Vacua of 5D, $N=2$ gauged Yang-Mills-Einstein-tensor supergravity: Abelian case

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We give a detailed study of the critical points of the potentials of the simplest nontrivial $\mathcal{N}=2$ gauged Yang-Mills-Einstein supergravity theories with tensor multiplets. The scalar field target space of these examples is $SO(1,1)\times SO(2,1)/SO(2)$. The possible gauge groups are $SO(2)\times U(1)_R$ and $SO(1,1)\times U(1)_R$, where $U(1)_R$ is a subgroup of the R-symmetry group $SU(2)_R$, and $SO(2)$ and $SO(1,1)$ are subgroups of the isometry group of the scalar manifold. The scalar potentials of these theories consist of a contribution from the $U(1)_R$ gauging and a contribution that is due to the presence of the tensor fields. We find that the latter contribution can change the form of the supersymmetric extrema from maxima to saddle points. In addition, it leads to novel critical points not present in the corresponding gauged Yang-Mills-Einstein supergravity theories *without* the tensor multiplets. For the $SO(2)\times U(1)_R$ gauged theory these novel critical points correspond to anti–de Sitter ground states. For the noncompact $SO(1,1)\times U(1)_R$ gauging, the novel ground states are de Sitter ground states. The analysis of the critical points of the potential carries over in a straightforward manner to the generic family of $\mathcal{N}=2$ gauged Yang-Mills-Einstein supergravity theories with tensor multiplets whose scalar manifolds are of the form $SO(1,1)\times SO(n-1,1)/SO(n-1)$.

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I. INTRODUCTION

In the last few years there has been a renewed intense interest in gauged supergravity theories. The work on AdS/CFT (anti–de Sitter/conformal field theory) dualities in recent years has reaffirmed the importance of gauged supergravity theories in various dimensions to the understanding of the dynamics of M or superstring theory [1–5]. The best studied example of this duality is between the type IIB superstring theory on the background manifold $AdS_5\times S^5$ with N units of five-form flux through the five-sphere and 4D, $\mathcal{N}=4$ super Yang-Mills theory with the gauge group $SU(N)$, which is a conformally invariant quantum field theory. In the limit of small string coupling and large N, the classical (i.e., tree level) type IIB supergravity approximation becomes valid. The lowest lying Kaluza-Klein modes of type IIB supergravity on $AdS_5\times S^5$ are believed to form a consistent nonlinear truncation[1] [6,7] which is described by five-dimensional $\mathcal{N}=8$ gauged supergravity [9–11]. Many aspects of the AdS/CFT correspondence, such as, e.g., renormalization group (RG) flows [12,13], can therefore be studied entirely within the framework of 5D gauged supergravity due to the lack of interference with the higher Kaluza-Klein modes.

On the other hand, five-dimensional, $\mathcal{N}=2$ gauged supergravity theories naturally occur as effective field theories in certain brane world scenarios based on heterotic M-theory compactifications [14–17]. Since gauged supergravity theories typically also allow for AdS ground states, they have recently been discussed as a potential framework for embedding the Randall/Sundrum scenario [18] into M or string theory.

Several attempts in this direction have been made (see, e.g., Refs. [19–24].) Many of them focused on what we will later call $\mathcal{N}=2$ ''gauged Maxwell-Einstein theories'' [19–23]. It was found, however, that the scalar potentials of these theories are not of the right form to admit a supersymmetric embedding of Randall-Sundrum-type models [21–23]. The question of whether this is a generic feature of all gauged supergravity theories provides one of the motivations to study the potentials of more general gauged supergravity theories in five dimensions.

Recently, we have constructed the general gaugings of 5D, $\mathcal{N}=2$ supergravity coupled to vector as well as *tensor* multiplets [25]. This was an extension and generalization of earlier work on the gaugings of $\mathcal{N}=2$ supergravity coupled to vector multiplets only [26–30].

Starting point of our construction were the ungauged Maxwell-Einstein supergravity theories (MESGT's) of Ref. [26], which describe the coupling of Abelian vector multiplets to supergravity. These theories have a global symmetry group of the form $SU(2)_R\times G$, where G is the subgroup of the isometry group of the scalar field target manifold that extends to a global symmetry group of the full Lagrangian,

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[1]The consistency of the nonlinear truncation for a subsector of the scalar manifold has been shown recently [8].

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and $SU(2)_R$ denotes the R-symmetry group of the $\mathcal{N}=2$ supersymmetry algebra. In general, there are various ways to turn a subgroup of $SU(2)_R\times G$ into a local gauge group. We will use different names for these different possibilities [27,25]. We refer to theories in which $U(1)_R\subset SU(2)_R$ is gauged as ''gauged Maxwell-Einstein supergravity theories.'' In order to gauge a subgroup K of G, a subset of the vector fields of the ungauged theory has to transform in the adjoint representation of K. If such a group K exists, there are two possibilities.

(i) There are additional vector fields outside the adjoint of K which transform nontrivially under K. These vector fields have to be dualized to ''self-dual'' antisymmetric tensor fields in order to perform the gauging of K in a supersymmetric way [25].[2]

(ii) If there are no vector fields outside the adjoint of K, or if the additional vectors are all singlets under K (''spectator vector fields''), the gauging of K proceeds in a straightforward way, and no tensor fields have to be introduced [27].

In order to distinguish between gaugings of $U(1)_R$ and K, we will refer to theories in which K is gauged as ''Yang-Mills-Einstein supergravity theories'' [''with or without tensor fields,'' depending on which of the possibilities (i) or (ii) is realized].[3]

The most general gauging in this framework is then obviously a simultaneous gauging of $U(1)_R$ and K. For consistency with our terminology, we will sometimes use the term ''gauged Yang-Mills-Einstein supergravity theories (with or without tensor multiplets)'' for this type of gauging.

As for the scalar potentials that are introduced by these different types of gaugings, one makes the following observation [27,25]: (i) The gauging of $U(1)_R$ introduces a scalar potential, which in all known cases (a) either has a maximum that corresponds to an anti–de Sitter space, (b) vanishes identically, or (c) has no critical points at all, (ii) the gauging of K introduces no potential when no vector fields have to be dualized to tensor fields; (iii) if tensor fields have to be introduced, the gauging of K introduces a scalar potential which is positive semidefinite and can therefore not lead to AdS vacua; (iv) the simultaneous gauging of $U(1)_R$ and K leads to a scalar potential which is simply the sum of the potentials that would result from the gaugings of $U(1)_R$ and K alone. The critical points of this combined potential have not yet been fully investigated.

The purpose of this paper is to give an explicit example of a gauged Yang-Mills-Einstein supergravity theory with tensor fields which is simple enough to admit a complete analysis of its scalar potential. The model we discuss describes the coupling of one vector multiplet and one self-dual tensor multiplet (which contains two real tensor fields) to supergravity. The three scalar fields from the vector-tensor multiplets parametrize the space $\mathcal{M}=SO(1,1)\times SO(2,1)/SO(2)$, and the possible gauge groups are $U(1)_R\times SO(2)$ and $U(1)_R\times SO(1,1)$. We will find that the structure of the resulting scalar potentials is much richer than for gaugings without tensor fields.

The organization of the paper is as follows. Section II briefly summarizes the most general form of a gauged Yang-Mills-Einstein supergravity theory with tensor fields. Section III discusses some general properties of the scalar potentials of these theories. The ungauged MESGT with scalar manifold $\mathcal{M}=SO(1,1)\times SO(2,1)/SO(2)$, its $U(1)_R\times SO(2)$ and $U(1)_R\times SO(1,1)$ gaugings and the resulting scalar potentials are analyzed in Sec. IV, which represents the main part of this paper. Section V discusses the generalization to the scalar manifolds $SO(1,1)\times SO(n-1,1)/SO(n-1)$, and Sec. VI finally ends with some conclusions. An appendix summarizes the ''very special geometry'' of the ungauged $\mathcal{M}=SO(1,1)\times SO(2,1)/SO(2)$ theory.

II. GAUGED YANG-MILLS-EINSTEIN SUPERGRAVITY WITH TENSOR FIELDS

In this section, we briefly review the most relevant features of $\mathcal{N}=2$ gauged Yang-Mills-Einstein supergravity theories coupled to tensor multiplets [25]. Unless otherwise stated, our conventions will coincide with those of Refs. [26,27,25], where further details can be found. In particular, we will use the metric signature $(-++++)$ and impose the ''symplectic'' Majorana condition on all fermionic quantities.

The fields of the $\mathcal{N}=2$ supergravity multiplet are the fünfbein e_μ^m, two gravitini Ψ_μ^i $(i=1,2)$ and a vector field A_μ. An $\mathcal{N}=2$ vector multiplet contains a vector field A_μ, two spin-1/2 fermions λ^i and one real scalar field φ. The fermions of each of these multiplets transform as doublets under the $USp(2)_R\cong SU(2)_R$ R-symmetry group of the $\mathcal{N}=2$ Poincaré superalgebra; all other fields are $SU(2)_R$ inert. A tensor field satisfying a five-dimensional ''self-duality'' condition must necessarily be complex [31]. We choose to work with the real and imaginary parts of the complex tensors. A self-dual $\mathcal{N}=2$ tensor multiplet contains such a pair of tensor fields, four spin-1/2 fermions [i.e., two $SU(2)_R$ doublets] and two scalars.

The general coupling of m self-dual tensor multiplets to $\mathcal{N}=2$ gauged Yang-Mills-Einstein supergravity was given in Ref. [25]. The field content of these theories is

$$\{e_\mu^m,\Psi_\mu^i,A_\mu^I,B_{\mu\nu}^M,\lambda^{i\tilde{a}},\varphi^{\tilde{x}}\},\tag{2.1}$$

where

[2]We should note that the gauging of $\mathcal{N}=8$ Poincaré supergravity in 5D requires the dualization of twelve of the vector fields of the $\mathcal{N}=8$ Poincaré supermultiplet to self-dual tensor fields [9–11] for completely analogous reasons.

[3]We will use the term ''Yang-Mills'' also when K is Abelian (as is the case for our examples in Sec. IV).

$$I=0,1,\ldots,n,$$

$$M=1,2,\ldots,2m,$$

$$\tilde{a}=1,\ldots,\tilde{n},$$

$$\tilde{x}=1,\ldots,\tilde{n},$$

with $\tilde{n}=n+2m$. Note that we have combined the ''graviphoton'' with the n vector fields of the n vector multiplets into a single $(n+1)$-plet of vector fields A_μ^I labeled by the index I. Also, the spinor and scalar fields of the vector and tensor multiplets are combined into $\tilde{n}$ tupels of spinor and scalar fields. The indices $\tilde{a},\tilde{b},\ldots$, and $\tilde{x},\tilde{y},\ldots$, are the flat and curved indices, respectively, of the $\tilde{n}$-dimensional target manifold $\mathcal{M}$ of the scalar fields. The metric, vielbein, and spin connection on $\mathcal{M}$ will be denoted by $g_{\tilde{x}\tilde{y}}$, $f_{\tilde{x}}^{\tilde{a}}$, and $\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}$, respectively. The $\mathrm{SU}(2)_R$ index i is raised and lowered with the antisymmetric metric $\varepsilon_{12}=\varepsilon^{12}=1$ according to

$$X^i=\varepsilon^{ij}X_j\,,\quad X_i=X^j\varepsilon_{ji}\,.$$

The fermions Ψ_μ^i and $\lambda^{i\tilde{a}}$ are $U(1)_R$ charged, whereas the fields $\varphi^{\tilde{x}}$, $\lambda^{i\tilde{a}}$, and $B_{\mu\nu}^M$ carry charge under K.

Denoting the $\mathrm{U}(1)_R$ and K coupling constants by g_R and g, respectively, the $[\mathrm{U}(1)_R\times K]$ gauge covariant derivatives of these fields are as follows (∇ denotes the ordinary space-time covariant derivative)

$$\begin{aligned}
\mathfrak{D}_\mu\Psi_\nu^i &\equiv \nabla_\mu\Psi_\nu^i+g_RV_IA_\mu^I\delta^{ij}\Psi_{\nu j}\,,\\
\mathfrak{D}_\mu\lambda^{i\tilde{a}} &\equiv \nabla_\mu\lambda^{i\tilde{a}}+g_RV_IA_\mu^I\delta^{ij}\lambda_j^{\tilde{a}}+gA_\mu^IL_I^{\tilde{a}\tilde{b}}\lambda^{i\tilde{b}},\\
\mathfrak{D}_\mu\varphi^{\tilde{x}} &\equiv \partial_\mu\varphi^{\tilde{x}}+gA_\mu^IK_I^{\tilde{x}}\,,\\
\mathfrak{D}_\mu B_{\nu\rho}^M &\equiv \nabla_\mu B_{\nu\rho}^M+gA_\mu^I\Lambda_{IN}^MB_{\nu\rho}^N\,.
\end{aligned}\tag{2.2}$$

Here, $K_I^{\tilde{x}}$ are the Killing vector fields on $\mathcal{M}$ that generate the subgroup K of its isometry group. The φ-dependent matrices $L_I^{\tilde{a}\tilde{b}}$ and the *constant* matrices Λ_{IN}^M are the K-transformation matrices of $\lambda^{i\tilde{a}}$ and $B_{\mu\nu}^M$, respectively. The V_I are some constants that define the linear combination of the vector fields A_μ^I that is used as the $\mathrm{U}(1)_R$-gauge field

$$A_\mu[U(1)_R]=V_IA_\mu^I\,.\tag{2.3}$$

They have to be constrained by

$$V_If_{JK}^I=0,\tag{2.4}$$

with f_{JK}^I being the structure constants of K.[4]

We denote the curls of the vector fields A_μ^I by $F_{\mu\nu}^I$. The non-Abelian field strengths $\mathcal{F}_{\mu\nu}^I\equiv F_{\mu\nu}^I+gf_{JK}^IA_\mu^JA_\nu^K$ $(I=0,1,\ldots,n)$ of the gauge group K and the self-dual tensor fields $B_{\mu\nu}^M$ $(M=1,2,\ldots,2m)$ are grouped together to define the tensorial quantity $\mathcal{H}_{\mu\nu}^{\tilde{I}}=(\mathcal{F}_{\mu\nu}^I,B_{\mu\nu}^M)$ with $\tilde{I}=0,1,\ldots,n+2m$.

The Lagrangian is then given by (up to four-fermion terms) [25]

$$\begin{aligned}
e^{-1}\mathcal{L}=&-\frac{1}{2}R(\omega)-\frac{1}{2}\bar{\Psi}_\mu^i\Gamma^{\mu\nu\rho}\mathfrak{D}_\nu\Psi_{\rho i}-\frac{1}{4}\overset{\circ}{a}_{\tilde{I}\tilde{J}}\mathcal{H}_{\mu\nu}^{\tilde{I}}\mathcal{H}^{\tilde{J}\mu\nu}-\frac{1}{2}\overline{\lambda^{i\tilde{a}}}(\Gamma^\mu\mathfrak{D}_\mu\delta^{\tilde{a}\tilde{b}}+\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}\Gamma^\mu\mathfrak{D}_\mu\varphi^{\tilde{x}})\lambda_i^{\tilde{b}}-\frac{1}{2}g_{\tilde{x}\tilde{y}}(\mathfrak{D}_\mu\varphi^{\tilde{x}})(\mathfrak{D}^\mu\varphi^{\tilde{y}})\\
&-\frac{i}{2}\overline{\lambda^{i\tilde{a}}}\Gamma^\mu\Gamma^\nu\Psi_{\mu i}f_{\tilde{x}}^{\tilde{a}}\mathfrak{D}_\nu\varphi^{\tilde{x}}+\frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\overline{\lambda^{i\tilde{a}}}\Gamma^\mu\Gamma^{\lambda\rho}\Psi_{\mu i}\mathcal{H}_{\lambda\rho}^{\tilde{I}}+\frac{i}{2\sqrt{6}}\left(\frac{1}{4}\,\delta_{\tilde{a}\tilde{b}}h_{\tilde{I}}+T_{\tilde{a}\tilde{b}\tilde{c}}h_{\tilde{I}}^{\tilde{c}}\right)\overline{\lambda^{i\tilde{a}}}\Gamma^{\mu\nu}\lambda_i^{\tilde{b}}\mathcal{H}_{\mu\nu}^{\tilde{I}}\\
&-\frac{3i}{8\sqrt{6}}h_{\tilde{I}}[\bar{\Psi}_\mu^i\Gamma^{\mu\nu\rho\sigma}\Psi_{\nu i}\mathcal{H}_{\rho\sigma}^{\tilde{I}}+2\bar{\Psi}^{\mu i}\Psi_i^\nu\mathcal{H}_{\mu\nu}^{\tilde{I}}]+\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}\Big\{F_{\mu\nu}^IF_{\rho\sigma}^JA_\lambda^K+\frac{3}{2}gF_{\mu\nu}^IA_\rho^J(f_{LF}^KA_\sigma^LA_\lambda^F)\\
&+\frac{3}{5}g^2(f_{GH}^JA_\nu^GA_\rho^H)(f_{LF}^KA_\sigma^LA_\lambda^F)A_\mu^I\Big\}+\frac{e^{-1}}{4g}\varepsilon^{\mu\nu\rho\sigma\lambda}\Omega_{MN}B_{\mu\nu}^M\mathfrak{D}_\rho B_{\sigma\lambda}^N+g\overline{\lambda^{i\tilde{a}}}\Gamma^\mu\Psi_{\mu i}W_{\tilde{a}}+g\overline{\lambda^{i\tilde{a}}}\lambda_i^{\tilde{b}}W_{\tilde{a}\tilde{b}}-g^2P\\
&-\frac{i\sqrt{6}}{8}g_R\bar{\Psi}_\mu^i\Gamma^{\mu\nu}\Psi_\nu^j\delta_{ij}P_0-\frac{1}{\sqrt{2}}g_R\overline{\lambda^{i\tilde{a}}}\Gamma^\mu\Psi_\mu^j\delta_{ij}P_{\tilde{a}}+\frac{i}{2\sqrt{6}}g_R\overline{\lambda^{i\tilde{a}}}\lambda^{j\tilde{b}}\delta_{ij}P_{\tilde{a}\tilde{b}}-g_R^2P^{(R)}.
\end{aligned}\tag{2.5}$$

[4]If there are spectator vector fields among the A_μ^I, the corresponding f_{IJ}^K are just zero.

The transformation laws are (to leading order in fermion fields)

$$\delta e_{\mu}^{m}=\frac{1}{2}\overline{\varepsilon^{i}}\Gamma^{m}\Psi_{\mu i},$$

$$\delta\Psi_{\mu}^{i}=\mathfrak{D}_{\mu}\varepsilon^{i}+\frac{i}{4\sqrt{6}}h_{\tilde{I}}(\Gamma_{\mu}^{\nu\rho}-4\delta_{\mu}^{\nu}\Gamma^{\rho})\mathcal{H}_{\nu\rho}^{\tilde{I}}\varepsilon^{i}+\frac{i}{2\sqrt{6}}g_{R}P_{0}\Gamma_{\mu}\delta^{ij}\varepsilon_{j},$$

$$\delta A_{\mu}^{I}=\vartheta_{\mu}^{I},$$

$$\delta B_{\mu\nu}^{M}=2\mathfrak{D}_{[\mu}\vartheta_{\nu]}^{M}+\frac{\sqrt{6}g}{4}\Omega^{MN}h_{N}\overline{\Psi}_{[\mu}^{i}\Gamma_{\nu]}\varepsilon_{i}+\frac{ig}{4}\Omega^{MN}h_{N\tilde{a}}\overline{\lambda^{i\tilde{a}}}\Gamma_{\mu\nu}\varepsilon_{i},$$

$$\delta\lambda^{i\tilde{a}}=-\frac{i}{2}f_{\tilde{x}}^{\tilde{a}}\Gamma^{\mu}(\mathfrak{D}_{\mu}\varphi^{\tilde{x}})\varepsilon^{i}+\frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\Gamma^{\mu\nu}\varepsilon^{i}\mathcal{H}_{\mu\nu}^{\tilde{I}}+gW^{\tilde{a}}\varepsilon^{i}+\frac{1}{\sqrt{2}}g_{R}P^{\tilde{a}}\delta^{ij}\varepsilon_{j},$$

$$\delta\varphi^{\tilde{x}}=\frac{i}{2}f_{\tilde{a}}^{\tilde{x}}\overline{\varepsilon^{i}}\lambda_{i}^{\tilde{a}}, \tag{2.6}$$

with

$$\vartheta_{\mu}^{\tilde{I}}\equiv-\frac{1}{2}h_{\tilde{a}}^{\tilde{I}}\overline{\varepsilon^{i}}\Gamma_{\mu}\lambda_{i}^{\tilde{a}}+\frac{i\sqrt{6}}{4}h^{\tilde{I}}\overline{\Psi}_{\mu}^{i}\varepsilon_{i}. \tag{2.7}$$

The various scalar field dependent quantities $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$, $h_{\tilde{I}}$, $h^{\tilde{I}}$, $h_{\tilde{I}}^{\tilde{a}}$, $h^{\tilde{I}\tilde{a}}$, and $T_{\tilde{a}\tilde{b}\tilde{c}}$ that contract the different types of indices are already present in the corresponding *ungauged* MESGT's and describe the ''very special''geometry of the scalar manifold $\mathcal{M}$ (see Ref. [26] for details). The ungauged MESGT's also contain a constant symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$. If the gauging of K involves the introduction of tensor fields, the coefficients of the type C_{MNP} and C_{IJM} have to vanish [25]. The only components that survive such a gauging are thus C_{IJK}, which appear in the Chern-Simons-like term of Eq. (2.5), and C_{IMN}, which are related to the transformation matrices of the tensor fields by

$$\Lambda_{IN}^{M}=\frac{2}{\sqrt{6}}\Omega^{MP}C_{IPN}.$$

Here Ω^{MN} is the inverse of Ω_{MN}, which is a (constant) invariant antisymmetric tensor of the gauge group K:

$$\Omega_{MN}=-\Omega_{NM},\quad \Omega_{MN}\Omega^{NP}=\delta_{M}^{P}. \tag{2.8}$$

The terms proportional to

$$W^{\tilde{a}}(\varphi)=-\frac{\sqrt{6}}{8}h_{M}^{\tilde{a}}\Omega^{MN}h_{N},$$

$$W^{\tilde{a}\tilde{b}}(\varphi)=-W^{\tilde{b}\tilde{a}}(\varphi)=ih^{J[\tilde{a}}K_{J}^{\tilde{b}]}+\frac{i\sqrt{6}}{4}h^{J}K_{J}^{[\tilde{a};\tilde{b}]} \tag{2.9}$$

(the semicolon denotes covariant differentiation on the target space $\mathcal{M}$) and the potential term

$$P(\varphi)=2W_{\tilde{a}}W^{\tilde{a}} \tag{2.10}$$

are due to the presence of the tensor fields.

The supersymmetric gauging of the $U(1)_R$ factor, on the other hand, introduces the terms proportional to

$$P^{\tilde{a}}(\varphi)=\sqrt{2}h^{\tilde{a}I}V_{I}, \tag{2.11}$$

$$P_{0}(\varphi)=2h^{I}V_{I}, \tag{2.12}$$

$$P_{\tilde{a}\tilde{b}}(\varphi)=\frac{1}{2}\delta_{\tilde{a}\tilde{b}}P_{0}+2\sqrt{2}T_{\tilde{a}\tilde{b}\tilde{c}}P^{\tilde{c}} \tag{2.13}$$

in Eqs. (2.5) and (2.6) and leads to the scalar potential contribution

$$P^{(R)}(\varphi)=-(P_{0})^{2}+P_{\tilde{a}}P^{\tilde{a}}. \tag{2.14}$$

III. SOME GENERAL PROPERTIES OF THE SCALAR POTENTIAL

As summarized in the previous section, the simultaneous gauging of $U(1)_R\subset SU(2)_R$ and a subgroup $K\subset G$ of the isometry group G of the vector-tensor multiplets moduli space $\mathcal{M}$ leads to a scalar potential of the form

$$e^{-1}\mathcal{L}_{pot}=-g^{2}P-g_{R}^{2}P^{(R)}, \tag{3.1}$$

where $P^{(R)}$ arises from the gauging of $U(1)_R$, whereas P is nonzero if and only if some K-charged vector fields A_{μ}^{M} had to be dualized to tensor fields $B_{\mu\nu}^{M}$ in order to perform the gauging of K in a supersymmetric way. In the remainder we will write

$$P_{\rm tot}:=P+\lambda P^{(R)},\quad \text{with } \lambda:=\frac{g_{R}^{2}}{g^{2}} \tag{3.2}$$

so that

$$e^{-1}\mathcal{L}_{\rm pot}=-g^{2}P_{\rm tot}. \tag{3.3}$$

The potentials P and $P^{(R)}$ are given by

$$P=2W_{\tilde{a}}W^{\tilde{a}}, \tag{3.4}$$

$$P^{(R)}=-(P_{0})^{2}+P_{\tilde{a}}P^{\tilde{a}}.$$

Using $h^{\tilde I}_{\tilde a}h^{\tilde a}_{\tilde J}=\delta^{\tilde I}_{\tilde J}-h^{\tilde I}h_{\tilde J}$ [26], it is easy to verify that $W^{\tilde a}$ and $P^{\tilde a}$ are orthogonal:

$$W_{\tilde a}P^{\tilde a}=0.$$

Contracting $\langle\delta\lambda^{i\tilde a}\rangle=0$ with $W^{\tilde a}$ and $P^{\tilde a}$ then shows that an $\mathcal{N}=2$ supersymmetric ground state requires

$$\langle W^{\tilde a}\rangle=\langle P^{\tilde a}\rangle=0. \tag{3.5}$$

This implies, in particular, that the cosmological constant of an $\mathcal{N}=2$ supersymmetric vacuum is given by $P^{(R)}(\varphi_c)$ alone, i.e., $P(\varphi_{c,\mathrm{SUSY}})=0$, as has also been pointed out in Ref. [22]. Nevertheless, P can still have a nontrivial effect on the *form* of a supersymmetric critical point, i.e., it can change it from a maximum to a saddle point. In addition, there might be critical points which do not preserve the full $\mathcal{N}=2$ supersymmetry and therefore *can* have $P(\varphi_c)\neq 0$. We will see examples for all this in the next section.

Using [26]

$$C_{\tilde I\tilde J\tilde K}h^{\tilde K}=h_{\tilde I}h_{\tilde J}-\frac{1}{2}h_{\tilde I\tilde a}h^{\tilde a}_{\tilde J},$$

P and $P^{(R)}$ can be expressed in a more compact form which will facilitate the analysis of the critical points:

$$P=\frac{3\sqrt{6}}{16}h^I\Lambda_I^{MN}h_Mh_N, \tag{3.6}$$

$$P^{(R)}=-4C^{IJ\tilde K}V_IV_Jh_{\tilde K}, \tag{3.7}$$

where we have defined

$$\Lambda_I^{MN}\equiv\Lambda_{IP}^{M}\Omega^{PN}=\frac{2}{\sqrt{6}}\Omega^{MR}C_{IRP}\Omega^{PN}, \tag{3.8}$$

$$C^{\tilde I\tilde J\tilde K}\equiv\mathring{a}^{\tilde I\tilde I'}\mathring{a}^{\tilde J\tilde J'}\mathring{a}^{\tilde K\tilde K'}C_{\tilde I'\tilde J'\tilde K'} \tag{3.9}$$

with $\mathring{a}^{\tilde I\tilde J}$ being the inverse of $\mathring{a}_{\tilde I\tilde J}$.

If $\mathcal{M}$ is associated with a Jordan algebra[5] [26], one has (componentwise)

$$C^{\tilde I\tilde J\tilde K}=C_{\tilde I\tilde J\tilde K}=\text{const.}$$

In this case, because of $C^{IJM}=C_{IJM}=0$, $P^{(R)}$ simplifies to

$$P^{(R)}=-4C^{IJK}V_IV_Jh_K \quad \text{(for the Jordan family)} \tag{3.10}$$

with constant $C^{IJK}=C_{IJK}$ and summation over K instead of $\tilde K$.

[5]We recall that the MESGT's associated with Jordan algebras are those for which the cubic form defined by the symmetric tensor $C_{\tilde I\tilde J\tilde K}$ can be identified with the norm form of a Jordan algebra of degree 3.

The critical points of $P^{(R)}$ have been analyzed in Ref. [27] for the purely $\mathrm{U}(1)_R$-gauged Maxwell-Einstein supergravity theories (MESGTs) of the Jordan type. It was found that they are characterized by the ''dual'' element

$$V^{\#\tilde I}\equiv\sqrt{\frac{2}{3}}C^{\tilde I\tilde J\tilde K}V_{\tilde J}V_{\tilde K} \tag{3.11}$$

of $V_{\tilde I}$. Three cases could be distinguished. (i) $V^{\#\tilde I}=0$. In this case, the scalar potential $P^{(R)}$ vanishes identically, leading to Minkowski ground states with broken supersymmetry. (ii) $V^{\#\tilde I}$ is in the ''domain of positivity'' of the corresponding Jordan algebra J. In this case, there exists precisely one critical point, which sits at the unique global maximum of the scalar potential $P^{(R)}$ and corresponds to an anti–de Sitter ground state with unbroken $\mathcal{N}=2$ supersymmetry and unbroken global $\mathrm{Aut}(J)$-invariance, where $\mathrm{Aut}(J)$ denotes the automorphism group of the Jordan algebra J. (iii) $V^{\#\tilde I}$ is nonzero and not in the domain of positivity of J. In this case, the scalar potential $P^{(R)}$ has no critical points at all. In order to get a better understanding as to whether and how the presence of the tensor field related potential P changes this picture, we will analyze the simplest nontrivial example of a gauged Yang-Mills-Einstein supergravity theory with tensor multiplets in full detail in the next section.

IV. THE SIMPLEST NONTRIVIAL EXAMPLE: $\mathcal{M}=\mathrm{SO}(1,1)\times\mathrm{SO}(2,1)/\mathrm{SO}(2)$

A. The ungauged theory

The *ungauged* MESGT with the scalar manifold $\mathcal{M}=\mathrm{SO}(1,1)\times\mathrm{SO}(2,1)/\mathrm{SO}(2)$ allows the construction of two of the simplest non-trivial examples of a gauged Yang-Mills-Einstein supergravity theory with tensor multiplets. Let us consider this *ungauged* theory first. It belongs to the generic Jordan family[6] and describes the coupling of three Abelian vector multiplets to supergravity. Consequently, the field content is

$$\{e_\mu^m,\Psi_\mu^i,A_\mu^{\tilde I},\lambda^{i\tilde a},\varphi^{\tilde x}\} \tag{4.1}$$

with

$$i=1,2,$$

$$\tilde I=0,1,\ldots,3,$$

$$\tilde a=1,\ldots,3,$$

$$\tilde x=1,\ldots,3,$$

where the three scalar fields $\varphi^{\tilde x}$ parametrize the target space $\mathcal{M}=\mathrm{SO}(1,1)\times\mathrm{SO}(2,1)/\mathrm{SO}(2)$. The latter can be described as the hypersurface

[6]The ''generic Jordan family'' consists of the MESGT's with scalar manifolds of the form $\mathcal{M}=\mathrm{SO}(1,1)\times\mathrm{SO}(n-1,1)/\mathrm{SO}(n-1)$.

$$N(\xi)=\left(\frac{2}{3}\right)^{3/2}C_{\tilde{I}\tilde{J}\tilde{K}}\xi^{\tilde{I}}\xi^{\tilde{J}}\xi^{\tilde{K}}=1$$

in a four-dimensional ambient vector space parametrized by coordinates $\xi^{\tilde{I}}$. In the case at hand, this vector space can be identified with the Jordan algebra

$$J=\mathbb{R}\oplus\Sigma_3,$$

where Σ_3 is the Jordan algebra of degree two corresponding to a quadratic form Q with signature $(+,-,-)$ [26]. In the most natural basis of this Jordan algebra, $N(\xi)$ takes on the following form:

$$N(\xi)=\sqrt{2}\xi^0[(\xi^1)^2-(\xi^2)^2-(\xi^3)^2],$$

where the normalization factor $\sqrt{2}$ ensures that the unique selfdual point $\xi^{\tilde{I}}=\xi^{\#\tilde{I}}$ (i.e. the ''base point'' $c^{\tilde{I}}$ of the Jordan algebra [26]) really lies on the hypersurface $N(\xi)=1$, or equivalently, that there is a point on $\mathcal{M}$ where $\overset{\circ}{a}_{\tilde{I}\tilde{J}}=\delta_{\tilde{I}\tilde{J}}$ [26].[7]

Hence, the nonvanishing $C_{\tilde{I}\tilde{J}\tilde{K}}$ are

$$C_{011}=\frac{\sqrt{3}}{2},$$

$$C_{022}=C_{033}=-\frac{\sqrt{3}}{2}. \tag{4.2}$$

The constraint $N=1$ can be solved by

$$\xi^0=\frac{1}{\sqrt{2}\|\varphi\|^2}, \tag{4.3}$$

$$\xi^1=\varphi^1, \tag{4.4}$$

$$\xi^2=\varphi^2, \tag{4.5}$$

$$\xi^3=\varphi^3 \tag{4.6}$$

with

$$\|\varphi\|^2\equiv(\varphi^1)^2-(\varphi^2)^2-(\varphi^3)^2.$$

Obviously, the hypersurface $N=1$ decomposes into three disconnected components: (i) $\|\varphi\|^2>0$ and $\varphi^1>0$, (ii) $\|\varphi\|^2<0$, (iii) $\|\varphi\|^2>0$ and $\varphi^1<0$. In the following, we will consider the ''positive timelike'' region (i) only, since in region (ii), $g_{\tilde{x}\tilde{y}}$ and $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ are not positive definite (see the Appendix), and region (iii) is isomorphic to region (i). All the scalar field dependent quantities in the Lagrangian and the supersymmetry transformation laws can be derived from $N(\xi)$, and they are listed in the Appendix.

[7]In our parametrization, $c^{\tilde{I}}=(1/\sqrt{2},1,0,0)$, which corresponds to $\varphi^{\tilde{x}}=(1,0,0)$ (see the Appendix).

B. The $U(1)_R\times SO(2)$ gauging

We will now turn the above ungauged SO(1,1)×SO(2,1)/SO(2) model into a gauged Yang-Mills-Einstein supergravity theory with tensor fields. The isometry group of the scalar manifold $\mathcal{M}$ is $G=SO(2,1)\times SO(1,1)$, which is simply the invariance group of $N(\xi)$. There are now two different ways to construct a Yang-Mills-Einstein supergravity theory with tensor multiplets: Either one gauges the compact subgroup $SO(2)\subset SO(2,1)$, or one gauges the noncompact subgroup $SO(1,1)\subset SO(2,1)$. We will focus on the compact gauging first and discuss the noncompact SO(1,1) gauging in the next subsection. The SO(2) subgroup of SO(2,1) rotates ξ^2 and ξ^3 into each other and therefore acts nontrivially on the vector fields A_μ^2 and A_μ^3. Hence, gauging this SO(2) requires the dualization of A_μ^2 and A_μ^3 to antisymmetric tensor fields. Accordingly, we decompose the index $\tilde{I}$ as follows:

$$\tilde{I}=(I,M)$$

with $I,J,K,\ldots=0,1$ and $M,N,P,\ldots=2,3$.

It is easy to verify that our $C_{\tilde{I}\tilde{J}\tilde{K}}$ in Eqs. (4.2) are consistent with the requirements $C_{IJM}=C_{MNP}=0$ for this type of gauging. Having a closer look at the $C_{\tilde{I}\tilde{J}\tilde{K}}$ of the type C_{IMN} we also see that C_{1MN} is zero, whereas C_{0MN} is nonvanishing. This means, because of $\Lambda_{IN}^M\sim\Omega^{MP}C_{IPN}$, that the vector field A_μ^0 plays the role of the SO(2)-gauge field, whereas A_μ^1 is just a ''spectator vector field'' with respect to the SO(2) gauging.

In addition to this SO(2) gauging, one can now use a linear combination $A_\mu[U(1)_R]=A_\mu^I V_I$ of the vector fields A_μ^0 and A_μ^1 as the $U(1)_R$-gauge field, and simultaneously gauge $U(1)_R$ and SO(2). The result is a *gauged* Yang-Mills-Einstein supergravity theory with tensor fields with the full gauge group $U(1)_R\times SO(2)$.

In our parametrization, the resulting potentials P and $P^{(R)}$ [see Eqs. (3.6), (3.10) and the Appendix] are found to be[8]

$$P=\frac{1}{8}\frac{[(\varphi^2)^2+(\varphi^3)^2]}{\|\varphi\|^6}, \tag{4.7}$$

$$P^{(R)}=-2\left[2\sqrt{2}\frac{\varphi^1}{\|\varphi\|^2}V_0V_1+\|\varphi\|^2(V_1)^2\right]. \tag{4.8}$$

For the functions $W_{\tilde{x}}$, $P_{\tilde{x}}$, and P_0 that enter the supersymmetry transformation laws of the fermions, one obtains

$$W_1=0, \tag{4.9}$$

$$W_2=\frac{1}{4}\frac{\varphi^3}{\|\varphi\|^4}, \tag{4.10}$$

[8]We are choosing $\Omega^{23}=-\Omega^{32}=-1$.

$$W_3=-\frac{1}{4}\frac{\varphi^2}{\|\varphi\|^4}, \tag{4.11}$$

respectively,

$$P_1=\sqrt{2}\left(\sqrt{2}\frac{\varphi^1}{\|\varphi\|^4}V_0-V_1\right), \tag{4.12}$$

$$P_2=-2\frac{\varphi^2}{\|\varphi\|^4}V_0, \tag{4.13}$$

$$P_3=-2\frac{\varphi^3}{\|\varphi\|^4}V_0, \tag{4.14}$$

and

$$P_0=\frac{2}{\sqrt{3}}\left(\frac{V_0}{\|\varphi\|^2}+\sqrt{2}\varphi^1V_1\right). \tag{4.15}$$

This shows that the necessary condition for an $\mathcal{N}=2$ supersymmetric critical point, $W_{\tilde{x}}(\varphi_c)=P_{\tilde{x}}(\varphi_c)=0$, is equivalent to

$$\langle\varphi^2\rangle=\langle\varphi^3\rangle=0, \tag{4.16}$$

$$\langle\varphi^1\rangle^3V_1=\sqrt{2}V_0. \tag{4.17}$$

Let us now analyze the critical points of the above scalar potentials. We will first investigate the critical points of P and $P^{(R)}$ separately and then consider the combined potential $P_{\rm tot}=P+\lambda P^{(R)}$.

The critical points of P. Taking the deriative of $P(\varphi)$ with respect to $\varphi^{\tilde{x}}$, one finds

$$P_{,1}=-\frac{3}{4}\frac{[(\varphi^2)^2+(\varphi^3)^2]}{\|\varphi\|^8}\varphi^1, \tag{4.18}$$

$$=-A\varphi^1+\frac{\varphi^1}{4\|\varphi\|^6}, \tag{4.19}$$

$$P_{,2}=A\varphi^2, \tag{4.20}$$

$$P_{,3}=A\varphi^3, \tag{4.21}$$

where

$$A\equiv\frac{3}{4}\frac{[(\varphi^2)^2+(\varphi^3)^2]}{\|\varphi\|^8}+\frac{1}{4\|\varphi\|^6}$$

has been introduced. There are now two possibilities.

Case 1: $A\neq0$. Then $P_{,2}=P_{,3}=0$ implies $\varphi_c^2=\varphi_c^3=0$ (which then also implies $P_{,1}=0$). But then $P(\varphi_c)=0$, and we have a Minkowski ground state, which, because of $W_{\tilde{x}}(\varphi_c)=0$, preserves the full $\mathcal{N}=2$ supersymmetry [as long as the $U(1)_R$ gauging is turned off].

Case 2: $A=0$. Then $P_{,1}=0$ implies $\varphi^1/4\|\varphi\|^6=0$, which is inconsistent with $\varphi^1>0$ and $\|\varphi\|^2>0$.

Summary for P. There exists a one parameter family of $\mathcal{N}=2$ supersymmetric Minkowski ground states, given by $\langle\varphi^2\rangle=\langle\varphi^3\rangle=0$ and arbitrary $\langle\varphi^1\rangle>0$. These vacua also preserve the SO(2)-gauge invariance. There are no other critical points.

The critical points of $P^{(R)}$. The gradient of $P^{(R)}$ is

$$P^{(R)}_{,1}=-B\varphi^1-\frac{4\sqrt{2}V_0V_1}{\|\varphi\|^2}, \tag{4.22}$$

$$P^{(R)}_{,2}=B\varphi^2, \tag{4.23}$$

$$P^{(R)}_{,3}=B\varphi^3, \tag{4.24}$$

where

$$B\equiv-8\sqrt{2}\frac{\varphi^1}{\|\varphi\|^4}V_0V_1+4(V_1)^2.$$

There are now two possibilities.

Case 1: $B=0$. $P^{(R)}_{,1}=0$ then requires $V_0V_1=0$. Thus either V_0 or V_1 (or both of them) have to be zero. If $V_0=0$, $B=0$ implies $V_1=0$. Thus, $B=0$ automatically implies $V_1=0$, and the potential $P^{(R)}$ vanishes identically [see Eq. (4.8)] resulting in a Minkowski vacuum. The $U(1)_R$ gauging is nontrivial only when at least one V_I is nonzero. Since $V_1=0$ in the case at hand, a nontrivial $U(1)_R$ gauging requires $V_0\neq0$, implying $P_1\neq0$, i.e., broken supersymmetry.

Case 2: $B\neq0$. The vanishing of $P^{(R)}_{,2}$ and $P^{(R)}_{,3}$ then requires $\langle\varphi^2\rangle=\langle\varphi^3\rangle=0$, i.e., $\langle\|\varphi\|^2\rangle=\langle\varphi^1\rangle^2$. Because $P^{(R)}_{,1}$ has to vanish, this implies $\langle\varphi^1\rangle^3(V_1)^2=\sqrt{2}V_0V_1$. Thus there are two possibilities: Either $V_1=0$, or V_0 and V_1 are both nonvanishing. The former case leads us back to the case of identically vanishing potential $P^{(R)}\equiv0$. The second possibility leads to a critical point with $\langle\varphi^2\rangle=\langle\varphi^3\rangle=0$ and

$$\langle\varphi^1\rangle^3=\sqrt{2}\,\frac{V_0}{V_1} \tag{4.25}$$

whenever $V_0V_1>0$ (since $\varphi^1>0$). It is easy to see that this critical point satisfies the necessary conditions (4.16), (4.17) for $\mathcal{N}=2$ supersymmetry. The value of the potential $P^{(R)}$ at this critical point is

$$P^{(R)}(\varphi_c)=-6(\varphi_c^1)^2(V_1)^2<0, \tag{4.26}$$

i.e., it corresponds to an anti–de Sitter ground state.

Summary for $P^{(R)}$. There are three possibilities: (a) $V_1=0$. This implies a flat potential $P^{(R)}\equiv0$ and Minkowski ground states with broken supersymmetry [supersymmetry is broken as long as the $U(1)_R$ gauging is nontrivial, i.e., if $V_0\neq0$]. (b) $V_0V_1>0$. In this case, there exists exactly one critical point. It is given by $\langle\varphi^2\rangle=\langle\varphi^3\rangle=0$ and $\langle\varphi^1\rangle^3=\sqrt{2}V_0/V_1$ and corresponds to an $\mathcal{N}=2$ supersymmetric AdS ground state whose cosmological constant can be read off from Eq. (4.26). This vacuum breaks the global symmetry group SO(1,1)$\times$SO(2,1) down to its maximal compact subgroup SO(2). (c) $V_0V_1<0$. No critical points exist in this case.

It is instructive to recover the characterization of the critical points in terms of the dual element $V^{\#\tilde{I}}$ [27] mentioned in Sec. III. Using Eq. (3.11), one finds

$$V^{\#\tilde{I}}=[(V_1)^2/\sqrt{2},\sqrt{2}V_0V_1,0,0].$$

This shows that $V^{\#\tilde{I}}=0$ is equivalent to $V_1=0$ and that $V^{\#\tilde{I}}$ is in the domain of positivity if $V_0V_1>0$ so that our cases (a)–(c) are equivalent to the cases (i), (ii), (iii), respectively, listed in Sec. III.

The critical points of the combined potential ($P_{\mathrm{tot}}=P+\lambda P^{(R)}$. The gradient of P_{tot} is given by

$$P_{\mathrm{tot},1}=-(A+\lambda B)\varphi^1+\frac{\varphi^1}{4\|\varphi\|^6}-\lambda 4\sqrt{2}\frac{V_0V_1}{\|\varphi\|^2}, \quad (4.27)$$

$$P_{\mathrm{tot},2}=(A+\lambda B)\varphi^2, \quad (4.28)$$

$$P_{\mathrm{tot},3}=(A+\lambda B)\varphi^3, \quad (4.29)$$

where A and B are again as defined above.

There are now two possibilities.

Case 1: $\langle\varphi^2\rangle=\langle\varphi^3\rangle=0$. In this case, $P_{,\tilde{x}}(\varphi_c)$ vanishes automatically (see the discussion of P above). This implies that $P^{(R)}_{,\tilde{x}}(\varphi_c)$ also has to vanish separately, i.e., we are dealing with critical points that are just simultaneous critical points of the individual potentials P and $P^{(R)}$. These have already been discussed above.

Case 2: $\langle\varphi^2\rangle^2+\langle\varphi^3\rangle^2>0$. This case involves a nontrivial interplay of the two potentials P and $P^{(R)}$. For $P_{\mathrm{tot},2}$ and $P_{\mathrm{tot},3}$ to vanish, one obviously needs $A+\lambda B=0$. $P_{\mathrm{tot},1}=0$ then implies

$$\frac{\varphi^1}{\|\varphi\|^4}=16\sqrt{2}\lambda V_0V_1. \quad (4.30)$$

This implies (remembering $\lambda>0$ and $\varphi^1>0$)

$$V_0V_1>0. \quad (4.31)$$

Inserting Eq. (4.30) into $A+\lambda B=0$, and reexpressing $(\varphi^2)^2+(\varphi^3)^2$ in terms of $\|\varphi\|^2$ and $(\varphi^1)^2$, one derives the additional condition

$$\frac{1}{\|\varphi\|^6}=\frac{1}{2}(16\sqrt{2}\lambda V_0V_1)^2+8\lambda(V_1)^2. \quad (4.32)$$

Now, by assumption, $\langle\varphi^2\rangle^2+\langle\varphi^3\rangle^2>0$. Hence

$$\frac{(\varphi^1)^2}{\|\varphi\|^8}>\frac{1}{\|\varphi\|^6},$$

so that in order for Eqs. (4.30) and (4.32) to be consistent, one needs

$$32\lambda(V_0)^2>1. \quad (4.33)$$

Thus, if V_0 is big enough such that Eq. (4.33) is satisfied and if $V_1V_0>0$ [see Eq. (4.31)], new non-trivial critical points exist. Equation (4.32) fixes $\|\varphi_c\|^2$ so that Eq. (4.30) fixes φ_c^1. This in turn fixes $[(\varphi_c^2)^2+(\varphi_c^3)^2]$, but not φ_c^2 and φ_c^3 individually. Hence, we obtain a one-parameter family of critical points, which, because of $[(\varphi_c^2)^2+(\varphi_c^3)^2]>0$, do not preserve the full $\mathcal{N}=2$ supersymmetry [see Eq. (4.16)] and spontaneously break the SO(2)-gauge invariance. Using Eqs. (4.30) and (4.32), one finds for the value of P_{tot} at these critical points

$$P_{\mathrm{tot}}(\varphi_c)=-\frac{3}{8}\frac{1}{\|\varphi\|^4}<0, \quad (4.34)$$

which again corresponds to an Anti-de Sitter solution. Putting everything together, we arrive at the following.

Summary for P_{tot}: Depending on the values of the V_I, the total potential $P_{\mathrm{tot}}=P+\lambda P^{(R)}$ admits the following types of critical points. (a) $V_1=0$. In this case, $P^{(R)}$ vanishes identically, and one has a one-parameter family of SO(2) gauge invariant Minkowski ground states. They are given by $\varphi_c^2=\varphi_c^3=0$ and an arbitrary $\varphi_c^1>0$. If $V_0\neq 0$ [i.e., if the $U(1)_R$-gauging is nontrivial], these ground states break the $\mathcal{N}=2$ supersymmetry. If $V_0=0$, the $U(1)_R$ gauging is switched off, and supersymmetry is unbroken, corresponding to case 1 in the discussion of P. (b1) $V_0V_1>0$, and $32\lambda(V_0)^2\leqslant 1$. In this case, there is precisely one ground state. It preserves the full $\mathcal{N}=2$ supersymmetry and the SO(2) gauge invariance. It corresponds to an anti–de Sitter solution, and is given by $\varphi_c^2=\varphi_c^3=0$ and $(\varphi_c^1)^3=\sqrt{2}V_0/V_1$ with $P_{\mathrm{tot}}(\varphi_c)=\lambda P^{(R)}(\varphi_c)=-6\lambda(\varphi_c^1)^2(V_1)^2$. Although the potential P due to the tensor fields does not contribute to this cosmological constant, it does have an effect on the form of the extremum of the total potential: It is now a saddle point, as opposed to the case of pure $U(1)_R$ gauging, where the supersymmetric critical point is always a maximum. (b2) $V_0V_1>0$, and in addition $32\lambda(V_0)^2>1$. In this case, there are two types of critical points. The first one is an isolated supersymmetric critical point which has exactly the same properties as the one described in (b1) above, with one exception: it is now a local *maximum* of the total scalar potential. Apart from this point, there is an additional one-parameter family of critical points. They are given by Eqs. (4.30) and (4.32), which fix φ_c^1, and $[(\varphi_c^2)^2+(\varphi_c^3)^2]$. They break the $\mathcal{N}=2$ supersymmetry and the SO(2)-gauge invariance and correspond to an anti–de Sitter solution with $P_{\mathrm{tot}}(\varphi_c)=-3/(8\|\varphi_c\|^4)$. These critical points are saddle points of the total potential. (c) $V_0V_1<0$. In this case, there are no critical points.

C. The $U(1)_R\times$SO(1,1) gauging

We now come to the noncompact version of the above theory. Since the analysis is very similar to the compact case, our presentation can be less detailed.

We choose the SO(1,1) subgroup of SO(2,1) to rotate the components ξ^1 and ξ^2 into each other. Consequently, this SO(1,1) acts nontrivially on the vector fields A_μ^1 and A_μ^2, and its gauging requires the dualization of A_μ^1 and A_μ^2 to antisymmetric tensor fields. Accordingly, we decompose the index $\tilde{I}$ as follows:

$$\tilde{I}=(I,M)$$

with $I,J,K,\ldots=0,3$ and $M,N,P,\ldots=1,2$.

Since $C_{0MN}\neq 0$ and $C_{3MN}=0$, A_μ^0 plays the role of the SO(1,1)-gauge field, whereas A_μ^3 is a "spectator vector field" with respect to the SO(1,1) gauging. Using a linear combination $A_\mu[U(1)_R]=A_\mu^I V_I$ of the vector fields A_μ^0 and A_μ^3 as the $U(1)_R$-gauge field, one can then simultaneously gauge $U(1)_R$ and SO(1,1), and obtains the $[U(1)_R\times SO(1,1)]$-gauged analog of the $[U(1)_R\times SO(2)]$ theory discussed in the previous subsection.

The scalar potentials P and $P^{(R)}$ are now (we use $\Omega^{12}=-\Omega^{21}=-1$)

$$P=\frac{1}{8}\frac{[(\varphi^1)^2-(\varphi^2)^2]}{\|\varphi\|^6}, \tag{4.35}$$

$$P^{(R)}=-2\left[2\sqrt{2}\frac{\varphi^3}{\|\varphi\|^2}V_0V_3-\|\varphi\|^2(V_3)^2\right]. \tag{4.36}$$

For the functions $W_{\tilde{x}}$, $P_{\tilde{x}}$, and P_0 that enter the supersymmetry transformation laws of the fermions, one obtains

$$W_1=-\frac{1}{4}\frac{\varphi^2}{\|\varphi\|^4}, \tag{4.37}$$

$$W_2=\frac{1}{4}\frac{\varphi^1}{\|\varphi\|^4}, \tag{4.38}$$

$$W_3=0, \tag{4.39}$$

respectively,

$$P_1=2\frac{\varphi^1}{\|\varphi\|^4}V_0, \tag{4.40}$$

$$P_2=-2\frac{\varphi^2}{\|\varphi\|^4}V_0, \tag{4.41}$$

$$P_3=-\sqrt{2}\left(\sqrt{2}\frac{\varphi^3}{\|\varphi\|^4}V_0+V_3\right), \tag{4.42}$$

and

$$P_0=\frac{2}{\sqrt{3}}\left(\frac{V_0}{\|\varphi\|^2}+\sqrt{2}\varphi^3V_3\right). \tag{4.43}$$

This already shows that there can be no $\mathcal{N}=2$ supersymmetric critical point, because W_2 can never vanish.

Let us now come to the critical points of the scalar potentials. We will again first analyze the critical points of P and $P^{(R)}$ separately and then consider the combined potential $P_{\rm tot}=P+\lambda P^{(R)}$.

The critical points of P. For the gradient of $P(\varphi)$ with respect to $\varphi^{\tilde{x}}$, one obtains

$$P_{,1}=\tilde{A}\varphi^1, \tag{4.44}$$

$$P_{,2}=-\tilde{A}\varphi^2, \tag{4.45}$$

$$P_{,3}=-\tilde{A}\varphi^3+\frac{\varphi^3}{4\|\varphi\|^6}, \tag{4.46}$$

with

$$\tilde{A}\equiv-\frac{3}{4}\frac{[(\varphi^1)^2-(\varphi^2)^2]}{\|\varphi\|^8}+\frac{1}{4\|\varphi\|^6}.$$

Since φ^1 cannot vanish, $P_{,1}=0$ requires $\tilde{A}=0$. But then $P_{,3}=0$ implies $\varphi^3=0$. The assumption $\tilde{A}=0$ then leads to the contradiction $1=3$, and is therefore inconsistent.

Summary for P. P alone has no critical points at all.

The critical points of $P^{(R)}$. The gradient of $P^{(R)}$ is

$$P_{,1}^{(R)}=\tilde{B}\varphi^1, \tag{4.47}$$

$$P_{,2}^{(R)}=-\tilde{B}\varphi^2, \tag{4.48}$$

$$P_{,3}^{(R)}=-\tilde{B}\varphi^3-\frac{4\sqrt{2}V_0V_3}{\|\varphi\|^2}, \tag{4.49}$$

where

$$\tilde{B}\equiv 8\sqrt{2}\frac{\varphi^3}{\|\varphi\|^4}V_0V_3+4(V_3)^2.$$

Since φ^1 cannot vanish, $P_{,1}^{(R)}=0$ implies $\tilde{B}=0$. The condition $P_{,3}^{(R)}=0$ then implies $V_0V_3=0$. Assume $V_3\neq 0$. Then $V_0=0$ would imply $V_3=0$ by virtue of $\tilde{B}=0$. Thus, V_3 has to vanish in any case if a critical point of $P^{(R)}$ is assumed to exist. However, $P^{(R)}$ then vanishes identically.

Summary for $P^{(R)}$. A critical point of $P^{(R)}$ exists if and only if $P^{(R)}$ vanishes identically (which is equivalent to $V_3=0$).

It is easy to recover the characterization of the critical points of $P^{(R)}$ in terms of the dual element $V^{\#\tilde{I}}$ [27] mentioned in Sec. III. In the case at hand, one finds

$$V^{\#\tilde{I}}=[-(V_3)^2/\sqrt{2},0,0,-\sqrt{2}V_0V_3].$$

This shows that $V^{\#\tilde{I}}=0$ is equivalent to $V_3=0$ and that $V^{\#\tilde{I}}$ can never be in the domain of positivity of $V_3\neq 0$. Thus, our results are consistent with the discussion given in Sec. III.

The critical points of the combined potential $P_{\rm tot}=P+\lambda P^{(R)}$. The gradient of $P_{\rm tot}$ is given by

$$P_{{\rm tot},1}=(\tilde{A}+\lambda\tilde{B})\varphi^1, \tag{4.50}$$

$$P_{{\rm tot},2}=-(\tilde{A}+\lambda\tilde{B})\varphi^2, \tag{4.51}$$

$$P_{{\rm tot},3}=-(\tilde{A}+\lambda\tilde{B})\varphi^3+\frac{\varphi^3}{4\|\varphi\|^6}-\lambda 4\sqrt{2}\frac{V_0V_3}{\|\varphi\|^2}, \tag{4.52}$$

where $\tilde{A}$ and $\tilde{B}$ are again as defined above.

Since φ^1 cannot vanish, $P_{\text{tot},1}=0$ requires $(\tilde{A}+\lambda\tilde{B})=0$. $P_{\text{tot},3}=0$ then implies

$$\frac{\varphi^3}{\|\varphi\|^4}=16\sqrt{2}\lambda V_0 V_3. \tag{4.53}$$

The analogous equation (4.30) in the compact gauging implied $V_0V_1>0$ [see Eq. (4.31)]. In the case at hand, however, Eq. (4.53) does not imply any constraint for V_0V_3, because $\varphi^3/\|\varphi\|^4$ does not have to be positive, as opposed to $\varphi^1/\|\varphi\|^4$, which is always greater than zero.

Inserting Eq. (4.53) into $\tilde{A}+\lambda\tilde{B}=0$, one derives the additional condition [i.e., the analogue of Eq. (4.32)]

$$\frac{1}{\|\varphi\|^6}=-\frac{1}{2}(16\sqrt{2}\lambda V_0V_3)^2+8\lambda(V_3)^2. \tag{4.54}$$

Since $1/\|\varphi\|^6>0$, the last equation implies the consistency conditions

$$V_3\neq 0,$$
$$32\lambda(V_0)^2<1. \tag{4.55}$$

[The analogous equation (4.33) in the compact gauging arose as a consistency condition of Eqs. (4.30) and (4.32). However, it is easy to see that Eqs. (4.53) and (4.54) do not imply any additional constraints on V_0 or V_3, so that Eqs. (4.55) remain the only constraints on the V_I.]

For a given set of V_I and λ subject to Eq. (4.55), $\|\varphi\|^2$ is fixed by Eq. (4.54). This in turn fixes φ^3 and $[(\varphi^1)^2-(\varphi^2)^2]$ by virtue of Eq. (4.53), but leaves the φ^1 and φ^2 otherwise undetermined. We thus obtain a one parameter family of critical points which can be viewed as the noncompact analog of the nontrivial nonsupersymmetric critical points found for the compact gauging [i.e., the ones mentioned in case (b2) in the discussion of P_{tot}]. However, for the noncompact gauging, these critical points have very different physical properties. In particular, the total scalar potential becomes

$$P_{\text{tot}}(\varphi_c)=3\lambda\|\varphi\|^2(V_3)^2[1-32\lambda(V_0)^2], \tag{4.56}$$

which is positive because of the condition (4.55) and therefore corresponds to a de Sitter rather than an anti–de Sitter spacetime.

Summary for P_{tot}. If $V_3\neq 0$ and $32\lambda(V_0)^2<1$, there exists a one parameter family of critical points given by Eqs. (4.53) and (4.54). They correspond to a de Sitter spacetime with $P_{\text{tot}}(\varphi_c)=3\lambda\|\varphi\|^2(V_3)^2[1-32\lambda(V_0)^2]>0$ and break the $\mathcal{N}=2$ supersymmetry and the SO(1,1) gauge invariance. There are no other critical points of the combined potential. In particular, neither the analogue of the $\mathcal{N}=2$ supersymmetric critical point mentioned in cases (b1) and (b2), nor the analogs of the Minkowski ground states mentioned in case (a) in the summary for P_{tot} in Sec. IV B, exist for the noncompact gauging.

V. THE GENERIC JORDAN FAMILY OF $\mathcal{N}=2$ GAUGED YANG-MILLS-EINSTEIN SUPERGRAVITY THEORIES COUPLED TO TENSOR MULTIPLETS

In the previous section we studied in detail the critical points of the potentials of the simplest nontrivial gauged Yang-Mills Einstein supergravity theories with tensor multiplets. The corresponding $\mathcal{N}=2$ MESGT belongs to the generic Jordan family and has the scalar manifold $SO(1,1)\times SO(2,1)/SO(2)$. The MESGT's of the generic Jordan family have the scalar manifold $SO(1,1)\times SO(n-1,1)/SO(n-1)$. From the results of Ref. [25] and the arguments given in the previous section it follows that any gaugeable subgroup K of the isometry group with K-charged vectors dualized to tensor fields must be Abelian. Since the vector field A_μ^0 must be the gauge field it follows that one can only gauge SO(2) or SO(1,1) and have some K-charged tensor fields under them. We should also note that the gaugeable SO(1,1) must be a subgroup of $SO(n-1,1)$ and cannot be the SO(1,1) factor in the isometry group since all the vector fields are charged under the latter SO(1,1). The SO(2) gauge group is some diagonal subgroup of the maximal Abelian subgroup $SO(2)_1\times SO(2)_2\times\cdots\times SO(2)_p$ of $SO(n-1,1)$ (for $n=2p+1$ or $n=2p+2$). The gaugeable SO(1,1) subgroup is unique modulo some $SO(n-1)$ rotation.

After the gauging of the Abelian subgroup of the isometry group with the charged vectors dualized to tensor fields, the remaining vector fields can be used to gauge some non-Abelian subgroup S of the full isometry group so long as they decompose as the adjoint plus some singlets of S. This non-Abelian gauging does not introduce any additional potential [27]. A linear combination of the remaining S singlet vector fields can then be used to gauge the $U(1)_R$ subgroup of the R-symmetry group $SU(2)_R$. The full potential of the $K\times U(1)_R\times S$ gauged Yang-Mills Einstein supergravity with K-charged tensor fields must have novel critical points of the type we discussed in the previous section since these theories can be truncated to the the simplest non-trivial model consistently.

There exist an infinite family of non-Jordan MESGT's with the scalar manifold $SO(n,1)/SO(n)$ [28]. For this family only the parabolic subgroup $SO(n-1)\times SO(1,1)\odot T_{(n-1)}$, which is simply an ''internal Euclidean group'' in $(n-1)$ dimensions times a dilatation factor, extends to a symmetry of the full action [32]. The analysis of the possible gauge groups K that involve a dualization of K-charged vectors to tensor fields is very similar to the generic Jordan case [25]. In this case too one finds that only a one dimensional Abelian subgroup K can be gauged with nontrivial tensor fields carrying charge under K. However, there is one crucial difference between the Jordan family and the non-Jordan family. For the non-Jordan family the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ is not an invariant tensor of the full isometry group $SO(n,1)$ of the scalar manifold. As a consequence one finds that

$$C_{\tilde{I}\tilde{J}\tilde{K}}\neq C^{\tilde{I}\tilde{J}\tilde{K}}, \tag{5.1}$$

and the $C^{\tilde{I}\tilde{J}\tilde{K}}$ are no longer constant tensors but depend on the scalars.

VI. CONCLUSIONS

In this paper we have analyzed the scalar potentials of the simplest examples of a gauged Yang-Mills-Einstein supergravity theory coupled to tensor multiplets. Although not all the results we have derived for these examples may carry over to the most general gauged Yang-Mills-Einstein supergravity theory with tensor fields, they show that the scalar potentials of these theories can exhibit a much richer structure than the purely $U(1)_R$-gauged supergravity theories or the gauged Yang-Mills-Einstein supergravity theories *without* tensor fields. Our analysis revealed that even though the total potential is just a sum of the potentials that appear in the separate gaugings of K and $U(1)_R$, there can be critical points of the total potential which would not be critical points of the individual potentials. In particular, we found that, for a certain parameter range [case (b2)] in Sec. IV B)], the $[U(1)_R\times SO(2)]$ gauging leads to a new one-parameter family of non-supersymmetric critical points, which are saddle points of the total potential. These are accompanied by an isolated $\mathcal{N}=2$ supersymmetric *maximum*, which is already present in the purely $U(1)_R$ gauged theory without tensor fields. In another parameter range [case (b1)], the novel nonsupersymmetric one-parameter family of critical points disappears and the $\mathcal{N}=2$ supersymmetric critical point becomes a *saddle point* (and remains supersymmetric). In yet another parameter range [case (a)], the theory has a one-parameter family of Minkowski ground states which break the $\mathcal{N}=2$ supersymmetry as long as the $U(1)_R$ gauging is nontrivial. If the $U(1)_R$ gauging is switched off, these critical points become supersymmetric.

The possible types of critical points are much more restricted for the noncompact $U(1)_R\times SO(1,1)$ gauging, which can have at most a one-parameter family of nonsupersymmetric *de Sitter* ground states (which are presumably unstable). This is consistent with the experience from compact and noncompact gaugings of the $\mathcal{N}=8$ theory [10] where a nonsupersymmetric de Sitter critical point was found in the SO(3,3)-gauged version of the $\mathcal{N}=8$ theory.

In this paper we have not studied the critical points of the potential when one gauges a *non-Abelian* subgroup K of the isometry group of the scalar manifold with tensor multiplets transforming in a nontrivial representation of K. Such gauge groups are possible for the magical Jordan $\mathcal{N}=2$ theories as well as for the infinite family of theories with $SU(n)$ isometries discussed in Ref. [25]. The study of the critical points of these theories as well as those of the non-Jordan family will be the subject of a future investigation.

APPENDIX A: THE "VERY SPECIAL GEOMETRY" OF THE SO(1,1)×SO(2,1)/SO(2)-MODEL

This Appendix contains a list of the basic scalar field dependent quantities that enter the Lagrangian and the transformation laws of the ungauged and gauged $SO(1,1)\times SO(2,1)/SO(2)$ theory. In our parametrization, the $h^{\tilde I}=\sqrt{\tfrac{2}{3}}\,\xi^{\tilde I}|_{N=1}$ are

$$h^0=\frac{1}{\sqrt{3}\|\varphi\|^2},\quad h^1=\sqrt{\frac{2}{3}}\varphi^1,\quad h^2=\sqrt{\frac{2}{3}}\varphi^2,\quad h^3=\sqrt{\frac{2}{3}}\varphi^3.$$

For the

$$h_{\tilde I}=\frac{1}{\sqrt{6}}\frac{\partial}{\partial\xi^{\tilde I}}N|_{N=1}$$

one obtains

$$h_0=\frac{1}{\sqrt{3}}\|\varphi\|^2,\quad h_1=\frac{2}{\sqrt{6}}\frac{\varphi^1}{\|\varphi\|^2},\quad h_2=-\frac{2}{\sqrt{6}}\frac{\varphi^2}{\|\varphi\|^2},$$

$$h_3=-\frac{2}{\sqrt{6}}\frac{\varphi^3}{\|\varphi\|^2}.$$

The vector-tensor field metric

$$\overset{\circ}{a}_{\tilde I\tilde J}=-\frac{1}{2}\frac{\partial}{\partial\xi^{\tilde I}}\frac{\partial}{\partial\xi^{\tilde J}}\ln N(\xi)|_{N=1}$$

turns out to be

$$\overset{\circ}{a}_{\tilde I\tilde J}=\begin{pmatrix}\|\varphi\|^4 & 0 & 0 & 0\\ 0 & 2(\varphi^1)^2\|\varphi\|^{-4}-\|\varphi\|^{-2} & -2\varphi^1\varphi^2\|\varphi\|^{-4} & -2\varphi^1\varphi^3\|\varphi\|^{-4}\\ 0 & -2\varphi^1\varphi^2\|\varphi\|^{-4} & 2(\varphi^2)^2\|\varphi\|^{-4}+\|\varphi\|^{-2} & 2\varphi^2\varphi^3\|\varphi\|^{-4}\\ 0 & -2\varphi^1\varphi^3\|\varphi\|^{-4} & 2\varphi^2\varphi^3\|\varphi\|^{-4} & 2(\varphi^3)^2\|\varphi\|^{-4}+\|\varphi\|^{-2}\end{pmatrix}.$$

This shows that the unique point with $\overset{\circ}{a}_{\tilde I\tilde J}=\delta_{\tilde I\tilde J}$ corresponds to $\varphi^{\tilde x}=(1,0,0)$, as has been mentioned earlier.[9]

Finally, the metric $g_{\tilde x\tilde y}$ on $\mathcal{M}$ reads

$$g_{\tilde x\tilde y}=\begin{pmatrix}4(\varphi^1)^2\|\varphi\|^{-4}-\|\varphi\|^{-2} & -4\varphi^1\varphi^2\|\varphi\|^{-4} & -4\varphi^1\varphi^3\|\varphi\|^{-4}\\ -4\varphi^1\varphi^2\|\varphi\|^{-4} & 4(\varphi^2)^2\|\varphi\|^{-4}+\|\varphi\|^{-2} & 4\varphi^2\varphi^3\|\varphi\|^{-4}\\ -4\varphi^1\varphi^3\|\varphi\|^{-4} & 4\varphi^2\varphi^3\|\varphi\|^{-4} & 4(\varphi^3)^2\|\varphi\|^{-4}+\|\varphi\|^{-2}\end{pmatrix}.$$

[9] If we had chosen another normalization for N, i.e., $N(\xi)=a\xi^0[(\xi^1)^2-(\xi^2)^2-(\xi^3)^2]$ for some $a\in\mathbb{R}$, $\overset{\circ}{a}_{00}$ would have been $a^2\|\varphi\|^4/2$ with the other components unchanged. It is easy to see that only $a=\sqrt{2}$ can lead to a point where $\overset{\circ}{a}_{\tilde I\tilde J}=\delta_{\tilde I\tilde J}$.

For the determinants of $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ and $g_{\tilde{x}\tilde{y}}$, one finds

$$\det \overset{\circ}{a}_{\tilde{I}\tilde{J}} = \|\varphi\|^{-2}, \tag{A1}$$

$$\det g_{\tilde{x}\tilde{y}} = 3\|\varphi\|^{-6}, \tag{A2}$$

which shows that $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ and $g_{\tilde{x}\tilde{y}}$ are positive definite and well behaved throughout the entire ''positive timelike'' region (i) and that both are not positive definite in region (ii), where $\|\varphi\|^2 < 0$.

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