Low-energy effective actions in $\mathcal{N}=2, 4$ superconformal theories in four dimensions

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We study some aspects of low-energy effective actions in $4d$ superconformal gauge theories on the Coulomb branch. We describe superconformal invariants constructed in terms of the $\mathcal{N}=2$ Abelian vector multiplet which play the role of building blocks for the $\mathcal{N}=2, 4$ low-energy effective actions. We compute the one-loop effective actions in a constant $\mathcal{N}=2$ field strength background in $\mathcal{N}=4$ SYM theory and in $\mathcal{N}=2$ SU(2) SYM theory with four hypermultiplets in the fundamental representation. Using a classification of superconformal invariants, we then find the manifestly $\mathcal{N}=2$ superconformal form of these effective actions. While our explicit computations are done in the one-loop approximation, our conclusions about the structure of the effective actions in $\mathcal{N}=2$ superconformal theories are general. We comment on some relations to supergravity-gauge theory duality in the description of D-brane interactions.

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I. INTRODUCTION

The study of the structure of low-energy effective actions in $d=4$ superconformal theories is an important subject from several points of view, in particular, in connection with the interactions of D-branes in string theory. Systems of D3-branes have complementary descriptions in terms of gauge theory and supergravity. As one of the consequences, the leading-order interaction potential between separated branes admits two equivalent representations: as a classical supergravity potential between a probe and a source, and as a leading term in the quantum gauge theory effective action. The agreement between the supergravity and the gauge theory expressions for the potential is possible because of the existence of certain nonrenormalization theorems on the gauge theory side (see [1,2] and references therein).

One may conjecture that not only the F^4/X^4 term but all higher terms

$$\sum_{n=1}^{\infty} c_n (g^2 N)^{n-1} \frac{F^{2n+2}}{X^{4n}} \tag{1.1}$$

in the Born-Infeld (BI) action for a D3-brane probe moving near the core of a multiple D3-brane source (or in AdS$_5$ ×S^5 space) may be reproduced by the leading low-energy, large N, part of the quantum $\mathcal{N}=4$ SU(N) super Yang-Mills (SYM) effective action. The latter is obtained by keeping the U(1) $\mathcal{N}=4$ vector multiplet as an external background and

integrating out massive SYM fields (see, e.g., [3–6] and references therein). This conjecture seems likely to be true at the first subleading order, i.e., for the F^6/X^8 term. Indeed, it is easy to show that this term is not present in the $\mathcal{N}=4$ SYM analogue [7] of the one-loop Schwinger effective action, and the result of [8] for the dimensionally reduced $0+1$ gauge theory suggests that this F^6 term should appear in the two-loop effective action with precisely the right coefficient to match the supergravity expression.

This conjecture seems, however, to run into a problem at the next order of the F^8/X^{12} term. According to the supergravity expression (1.1), it should appear in the SYM action only at the three-loop order, but the 1-loop SYM effective action already contains the $O(F^8)$ term. One may hope that the F^8 term does not receive corrections beyond the three-loop order, so that the three-loop correction dominates over the one-loop and two-loop terms in the supergravity limit ($g^2 N \gg 1$). Still, this may not be enough for agreement since the F^8 invariants in the one-loop SYM effective action and in the Born-Infeld D3-brane action happen to have different Lorentz index structure.

In order to shed more light on this problem of the supergravity-SYM correspondence one may study the constraints imposed by the superconformal invariance (which is a natural symmetry of the supergravity ''D3-brane in AdS$_5$ ×S^5'' action [9–11]) on the structure of the SYM effective action.[1] A possible strategy is to start with the one-loop expression for the low-energy effective action on the Coulomb branch written in a manifestly superconformally invariant

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[1]Some implications of special conformal transformations in $\mathcal{N}=4$ SYM theory in the context of the AdS conformal field theory (CFT) correspondence were considered in [12].

form and try to draw some general lessons about the form of the effective action which may go beyond the one-loop order.

In this paper we shall consider two superconformal theories in four dimensions—the $\mathcal{N}=4$ SU(2) SYM model and the $\mathcal{N}=2$ SU(2) SYM model with four hypermultiplets in the fundamental representation of SU(2), with the gauge group spontaneously broken to its U(1) subgroup. We will be mainly interested in the part of their low-energy effective actions of $\mathcal{N}=2, 4$ superconformal theories which involves the physical bosonic fields of $\mathcal{N}=2$ vector multiplet (vector field strength and scalars). We will compute the one-loop effective actions in a constant field background:

$$\mathcal{W}|_{\theta=0}=X=\text{const}, \quad D^i_\alpha \mathcal{W}|_{\theta=0}=\psi^i_\alpha=\text{const},$$

$$D^i_{(\alpha}D_{\beta)\,i}\mathcal{W}|_{\theta=0}=8\,F_{\alpha\beta}=\text{const}, \quad D^{\alpha(i}D^{j)}_\alpha\mathcal{W}|_{\theta=0}=0, \tag{1.2}$$

which is a special supersymmetric solution of the equations of motion of the Abelian $\mathcal{N}=2$ vector multiplet ($\mathcal{W}$ is the $\mathcal{N}=2$ gauge superfield strength). The fact that the theories under consideration are superconformal will allow us to use the classification of superconformal invariants constructed in terms of the Abelian $\mathcal{N}=2$ vector multiplet (Sec. II). As a result, we will be able to restore not only the known F^4-type quantum corrections

$$\int \mathrm{d}^{12}z\,\mathcal{H}(\mathcal{W},\bar{\mathcal{W}}), \quad \mathcal{H}(\mathcal{W},\bar{\mathcal{W}})\propto\ln\mathcal{W}\ln\bar{\mathcal{W}}, \tag{1.3}$$

computed previously (for $\mathcal{N}=4$ SYM) using supergraph techniques [13,14] (see also [15–17]), but also all terms in the effective action

$$\Gamma=c\int \mathrm{d}^{12}z\,\ln\mathcal{W}\ln\bar{\mathcal{W}}+\int \mathrm{d}^{12}z\,\ln\mathcal{W}\Lambda(\bar{\mathcal{W}}^{-2}D^4\ln\mathcal{W})$$

$$+\text{c.c.}+\int \mathrm{d}^{12}z\,\Upsilon(\bar{\mathcal{W}}^{-2}D^4\ln\mathcal{W},\mathcal{W}^{-2}\bar{D}^4\ln\bar{\mathcal{W}}), \tag{1.4}$$

which generate quantum corrections of the form (1.1) in components (Λ and Υ are specific functions of their arguments). While our explicit computations will be done in the one-loop approximation, our conclusions about the general structure of the effective action in superconformal theories have a universal, loop-independent, character.

The paper is organized as follows. In Sec. II we describe superconformal invariants of the $\mathcal{N}=2$ Abelian vector multiplet, which appear as building blocks for the effective actions of $\mathcal{N}=2, 4$ superconformal theories.

In Sec. III we start with the one-loop effective action of $\mathcal{N}=4$ SYM theory computed for the constant field background (1.2), and then restore its general $\mathcal{N}=2$ superfield form using superconformal invariance considerations.

In Sec. IV the analysis of Sec. III is extended to the case of $\mathcal{N}=2$ SU(2) SYM effective action with four fundamental hypermultiplets. We find that a specific feature of the $\mathcal{N}=4$

super Yang-Mills theory, that there are no Λ-type quantum corrections [second term in Eq. (1.4)] in the one-loop effective action (in particular, the absence of the induced F^6 term), is not shared by generic $\mathcal{N}=2$ superconformal theories. This unique property of the $\mathcal{N}=4$ theory should be a consequence of a hidden $\mathcal{N}=4$ superconformal symmetry.

Section V contains the concluding remarks. Some useful facts about $\mathcal{N}=1, 2$ superconformal transformations are collected in the Appendix.

II. SUPERCONFORMAL INVARIANTS OF THE $\mathcal{N}=2$ VECTOR MULTIPLET

In this section we present superconformal invariants of an Abelian $\mathcal{N}=2$ vector multiplet described by a chiral superfield $\mathcal{W}(z)$ and its conjugate $\bar{\mathcal{W}}(z)$ which are subject to the standard off-shell constraints [18]

$$\bar{D}_{\dot{\alpha}\,i}\mathcal{W}=D^i_\alpha\bar{\mathcal{W}}=0, \quad i=\underline{1},\underline{2},$$

$$D^{ij}\mathcal{W}=\bar{D}^{ij}\bar{\mathcal{W}}, \quad D^{ij}\equiv D^{\alpha(i}D^{j)}_\alpha, \quad \bar{D}^{ij}\equiv\bar{D}^{(i}_{\dot{\alpha}}\bar{D}^{j)\,\dot{\alpha}}. \tag{2.1}$$

The $\mathcal{N}=2$ superconformal transformation law of $\mathcal{W}$ reads

$$\delta\mathcal{W}=-\xi\mathcal{W}-2\sigma\mathcal{W}. \tag{2.2}$$

Here $\xi=\xi^A D_A$ is a superconformal Killing vector, and the chiral scalar σ is defined by Eq. (A8), see the Appendix for more details. It follows then that the classical vector multiplet action

$$S_{\text{vm}}=\frac{1}{4}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\;\mathcal{W}^2 \tag{2.3}$$

is, of course, superconformal invariant.

Let us assume that $\mathcal{W}$ possesses a nonvanishing expectation value, as is the case in $\mathcal{N}=2, 4$ superconformal models with the gauge group spontaneously broken to its maximal compact subgroup. Then, using the results of the Appendix, one can check that the (anti)chiral combinations[2]

$$\bar{\Psi}^2=\frac{1}{16\bar{\mathcal{W}}^2}D^4\ln\frac{\mathcal{W}}{\mu}, \quad D^4=(D^{\underline{1}})^2\,(D^{\underline{2}})^2,$$

$$\Psi^2=\frac{1}{16\mathcal{W}^2}\bar{D}^4\ln\frac{\bar{\mathcal{W}}}{\mu}, \quad \bar{D}^4=(\bar{D}_{\underline{1}})^2\,(\bar{D}_{\underline{2}})^2, \tag{2.4}$$

transform as scalars with respect to the $\mathcal{N}=2$ superconformal group:

$$\delta\bar{\Psi}^2=-\xi\bar{\Psi}^2, \quad \delta\Psi^2=-\xi\Psi^2. \tag{2.5}$$

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[2]Here μ is a formal scale which is introduced to make the argument of the logarithm dimensionless. It drops out from all superconformal structures listed below.

Using the fact that $\mathcal{N}=2$ superconformal transformations preserve the $\mathcal{N}=2$ superspace measure $d^{12}z = d^4x\, d^4\theta\, d^4\bar\theta$,

$$(-1)^A D_A \xi^A = 0, \tag{2.6}$$

one can construct three types of $\mathcal{N}=2$ superconformal invariants:[3]

$$\mathbf{S}_1 = \int d^{12}z\, \ln\frac{\mathcal{W}}{\mu} \ln\frac{\bar{\mathcal{W}}}{\mu}, \tag{2.7}$$

$$\mathbf{S}_2 = \int d^{12}z\, \Lambda(\bar{\mathbf{\Psi}}^2)\, \ln\frac{\mathcal{W}}{\mu} + \text{c.c.}, \tag{2.8}$$

$$\mathbf{S}_3 = \int d^{12}z\, \Upsilon(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2), \tag{2.9}$$

where Λ and Υ are arbitrary holomorphic and real analytic functions, respectively.[4] These functionals are the main data describing quantum corrections of the form (1.1) (along with special contributions with derivatives of the fields required by supersymmetry) which appear in the low-energy effective actions of $\mathcal{N}=2, 4$ superconformal theories.

There exist additional superconformal invariants constructed in terms of

$$\mathbf{\Sigma}^{ij} = \frac{1}{\mathcal{W}\bar{\mathcal{W}}} D^{ij}\mathcal{W} = \frac{1}{\mathcal{W}\bar{\mathcal{W}}} \bar{D}^{ij}\bar{\mathcal{W}}, \tag{2.10}$$

where the primary field $D^{ij}\mathcal{W}$ transforms as follows:

$$\delta D^{ij}\mathcal{W} = -\xi D^{ij}\mathcal{W} - 2i\,\hat\Lambda_k{}^{(i} D^{j)k}\mathcal{W} - 2(\sigma+\bar\sigma) D^{ij}\mathcal{W}. \tag{2.11}$$

However, $\mathbf{\Sigma}^{ij}$ involves the free equation of motion of the $\mathcal{N}=2$ vector multiplet. As is well known, contributions to the effective action, which contain the classical equations of motion factor, are ambiguous (in particular, gauge dependent). For that reason we will ignore $\mathbf{\Sigma}$-dependent quantum corrections in what follows.

A large number of nontrivial superconformal invariants can be obtained by noting that for a primary superfield $\Gamma_{ij} = \Gamma_{ji}$ with the transformation law

$$\delta\Gamma_{ij} = -\xi\Gamma_{ij} + 2\sigma\Gamma_{ij} + 2i\,\hat\Lambda_{(i}{}^k \Gamma_{j)\,k}, \tag{2.12}$$

its descendant $D^{ij}\Gamma_{ij}$ is also primary:

$$\delta D^{ij}\Gamma_{ij} = -\xi D^{ij}\Gamma_{ij} + 2(\sigma-\bar\sigma)D^{ij}\Gamma_{ij}. \tag{2.13}$$

Given an arbitrary function $f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2)$, the superfield $\mathcal{W}f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2)$ transforms like $\mathcal{W}$, and therefore Γ_{ij}

$\equiv (\mathcal{W}^2\bar{\mathcal{W}})^{-1} D_{ij}(\mathcal{W}f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2))$ has the superconformal transformation law (2.12). As a consequence, the following combinations

$$\mathcal{W}D^{ij}\left\{\frac{1}{\mathcal{W}^2\bar{\mathcal{W}}^2} D_{ij}(\mathcal{W}f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2))\right\},$$

$$\bar{\mathcal{W}}\bar{D}^{ij}\left\{\frac{1}{\mathcal{W}^2\bar{\mathcal{W}}^2} D_{ij}(\mathcal{W}f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2))\right\} \tag{2.14}$$

are superconformal scalars. One more possibility to generate superconformal scalars is to take SU(2) invariant products of several superfields of the form

$$\frac{1}{\mathcal{W}\bar{\mathcal{W}}} D^{ij}(\mathcal{W}f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2)) \tag{2.15}$$

and their conjugates which transform similar to $\mathbf{\Sigma}^{ij}$. Then one can repeat the construction of superconformal invariants discussed above by replacing the arguments of $f(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2)$ by other superconformal scalars, etc.

In this paper, we are mainly interested in the part of the low-energy effective action of $\mathcal{N}=2, 4$ superconformal theories which involves the physical bosonic fields of the $\mathcal{N}=2$ *vector multiplet*, i.e., the U(1) field strength and its scalar superpartners, without higher derivatives. The crucial point is that all relevant component structures are then generated by the superconformal invariants of the three types given in Eqs. (2.7), (2.8), (2.9). It should be noted that while many component structures of interest can be also obtained from the superconformal invariants generated by Eqs. (2.14), (2.15) and their descendants, the difference between the two descriptions is only in terms which involve higher derivatives of the fields.

Let us represent $\mathcal{W}$ in terms of its $\mathcal{N}=1$ superfield parts:[5]

$$\mathcal{W}| = \Phi, \qquad D_{\dot\alpha}^2\mathcal{W} = 2i\,W_\alpha, \tag{2.16}$$

where we used the notation $U| = U(z)|_{\theta_2 = \bar\theta^2 = 0}$, for any $\mathcal{N}=2$ superfield U. Then

$$\bar{\mathbf{\Psi}}^2| = \frac{1}{4\bar\Phi^2} D^2\left(\frac{W^\alpha W_\alpha}{\Phi^2} + \frac{1}{4\Phi}\bar{D}^2\bar\Phi\right) \equiv \bar\Psi^2 + \frac{1}{16\bar\Phi^2}D^2\bar{D}^2\frac{\bar\Phi}{\Phi}. \tag{2.17}$$

From the $\mathcal{N}=1$ superconformal transformations

$$\delta = -\xi\Phi - 2\sigma\Phi,$$

$$\delta W_\alpha = -\xi W_\alpha + \hat\omega_\alpha{}^\beta W_\beta - 3\sigma W_\alpha, \tag{2.18}$$

it follows that the (anti)chiral combinations

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[3]Chiral-like superconformal invariants $\int d^4x\, d^4\theta\, \mathcal{W}^2 H(\mathbf{\Psi}^2) + \text{c.c.}$ are equivalent to $\mathbf{S}_2$.

[4]$\Upsilon(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2)$ is defined modulo Kähler-like shifts $\Upsilon(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2) \to \Upsilon(\mathbf{\Psi}^2, \bar{\mathbf{\Psi}}^2) + \Xi(\mathbf{\Psi}^2) + \bar\Xi(\bar{\mathbf{\Psi}}^2)$.

[5]Our $\mathcal{N}=1$ conventions correspond to [19].

$$\bar{\Psi}^2 = \frac{1}{4\bar{\Phi}^2} D^2 \left(\frac{W^2}{\Phi^2} \right), \quad \Psi^2 = \frac{1}{4\Phi^2} \bar{D}^2 \left(\frac{\bar{W}^2}{\bar{\Phi}^2} \right) \quad (2.19)$$

transform as *scalars* with respect to the $\mathcal{N}=1$ superconformal group.

III. $\mathcal{N}=4$ SUPER YANG-MILLS THEORY

In this section we analyze the low-energy effective action of the $\mathcal{N}=4$ SU(2) super Yang-Mills theory with the gauge group broken to U(1). A generalization to the case of an arbitrary semisimple gauge group spontaneously broken to its maximal Abelian subgroup is straightforward and can be done as in Refs. [15–17] where the leading superfield correction to the low-energy action was computed.

Our aim will be to find the manifestly superconformal invariant generalization of the well-known Schwinger-type expression for the bosonic part of the one-loop effective action of $\mathcal{N}=4$ SYM theory in the purely bosonic F_{mn} = const background. The use of the superconformal invariance requirement may allow us, in principle, to go beyond the constant field approximation.

For example, in the SU(2) $\mathcal{N}=4$ theory with the classical scalar field value producing the mass parameter $X^2 = |\Phi|^2$, the action in the background $F_{mn} = F_{mn}\sigma_3/2$, with F_{mn} having eigenvalues f_1 and f_2, is given by [6] [7,20]

$$\Gamma = \frac{4V_4}{(4\pi)^2} \int_0^\infty \frac{dt}{t^3} \, e^{-tX^2} \frac{f_1 t}{\sinh f_1 t} \frac{f_2 t}{\sinh f_2 t}$$

$$\times (\cosh f_1 t - \cosh f_2 t)^2. \quad (3.1)$$

Expanding in powers of $f_n \sim F$ one finds that there is no F^6/X^8 term, while the F^8/X^{12} term has a structure *different* from the one that appears in the expansion of the Abelian BI action (with the scale set up by X):

$$L_{BI} = X^4 [\sqrt{(1+f_1^2/X^4)(1+f_2^2/X^4)} - 1].$$

The F^8 terms in the BI and SYM actions are thus different combinations of the F^8-type superinvarints.

Below we shall find how to "supersymmetrize" the bosonic expression (3.1). Using the background field formulation [21] for general $\mathcal{N}=2$ super Yang-Mills theories in $\mathcal{N}=2$ harmonic superspace [22], it was shown [13] that under some restrictions on the background $\mathcal{N}=2$ vector multiplet $\mathcal{W} = \{W_\alpha, \Phi\}$, the one-loop effective action of $\mathcal{N}=4$ SYM theory admits a simple functional representation in terms of $\mathcal{N}=1$ superfields:

$$\exp(i\Gamma) = \int D\bar{V} DV \exp \left\{ i \int d^8 z \, \bar{V} \Delta V \right\}, \quad (3.2)$$

where the operator Δ is defined by

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[6]Here we consider the action in Minkowski space and hence the sign of Γ is opposite to that in [7,20].

$$\Delta = \mathcal{D}^a \mathcal{D}_a + W^\alpha \mathcal{D}_\alpha - \bar{W}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - |\Phi|^2. \quad (3.3)$$

The integration in Eq. (3.2) is carried out over complex *unconstrained* $\mathcal{N}=1$ superfields $V, \bar{V}$. The algebra of $\mathcal{N}=1$ gauge-covariant derivatives is

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = -2i \mathcal{D}_{\alpha\dot{\alpha}}, \quad [\mathcal{D}_a, \mathcal{D}_b] = i F_{ab},$$

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_\beta] = -2i \varepsilon_{\alpha\beta} \bar{W}_{\dot{\alpha}}, \quad [\mathcal{D}_{\alpha\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}] = -2i \varepsilon_{\dot{\alpha}\dot{\beta}} W_\alpha, \quad (3.4)$$

where

$$F_{\alpha\dot{\alpha}, \beta\dot{\beta}} = (\sigma^a)_{\alpha\dot{\alpha}} (\sigma^b)_{\beta\dot{\beta}} F_{ab}$$

$$= -i\varepsilon_{\alpha\beta}\bar{D}_{(\dot{\alpha}}\bar{W}_{\dot{\beta})} - i\varepsilon_{\dot{\alpha}\dot{\beta}}D_{(\alpha}W_{\beta)}. \quad (3.5)$$

For a simple superfield background

$$D_\alpha W_\beta = D_{(\dot{a}}W_{\beta)} = \text{const}, \quad \Phi = \text{const}, \quad (3.6)$$

the effective action can be exactly computed using the superfield proper-time technique (see [19] for a review), and the result is [23]

$$\Gamma = \frac{1}{64\pi^2} \int d^8 z \int_0^\infty \frac{dt}{t^3} \, W^2 \bar{W}^2 \exp(-t|\Phi|^2) \text{tr} \left[\left(\frac{e^{tM} - 1}{M} \right) \right.$$

$$\times \left(\frac{e^{-tM} - 1}{M} \right) \right] \text{tr} \left[\left(\frac{e^{t\bar{M}} - 1}{\bar{M}} \right) \left(\frac{e^{-t\bar{M}} - 1}{\bar{M}} \right) \right] \det \left(\frac{tF}{\sin(tF)} \right)^{1/2}$$

$$= \frac{1}{16\pi^2} \int d^8 z \int_0^\infty \frac{dt}{t^3} \, W^2 \bar{W}^2 \exp(-t|\Phi|^2)$$

$$\times \det \left(\frac{e^{tM} - 1}{M} \right) \det \left(\frac{e^{t\bar{M}} - 1}{\bar{M}} \right) \det \left(\frac{tF}{\sin(tF)} \right)^{1/2}, \quad (3.7)$$

where

$$M_\alpha{}^\beta = D_\alpha W^\beta = 2i F_\alpha{}^\beta, \quad \bar{M}_{\dot{\alpha}}{}^{\dot{\beta}} = -\bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\beta}} = -2i\bar{F}_{\dot{\alpha}}{}^{\dot{\beta}}. \quad (3.8)$$

The effective action is ultraviolet and infrared finite.

To bring Eq. (3.7) to a more useful form, we first note

$$\text{tr} \left[\left(\frac{e^{tM} - 1}{M} \right) \left(\frac{e^{-tM} - 1}{M} \right) \right] = \frac{4}{B^2} [1 - \cosh(tB)], \quad (3.9)$$

where

$$B^2 \equiv \frac{1}{2} \text{tr}(M^2) = \frac{1}{4} D^2 W^2. \quad (3.10)$$

In terms of the two invariants of the electromagnetic field,

$$\mathcal{F} = \frac{1}{4} F^{ab} F_{ab}, \quad \mathcal{G} = \frac{1}{4} *F^{ab} F_{ab}, \quad (3.11)$$

we find that for the background under consideration one has

$$B^2 = 2(\mathcal{F} + i\,\mathcal{G}) \tag{3.12}$$

and

$$\frac{1}{16} D^2 \bar{D}^2 (W^2 \bar{W}^2) = \frac{1}{16} D^2 W^2 \bar{D}^2 \bar{W}^2 = B^2 \bar{B}^2 = 4(\mathcal{F}^2 + \mathcal{G}^2). \tag{3.13}$$

Then [24]

$$\det\left(\frac{tF}{\sin(tF)}\right)^{1/2} = \frac{2i\,t^2 \mathcal{G}}{\cosh t \sqrt{2(\mathcal{F} + i\,\mathcal{G})} - \cosh t \sqrt{2(\mathcal{F} - i\,\mathcal{G})}}$$

$$= \frac{1}{2} \frac{t^2 (B^2 - \bar{B}^2)}{\cosh(tB) - \cosh(t\bar{B})}, \tag{3.14}$$

and therefore the component form of the effective action is [which is equivalent to the one in Eq. (3.1)]

$$\Gamma = \frac{1}{4\pi^2} \int d^4x \int_0^\infty \frac{dt}{t^3} \exp(-t|\Phi|^2)$$

$$\times [\cosh t \sqrt{2(\mathcal{F} + i\,\mathcal{G})} - 1][\cosh t \sqrt{2(\mathcal{F} - i\,\mathcal{G})} - 1]$$

$$\times \frac{2i\,t^2 \mathcal{G}}{\cosh t \sqrt{2(\mathcal{F} + i\,\mathcal{G})} - \cosh t \sqrt{2(\mathcal{F} - i\,\mathcal{G})}}. \tag{3.15}$$

The superfield effective action is

$$\Gamma = \frac{1}{8\pi^2} \int d^8z \int_0^\infty dt\, t W^2 \bar{W}^2 \exp(-t|\Phi|^2)$$

$$\times \frac{\cosh(tB) - 1}{t^2 B^2} \frac{\cosh(t\bar{B}) - 1}{t^2 \bar{B}^2} \frac{t^2 (B^2 - \bar{B}^2)}{\cosh(tB) - \cosh(t\bar{B})}. \tag{3.16}$$

After a simple rescaling of the proper-time integral, we can rewrite the action as follows:

$$\Gamma = \frac{1}{8\pi^2} \int d^8z \int_0^\infty dt\, t e^{-t} \frac{W^2 \bar{W}^2}{\Phi^2 \bar{\Phi}^2}$$

$$\times \frac{\cosh(t\Psi) - 1}{t^2 \Psi^2} \frac{\cosh(t\bar{\Psi}) - 1}{t^2 \bar{\Psi}^2} \frac{t^2 (\Psi^2 - \bar{\Psi}^2)}{\cosh(t\Psi) - \cosh(t\bar{\Psi})}, \tag{3.17}$$

with Ψ and $\bar{\Psi}$ defined in Eq. (2.19).

Let us introduce the following function:

$$\omega(x,y) = \omega(y,x) = \frac{\cosh x - 1}{x^2} \frac{\cosh y - 1}{y^2}$$

$$\times \frac{x^2 - y^2}{\cosh x - \cosh y} - \frac{1}{2},$$

$$\omega(0,y) = \omega(x,0) = 0. \tag{3.18}$$

Then the effective action can be rewritten in the form

$$\Gamma = \frac{1}{16\pi^2} \int d^8z \frac{W^2 \bar{W}^2}{\Phi^2 \bar{\Phi}^2}$$

$$+ \frac{1}{8\pi^2} \int d^8z \int_0^\infty dt\, t e^{-t} \frac{W^2 \bar{W}^2}{\Phi^2 \bar{\Phi}^2} \omega(t\Psi, t\bar{\Psi}). \tag{3.19}$$

Now we come to the key point. Until now we have used the constant field approximation (3.6). However, in Eq. (3.19) we may no longer assume such an approximation. The effective action of $\mathcal{N}=4$ SYM theory should be superconformal invariant, but Ψ and $\bar{\Psi}$ are basically the only superconformal scalars constructed from both W_α and Φ (modulo contributions involving the free equations of motion terms $D^\alpha W_\alpha$ and $D^2 \Phi$ and higher derivative invariants; see Sec. II). Thus the effective action (3.19) is manifestly invariant under $\mathcal{N}=1$ superconformal transformations.

Of course, the effective action should not only be manifestly $\mathcal{N}=1$ superconformal, but $\mathcal{N}=2$ superconformal as well. One can restore an $\mathcal{N}=2$ superconformal form of Γ simply by noting that Ψ is a part (2.17) of the leading $\mathcal{N}=1$ component of $\mathbf{\Psi}$.

As follows from Eqs. (3.18) and (3.19), Γ contains contributions of the two types

$$S_1 = \int d^8z \frac{W^2 \bar{W}^2}{\Phi^2 \bar{\Phi}^2}, \tag{3.20}$$

$$S_3^{(m,n)} = \int d^8z \frac{W^2 \bar{W}^2}{\Phi^2 \bar{\Phi}^2} \Psi^{2m} \bar{\Psi}^{2n}, \quad m,n \neq 0. \tag{3.21}$$

Using the identities

$$\frac{1}{16} D^4 \ln \mathcal{W} = \frac{1}{4} D^2 \left(\frac{W^\alpha W_\alpha}{\Phi^2}\right) + \cdots,$$

$$\frac{1}{4} (D^2)^2 \frac{1}{\mathcal{W}^{2m}}\bigg| = -\frac{2m(2m+1)}{\Phi^{2m}} \frac{W^\alpha W_\alpha}{\Phi^2} + \cdots, \tag{3.22}$$

where ellipses denote terms involving derivatives of Φ, we observe that the $\mathcal{N}=2$ extensions of S_1 and S_3 are

$$\mathbf{S}_1 = \int d^{12}z \ln \frac{\mathcal{W}}{\mu} \ln \frac{\bar{\mathcal{W}}}{\mu}, \tag{3.23}$$

$$\mathbf{S}_3^{(m,n)} = \frac{1}{2m(2m+1)2n(2n+1)} \int d^{12}z\, \mathbf{\Psi}^{2m} \bar{\mathbf{\Psi}}^{2n}. \tag{3.24}$$

Let $\Omega(x,y) = \Omega(y,x)$ be the analytic function related to $\omega(x,y)$ as follows: if

$$(x,y) = \sum_{m,n=1}^{\infty} c_{m,n} x^{2m} y^{2n}, \qquad (3.25)$$

then

$$\Omega(x,y) = \frac{1}{4} \sum_{m,n=1}^{\infty} \frac{c_{m,n}}{m(2m+1)n(2n+1)} x^{2m} y^{2n}. \qquad (3.26)$$

Then the manifestly $\mathcal{N}=2$ superconformal form of Γ is

$$\Gamma = \frac{1}{16\pi^2} \int d^{12}z \ln\frac{\mathcal{W}}{\mu} \ln\frac{\bar{\mathcal{W}}}{\mu}$$

$$+ \frac{1}{8\pi^2} \int d^{12}z \int_0^{\infty} dt\, t e^{-t}\; \Omega(t\mathbf{\Psi}, t\bar{\mathbf{\Psi}}). \qquad (3.27)$$

Here the first term was computed in [13,14] (see also [15–17]).

As is seen from Eq. (3.27), the one-loop effective action of $\mathcal{N}=4$ SYM theory does not contain terms described by the "second" superconformal invariant (2.8). In particular, there are no F^6-type corrections generated by

$$\int d^{12}z \frac{1}{\bar{\mathcal{W}}^2} \ln\frac{\mathcal{W}}{\mu} D^4 \ln\frac{\mathcal{W}}{\mu}. \qquad (3.28)$$

Such terms are expected to appear at the two-loop order.

The absence of this "F^6" correction at the one-loop order is a unique feature of the maximally supersymmetric $\mathcal{N}=4$ super Yang-Mills theory (which, as discussed in the Introduction, is crucial for supergravity-SYM correspondence at the subleading order). As we are going to demonstrate in the next section, this property is no longer true in generic $\mathcal{N}=2$ superconformal models.

It may be instructive to compare the low-energy action (3.19) with the $\mathcal{N}=1$ supersymmetric Born-Infeld action [25]

$$S_{\mathrm{BI}} = \frac{1}{4} \int d^6z\, W^2 + \frac{1}{4} \int d^6\bar{z}\, \bar{W}^2$$

$$+ \frac{1}{X^4} \int d^8z \frac{W^2 \bar{W}^2}{1 + \frac{1}{2} a + \sqrt{1 + a + \frac{1}{4} b^2}}, \qquad (3.29)$$

$$a = \frac{1}{2X^4}(D^2 W^2 + \bar{D}^2 \bar{W}^2),$$

$$b = \frac{1}{2X^4}(D^2 W^2 - \bar{D}^2 \bar{W}^2),$$

where we used $1/X$ as a scale parameter. The nontrivial last term here has a structure similar to that of Γ in Eq. (3.19), with X^2 playing the role of $|\Phi|^2$. While the two actions coincide at the leading $W^2\bar{W}^2$ order, they contain different combinations of invariants at higher orders (see also the discussion in the Introduction). In particular, the subleading

"F^6" term which was absent in the one-loop $\mathcal{N}=4$ SYM effective action is present in the BI action (3.29) and has the form

$$-\frac{1}{8X^8} \int d^8z\, W^2\, \bar{W}^2 (D^2 W^2 + \bar{D}^2 \bar{W}^2). \qquad (3.30)$$

IV. $\mathcal{N}=2$ SUPERCONFORMAL MODELS

In this section we shall consider a special $\mathcal{N}=2$ superconformal theory—the $\mathcal{N}=2$ SU(N) super Yang-Mills model with $2N$ hypermultiplets in the fundamental representation; the effective action of generic $\mathcal{N}=2$ superconformal models [26] can be analyzed in a similar fashion. For simplicity, only the case of $\mathcal{N}=2$ will be discussed, with the gauge group SU(2) spontaneously broken to its U(1) subgroup.

Both $\mathcal{N}=2$ SYM and hypermultiplet models are superconformal invariant at the classical level. Their quantum effective actions include the scale-independent nonholomorphic terms besides standard divergent and holomorphic scale-dependent contributions. For special combinations of these models divergent and holomorphic contributions cancel out and the full quantum effective action is superconformal invariant.

For computing the one-loop low-energy effective action of a hypermultiplet coupled to a background Abelian $\mathcal{N}=2$ vector multiplet it is sufficient to make use of the simplest realization of the hypermultiplet in terms of two $\mathcal{N}=1$ covariantly chiral superfields ϕ_1 and ϕ_1 with opposite U(1) charges $e = \pm 1$, with the action

$$S = \int d^8z\, (\bar{\phi}_1\phi_1 + \bar{\phi}_2\phi_2) + \left\{ \mathrm{i} \int d^6z\, \Phi\, \phi_1\, \phi_2 + \mathrm{c.c.} \right\},$$

$$\bar{\mathcal{D}}_{\dot{\alpha}}\phi_{1,2} = 0. \qquad (4.1)$$

In the constant field approximation (3.6), the effective action is given by a functional determinant of the D'Alambertian

$$\Delta_{\mathrm{c}} = \mathcal{D}^a\mathcal{D}_a + W^\alpha\mathcal{D}_\alpha - |\Phi|^2, \qquad (4.2)$$

which acts on the space of covariantly chiral superfields. The effective action is [27,28,23]

$$\Gamma_{\mathrm{hm}} = \frac{1}{16\pi^2} \int_{\epsilon^2}^{\infty} \frac{dt}{t} \exp(-t|\Phi|^2)\; \int d^6z\, W^2$$

$$\times \frac{\cosh(tB)-1}{t^2 B^2}\, \frac{(B^2-\bar{B}^2)t^2}{\cosh(tB)-\cosh(t\bar{B})}, \qquad (4.3)$$

where $\epsilon \to 0$ is a UV cutoff.

The form of Γ_{hm} is determined by the function

$$\lambda(x,y) = \frac{\cosh x - 1}{x^2}\, \frac{x^2 - y^2}{\cosh x - \cosh y}, \quad \lambda(x,0) = 1. \qquad (4.4)$$

It is useful to introduce a new function $\zeta(x,y)$ related to λ by

$$\lambda(x,y)-1=-y^2\zeta(x,y),$$

$$\zeta(x,y)=\zeta(y,x)=\frac{y^2(\cosh x-1)-x^2(\cosh y-1)}{x^2y^2(\cosh x-\cosh y)}.$$
(4.5)

Recalling the definition $\bar{B}^2=\frac{1}{4}\bar{D}^2\bar{W}^2$, we can rewrite the effective action as follows:

$$\Gamma_{\text{hm}}=\frac{1}{16\pi^2}\int_{\epsilon^2}^{\infty}\frac{dt}{t}\exp(-t|\Phi|^2)\int d^6z\,W^2-\frac{1}{16\pi^2}\int_0^{\infty}\frac{dt}{t}$$

$$\times\exp(-t|\Phi|^2)\int d^6z\,W^2\,t^2\,\bar{B}^2\zeta(tB,t\bar{B}),$$
(4.6)

i.e.,

$$\Gamma_{\text{hm}}=-\frac{1}{16\pi^2}\int d^6z\,W^2\ln\frac{\Phi}{\mu}+\text{c.c.}$$

$$+\frac{1}{16\pi^2}\int d^8z\int_0^{\infty}dt\,t\,e^{-t}\,\frac{W^2\bar{W}^2}{\Phi^2\bar{\Phi}^2}\,\zeta(t\bar{\Psi},t\Psi),$$
(4.7)

where we have absorbed the UV cutoff into the renormalization scale μ. Here the first term (holomorphic contribution) may be derived also by other well-known methods[7] (see, e.g., [30]).

In the $\mathcal{N}=2$ superconformal theories holomorphic contributions cancel out. Let us recall how this happens for the present model with four fundamental hypermultiplets. Each hypermultiplet has two SU(2) components, so that altogether we have eight Abelian hypermultiplets with charges $e=\pm\frac{1}{2}$ with respect to the unbroken U(1) generated by $\frac{1}{2}\sigma_3$. In addition, we have the adjoint ghost superfields or two hypermultiplets with U(1) charges $e=\pm1$. The charges may be accounted for by replacing W_α and Φ in the effective action by

$$W_\alpha\to eW_\alpha,\quad \Phi\to e\Phi.$$
(4.8)

Then the complete effective action is

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[7]In obtaining Eq. (4.7), we concentrated on the quantum corrections involving the vector multiplet strength and did not take into account the effective Kähler potential $K(\Phi,\bar{\Phi})=-(1/16\pi^2)\Phi\bar\Phi\ln(\bar\Phi\Phi/\mu^2)=\bar\Phi\,\mathcal{F}'(\Phi)+\Phi\,\bar{\mathcal{F}}'(\bar\Phi)$ generated by the holomorphic Seiberg potential $\mathcal{F}(\Phi)=-(1/32\pi^2)\Phi^2\ln(\Phi/\mu)$. A derivation of $K(\Phi,\bar{\Phi})$ in the framework of the superfield proper time technique, which we used in this paper, can be found in [29,19].

$$\Gamma=8\times\frac{1}{16\pi^2}\int d^8z\int_0^{\infty}dt\,t e^{-t}\,\frac{W^2\bar{W}^2}{\Phi^2\bar{\Phi}^2}\,\zeta(2t\Psi,2t\bar{\Psi})-2$$

$$\times\frac{1}{16\pi^2}\int d^8z\int_0^{\infty}dt\,t e^{-t}\,\frac{W^2\bar{W}^2}{\Phi^2\bar{\Phi}^2}\,\zeta(t\Psi,t\bar{\Psi})$$

$$+\frac{1}{8\pi^2}\int d^8z\int_0^{\infty}dt\,t e^{-t}\,\frac{W^2\bar{W}^2}{\Phi^2\bar{\Phi}^2}\left\{\omega(t\Psi,t\bar{\Psi})+\frac{1}{2}\right\},$$
(4.9)

with the function $\omega(x,y)$ defined in Eq. (3.18). Here the last term coincides with the effective action (3.19) of $\mathcal{N}=4$ SYM theory.[8]

Note that since

$$\zeta(x,0)=\frac{\cosh x-1-\frac{1}{2}x^2}{x^2(\cosh x-1)},$$
(4.10)

the effective action now contains the $\mathcal{N}=2$ superconformal invariants of the type (2.8) [and, in particular, the "F^6" contributions (3.28)] which were absent in the $\mathcal{N}=4$ case.

V. CONCLUSIONS

Let us summarize the results obtained. We described the superconformal invariants which are constructed in terms of the $\mathcal{N}=2$ Abelian vector multiplet and play the role of building blocks for the low-energy effective actions of $\mathcal{N}=2$ or $\mathcal{N}=4$ superconformal theories on the Coulomb branch. We then computed the one-loop effective actions in constant $\mathcal{N}=2$ field strength background in the $\mathcal{N}=4$ SYM theory and in a particular $\mathcal{N}=2$ SU(2) gauge theory.

The fact that the theories under consideration are *superconformal* allowed us to go beyond the constant field approximation and to restore, with the aid of the classification of superconformal invariants, the one-loop effective actions (1.4) (with Λ and Υ being special model-dependent functions). These actions generate contributions which in components have the form (1.1) (with no coupling constant prefactors since we consider the one-loop approximation).

The crucial difference between the $\mathcal{N}=4$ SYM theory and generic $\mathcal{N}=2$ superconformal models is that the second term in Eq. (1.4) is absent at the one-loop level in $\mathcal{N}=4$ SYM. The first term in Eq. (1.4), which generates F^4 corrections, is known to be one-loop exact [1]. It would be of interest to study if there are possible nonrenormalization theorems for the quantum corrections which are given by the second and the third terms in Eq. (1.4) for particular choices of the function Λ and Υ.

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[8]The $\mathcal{N}=4$ SYM theory is equivalent to the $\mathcal{N}=2$ SYM theory coupled to a single hypermultiplet in the adjoint representation; in this case, the hypermultiplet and the ghost contributions cancel each other, and the effective action is given by the last term in Eq. (4.9).

APPENDIX: SUPERCONFORMAL TRANSFORMATIONS

In this Appendix we collect basic properties of $\mathcal{N}=1,2$ superconformal transformations (see, for instance, Refs. [19,31] for more details). In $\mathcal{N}=1,2$ global superspace $\mathbf{R}^{4|4\mathcal{N}}$ parametrized by $z^A=(x^a,\theta_i^\alpha,\bar\theta_\alpha^i)$, infinitesimal superconformal transformations

$$z^A \to z^A + \xi^A \tag{A1}$$

are generated by superconformal Killing vectors

$$\xi=\bar{\bar\xi}=\xi^A D_A=\xi^a(z)\partial_a+\xi_i^\alpha(z)D_\alpha^i+\bar\xi_\alpha^i(z)\bar{D}_i^{\dot\alpha} \tag{A2}$$

defined to satisfy

$$[\xi,D_\alpha^i]\propto D_\beta^j. \tag{A3}$$

From here one gets

$$\xi_i^\alpha=-\frac{\mathrm{i}}{8}\bar{D}_{\dot\beta i}\xi^{\dot\beta\alpha}, \quad \bar{D}_{\dot\beta j}\xi_i^\alpha=0, \tag{A4}$$

while the vector parameters satisfy the equation

$$D_{(\alpha}^i\xi_{\beta)\dot\beta}=\bar{D}_{i(\dot\alpha}\xi_{\beta\dot\beta)}=0, \tag{A5}$$

implying, in turn, the conformal Killing equation

$$\partial_a\xi_b+\partial_b\xi_a=\frac{1}{2}\eta_{ab}\,\partial_c\xi^c. \tag{A6}$$

From Eqs. (A4) and (A5) one gets

$$[\xi,D_\alpha^i]=-(D_\alpha^i\xi_j^\beta)D_\beta^j$$

$$=\hat\omega_\alpha{}^\beta D_\beta^i-\frac{1}{N}[(N-2)\sigma+2\bar\sigma]D_\alpha^i-\mathrm{i}\hat\Lambda_j{}^i\;D_\alpha^j. \tag{A7}$$

Here the parameters of ''local'' Lorentz $\hat\omega$ and scale-chiral σ transformations are

$$\hat\omega_{\alpha\beta}(z)=-\frac{1}{N}\;D_{(\alpha}^i\xi_{\beta)i},$$

$$\sigma(z)=\frac{1}{N(N-4)}\left(\frac{1}{2}(N-2)D_\alpha^i\xi_i^\alpha\right.$$

$$\left.-\bar{D}_i^{\dot\alpha}\bar\xi_{\dot\alpha}^i\right) \tag{A8}$$

and turn out to be chiral:

$$\bar{D}_{\dot\alpha i}\hat\omega_{\alpha\beta}=0, \quad \bar{D}_{\dot\alpha i}\sigma=0. \tag{A9}$$

The parameters $\hat\Lambda_j{}^i$,

$$\hat\Lambda_j{}^i(z)=-\frac{1}{32}\left([D_\alpha^i,\bar{D}_{\dot\alpha j}]-\frac{1}{N}\delta_j{}^i[D_\alpha^k,\bar{D}_{\dot\alpha k}]\right)\xi^{\dot\alpha\alpha},$$

$$\hat\Lambda^\dagger=\hat\Lambda, \quad \mathrm{tr}\,\hat\Lambda=0, \tag{A10}$$

appear only in the $\mathcal{N}=2$ case and correspond to ''local'' SU(2) transformations. One can readily check the identities

$$D_\alpha^k\hat\Lambda_j{}^i=2\mathrm{i}\left(\delta_j^k D_\alpha^i-\frac{1}{N}\delta_j^i D_\alpha^k\right)\sigma,$$

$$D_\alpha^i\hat\omega_{\beta\gamma}=2\varepsilon_{\alpha(\beta}D_{\gamma)}^i\,\sigma, \tag{A11}$$

along with

$$D_\alpha^i D_\beta^j\sigma=0. \tag{A12}$$

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