S-wave absorption of scalars by noncommutative D3-branes

Y. S. Myung, Gungwon Kang, and H. W. Lee
Department of Physics, Inje University, Kimhae 621-749, Korea
(Received 25 February 2000; published 24 July 2000)

On the supergravity side, we study the propagation of the Ramond-Ramond (RR) scalar and the dilaton in the D3-branes with a NS B field. To obtain the noncommutative effect in a simple way, we consider the interesting case of $B\to\infty(\theta\to\pi/2)$. We represent this as the smeared D1-brane background with $\bar{F}_5=\bar{H}=0$ approximately. In this background, considering the RR scalar equation only, it induces an instability of the near-horizon geometry. However, it turns out that, considering all coupled equations, the RR scalar is non-propagating. On the other hand, the dilaton is a physically propagating mode. We calculate the s-wave absorption cross section of the dilaton. One finds $\sigma_0^\phi|_{B\to\infty}\sim(\tilde{\omega}\tilde{R}_{\pi/2})^{8.9}/\omega^5$, while $\sigma_0^\phi|_{B=0}\sim(\tilde{\omega}R_0)^8/\omega^5$ in the leading-order calculation. This means that although the dilaton belongs to a minimally coupled scalar in the absence of a B field, it becomes a sort of fixed scalar in the limit of $B\to\infty$.

PACS number(s): 04.70.Dy, 04.50.+h, 11.25.Db

I. INTRODUCTION

Recently noncommutative geometry has attracted much interest in the study of string and M theory in the B field [1–7]. For example, we consider the classical solution which arises from D3-branes with a Neveu-Schwarz (NS) B field. According to the anti–de Sitter (AdS) conformal field theory (CFT) correspondence [8], we expect that classical physics based on the near-horizon AdS_5 of a $D=7$ black hole solution can describe the large-N limit of noncommutative super Yang-Mills (NCSYM) theory on the boundary. In this direction we take a decoupling limit to isolate the near horizon geometry from the remaining one. Then the correspondence naturally arises.

It turns out that the noncommutative effects are relevant for physics in the ultraviolet (UV) regime, leaving that in the infrared (IR) regime unchanged. On the other hand, the NCSYM theory is not useful for studying the UV theory at short distances. An NCSYM with the noncommutativity scale Δ on a torus of size Σ is equivalent to an ordinary supersymmetric Yang-Mills (OSYM) theory with a magnetic flux, provided that $\Theta=\Delta^2/\Sigma^2$ is a rational number [9]. The equivalence between the NCSYM and the OSYM can be understood from the T duality of the corresponding string theory. Hence the OSYM with B field is the proper description for the UV region, while the NCSYM takes over the IR region. Actually, the noncommutative effect comes from the $B\to\infty$ limit of the ordinary theories [5,6,10,11].

We remind the reader that, aside from the entropy, there exists an important dynamical quantity, ''the greybody factor (absorption cross section)'' to extract information for the quantum black hole [12–15]. It is well known that the cross section can be obtained from the solution to the linearized equation upon the diagonalization. On the string side, there was a calculation for the absorption of scalars into the noncommutative D3 branes [16].

Myung, Kang, and Lee [17] have studied the quantum aspects of the D3-brane black hole in the B_{23} field background using a minimally coupled scalar. Such a field might describe a fluctuation of the off-diagonal gravitons polarized parallel to the brane ($h_{ab},a,b=0,1,2,3$). We derived the exact form of the absorption cross section (σ_l) in a B field on the supergravity side. It turns out that $\sigma_l^{B\neq0}>\sigma_l^{B=0}$. This implies that the presence of the B field suppresses the curvature effect surrounding the $D=7$ black hole. Consequently, this leads to an increase of the greybody factor.

Recently, Kaya [18] has calculated the absorption cross section by considering only the Ramond-Ramond (RR) scalar equation (26) in the D3-brane with the large B field. He showed in the first version that the greybody factor is not changed even if the B field is large. This means that the RR scalar becomes a minimally coupled scalar even for the presence of the large B field. However, in the third version, he commented that the RR scalar holds with $\sigma_0^{B\neq0}>\sigma_0^{B=0}$.

In this paper we study the propagations of the RR scalar (χ) and the dilaton (Φ) by D3-branes with a B field along their world volume directions (x_2,x_3). Especially, we are interested in the case of $B\to\infty$. In this sense our spacetime background is different from Kaya's case. Here we use all information including all equations of motion [especially Eqs. (26) and (28)], the Bianchi identities, and the gauge condition for graviton. In the absence of a B field, all of χ,Φ,h_{ab} belong to minimally coupled scalars. However, in the presence of a B field, these fields are coupled to the background *nonminimally*. In this sense, we may regard these as the fixed scalars. Actually, in the smeared D1-brane background, the dilaton (RR scalar) turn out to be (non)propagating modes. Also we wish to test whether or not there is a change in the absorption cross sections of the dilaton when $B\to\infty$.

The organization of this paper is as follows. In Sec. II, we briefly review the field equations which are relevant for our study. Here we wish to study the $B\to\infty$ limit carefully and introduce the smeared D1-brane black hole. Section III is devoted to analyzing the perturbations around the smeared D1-brane background. Because the linearized equation for the RR scalar is completely decoupled, the propagation of the RR scalar is investigated in Sec. IV. This induces an instability of the near-horizon geometry of the smeared D1-brane black hole. In Sec. V we correct the propagation of the

RR scalar in view of all linearized equations. And we deal with the propagation of the dilaton with the dilaton gauge. In Sec. VI, we study the propagation of the dilaton with the harmonic gauge and obtain its absorption cross section. We discuss our results in Sec. VII. Finally, we present the smeared D1-brane solution in the Appendix.

II. FORMALISM

We start with the low-energy limit of a type-IIB superstring action in the Einstein frame ($g_{MN}=e^{-\Phi/2}G_{MN}$) [12]:

$$S_{10}^{E}=\frac{1}{2\kappa_{10}^{2}}\int d^{10}x\left[\sqrt{-g}\left\{R-\frac{1}{2}(\nabla\Phi)^{2}-\frac{1}{12}e^{-\Phi}(\partial B_{2})^{2}-\frac{1}{2}e^{2\Phi}(\partial\chi)^{2}-\frac{1}{12}e^{\Phi}(\partial C_{2}-\chi\partial B_{2})^{2}-\frac{1}{4\cdot 5!}F_{5}^{2}\right\}-\frac{1}{2\cdot 4!\cdot(3!)^{2}}\epsilon_{10}C_{4}\partial C_{2}\partial B_{2}\right], \tag{1}$$

where Φ is the dilaton, χ is the RR scalar, B_2 is the NS two form, C_2 is the RR two form, and C_4 is the RR four form. And one has

$$H_{MNP}=(\partial B_{2})_{MNP}=3\partial_{[M}B_{NP]},$$
$$F_{3MNP}=(\partial C_{2})_{MNP}=3\partial_{[M}C_{2NP]},$$
$$(\partial C_{4})_{MNPQR}=5\partial_{[M}C_{4NPQR]},$$
$$F_{5}=\partial C_{4}+5(B_{2}\partial C_{2}-C_{2}\partial B_{2}), \tag{2}$$

where the self-duality constraint $F_5=\tilde{F}_5$ is required at the level of the equations of motion. The relevant equations of motion lead to [19]

$$\nabla^{2}\chi+2\nabla\Phi\nabla\chi+\frac{e^{-\Phi}}{6}(F_{3}-\chi H)\cdot H=0, \tag{3}$$

$$\nabla^{2}\Phi+\frac{1}{12}\{e^{-\Phi}H^{2}-e^{\Phi}(F_{3}-\chi H)^{2}\}-e^{2\Phi}(\nabla\chi)^{2}=0, \tag{4}$$

$$\nabla_{M}(e^{-\Phi}H^{MPQ})-\nabla_{M}\{\chi e^{\Phi}(F_{3}-\chi H)^{MPQ}\}+\frac{2}{3}F^{PQRST}F_{3RST}=0, \tag{5}$$

$$\nabla_{M}\{e^{\Phi}(F_{3}-\chi H)^{MPQ}\}-\frac{2}{3}F^{PQRST}H_{RST}=0, \tag{6}$$

$$\nabla_{M}F^{MPQRS}=0. \tag{7}$$

In the string frame, Eqs. (3)–(7) take the forms

$$\nabla_{s}^{2}\chi+\frac{1}{6}(F_{3s}-\chi H_{s})\cdot H_{s}=0, \tag{8}$$

$$\nabla_{s}^{2}\Phi-2(\nabla_{s}\Phi)^{2}+\frac{1}{12}\{H_{s}^{2}-e^{2\Phi}(F_{3s}-\chi H_{s})^{2}\}-e^{2\Phi}(\nabla_{s}\chi)^{2}=0, \tag{9}$$

$$\nabla_{sM}(e^{-2\Phi}H_{s}^{MPQ})-\nabla_{sM}\{\chi(F_{3s}-\chi H_{s})^{MPQ}\}+\frac{2}{3}F_{s}^{PQRST}F_{3sRST}=0, \tag{10}$$

$$\nabla_{sM}(F_{3s}-\chi H_{s})^{MPQ}-\frac{2}{3}F_{s}^{PQRST}H_{sRST}=0, \tag{11}$$

$$\nabla_{sM}F_{s}^{MPQRS}=0. \tag{12}$$

In addition, we require the remaining Maxwell equations as three Bianchi identities,

$$\partial_{[M}H_{NPQ]}=\partial_{[M}F_{NPQ]}=\partial_{[M}F_{NPQRS]}=0. \tag{13}$$

The solution of the D=7 extremal black hole for the D3-branes with a nonzero B_{23} field is given in the D=10 string frame [5] by

$$ds_{s}^{2}=f^{-1/2}\{-dx_{0}^{2}+dx_{1}^{2}+h(dx_{2}^{2}+dx_{3}^{2})\}+f^{1/2}(dr^{2}+r^{2}d\Omega_{5}^{2}),$$

$$f=1+\frac{R_{\theta}^{4}}{r^{4}},\quad h^{-1}=\sin^{2}\theta f^{-1}+\cos^{2}\theta,$$

$$\bar{B}_{s23}=\tan\theta f^{-1}h,\quad e^{2\bar{\Phi}}=g^{2}h,$$

$$\bar{F}_{s01r}=\frac{1}{g}\sin\theta\partial_{r}(f^{-1}),\quad \bar{F}_{s0123r}=\frac{1}{g}\cos\theta h\partial_{r}(f^{-1}). \tag{14}$$

From now on we work in the string frame and thus neglect the subscript "s." Here the asymptotic value of the B field is $\bar{B}_{23}^{\infty}=\tan\theta$ and the parameter R_θ is defined by $\cos\theta R_{\theta}^{4}=R_{0}^{4}(=4\pi gN\alpha'^{2})$. N is the number of the D3 branes and $g=g_\infty$ is the string coupling constant. It is obvious that for $\theta=0\,(h=1)$ one recovers the ordinary D3-brane black hole with the standard $AdS_5\times S^5$ geometry in the near horizon. In this case we have $\bar{F}_{0123r}=(1/g)\partial_r(f^{-1})$, its dual ($\tilde{\bar{F}}_5$), and $e^{2\bar{\Phi}}=g^2$.

For $\theta\to\pi/2\,(h\to f)$, however, one finds the D3-brane black hole in the very large B-field and thus the effect of noncommutativity appears. Here one observes an apparent deviation from $AdS_5\times S^5$ in the near horizon. However, it is known that in order to make a connection to noncommutative geometry, the $\theta\to\pi/2\,(B\to\infty)$ limit must be carefully taken. In addition, we require a double scaling limit of $gN\to\infty,\omega^4\alpha'^2\to 0$ to keep the expansion parameter $\omega^4R_0^4$ very small in the calculation of the absorption cross section. This implies that one has to take both the decoupling limit of $g\to 0,\alpha'\to 0,gN\gg 1$ and the low-energy limit ($\omega\to 0$). Here we wish to take into account all of these limits mentioned the above by taking $\alpha'\to 0$ only:

$$\tan\,\theta=\frac{\tilde b}{\alpha'},\quad g=\alpha'\tilde g,\quad N=\frac{\tilde N}{\alpha'^2}, \tag{15}$$

where $\tilde b,\tilde g,\tilde N$ stay fixed [5]. Then we have the relation

$$\lim_{\theta\to\pi/2}\left(R_\theta^4=\frac{R_0^4}{\cos\,\theta}\right)=4\pi\tilde g\tilde b\tilde N\equiv R_{\pi/2}^4. \tag{16}$$

This means that $\lim_{\theta\to\pi/2}R_\theta^4$ is nearly independent of θ and is finite with $R_{\pi/2}^4\gg R_0^4$. But we must choose the low-energy limit ($\omega\to0$) to keep the new expansion parameter $\tilde\omega^4R_{\pi/2}^4$ small. Under this condition, one finds

$$\bar H_{r23}=\tan\,\theta\partial_r(f^{-1}h)\quad\xrightarrow[\alpha'\to0]{\theta\to\pi/2}\quad\alpha'\to0, \tag{17}$$

$$\bar F_{01r}=\frac{1}{g}\sin\,\theta\partial_r(f^{-1})\quad\xrightarrow[\alpha'\to0]{\theta\to\pi/2}\quad\frac{1}{\alpha'}\to\infty, \tag{18}$$

$$\bar F_{0123r}=\frac{1}{g}\cos\,\theta\partial_r(f^{-1})\quad\xrightarrow[\alpha'\to0]{\theta\to\pi/2}\quad\to\text{finite}. \tag{19}$$

Here one obtains a sequence of the background values: $\bar F_{01r}\gg\bar F_{0123r}\gg\bar H_{r23}$. Although the flux of the RR five-form ($\bar F_5$) counts the rank of the noncommutative gauge group, we have to recognize that this is very small in comparison with the RR three-form ($\bar F_3$) in the limit of $\theta\to\pi/2(B\to\infty)$. Hence we can neglect the effect of $\bar F_5$ and $\bar H$ on the absorption cross section in favor of $\bar F_3$. In this case of $\bar F_5=\bar H=0$, $e^{2\bar\Phi}=g^2f$, $\bar F_{01r}=(1/g)\partial_r(f^{-1})$, one finds the smeared D1-brane solution in the Appendix. We regard this solution as the simple one to include the noncommutative effects through $\tilde R_{\pi/2}^4\gg R_0^4$ in $\tilde f=1+\tilde R_{\pi/2}^4/r^4$. Here $\tilde R_{\pi/2}^4=(1+\epsilon)R_{\pi/2}^4$ with $\epsilon=k^2/\tilde\omega^2=k^2/\omega^2(1-k^2/\omega^2)^{-1}$.

III. PERTURBATIONS

For the perturbation analysis, we keep the background symmetry up to the linearized level. Here we introduce the perturbations to derive the greybody factor as [20]

$$G_{MN}=\bar G_{MN}+h_{MN}, \tag{20}$$

$$\chi=0+\eta, \tag{21}$$

$$\Phi=\bar\Phi+\phi, \tag{22}$$

$$F_{01r}=\bar F_{01r}+f_{01r}=\bar F_{01r}(1+f_3), \tag{23}$$

$$H_{r23}=\bar H_{r23}+h_{r23}=\bar H_{r23}(1+h_3), \tag{24}$$

$$F_{0123r}=\bar F_{0123r}+f_{0123r}=\bar F_{0123r}(1+f_5) \tag{25}$$

with setting all other perturbations to be zero. General fluctuations give us a coupled system of differential equations:

$$\nabla^2\eta-\frac{1}{6}\bar H^2\eta=0, \tag{26}$$

$$-h^{MN}\nabla_M\nabla_N\bar\Phi-\bar G^{MN}\delta\Gamma^P_{MN}\nabla_P\bar\Phi+\nabla^2\phi-4\nabla\bar\Phi\cdot\nabla\phi+2\nabla_M\bar\Phi\nabla_N\bar\Phi h^{MN}+\frac{1}{12}(2\bar H^2h_3-3\bar H_{MNQ}\bar H^{PNQ}h^M{}_P)$$
$$-\frac{e^{2\bar\Phi}}{12}\{2\bar F_3^2(\phi+f_3)-3\bar F_{MNQ}\bar F^{PNQ}h^M{}_P\}=0, \tag{27}$$

$$e^{-2\bar\Phi}\{(\nabla_M-2\nabla_M\bar\Phi)(\bar H^{MNP}h_3)-(\nabla_Mh_Q{}^N)\bar H^{MQP}+(\nabla_Mh^P{}_Q)\bar H^{MQN}-(\nabla_M\hat h^M{}_Q)\bar H^{QNP}-h^M{}_Q\nabla_M\bar H^{QNP}-2(\nabla_M\phi)\bar H^{MNP}\}$$
$$-2\nabla_M(e^{-2\bar\Phi}\bar H^{MNP})\phi-\nabla_M(\bar F^{MPQ}\eta)+\frac{2}{3}\bar F^{PQRST}\bar F_{RST}(f_3+f_5)=0, \tag{28}$$

$$\nabla_M(\bar F^{MNP}f_3)-(\nabla_Mh_Q{}^N)\bar F^{MQP}+(\nabla_Mh^P{}_Q)\bar F^{MQN}-h^M{}_Q\nabla_M\bar F^{QNP}$$
$$-(\nabla_M\hat h^M{}_Q)\bar F^{QNP}-\nabla_M(\bar H^{MPQ}\eta)-\frac{2}{3}\bar F^{PQRST}\bar H_{RST}(h_3+f_5)=0, \tag{29}$$

$$\nabla_M(\bar F^{MNPQR}f_5)-4\bar F^{MT[PQR}\nabla_Mh_T{}^{N]}-(\nabla_M\hat h^M{}_T)\bar F^{TNPQR}-h^M{}_T\nabla_M\bar F^{TNPQR}=0 \tag{30}$$

with $\delta\Gamma^P_{MN}=\frac{1}{2}\bar G^{PQ}(\nabla_Mh_{NQ}+\nabla_Nh_{MQ}-\nabla_Qh_{MN})$. Here we have a relation of $\bar G^{MN}\delta\Gamma^P_{MN}=\nabla_M\hat h^{MP}$, $\hat h^{MP}=h^{MP}-\frac{1}{2}\bar G^{MP}h$ with $h=h^T{}_T$. As a simple check, let us calculate the order of g in each equation. To obtain all consistent linearized equations, we should scale η in Eqs. (26), (28), and (29) as η/g. Furthermore, we find from three Bianchi identities in Eq. (13) with Eqs. (23)–(25) that

$$f_3, f_5 \rightarrow \text{propagating modes,}$$

$$h_3 \rightarrow \text{nonpropagating mode.}$$

This means that the NS B field (H_{MNP}) plays the role of a tool for giving the noncommutative effect but it is not the physically propagating field. For the graviton modes, we may use either the dilaton gauge [21]

$$\nabla_M \hat{h}^{MP} = h^{MN}\Gamma^P_{MN} \tag{31}$$

or the harmonic gauge [22]

$$\nabla_M \hat{h}^{MP} = 0. \tag{32}$$

Although a choice of gauge condition does not eliminate all of the gauge freedom, it simplifies the perturbation equations. We remark that, although Eq. (26) by itself is a decoupled one, Eqs. (28) and (29) contain η. Kaya considered Eq. (26) only in Ref. [18]. The dilaton equation (27) takes a very complicated form which is coupled with various other fields. To decouple ϕ from the other fields, we need some further work. Hence we separate the RR scalar from the dilaton. Let us first investigate the RR scalar.

IV. RR SCALAR PROPAGATION

Because the RR scalar equation (26) is completely decoupled from others, we start with an arbitrary $B(\theta)$. A simple way to obtain the noncommutative effect is to include the momenta along the world volume directions [5]. This is because the B_{23} field is set up along these directions. Hence x_2, x_3 become the noncommuting coordinates. Upon rescaling the coordinates $x_i \rightarrow (\tilde{b}/\alpha')\tilde{x}_i$ and keeping the new coordinates fixed in the limit of $\alpha' \rightarrow 0$, one gets $[\tilde{x}_2, \tilde{x}_3] = i\tilde{b}$. Now let us consider the spacetime dependence

$$\eta(t, x_1, x_2, x_3, r, \theta_i) = e^{-i\omega t} e^{i(k_1x_1 + k_2x_2 + k_3x_3)} Y_l(\theta_1, \theta_2, \ldots, \theta_5)\, \eta^l(r) \tag{33}$$

with $\bar{\nabla}^2_{\theta_i} Y_l(\theta_i) = -l(l+4)Y_l(\theta_i)$. $Y_l(\theta_i)$ denotes spherical harmonics on S^5 with the unit radius. Here $\eta^l(r)$ is the radial part of the lth-partial wave of energy ω. Then Eq. (26) takes the form

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \frac{h'}{h}\frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + (\omega^2 - k_1^2)f - \frac{(k_2^2 + k_3^2)f}{h} - \frac{f'^2 \sin^2\theta \cos^2\theta h^2}{f^3} \right\} \eta^l = 0 \tag{34}$$

with $f' = (d/dr)f$. If $k_1 = k_2 = k_3 = 0$, this is exactly the equation that Kaya has considered in the first version of Ref. [18].

If $\theta = 0$ (B field is turned off) and $l = 0$, one finds that Eq. (34) reduces to the s-wave minimally coupled scalar (φ) equation in the D=7 black hole background [12]

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \tilde{\omega}^2\left(1 + \frac{R_0^4}{r^4}\right) \right\} \varphi^0 = 0 \tag{35}$$

with $\tilde{\omega} = \sqrt{\omega^2 - k_1^2 - k_2^2 - k_3^2} \simeq \omega(1 - k^2/2\omega^2)$, $k^2 = k_1^2 + k_2^2 + k_3^2$, $\omega^2 > k^2$. The s-wave absorption cross section for Eq. (35) can be obtained from the solution to the Mathieu's equation as [13]

$$\sigma_0^{\varphi}|_{B=0} = \frac{\pi^4(\tilde{\omega}R_0)^8}{8\omega^5} \tag{36}$$

in the leading-order calculation. We note here that $\sigma_0^{\eta}|_{B=0} = \sigma_0^{\phi}|_{B=0} = \sigma_0^{\varphi}|_{B=0}$, because both the RR scalar and the dilaton belong to minimally coupled scalars when the B field is absent. For an arbitrary B, the corresponding equation for a minimally coupled field φ is given by

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \tilde{\omega}^2\left(1 + \frac{\tilde{R}_\theta^4}{r^4}\right) \right\} \varphi_B^0 = 0, \tag{37}$$

where $\tilde{R}_\theta^4 = (1+\epsilon)R_\theta^4$ with $\epsilon(\theta) = (k^2/\tilde{\omega}^2)\sin^2\theta < 1$. In the limit of $\theta \rightarrow \pi/2$, one finds $\tilde{R}_{\pi/2}^4 = (1 + k^2/\tilde{\omega}^2)R_{\pi/2}^4$. The above equation is exactly the same form as in Eq. (35) with different "R." Thus the absorption cross section can be read off from Eq. (36) by substituting R_0 with $\tilde{R}_\theta$ [17]

$$\sigma_0^{\varphi}|_B = \sigma_0^{\varphi}|_{B=0}(R_0 \rightarrow \tilde{R}_\theta) = \frac{\pi^4(\tilde{\omega}\tilde{R}_\theta)^8}{8\omega^5}. \tag{38}$$

For an arbitrary B field, one always finds that $\sigma_0^{\varphi}|_{B\neq 0} > \sigma_0^{\varphi}|_{B=0}$ with $\tilde{R}_\theta > R_0$.

In order to transform Eq. (34) into a familiar equation, such as Eq. (37), we redefine η^0 as $\eta^0 = h^{-1/2}\hat{\eta}$. Then this leads to

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \tilde{\omega}^2\left(1 + \frac{\tilde{R}_\theta^4}{r^4}\right) + \frac{4\sin^4\theta R_\theta^8 h^2}{r^{10}f^4} \right\} \hat{\eta} = 0. \tag{39}$$

We can rewrite the last term in Eq. (39) in terms of $\tilde{R}_\theta$, $\tilde{f} = 1 + \tilde{R}_\theta^4/r^4$, $\tilde{h}^{-1} = \sin^2\theta\tilde{f}^{-1} + \cos^2\theta$ as

$$\frac{\sin^4\theta R_\theta^8 h^2}{r^{10}f^4} = \frac{\sin^4\theta \tilde{R}_\theta^8 \tilde{h}^2}{r^{10}\tilde{f}^4}\left\{ 1 - 2\epsilon\left(1 - \frac{\tilde{R}_\theta^4}{r^4 + \tilde{R}_\theta^4} - \frac{\tilde{R}_\theta^4\cos^2\theta}{r^4 + \tilde{R}_\theta^4\cos^2\theta}\right) + \mathcal{O}(\epsilon^2) \right\}. \tag{40}$$

For the leading-order calculation, it is sufficient to keep the first term of the rhs of Eq. (40) only. Using $\hat{\eta} = r^{-5/2}\hat{\hat{\eta}}$, Eq. (39) leads to the Schrödinger-like equation as

$$\left(\frac{\partial^2}{\partial r^2} + \tilde{\omega}^2 - \tilde{V}_\theta \right)\hat{\hat{\eta}} = 0, \tag{41}$$

where

$$\tilde{V}_\theta = -\tilde{\omega}^2(\tilde{f}-1) + \frac{15}{4r^2} - \frac{4\sin^4\theta R_\theta^8 h^2}{r^{10}f^4}. \tag{42}$$

As will be shown in Eq. (65), the first term in Eq. (42) plays the role of an energy term with $E=1$ in the near horizon of $r<R_\theta$. For $r>R_\theta$, the first term can be ignored. Thus we can approximate $\tilde{V}_\theta$ as V_θ

$$V_\theta = \frac{15}{4r^2} - \frac{4\sin^4\theta R_\theta^8 h^2}{r^{10}f^4}. \tag{43}$$

For an arbitrary $\theta(B)$, it is very difficult to solve Eq. (41). Thus, let us discuss two interesting cases. If $\theta \simeq 0$, $h \simeq 1$. In this case, the last term of Eq. (43) can be neglected, compared with the first one. Then the RR scalar cross section takes the same form as that of the minimally coupled scalar in Eq. (37). For $\theta \to \pi/2$, $h \to f$. In this case, the last term of Eq. (43) plays an important role in the near horizon. In the near horizon, one finds that $V^{NH}_{\theta=0} = 15/4r^2$ for $\theta=0$ and $V^{NH}_{\theta\to\pi/2} = -1/4r^2$ for $\theta \to \pi/2$. The latter case induces an instability of the near-horizon geometry in the smeared D1-brane background because the potential well allows us the scattering state ($\omega=$real) as well as the exponentially growing state ($\omega = i\Omega$). Also the same situation is recovered if one uses $\nabla^2\eta=0$ instead of Eq. (26). Hence the instability appears even for $\bar{H}_{MNP}=0$. As is shown in Fig. 1, the singular behaviors of $V_{\theta=0,\pi/2}$ seem to appear as $r\to 0$. However, this is a coordinate artifact. Using the coordinate z in Sec.VI, instead of r, one cannot find the singular behaviors in the near horizon.

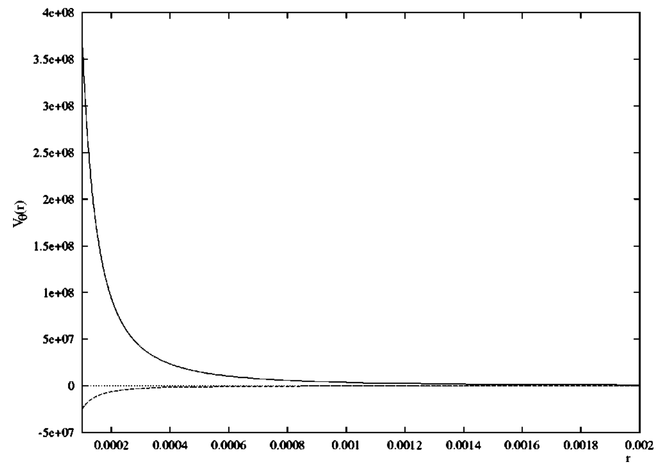

FIG. 1. The graphs of the RR scalar potential in the near horizon. For $\theta \to \pi/2$, one finds a potential well (dashed line) and for $\theta=0$, one finds a potential barrier (solid line). The horizon is located at $r=0$.

V. DILATON PROPAGATION WITH THE DILATON GAUGE

In this section, we wish to study the propagation of the dilaton with the dilaton gauge in Eq. (31). Under this gauge the dilaton equation takes a more simple form than the harmonic gauge [21]. Assuming

$$\phi(t,x_1,x_2,x_3,r,\theta_i) = e^{-i\omega t} e^{i(k_1x_1+k_2x_2+k_3x_3)} Y_l(\theta_1,\theta_2,\ldots,\theta_5)\phi^l(r), \tag{44}$$

the dilaton equation (27) leads to

$$\left\{\frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} - \frac{h'}{h}\frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + (\omega^2-k_1^2)f - \frac{(k_2^2+k_3^2)f}{h} + \frac{16\sin^2\theta\cos^2\theta R_\theta^8 h^2}{r^{10}f^3} + \frac{16\sin^4\theta R^8{}_\theta h^2}{r^{10}f^4}\right\}\phi^l + \frac{16\sin^2\theta\cos^2\theta R_\theta^8 h^2}{r^{10}f^3}\left\{f_3 - \frac{1}{2}(h^0{}_0 + h^1{}_1 + h^2{}_2 + h^3{}_3)\right\} + \frac{16\sin^4\theta R_\theta^8 h^2}{r^{10}f^4}\left\{f_3 - \frac{1}{2}(h^0{}_0 + h^1{}_1 + h^r{}_r)\right\} - \frac{10\sin^2\theta R_\theta^4 h}{r^6 f^2} h^r{}_r = 0. \tag{45}$$

Our strategy is to disentangle the last three terms. For this purpose we have to use the dilaton gauge condition (31) and the linearized equations for H, F_3, F_5 in Eqs. (28)–(30). Because this is a nontrivial task for an arbitrary θ, we only consider a simple and physically interesting case of $\theta \to \pi/2 (B\to\infty)$. For our purpose we choose the smeared D1-brane background with $\bar{H}_{MNP} = \bar{F}_{MNPQR} = 0$, which implies also that $h_3 = f_5 = 0$. Then Eq. (28) with this leads to a crucial equation

$$(\nabla_M \eta)\bar{F}^{MPQ} = 0. \tag{46}$$

Since $\bar{F}^{MPQ} \neq 0$, Eq. (46) implies $\eta=0$. This means that η is a nonpropagating mode in the smeared D1-brane background. In the previous section we are confronted with the instability of the smeared D1-brane background. However, our previous analysis comes from considering solely the linearized equation for the RR scalar. Actually, there exists other equations (28) and (29) which include the RR scalar. In the smeared D1-brane black hole, the relevant equation is Eq. (28). It requires that η satisfy both Eqs. (26) and (28). This leads to $\eta=0$. Hence the instability problem is cured.

The remaining ones are the s-wave dilaton equation and F_3 equation

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+(\omega^2-k_1^2)f-(k_2^2+k_3^2)+\frac{4R^8_{\pi/2}}{r^{10}f^2}\right\}\hat{\phi}-\frac{10R^8_{\pi/2}h^r{}_r}{r^6 f}+\frac{16R^8_{\pi/2}}{r^{10}f^2}\left\{\left(f_3-\frac{1}{2}(h^0{}_0+h^1{}_1+h^r{}_r)\right\}=0, \quad (47)$$

$$\nabla_M f^{MNP}-(\nabla_M h_Q{}^N)\bar{F}^{MQP}+(\nabla_M h^P{}_Q)\bar{F}^{MQN}-(\nabla_M\hat{h}^M{}_Q)\bar{F}^{QNP}-h^M{}_Q(\nabla_M\bar{F}^{QNP})=0. \quad (48)$$

Equation (47) is derived from Eq. (45) with $\phi^0=h^{1/2}\hat{\phi}$ and $\theta=\pi/2$. In order to decouple the last term in Eq. (47), we have to use both Eq. (48) and the dilaton gauge Eq. (31). When $N=0,P=1$, solving Eq. (48) leads to [21]

$$\partial_r(f_3-h^0{}_0-h^1{}_1)+\partial_0 h^0{}_r+\partial_1 h^1{}_r+\left(\frac{5}{r}+\frac{f'}{f}\right)h^r{}_r=0. \quad (49)$$

Using the dilaton gauge, the last three terms turn out to be $\partial_r(-h^r{}_r+\frac{1}{2}h)$. Then Eq. (49) gives us a crucial relation

$$f_3-\frac{1}{2}(h^0{}_0+h^1{}_1+h^r{}_r)+\frac{1}{2}(h^2{}_2+h^3{}_3+h^{\theta_i}{}_{\theta_i})=0. \quad (50)$$

We point out that the same relation as in Eq. (50) can be found if one uses the harmonic gauge (32) [20]. For simplicity, we can set $h^{\theta_i}{}_{\theta_i}=0$ and $h^r{}_r=0$. Then, Eq. (47) leads to

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+(\omega^2-k_1^2)f-(k_2^2+k_3^2)+\frac{4R^8_{\pi/2}}{r^{10}f^2}\right\}\hat{\phi} -\frac{8R^8_{\pi/2}}{r^{10}f^2}(h^2{}_2+h^3{}_3)=0. \quad (51)$$

If the last term is absent, Eq. (51) reduces to the RR scalar equation (39) with $\theta=\pi/2$. It is easily proved that, considering Eqs. (21) and (22) only, one finds that the dilaton equation (27) leads to Eq. (39). Hence the presence of the last term is important to distinguish the dilaton from the RR scalar. Without the B field, the fixed scalar λ is given by [14]

$$\lambda=\frac{D-7}{2\beta}\Phi-\frac{1}{2\beta}\log V, \quad (52)$$

where V is the world volume measured in terms of g_{MN}. This implies that a trace of gravitons ($h^a{}_a$) polarized parallel in the world volume plays the role of a fixed scalar. With the B field, we may assume a relation between the dilaton and $h^a{}_a$. However, although we have a simple dilaton equation with the dilaton gauge, one cannot determine the relation between ϕ and $h^2{}_2+h^3{}_3$. This is so because we do not have further information for $h^2{}_2+h^3{}_3$. This can be obtained from the linearized Einstein equation. Hence we have to use the Einstein equation (A2) in the smeared D1-brane background of the Appendix. Using Eqs. (A2) and (A3), one obtains a scalar equation

$$R-4(\nabla\Phi)^2+4\nabla^2\Phi=0. \quad (53)$$

For an s-wave propagation, it is sufficient to consider the scalar equation (53) instead of the tensorlike equation (A2). Its linearized equation takes the form

$$\bar{G}^{MN}\delta R_{MN}(h)-h^{MN}\bar{R}_{MN}-8(\nabla\bar{\Phi})\cdot\nabla\phi-4\nabla_M\bar{\Phi}\nabla_N\bar{\Phi}h^{MN} -4h^{MN}\nabla_M\nabla_N\bar{\Phi}-4\bar{G}^{MN}\delta\Gamma^P_{MN}\nabla_P\bar{\Phi}+4\nabla^2\phi=0 \quad (54)$$

with the Lichnerowitz operator [22]

$$\delta R_{MN}(h)=-\frac{1}{2}\nabla^2 h_{MN}-\frac{1}{2}\nabla_M\nabla_N h+\frac{1}{2}\nabla^P\nabla_N h_{PM} +\frac{1}{2}\nabla^P\nabla_M h_{PN} \quad (55)$$

$$=-\frac{1}{2}\nabla^2 h_{MN}-\bar{R}_{Q(M}h^Q{}_{N)}+\bar{R}_{PMQN}h^{PQ} +\nabla_{(M}\nabla_{|P|}\hat{h}^P{}_{N)}. \quad (56)$$

From Eq. (55) we obtain

$$G^{MN}\delta R_{MN}=-\nabla^2 h+\nabla_P\nabla_N h^{PN}. \quad (57)$$

The last term in Eq. (57) with the dilaton gauge gives rise to a difficult relation for h_{MN} to solve Eq. (54). Hence we would be better to use the harmonic gauge condition (32) to obtain

$$G^{MN}\delta R_{MN}=-\frac{1}{2}\nabla^2 h, \quad (58)$$

which is also recovered from Eq. (56) with Eq. (32).

VI. DILATON PROPAGATION WITH HARMONIC GAUGE

Equation (54) leads to

$$\nabla^2\left(4\phi-\frac{h}{2}\right)-\frac{4f'}{f^{3/2}}\phi'-\frac{1}{f^{5/2}}(2ff''-f'^2)h^r{}_r-\frac{5f'}{2rf^{3/2}}h^{\theta_i}{}_{\theta_i} +\frac{f'^2(h^0{}_0+h^1{}_1-h^2{}_2-h^3{}_3+h^r{}_r-h^{\theta_i}{}_{\theta_i})}{2f^{5/2}}=0. \quad (59)$$

Also the dilaton equation (27) takes the form

$$\nabla^2\phi - \frac{2f'}{f^{3/2}}\phi' - \frac{f''}{2f^{3/2}}h^r{}_r - \frac{2f'}{rf^{3/2}}h^{\theta_i}{}_{\theta_i} + \frac{f'^2\{h^0{}_0+h^1{}_1-5(h^2{}_2+h^3{}_3+h^{\theta_i}{}_{\theta_i})\}}{8f^{5/2}}=0. \quad (60)$$

Choosing Eq. (32) with $h=0$ (D=10 transverse-traceless gauge), one has $h^0{}_0+h^1{}_1=-(h^2{}_2+h^3{}_3)$ with $h^r{}_r=h^{\theta_i}{}_{\theta_i}=0$. Then the above two equations become, respectively,

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+\tilde{\omega}^2\left(1+\frac{\tilde{R}^4_{\pi/2}}{r^4}\right)\right\}\hat{\phi}-\frac{4R^8_{\pi/2}(h^2{}_2+h^3{}_3)}{r^{10}f^2}=0, \quad (61)$$

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+\tilde{\omega}^2\left(1+\frac{\tilde{R}^4_{\pi/2}}{r^4}\right)+\frac{4R^8_{\pi/2}}{r^{10}f^2}\right\}\hat{\phi} - \frac{12R^8_{\pi/2}(h^2{}_2+h^3{}_3)}{r^{10}f^2}=0. \quad (62)$$

The two equations (61) and (62) should be the same. Here we assume $h^2{}_2+h^3{}_3=a\phi$. Then one finds a relation

$$4-12a=-4a, \quad (63)$$

which gives us $a=1/2$. Hence one obtains the correct dilaton equation as

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+\tilde{\omega}^2\left(1+\frac{\tilde{R}^4_{\pi/2}}{r^4}\right)-\frac{2R^8_{\pi/2}}{r^{10}f^2}\right\}\hat{\phi}=0. \quad (64)$$

This can be approximated by using Eq. (40) as

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{5}{r}\frac{\partial}{\partial r}+\tilde{\omega}^2\left(1+\frac{\tilde{R}^4_{\pi/2}}{r^4}\right)-\frac{2\tilde{R}^8_{\pi/2}}{r^{10}\tilde{f}^2}\right\}\hat{\phi}\simeq 0 \quad (65)$$

for the leading-order calculation. Finally, it remains to find an approximate solution to Eq. (65) for low energies ($\tilde{\omega}\to 0$) and derive its absorption cross section. We divide the space into three regions (I,II,III) and then match solutions in them together. In the near horizon region (I) the equation takes the form

$$\left\{\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}+\frac{(\tilde{\omega}\tilde{R}_{\pi/2})^4}{\rho^4}-\frac{2}{\rho^2}\right\}\hat{\phi}_I(\rho)=0, \quad (66)$$

where $\rho=\tilde{\omega}r$. Defining $\rho=(\tilde{\omega}\tilde{R}_{\pi/2})^2/z$ and $\hat{\phi}_I(\rho)=z^{3/2}\hat{\hat{\phi}}_I$, this leads to

$$\left\{\frac{\partial^2}{\partial z^2}+1-\frac{23}{4z^2}\right\}\hat{\hat{\phi}}_I(z)=0, \quad (67)$$

which is nothing but the standard Bessel equation for $\hat{\hat{\phi}}_I(z)=H_{\sqrt{6}}(z)$. The above equation can be interpreted as the Schrödinger-like equation with the energy $E=1$ which is valid for large z (in the near horizon of $r\to 0$). The solution is given by

$$\hat{\phi}_I(z)=z^2H_{\sqrt{6}}(z). \quad (68)$$

In the intermediate zone (II), the $\tilde{\omega}$ term can be ignored. Thus one finds the solution

$$\hat{\phi}_{II}(\rho)=C\left\{\frac{\rho^4}{(\tilde{\omega}\tilde{R}_{\pi/2})^4+\rho^4}\right\}^{1/2\sqrt{3/2}-1/2}. \quad (69)$$

In the far infinity region (III) we have the equation

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{d}{d\rho}+\tilde{\omega}^2\right)\hat{\phi}_{III}(\rho)=0. \quad (70)$$

Its solution is given by

$$\hat{\phi}_{III}(\rho)=D\frac{J_2(\rho)}{\rho^2}. \quad (71)$$

Matching III to II leads to

$$D=8C. \quad (72)$$

Also matching I to II gives

$$C=\frac{2^{\sqrt{6}}}{\pi}\Gamma(\sqrt{6})(\tilde{\omega}\tilde{R}_{\pi/2})^{2-\sqrt{6}}. \quad (73)$$

Considering the ratio of the flux at the horizon ($r=0$) to the incoming flux at infinity leads to the absorption probability as

$$P_\phi=\frac{4}{|D|^2}(\tilde{\omega}\tilde{R}_{\pi/2})^8=\frac{1}{16}\frac{\pi^2}{2^{2\sqrt{6}}}\frac{(\tilde{\omega}\tilde{R}_{\pi/2})^{2\sqrt{6}+4}}{\Gamma(\sqrt{6})^2}. \quad (74)$$

Finally, we obtain the *s*-wave absorption cross section of the dilaton in the limit of $B\to\infty$ as

$$\sigma_0^\phi|_{B\to\infty}=\frac{2^5\pi^2}{\omega^5}P_\phi=\frac{\pi^4}{2^{2\sqrt{6}-1}\Gamma(\sqrt{6})^2}\frac{(\tilde{\omega}\tilde{R}_{\pi/2})^{4+2\sqrt{6}}}{\omega^5} \simeq\frac{\pi^4}{2^{2\sqrt{6}-1}\Gamma(\sqrt{6})^2}\frac{(\tilde{\omega}\tilde{R}_{\pi/2})^{8.9}}{\omega^5}. \quad (75)$$

VII. DISCUSSIONS

First we discuss the propagation of fields in the smeared D1-brane background that can give us the effect of the $B\to\infty$ limit approximately. We have shown that, considering $h=h^r{}_r=h^{\theta_i}{}_{\theta_i}=0$, the dilaton ϕ, f_3, and $h^2{}_2+h^3{}_3=-h^1{}_1-h^2{}_2=\phi/2$ are physically propagating modes whereas the RR scalar η, h_3, and f_5, are nonpropagating modes. Interestingly, it turns out that the absorption cross section of the dilaton in the limit of $B\to\infty$ is given by the replacement of

$R_0 \to \tilde{R}_{\pi/2}$ and $8 \to 8.9$ in Eqs. (36) and (75). The $R_0 \to \tilde{R}_{\pi/2}(8 \to 8.9)$ arise from the presence of the B field (the coupling is changed: minimal one $(B=0) \to$complicated form $(B \to \infty)$). We note that the RR scalar is a propagating mode in the large B field [18]. If one considers the decoupled equation (26) in the smeared D1-brane background of $B \to \infty$ limit, this induces an instability. Explicitly, the RR scalar has a negative potential as shown in Fig. 1. This induces an instability of the near horizon geometry in the smeared D1-brane background. However, considering all of Eqs. (26), (28), and (29) including the RR scalar (η), we find that $\eta = 0$.

For a general analysis, let us consider the following equation with the parameter s upon the diagonalization:

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \tilde{\omega}^2 \left(1 + \frac{\tilde{R}^4_{\pi/2}}{r^4} \right) - \frac{s \tilde{R}^8_{\pi/2}}{r^{10}\tilde{f}^2} \right\} \psi_s = 0. \quad (76)$$

For $s > -4$, its absorption cross section is given by

$$\sigma_0^{\psi_s}\big|_{B\to\infty} = \frac{\pi^4}{2^{2\sqrt{4+s}-1}\Gamma(\sqrt{4+s})^2} \frac{(\tilde{\omega}\tilde{R}_{\pi/2})^{2\sqrt{4+s}+4}}{\omega^5}. \quad (77)$$

For the case of $s = 32$, one finds an interesting cross section

$$\sigma_0^{\psi_{s=32}}\Big|_{B\to\infty} = \frac{\pi^4}{2^{14}\times 15} \frac{(\tilde{\omega}\tilde{R}_{\pi/2})^{16}}{\omega^5}, \quad (78)$$

which is the same order as in $h^a{}_a$ in the absence of a B field and $k^2 = 0$ [14]

$$\sigma_0^{h^a{}_a}\Big|_{B=0} = \frac{\pi^4}{2^{17}\times 3^4} \frac{(\omega R_0)^{16}}{\omega^5}. \quad (79)$$

Hence we expect that the new scalar appearing in the limit of $B \to \infty$ may take a value of $-4 \leqslant s \leqslant 32$. Exceptionally, the RR scalar with $s = -4$ is not allowed for matching procedure, which confirms that it cannot be a propagating mode. The dilaton has $s = 2$ and its absorption cross section is given by Eq. (75). For the $s = 0$ case (minimally coupled scalar), one can recover Eq. (38) with $\theta = \pi/2$ from Eq. (77).

In conclusion, the way to take the $B \to \infty (\theta \to \pi/2)$ limit is a delicate issue. Here for a simple calculation of the absorption cross section, we take only the limit of $\alpha' \to 0$. In this case one finds $\bar{H} \propto \alpha'$, $\bar{F}_3 \propto 1/\alpha'$, $\bar{F}_5 \propto$finite. It is known that $\bar{F}_5$ counts the rank of the noncommutative group. However, if $\bar{F}_5 \neq 0$ and $\bar{H} \neq 0$, we find the complicated coupled equations. Solving these coupled equations is a formidable task. We remind the reader that the fluxes of $\bar{F}_5$ and $\bar{H}$ can be neglected in comparison with that of $\bar{F}_3$. Hence we choose the simple smeared D1-brane background by setting $\bar{F}_5 = \bar{H} = 0$. At first sight, this action seems to be eliminating all connections with the noncommutative effects. However, although we do not count the fluxes of F_5 and H correctly, we still give the effects of the noncommutativity on the absorption cross section through $\tilde{R}^4_{\pi/2} \geqslant R^4_0$ in $f = 1 + \tilde{R}^4_{\pi/2}/r^4$ and the important coupling of F_3. If $\bar{H} \neq 0$ and $\bar{F}_5 \neq 0$, we expect that there will be a change in s: $2 \to s(2 \leqslant s \leqslant 32)$. This is so because the coupling scheme of $\bar{H} \neq 0, \bar{F}_5 \neq 0$ will change "s" eventually.

Finally, we summarize the ways to account for the noncommutative effect on the cross section of the dilaton on the supergravity side. These are $R^4_{\pi/2} \geqslant R^4_0$, $k^2 \neq 0$, and the couplings to all other fields. Here we include the expansion of the parameter $R^4_{\pi/2}(\geqslant R^4_0)$, the presence of momenta along the world volume directions (k_2, k_3) to detect the B_{23} field, and the coupling of F_3 with $H = F_5 = 0$. Analysis for an arbitrary $\theta(B)$ remains unexplored. A similar work on D6 branes with B fields appears in Ref. [23].

ACKNOWLEDGMENTS

The authors would like to thank Sangmin Lee for a helpful discussion. This work was supported by the Brain Korea 21 Program, Ministry of Education, Project No. D-0025.

APPENDIX: THE SMEARED D1-BRANE SOLUTION

In the case of $F_5 = H_3 = \chi$, the string frame action takes the form

$$S_{10}^{\rm SD1} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\Phi} \left\{ R + 4(\nabla\Phi)^2 - \frac{1}{12}F_3^2 \right\} \right], \quad (A1)$$

which leads to the equations of motion

$$R_{MN} = -2\nabla_M \nabla_N \Phi + \frac{1}{4} e^{2\Phi} F_{MPQ} F_N{}^{PQ} - \frac{1}{24} e^{2\Phi} F_3^2 G_{MN}, \quad (A2)$$

$$\nabla^2 \Phi - 2(\nabla\Phi)^2 - \frac{1}{12} e^{2\Phi} F^2 = 0, \quad (A3)$$

$$\nabla_M F_3^{MPQ} = 0. \quad (A4)$$

The smeared D1-brane solution is given by

$$ds^2_{\rm SD1} = f^{-\frac{1}{2}} \{ -dx_0^2 + dx_1^2 + f(dx_2^2 + dx_3^2) \} + f^{1/2}(dr^2 + r^2 d\Omega_5^2),$$

$$f = 1 + \frac{C}{r^4}, \quad e^{2\bar{\Phi}} = g^2 f, \quad \bar{F}_{01r} = \frac{1}{g} \partial_r (f^{-1}). \quad (A5)$$

Here C is an arbitrary constant, but in order to make connection to the noncommutative geometry we have to choose $C = R^4_{\pi/2} = 4\pi \tilde{g}\tilde{b}\tilde{N} \geqslant R^4_0$.

[1] A. Connes and M. Douglas, J. High Energy Phys. **02**, 003 (1998).
[2] M. Douglas and C. Hull, J. High Energy Phys. **02**, 008 (1998).
[3] M. M. Sheikh-Jabbari, Phys. Lett. B **450**, 119 (1999).
[4] A. Hashimoto and N. Itzhaki, Phys. Lett. B **465**, 142 (1999).
[5] J. Maldacena and J. Russo, J. High Energy Phys. **09**, 025 (1999).
[6] N. Seiberg and E. Witten, J. High Energy Phys. **09**, 032 (1999).
[7] D. Bigatti and L. Susskind, Phys. Rev. D (to be published), hep-th/9908056.
[8] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[9] A. Hashimoto and N. Itzhaki, J. High Energy Phys. **12**, 007 (1999).
[10] Y. S. Myung and H. W. Lee, hep-th/9911031; hep-th/9910083.
[11] R. G. Cai and N. Ohta, Phys. Rev. D **61**, 124012 (2000).
[12] S. Gubser, I. Klebanov, and A. Tseytlin, Nucl. Phys. **B499**, 217 (1997); S. Gubser and I. Klebanov, Phys. Lett. B **413**, 41 (1997); S. Gubser, I. Klebanov, and A. Polyakov, *ibid.* **428**, 105 (1998).
[13] S. Gubser and A. Hashimoto, Commun. Math. Phys. **203**, 325 (1999).
[14] S. Gubser, A. Hashimoto, I. Klebanov, and M. Krasnitz, Nucl. Phys. **B526**, 393 (1998).
[15] H. W. Lee and Y. S. Myung, Phys. Rev. D **58**, 104013 (1998); **61**, 024031 (2000); H. W. Lee, N. J. Kim, and Y. S. Myung, Class. Quantum Grav. **17**, 113 (2000).
[16] S. Hyun, Y. Kiem, S. Lee, and C. Y. Lee, Nucl. Phys. **B569**, 262 (2000).
[17] Y. S. Myung, G. Kang, and H. W. Lee, Phys. Rev. D (to be published), hep-th/9911193.
[18] A. Kaya, J. High Energy Phys. **01**, 012 (2000).
[19] S. R. Das, S. K. Rama, and S. P. Trivedi, J. High Energy Phys. **03**, 004 (2000).
[20] H. W. Lee, Y. S. Myung, and J. Y. Kim, J. High Energy Phys. **10**, 014 (1999); Phys. Lett. B **466**, 211 (1999).
[21] H. W. Lee, N. J. Kim, Y. S. Myung, and J. Y. Kim, Phys. Rev. D **57**, 7361 (1998).
[22] R. Gregorgy and R. Laflamme, Phys. Rev. Lett. **70**, 2837 (1993); Nucl. Phys. **B428**, 399 (1994).
[23] M. Alishahiha, H. Ita, and Y. Oz, hep-th/0004011.