows

$$\tau_{+\alpha\beta}=\tau_{-\alpha\beta}, \quad \gamma_{+\alpha\beta}=\gamma_{-\alpha\beta}, \tag{3.17}$$

and consequently, from now we will just call them $\tau_{\alpha\beta}$ and $\gamma_{\alpha\beta}$, so that Eq. (3.12) becomes

$$P_{\pm\alpha\beta}=2\kappa(\tau_{\alpha\beta}\pm\tfrac{1}{2}\gamma_{\alpha\beta}). \tag{3.18}$$

With the help of Eqs. (3.6) and (3.11) we recognize $E_{\pm\alpha}{}^{a}$ as vielbeins on the group manifold, and $\gamma_{\alpha\beta}$ as the Cartan metric in coordinate basis.

Equation (3.15) is a linear combination of equations (3.13), (3.14), and (3.16), and does not give anything new. We will discus Eq. (3.14) soon.

To make the geometric interpretation clearer we introduce a differential form notation. Let us define the pair of Lie-algebra valued 1-forms

$$v_{\pm}=t_a E_{\pm}{}^{a}{}_{\alpha} dq^{\alpha}, \tag{3.19}$$

and the 2-form

$$\tau=\frac{1}{2}\tau_{\alpha\beta}dq^{\alpha}dq^{\beta}. \tag{3.20}$$

Then Eqs. (3.6) become the Maurer-Cartan (MC) equations

$$dv_{\pm}+v_{\pm}^2=0. \tag{3.21}$$

They have a simple solutions in which the MC forms $v_{\pm}$ are expressed in terms of group-valued fields $g_{\pm}$ as

$$v_{\pm}=g_{\pm}{}^{-1}dg_{\pm}. \tag{3.22}$$

In new notation Eq. (3.13) with the help of Eq. (3.17) obtains the form

$$d\tau=\mp\frac{1}{3!}(v_{\pm},v_{\pm}^2), \tag{3.23}$$

where (X,Y) is the Cartan inner product defined as $(t_a,t_b)=\delta_{ab}$, so that in our normalization $(t_a,t_b)=-2\,\mathrm{tr}\{t_a t_b\}$.

From the second Eq. (3.17) and Eq. (3.11) we have

$$(v_+,v_+)=(v_-,v_-), \tag{3.24}$$

and from Eq. (3.23)

$$(v_+,v_+^2)=-(v_-,v_-^2). \tag{3.25}$$

The Cartan-Killing form (X,Y) is invariant under the adjoint action of the group element g: $(g^{-1}Xg,g^{-1}Yg)=(X,Y)$, so we can conclude that v_+ and v_- are connected as

$$v_-=-g^{-1}v_+g, \tag{3.26}$$

where g is a group valued field. Substituting Eq. (3.22) in Eq. (3.26) we easily obtain that $g=g_-=g_+^{-1}$ and finally

$$v_+=gdg^{-1},\quad v_-=g^{-1}dg. \tag{3.27}$$

Let us now come back to Eq. (3.14). After some transformations we can write it as

$$dv_-=g^{-1}dv_+g, \tag{3.28}$$

or with the help of Eq. (3.21) as $v_-^2=g^{-1}v_+^2g$. Therefore, it is not a new relation but the consequence of Eq. (3.26).

The final result for the current components is

$$J_{\pm a}=-E_{\pm a}{}^{\alpha}\left[\pi_{\alpha}+2\kappa\left(\tau_{\alpha\beta}\pm\frac{1}{2}\gamma_{\alpha\beta}\right)q^{\beta\prime}\right]. \tag{3.29}$$

We still need to "solve" the last two equations (3.1). Because all expressions with the opposite chirality commute, we will take $\Theta_{\pm}=\Theta_{\pm}(J_{\pm a})$. The group invariant expressions

$$\Theta_{\pm}=\pm\frac{1}{4\kappa}\delta^{\alpha\beta}J_{\pm a}J_{\pm b}, \tag{3.30}$$

representing the components of energy-momentum tensor, is the solution we are looking for. It is enough to use only the first two relations (3.1) and not the expressions (3.29), to check that Eq. (3.30) satisfies the last two equations (3.1).

If we try to add a new term for the energy-momentum tensor, it must commute with both J_{+a} and J_{-a}, and consequently should be trivial. For example, the term proportional to $J'_{\pm a}$, which is often a part of the energy-momentum tensor is not a solution of Eq. (3.1). This term, as a total space derivative, will not contribute to the effective action. The solutions for the currents, where they are not limited to be linear in the momenta, would lead to serious technical complications.

B. Effective action

We are ready to construct the effective action based on the general canonical formalism. It takes the standard form

$$W(q,\pi,A,B)=\int d^2\xi(\pi_{\alpha}\dot{q}^{\alpha}-\mathcal{H}_T), \tag{3.31}$$

where the total Hamiltonian $\mathcal{H}_T=\Theta_+-\Theta_-+\sqrt{2}(A_+^aJ_{-a}+B_-^aJ_{+a})$, is defined in Eq. (3.2). The expressions for the current components and for the energy-momentum tensor components in terms of canonical pairs $(q^{\alpha},\pi_{\alpha})$ are defined in Eqs. (3.29) and (3.30).

To find the usual second-order form of the action, we will eliminate the momentum variables π_{α} on their equations of motion

$$\dot{q}^{\alpha}-\frac{1}{2\kappa}\gamma^{\alpha\beta}(J_{+\beta}+J_{-\beta})+\sqrt{2}(A_+^{\alpha}+B_-^{\alpha})=0, \tag{3.32}$$

where $A_+^{\alpha}=E_+{}^{\alpha}{}_a A_+^a$, $B_-^{\alpha}=E_-{}^{\alpha}{}_a B_-^a$ and $J_{\pm\alpha}=-E_{\pm\alpha}{}^{a}J_{\pm a}$. With the help of Eq. (3.29) we have on the equations of motion

$$J_{\pm}^{\alpha}=\sqrt{2}\kappa(\partial_{\pm}q^{\alpha}+A_+^{\alpha}+B_-^{\alpha}). \tag{3.33}$$

Substituting this in Eq. (3.31), after some calculations we find

$$W(q,A,B)=W_0(q)+W_1(q,A,B), \tag{3.34}$$

where

$$W_0(q)=-\int d^2\xi P_{-\alpha\beta}\partial_-q^{\alpha}\partial_+q^{\beta}$$
$$=\kappa\int d^2\xi(\gamma_{\alpha\beta}-2\tau_{\alpha\beta})\partial_-q^{\alpha}\partial_+q^{\beta},$$

$$W_1(q,A,B)=2\kappa\int d^2\xi\Big[\partial_-q^{\alpha}E_{+\alpha}{}^{a}A_{+a}$$
$$+\partial_+q^{\alpha}E_{-\alpha}{}^{a}B_{-a}+A_+^aE_{+a}{}^{\alpha}E_{-\alpha}{}^{b}B_{-b}$$
$$+\frac{1}{2}(A_+^aA_{+a}+B_-^aB_{-a})\Big]. \tag{3.35}$$

It is possible to add to the effective action some local functional, depending on the fields A_+ and B_-. In order to cancel the last term in Eq. (3.35) we add

$$\Delta W = -\kappa \int d^2\xi [A_+^a A_{+a} + B_-^a B_{-a}], \tag{3.36}$$

and get

$$W_{l,r} = W + \Delta W, \tag{3.37}$$

where the meaning of the indices l,r will be clear in Sec. V.

In the differential form notation, with Cartan inner product normalization, using the Stoke's theorem we have

$$W_0(v) = \frac{1}{2}\kappa \int_\Sigma (*v_\pm, v_\pm) \pm \frac{1}{3}\kappa \int_M (v_\pm, v_\pm^2),$$

$$(W_1 + \Delta W)(v,A,B,) = \kappa \int_\Sigma \Big[(v_+, A - *A) - (v_-, B + *B) - \frac{1}{2}(B + *B, g^{-1}(A - *A)g) \Big]. \tag{3.38}$$

Here, $W_0(v)$ is the well-known WZNW model [2,9]. As defined in Eq. (3.19), $v_\pm$ are the Lie algebra valued 1-forms and $*v_\pm$ are the dual of $v_\pm$. Both expressions for $W_0(v)$ are equal on the basis of Eqs. (3.24) and (3.25). The first term is the action of the nonlinear σ model, while the second one is the topological Wess-Zumino term, defined over a three manifold M whose boundary is the spacetime: $\partial M = \Sigma$. For our value of $\kappa = -\hbar/8\pi$ it reads

$$\frac{\hbar}{24\pi}\int_M (g^{-1}dg, g^{-1}dg g^{-1}dg), \tag{3.39}$$

so that it is well-defined modulo $2\pi\hbar n$, where $n \in Z$ is a winding number. We want to stress that the proper value of κ for which quantum theory is single valued is determined from the central charge, which we obtained using just the normal ordered prescription in quantum fermionic theory.

The expression for $W_1 + \Delta W$ is the regular part of the gauge extension of the WZNW action. It has been obtained in the process of consistent gauge invariant extension of the WZNW model [10].

Starting with the PB algebra (3.1) and Eq. (3.2), we obtained the action (3.38) for any value of the constant κ. But, the uniqueness of the irreducible representation of the KM algebra, as well as topological arguments [2] force it just to the same value as in our normal-order approach.

IV. BOSONIZATION

A. The chiral currents bosonization rules

In the previous section we obtained the effective action W for the massless Fermi theory in the external gauge fields. This is equivalent, to solving the functional integral

$$Z_F(A,B) = \int d\psi_+ d\psi_- e^{iS(\psi_+,\psi_-,A,B)}. \tag{4.1}$$

The final result depends only on the background fields A and B. The bosonic expression for W depends not only on A and B, but also on some auxiliary fields, q^α and π_α in the Hamiltonian and g in the Lagrangian case. So, after integration over auxiliary fields we can eliminate them and obtain

$$Z_B(A,B) = \int d\pi dq e^{iW(q,\pi,A,B)},$$
$$Z_B(A,B) = \int dg e^{iW_{l,r}(v,A,B)}, \tag{4.2}$$

for the Hamiltonian and the Lagrangian approach, respectively. We are not going to do the integrations explicitly, because they lead to nonlocal expression in terms of A and B, even in the Abelian case. We only conclude that Z_F should be proportional to Z_B

$$Z_F \sim Z_B. \tag{4.3}$$

This functional integral identity admits interpretation in terms of bosonization. Differentiating Eq. (4.3) with respect to B_-^a and A_+^a and setting $A_+ = 0 = B_-$, we obtain the bosonization rules for chiral currents in non-Abelian theory.

If we choose the expression $W(q,\pi,A,B)$ from Eq. (3.31) as an effective action we get the Hamiltonian bosonization rules

$$i\psi_\pm^* t_a \psi_\pm \to -E_{\pm a}{}^\alpha(\pi_\alpha + P_{\pm\alpha\beta} q^{\beta\prime}), \tag{4.4}$$

and if we choose $W_{l,r}(v,A,B)$ from Eq. (3.38), we get the well-known Lagrangian bosonization rules

$$t^a i\psi_+^* t_a \psi_+ \to -\sqrt{2}\kappa g \partial_+ g^{-1}$$
$$t^a i\psi_-^* t_a \psi_- \to -\sqrt{2}\kappa g^{-1}\partial_- g. \tag{4.5}$$

B. The chiral densities bosonization rules

It is possible to add the mass term

$$\bar{\psi}\psi = \psi_+^* \psi_- + \psi_-^* \psi_+ \tag{4.6}$$

to the action (2.1) and find the corresponding expression in terms of the bosonic variables. Let us introduce the chiral densities

$$\rho_\pm = \bar{\psi}\,\frac{1 \pm \gamma_5}{2}\,\psi = \psi_\pm^* \psi_\mp = -i\pi_\pm \psi_\mp. \tag{4.7}$$

All expressions are matrices with indices i,j (for example $\rho_\pm^{ij} = -i\pi_\pm^j \psi_\mp^i$) but we will omit them for simplicity.

It is easy to find the PB between the chiral densities and the currents $j_{\pm a}$, defined in Eq. (2.3)

$$\{j_{\pm a}, \rho_\pm\} = (\rho_\pm t_a)\delta, \quad \{j_{\pm a}, \rho_\mp\} = -(t_a \rho_\mp)\delta. \tag{4.8}$$

In the quantum theory the central term does not appear, so the commutation relations, up to $i\hbar$, are the same as PB Eq. (4.8). The commutators with the currents $\hat{j}_{\pm a}$ completely define the expressions for $\hat{\rho}_{\pm}$.

For the bosonic representations, instead of $\hat{\rho}_{\pm}$ we introduce the corresponding matrix valued expressions $\Upsilon_{\pm}$, depending on the bosonic variables. Their PB algebra with $J_{\pm a}$ should be isomorphic to the operators algebra of $\hat{\rho}_{\pm}$ with $\hat{j}_{\pm a}$

$$\{J_{\pm a},\Upsilon_{\pm}\}=(\Upsilon_{\pm}t_a)\delta, \quad \{J_{\pm a},\Upsilon_{\mp}\}=-(t_a\Upsilon_{\mp})\delta. \tag{4.9}$$

Both relations (4.9) will give the same result, so we will solve only the first one.

Assuming that $\Upsilon_{\pm}$ does not depend on the momenta, and using the expression (3.29) for the current, we obtain the equation

$$E_{\pm a}{}^{\alpha}\partial_{\alpha}\Upsilon_{\pm}=\Upsilon_{\pm}t_a. \tag{4.10}$$

Multiplying it with $E_{\pm}{}^{a}{}_{\beta}dq^{\beta}$, with the help of Eq. (3.19) we get

$$\Upsilon_{\pm}^{-1}d\Upsilon_{\pm}=v_{\pm}. \tag{4.11}$$

Comparing last equation with Eq. (3.27), we conclude that

$$\Upsilon_{+}=Mg^{-1}, \quad \Upsilon_{-}=Mg, \tag{4.12}$$

where M is a constant. Because g is dimensionless, M has dimension of mass. At the end, we can complete our bosonization formulas for the chiral densities

$$\begin{aligned}\bar{\psi}\psi&\rightarrow\Upsilon_{+}+\Upsilon_{-}=M(g+g^{-1}),\\ \bar{\psi}\gamma_5\psi&\rightarrow\Upsilon_{+}-\Upsilon_{-}=M(g^{-1}-g).\end{aligned} \tag{4.13}$$

They are the same for the Hamiltonian and Lagrangian case, because they do not depend on the momenta and agree with those of Ref. [2].

V. CANONICAL APPROACH TO ANOMALIES

A. From Schwinger term to anomalies

The Schwinger term breaks the symmetries, changing the generators from the FCC's to the SCC's. It is interesting to investigate its influence on the transformation properties of the effective action.

Let us extend the previously described general canonical method for constructing the effective action, from the known PB algebra to the case with the central term. The basic idea of that approach was that the action

$$W=\int d^2\xi[\pi_{\alpha}\dot{q}^{\alpha}-\mathcal{H}_c-\lambda^m G_m] \tag{5.1}$$

is invariant under gauge transformations

$$\delta F(\sigma)=\left\{F(\sigma),\int d\bar{\sigma}\varepsilon^m(\bar{\sigma})G_m(\bar{\sigma})\right\} \tag{5.2}$$

of any quantity $F(\pi,q)$, and

$$\delta\lambda^m=\dot{\varepsilon}^m-\varepsilon^{n\prime}h_n^m-\varepsilon^n\lambda^k f_{kn}{}^m \tag{5.3}$$

of the Lagrange multipliers λ^m, if G_m are the FCC's,

$$\{G_m,G_n\}=f_{mn}{}^pG_p\delta, \quad \{\mathcal{H}_c,G_m\}=h_m{}^nG_n\delta'. \tag{5.4}$$

Here we adopted the notation appropriate for field theory.

In the case where the central term is present the first equation (5.4) becomes

$$\{G_m,G_n\}=f_{mn}{}^pG_p\delta+\Delta_{mn}(\sigma,\bar{\sigma}). \tag{5.5}$$

We want to preserve the gauge transformations of the fields (5.2) and (5.3). Then the Schwinger term appears in the variation of the effective action

$$\delta W=-\int d\tau d\sigma\int d\bar{\sigma}\lambda^m(\sigma)\Delta_{mn}(\sigma,\bar{\sigma})\varepsilon^n(\bar{\sigma}). \tag{5.6}$$

The method we have used works only for actions linear in the gauge fields λ^m. We can add arbitrary local functional $\Delta W(\lambda^m)$ to the effective action and obtain the general expression for the anomaly

$$\begin{aligned}\mathcal{A}_n(\sigma,\tau)&=\frac{\delta W}{\delta\varepsilon^n(\sigma,\tau)}\\ &=\int d\bar{\sigma}\Delta_{nm}(\sigma,\bar{\sigma})\lambda^m(\bar{\sigma},\tau)+\frac{\delta}{\delta\varepsilon^n(\sigma,\tau)}\Delta W.\end{aligned} \tag{5.7}$$

The nontrivial part of the anomaly is proportional to the Schwinger term, because it breaks the symmetry and measures noninvariance of the effective action.

We want to emphasize, that we do not need the expression for the generators G_m in terms of π_{α} and q^{α} to obtain Eqs. (5.6) and (5.7). It is enough to know only the central term, and the anomaly will depend on the gauge field λ^m, but not on the phase-space coordinate.

B. The left-right and the axial anomalies

In the previous section, we used the canonical method just as a useful technical tool in order to obtain gauge transformations for the Lagrange multipliers λ^m. We shall now take the currents to be our FCC's G_m, and the gauge fields A_+^a and B_-^a instead of the Lagrangian multipliers λ^m. From this point on, we will continue considering the gauge fields as the background fields with the same gauge transformations. The case where the gauge fields are dynamical variables will be discussed in the conclusion.

We can apply this method to the classical fermionic theory with

$$G_m=j_{+a}, \quad j_{-a},$$

$$\lambda^m=\sqrt{2}B_-^a, \quad \sqrt{2}A_+^a,$$

$$\varepsilon^m=\beta^a, \quad \alpha^a,$$

and obtain the well-known transformations under the local gauge group $G_l \times G_r$:

$$\delta\psi_- = -\alpha\psi_-, \quad \delta A_+ = \partial_+\alpha - [A_+, \alpha],$$
$$\delta\psi_+ = -\beta\psi_+, \quad \delta B_- = \partial_-\beta - [B_-, \beta], \tag{5.8}$$

where $\alpha = \alpha^a t_a$ and $\beta = \beta^a t_a$.

In the case of the bosonic theory we have

$$G_m = J_{+a}, \quad J_{-a},$$
$$\lambda^m = \sqrt{2} B_-^a, \quad \sqrt{2} A_+^a,$$
$$\varepsilon^m = \beta^a, \quad \alpha^a,$$

and

$$\Delta_{mn}(\sigma, \bar{\sigma}) \to \pm 2\kappa \delta_{ab} \delta'(\sigma - \bar{\sigma}) \quad [m \to (a, \pm), n \to (b, \pm)]. \tag{5.9}$$

The local gauge transformations for the fields A_+ and B_- are the same as in Eq. (5.8) and for matter fields we have

$$\delta q^\alpha = -\beta^a E_{+a}{}^\alpha - \alpha^a E_{-a}{}^\alpha, \tag{5.10}$$

which yields

$$\delta g = \beta g - g\alpha \tag{5.11}$$

(see [8] and the second Ref. [7]).

Under these transformations we have, from Eqs. (5.6) and (5.9),

$$\delta W = -4\sqrt{2}\kappa \int d^2\xi \operatorname{tr}\{\beta B'_- - \alpha A'_+\}. \tag{5.12}$$

When we include the transformation of the ΔW Eq. (3.36)

$$\delta(\Delta W) = -4\kappa \int d^2\xi \operatorname{tr}\{\alpha\partial_+ A_+ + \beta\partial_- B_-\}, \tag{5.13}$$

we obtain

$$\delta W_{l,r} = -4\kappa \int d^2\xi \operatorname{tr}\{\alpha\partial_- A_+ + \beta\partial_+ B_-\},$$
$$\mathcal{A}_l = 2\kappa\partial_- A_+, \quad \mathcal{A}_r = 2\kappa\partial_+ B_-, \tag{5.14}$$

where $W_{l,r} = W + \Delta W$ as in Eq. (3.37). In this case both left and right symmetries are anomalous, which we denoted by indices l,r.

We can add the finite local counterterm

$$\Delta W_{ax} = -4\kappa \int d^2\xi \operatorname{tr}\{A_+ B_-\}, \tag{5.15}$$

and shift the anomaly from left-right to an axial one. The redefined effective action $W_{ax} = W_{l,r} + \Delta W_{ax}$ is invariant under the vector gauge transformation: $\alpha = \beta = \varepsilon_v$,

$$\delta_v W_{ax} = 0, \tag{5.16}$$

but not under the axial one: $\alpha = -\beta = \varepsilon_{ax}$,

$$\delta_{ax} W_{ax} = -8\kappa \int d^2\xi \operatorname{tr}\{\varepsilon_{ax} {}^*F\}, \quad \mathcal{A}_{ax} = 4\kappa {}^*F, \tag{5.17}$$

where

$${}^*F = \frac{1}{2}\varepsilon^{\mu\nu} F_{\mu\nu}, \tag{5.18}$$

and

$$F_{-+} = \partial_- A_+ - \partial_+ B_- + [A_+, B_-]. \tag{5.19}$$

The second equation (5.17) is the well-known result for axial anomaly.

The noninvariance of the effective action is a consequence of the nonconservation of the currents. The Hamiltonian equations of motion have anomalous divergent currents, instead of the conserved one in Eq. (2.11). Taking into account ΔW and ΔW_{ax}, the same expressions for $\mathcal{A}_l$, $\mathcal{A}_r$ Eq. (5.14) and for $\mathcal{A}_{ax}$ Eq. (5.17) can be obtained.

VI. CONCLUDING REMARKS

We presented here a complete and independent derivation of the two-dimensional gauged WZNW model, using the Hamiltonian methods. We also obtained Hamiltonian and Lagrangian non-Abelian bosonization rules and the expression for the anomalies.

We started with canonical analysis of the theory of massless chiral fermions coupled to the external gauge field. We found that there are FCC's $j_{\pm a}$, whose PB satisfies two independent copies of KM algebras without central charges. In passing to the quantum theory, the central term appears in the commutation relations of the operators $\hat{j}_{\pm a}$, which changes the nature of constraints: they become SCC's instead of FCC's.

We define the new effective theory, postulating the PB of the constraints and Hamiltonian density. Particularly, we require that the PB algebra of the classical bosonic theory should be isomorphic to the commutator algebra of the quantum fermionic theory. Then we found the representation for the currents and Hamiltonian density in terms of phase-space coordinates. Finally, we derived effective action using general canonical formalism and obtained the gauged WZNW model. We want to stress that we also got the topological Wess-Zumino term. The tensor $\tau_{\pm\alpha\beta}$, as its origin, appears in our approach as a general solution of the homogeneous part of Eq. (3.9). The coefficient in front of the Wess-Zumino term is defined by the numerical value of the central charge and gives a correct expression for the winding number.

Once we established the connection between the fermionic and the bosonic theories, it was easy to find the bosonization rules, just differentiating generating functionals with respect to the background fields. Beside the usual bosonization rules, we also got the Hamiltonian ones, expressing the currents $J_{\pm a}$ in terms of both coordinate q^α and momentum π_α. After elimination of momenta on the equations of motion, we came back to the conventional bosonization rules.

The algebra of the currents $J_{\pm a}$ is the basic PB algebra. Knowing its representation in terms of q^α and π_α we can find the representation for all other quantities from their PB algebra with the currents. As an example we found the bosonization rules for the chiral densities.

The canonical approach is very suitable for the calculation of the anomaly. The general formula (5.7) expresses the anomaly as a function of the Schwinger term. The normal ordering prescription for the quantum operators takes the role of left-right symmetric regularization scheme. So, we obtain both left and right anomalies. By adding the finite local counterterm we in fact changed the regularization scheme, and shifted the anomaly from the left-right symmetric to the axial one.

The Schwinger term, and consequently the WZNW model and the anomaly have the correct dependence on Planck's constant $\hbar$, because κ is proportional to $\hbar$. The fact that $\hbar$ arises in the classical effective theory, shows its quantum origin.

In the case when the gauge fields are considered as dynamical variables, integration over them yields constraints which reduce the number of degrees of freedom, removing components of the currents corresponding to the anomaly free subgroup H of the group G. In fact it leads to the Goddard-Kent-Olive (GKO) coset construction [12], when the energy-momentum tensor of the coset G/H takes the form $\Theta_\pm^{G/H} = \Theta_\pm^G - \Theta_\pm^H$ and both $\Theta_\pm^G$ and $\Theta_\pm^H$ have the same structure as in Sec. III A.

The canonical approach of this paper can be applied to the other symmetries. If we take the diffeomorphism transformations instead of the non-Abelian ones, then we will get Virasoro algebra, 2D induced gravity, and conformal anomaly instead of KM algebra, WZNW model, and axial anomaly. The work of this program is in progress and will be published separately.

ACKNOWLEDGMENTS

This work was supported in part by the Serbian Science Foundation, Yugoslavia.

APPENDIX: NORMAL ORDERING AND SCHWINGER TERM

In this appendix we will derive the expression for the central terms in the commutation relations (2.13).

We define currents $\hat{j}_{\pm a}$ as a quantum operators and introduce the normal ordering prescription. Usually, it is convenient to employ the Fourier expansion of the fields, identifying the modes as creation and annihilation operators with respect to Fock vacuum state.

Following [11] we prefer to decompose operators in positive and negative frequencies in the position space. We introduce two parts of the delta function

$$\delta^{(\pm)}(\sigma) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \theta(\mp k) e^{ik(\sigma \mp i\varepsilon)} = \frac{\pm i}{2\pi(\sigma \pm i\varepsilon)}, \quad (\varepsilon > 0), \tag{A1}$$

so that $\delta(\sigma) = \delta^{(+)}(\sigma) + \delta^{(-)}(\sigma)$ with the following properties:

$$\delta^{(\pm)}(-\sigma) = \delta^{(\mp)}(\sigma), \quad (\delta^{(+)})^2 - (\delta^{(-)})^2 = \frac{-i}{2\pi}\delta'. \tag{A2}$$

Then for any operator $\hat{\Omega}(\tau,\sigma)$ we can perform the splitting

$$\hat{\Omega}^{(\pm)}(\tau,\sigma) = \int_{-\infty}^{\infty} \delta\bar{\sigma}\,\delta^{(\pm)}(\sigma - \bar{\sigma})\hat{\Omega}(\tau,\bar{\sigma}), \tag{A3}$$

where $\hat{\Omega} = \hat{\Omega}^{(+)} + \hat{\Omega}^{(-)}$.

Now, we adopt $\hat{\pi}_+^{(-)}$ and $\hat{\psi}_+^{(-)}$ as creation operators and $\hat{\pi}_+^{(+)}$ and $\hat{\psi}_+^{(+)}$ as annihilation operators

$$\hat{\pi}_+^{(+)}|0\rangle = \hat{\psi}_+^{(+)}|0\rangle = 0, \quad \langle 0|\hat{\pi}_+^{(-)} = \langle 0|\hat{\psi}_+^{(-)} = 0. \tag{A4}$$

To preserve symmetry under parity transformations, we define creation and annihilation operators for $\hat{\pi}_-$ and $\hat{\psi}_-$ in an opposite way [with the index $(+)$ for a creation and index $(-)$ for an annihilation operator]. Then in the both cases, the normal order for products of operators means that annihilation operators are placed to the right of the creations one.

From the basic commutation relations $[\hat{\psi}^i_\pm, \hat{\pi}^j_\pm] = i\hbar\,\delta^{ij}\delta$, we can conclude that the only nontrivial parts are

$$\begin{aligned} [\hat{\psi}_+^{(\pm)}(\sigma), \hat{\pi}_+^{(\mp)}(\bar{\sigma})] &= i\hbar\,\delta^{(\pm)}(\sigma - \bar{\sigma}), \\ [\hat{\psi}_-^{(\pm)}(\sigma), \hat{\pi}_-^{(\mp)}(\bar{\sigma})] &= i\hbar\,\delta^{(\pm)}(\sigma - \bar{\sigma}). \end{aligned} \tag{A5}$$

After some calculation it is possible to check the commutator algebra (2.13). Because the central term is the only possible difference compared to the PB algebra, the easiest way to confirm Eq. (2.13) is to start with expression

$$[\hat{j}_{\pm a}, \hat{j}_{\pm b}] = i\hbar(f_{ab}{}^c \hat{j}_{\pm c} \pm \Delta_{ab}), \tag{A6}$$

and find its vacuum expectation value. With the help of the Eq. (A2) and with the convention $\mathrm{tr}\{t_a t_b\} = -\frac{1}{2}\delta_{ab}$, we obtain

$$\Delta_{ab} = \frac{\hbar}{2\pi}\,\mathrm{tr}\{t_a t_b\}\,\delta' = 2\kappa\,\delta_{ab}\delta', \quad \left(\kappa = \frac{-\hbar}{8\pi}\right) \tag{A7}$$

proving the first relation (2.13).

Commutators $[\hat{j}_+, \hat{j}_-]$ and $[\hat{\theta}, \hat{j}]$ do not have central extensions.

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