Warp drive space-time

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In this paper the problem of the quantum stability of the two-dimensional warp drive spacetime moving with an apparent faster than light velocity is considered. We regard as a maximum extension beyond the event horizon of that spacetime its embedding in a three-dimensional Minkowskian space with the topology of the corresponding Misner space. It is obtained that the interior of the spaceship bubble becomes then a multiply connected nonchronal region with closed spacelike curves and that the most natural vacuum allows quantum fluctuations which do not induce any divergent behavior of the renormalized stress-energy tensor, even on the event (Cauchy) chronology horizon. In such a case, the horizon encloses closed timelike curves only at scales close to the Planck length, so that the warp drive satisfies Ford's negative energy-time inequality. Also found is a connection between the superluminal two-dimensional warp drive space and two-dimensional gravitational kinks. This connection allows us to generalize the considered Alcubierre metric to a standard, nonstatic metric which is only describable on two different coordinate patches.

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I. INTRODUCTION

General relativity admits many rather unexpected solutions, most of which represent physical situations which have been thought to be pathological for a variety of respects, as they correspond to momentum-energy tensors which violate classical conditions and principles considered as sacrosant by physicists for many years [1]. Among these solutions you can find Lorentzian wormholes [2], ringholes [3], Klein bottle holes [4], the Gott-Grant's double string [5,6], the Politzer time machine [7], the multiply connected de Sitter space [8,9], a time machine in superfluid ^{3}He [10], etc., all of which allow the existence of closed timelike curves (CTC's), so as spacetimes where superluminal, though not into the past, travels are made possible, such as happens in the Alcubierre warp drive [11]. Most of the solutions that contain CTC's are nothing but particular modifications or generalizations from Misner space which can thereby be considered as the prototype of the nonchronal pathologies in general relativity [12].

Time machines constructed from the above-mentioned spacetimes contain nonchronal regions that are generated by shortcutting the spacetime and can allow traveling into the past and future at velocities that may exceed the speed of light. A solution to Einstein equations that has also fascinated and excited relativists is the so-called warp drive spacetime discovered by Alcubierre [11]. What is violated in this case is only the relativistic principle that a space-going traveller may move with any velocity up to, but not including or overcoming, the speed of light. Alcubierre's construct corresponds to a warp drive of science fiction in that it causes spacetime to contract in front of a spaceship bubble and expand behind, thus providing the spaceship with a velocity that can be much greater than the speed of light relative to distant objects, while the spaceship never locally travels faster than light. Thus, the ultimate relativistic reason making possible such a warp drive within the context of general relativity is that, whereas two observers at the same point cannot have relative velocities faster than light, when such observers are placed at distant locations their relative velocity may be arbitrarily large while the observers are moving in their respective light cones.

The properties of a warp drive and those of spacetimes with CTC's can actually be reunited in a single spacetime. Everett has in fact considered [13] the formation of CTC's which arise in the Alcubierre warp drive whenever it is modified so that it can contain two sources for the gravitational disturbance which are allowed to move past one another on parallel, noncolinear paths. The resulting time machine can then be regarded to somehow be a three-dimensional analogue of the two-dimensional Gott's time machine [5] in which the CTC's encircle pairs of infinite, straight, parallel cosmic strings which also move noncolinearly. In the present paper we consider the physical and geometric bases which allow us to construct a two-dimensional single-source Alcubierre warp drive moving at a constant faster than light apparent speed, which at the same time can be viewed as a time machine for the astronaut travelling in the spaceship. This can be achieved by converting the interior of the spaceship bubble in a multiply connected space region satisfying the identification properties of three-dimensional Misner space [14]. We shall also show how in doing so some of the acutest problems of the Alcubierre warp drive can be solved.

It was Alcubierre himself who raised [11] doubts about whether his spacetime is a physically reasonable one. Energy density is in fact negative within the spaceship bubble. This may in principle be allowed by quantum mechanics, provided the amount and time duration of the negative energy satisfy Ford like inequalities [15]. However, Ford and Pfenning later showed [16] that even for bubble wall thickness of the order of only a few hundred Planck lengths, it turns out that the integrated negative energy density is still physically unreasonable for macroscopic warp drives. This problem has

been recently alleviated, but not completely solved, by Van Den Broeck [17] who, by only slightly modifying the Alcubierre spacetime, succeeded in largely reducing the amount of negative energy density of the warp. A true solution of the problem comes only about if the warp bubble is microscopically small [16], a situation which will be obtained in this paper when CTC's are allowed to exist within the warp bubble.

On the other hand, Hiscock has computed [18] for the two-dimensional Alcubierre warp drive the stress-energy tensor of a conformally invariant field and shown that it diverges on the event horizon appearing when the apparent velocity exceeds the speed of light, thus rendering the warp drive instable. A similar calculation performed in this paper for the case where the interior spacetime of the spaceship bubble is multiply connected yields a vanishing stress-energy tensor even on the event horizon which becomes then a (Cauchy) chronology horizon, both for the self consistent Li-Gott vacuum [19] and when one uses a modified three-dimensional Misner space [20] as the embedding of the two-dimensional warp drive spacetime. It is worth noticing that if the astronaut inside the spaceship is allowed to travel into the past, then most of the Alcubierre's warp drive problems derived from the fact that an observer at the center of the warp bubble is causally separated from the bubble exterior [21] can be circumvented in such a way that the observer might now contribute the creation of the warp bubble and control one once it has been created, by taking advantage of the causality violation induced by the CTC's.

We outline the rest of the paper as follows. In Sec. II we biefly review the spacetime geometry of the Alcubierre warp drive, discussing in particular its two-dimensional metric representation and extension when the apparent velocity exceeds the speed of light. The visualization of the two-dimensional Alcubierre spacetime as a three-hyperboloid embedded in E^3 and its connection with the three-dimensional Misner space is dealt with in Sec. III, where we also covert the two-dimensional warp drive spacetime into a multiply connected space by making its coordinates satisfy the identification properties of the three-dimensional Misner space. By replacing the Kruskal maximal extension of the geodesically incomplete warp drive space with the above embedding we study in Sec. IV the Euclidean continuation of the multiply connected warp drive, identifying the periods of the variables for two particular *Ansätze* of interest. We also consider the Hadamard function and the resulting renormalized stress-energy tensor for each of these *Ansätze*, comparing their results. Finally, we conclude in Sec. V. Throughout the paper units so that $G=c=\hbar=1$ are used.

II. ALCUBIERRE WARP DRIVE SPACETIME

The Alcubierre spacetime having the properties associated with the warp drive can be described by a metric of the form [11]

$$ds^2=-dt^2+(dx-vf(r)dt)^2+dy^2+dz^2, \qquad (2.1)$$

with $v=dx_s(t)/dt$ the apparent velocity of the warp drive spaceship, $x_s(t)$ the trajectory of the spaceship along coordinate x, the radial coordinate being defined by

$$r=\{[x-x_s(t)]+y^2+z^2\}^{1/2}, \qquad (2.2)$$

and $f(r)$ an arbitrary function subjected to the boundary conditions that $f=1$ at $r=0$ (the location of the spaceship) and $f=0$ at infinity.

Most of the physics in this spacetime concentrates on the two-dimensional space resulting from setting $y=z=0$, defining the axis about which a cylindrically symmetric space develops; thus, the two-dimensional Alcubierre space still contains the entire world line of the spaceship. If the apparent velocity of the spaceship is taken to be constant, $v=v_0$, then the metric of the two-dimensional Alcubierre space becomes [18]

$$ds^2=-[1-v_0^2f(r)^2]dt^2-2v_0f(r)dtdx+dx^2, \qquad (2.3)$$

with r now given by $r=\sqrt{(x-v_0t)^2}$, which in the past of the spaceship $(x>v_0t)$ can simply be written as $r=x-v_0t$. Metric (2.3) can still be represented in a more familiar form when one chooses as coordinates (t,r), instead of (t,x). This can obviously be achieved by the replacement $dx=dr+v_0dt$, with which metric (2.3) transforms into

$$ds^2=-A(r)\left[dt-\frac{v_0[1-f(r)]}{A(r)}dr\right]^2+\frac{dr^2}{A(r)}, \qquad (2.4)$$

where the Hiscock function [18]

$$A(r)=1-v_0^2[1-f(r)]^2 \qquad (2.5)$$

has been introduced. Metric (2.4) can finally be brought into a comoving form, in terms of the proper time $d\tau=dt-v_0[1-f(r)]dr/A(r)$:

$$ds^2=-A(r)d\tau^2+\frac{dr^2}{A(r)}. \qquad (2.6)$$

As pointed out by Hiscock [18], this form of the metric is manifestly static. The case of most interest corresponds to apparent velocities $v_0>1$ (superluminal velocity) where the metrics (2.4) and (2.6) turn out to be singular with a coordinate singularity (apparent event horizon) at a given value of $r=r_0$ such that $A(r_0)=0$, i.e. $f(r_0)=1-1/v_0$.

We note that metric (2.4) can be regarded as the kinked metric (describing a gravitational topological defect [22]) that corresponds to the static metric (2.6). To see this, let us first redefine the coordinates t and r as $dt'=A^{1/4}dt$ and $dr'=A^{-1/4}dr$, so that metric (2.4) can be re-written along the real intervals $0\leqslant r'\leqslant\infty$ (i.e. $0\leqslant r\leqslant r_0$) and $0\leqslant t'\leqslant\infty$ as

$$ds^2=-\sqrt{A(r)}[(dt')^2-(dr')^2]+2v_0[1-f(r)]dt'dr'. \qquad (2.7)$$

Now, metric (2.7) can be viewed as the metric describing at least a given part of a two-dimensional one-kink (gravitational topological charge ±1) if we take $\sin(2\alpha)=\pm v_0(1-f(r))$, i.e. $\cos(2\alpha)=\sqrt{A}$, so that this metric can be transformed into

$$ds^2 = -\cos(2\alpha)[(dt')^2 - (dr')^2] \pm 2\sin(2\alpha)dt'dr', \tag{2.8}$$

where α is the tilt angle of the light cones tipping over the hypersurfaces [22,23], and the choice of sign in the second term depends on whether a positive (upper sign) or negative (lower sign) gravitational topological charge is considered.

The existence of the complete one-kink is allowed whenever one lets α monotonously increase from 0 to π, starting with $\alpha(0)=0$, with the support of the kink being the region inside the event horizon. Then metric (2.8) converts into metric (2.6) if we introduce the substitution $\sin\alpha = \sqrt{[1 \pm \sqrt{A(r)}]/2}$ and the change of time variable $\tau = t' + g(r)$, with $dg(r)/dr' = \tan(2\alpha)$. The region inside the warp drive bubble supporting the kink is the only region which can actually be described by metric (2.8) because $\sin\alpha$ cannot exceed unity and hence $0 \leqslant r \leqslant r_0$. In order to have a comple description of the one-kink and therefore of the warp drive one needs a second coordinate patch where the other half of the α interval, $\pi/2 \leqslant \alpha \leqslant \pi$, can be described. This can be achieved by introducing a new time coordinate [23] $\bar{t}' = t' + h(r)$, with $dh(r)/dr = [v_0(1 - f(r)) - k]/A^{3/4}$, in which $k = \pm 1$, the upper sign for the first patch and the lower one for the second patch. This choice is adopted for the following reason. The zeros of the denominator of $dh/dr' = [\sin(2\alpha) \mp 1]/\sqrt{A}$ correspond to the two event horizons where $r = r_0$ and $f(r_0) = 1 - 1/r_0$, one per patch. For the first patch, the horizon occurs at $\alpha = \pi/4$ and therefore the upper sign is selected so that both dh/dr and h remain well defined and the kink is preserved on this horizon. For the second patch the horizon occurs at $\alpha = 3\pi/4$ and therefore the lower sign is selected for it. The two-dimensional metric for a (in this sense) complete warp drive will be then [23]

$$ds^2 = -A(r)d\bar{t}^2 \mp 2kd\bar{t}dr \tag{2.9}$$

or, in terms of the $(\bar{t}, x)$ coordinates in the four-dimensional manifold,

$$ds^2 = -[A(r) \mp 2kv_0]d\bar{t}^2 \mp 2kd\bar{t}dx + dy^2 + dz^2, \tag{2.10}$$

with r as defined by Eq. (2.2).

Having shown the existence of a connection between warp drive spacetime and topological gravitational kinks and hence extended the warp drive metric to that described by Eq. (2.10), we now return to analyze metric (2.6) by noting that the geodesic incompleteness of this metric at $r = r_0$ can, as usual, be avoided by maximally extending it according to the Kruskal technique. For this to be achieved we need to define first the quantity $r^* = \int dr/A(r)$. Using for $f(r)$ the function suggested by Alcubierre,

$$f(r) = \frac{\tanh[\sigma(r+R)] - \tanh[\sigma(r-R)]}{2\tanh(\sigma R)}, \tag{2.11}$$

where R and σ are positive arbitrary constants, we obtain

$$\begin{aligned} r^* = {} & \frac{(1+2v_0)r}{1-v_0^2} + \frac{(3+2v_0-v_0^2)v_0^2}{\sigma(1-v_0^2)^2\sqrt{3v_0^2+2v_0-1}} \\ & \times \ln\left[\frac{2v_0(1+v_0)\tanh(\sigma r) - \sqrt{3v_0^2+2v_0-1}}{2v_0(1+v_0)\tanh(\sigma r) + \sqrt{3v_0^2+2v_0-1}}\right] \\ & - \frac{v_0(1+3v_0)}{4\sigma(1+v_0)\sqrt{2v_0(1+v_0)}} \\ & \times \ln\left[\frac{1+v_0-\sqrt{2v_0(1+v_0)}\tanh(\sigma r)}{1+v_0+\sqrt{2v_0(1+v_0)}\tanh(\sigma r)}\right] \\ & + \frac{v_0}{2\sigma(1+v_0)}\tanh(\sigma r), \end{aligned} \tag{2.12}$$

where we have specialized to the allowed particular case for which $\sinh^2(\sigma R) = v_0$. Introducing then the usual coordinates $V = t + r^*$ and $W = t - r^*$, so that

$$ds^2 = -\left[1 - \frac{v_0^2\sinh^4(\sigma r)}{(\cosh^2(\sigma r) + v_0)^2}\right]dVdW, \tag{2.13}$$

and hence the new coordinates

$$\tanh V' = \exp\left(\frac{\sigma V}{4\sinh^{-1}\sqrt{\dfrac{v_0+1}{v_0-1}}}\right)$$

$$\tanh W' = -\exp\left(\frac{-\sigma W}{4\sinh^{-1}\sqrt{\dfrac{v_0+1}{v_0-1}}}\right),$$

we finally obtain the maximally extended, geodesically complete metric

$$\begin{aligned} ds^2 = {} & \frac{64}{\sigma^2}\left[1 - \frac{v_0^2\sinh^4(\sigma r)}{(\cosh^2(\sigma r) + v_0)^2}\right] \\ & \times \frac{\left(\sinh^{-1}\sqrt{\dfrac{v_0+1}{v_0-1}}\right)^2 dV'dW'}{\sin(2V')\sin(2W')}, \end{aligned} \tag{2.14}$$

where r is implicitly defined by

$$\tan V' \tan W' = -\exp\left(\frac{\sigma r^*}{2\sinh^{-1}\sqrt{\dfrac{v_0+1}{v_0-1}}}\right). \tag{2.15}$$

The maximal extension of metric (2.6) is thus obtained by taking expression (2.14) as the metric of the largest manifold which metrics given only either in terms of V or in terms of W can be isometrically embedded. There will be then a maximal manifold on which metric (2.14) is C^2 [24].

III. WARP DRIVE WITH INTERNAL CTC

In this section we investigate a property of the two-dimensional Alcubierre spacetime which will allow us to avoid the complicatedness of Kruskal extension in order to study its Euclidean continuation and hence its stability against quantum vacuum fluctuations. Thus, taking advantage from the similarity between metric (2.6) and the de Sitter metric in two dimensions, we can visualize the dimensionally reduced Alcubierre spacetime as a three-hyperboloid defined by

$$-v^2+w^2+x^2=v_0^{-2}, \tag{3.1}$$

where $v_0>1$. This hyperboloid is embedded in E^3 and the most general expression for the two-dimensional metric of Alcubierre space for $v_0>1$ is then that which is induced in this embedding, i.e.

$$ds^2=-dv^2+dw^2+dx^2, \tag{3.2}$$

which has topology $R\times S^2$ and invariance group $SO(2,1)$.

Metric (3.2) can in fact be conveniently exhibited in static coordinates $\tau\in(-\infty,\infty)$ and $r\in(0,r_0)$, defined by

$$\begin{aligned} v&=v_0^{-1}\sqrt{A(r)}\sinh(v_0\tau)\\ w&=v_0^{-1}\sqrt{A(r)}\cosh(v_0\tau)\\ x&=F(r), \end{aligned} \tag{3.3}$$

where

$$[F(r)']^2=-\left[\frac{(d\rho/dr)^2-4v_0^2}{4v_0^2(1+\rho)}\right], \tag{3.4}$$

the prime denoting a derivative with respect to the radial coordinate r, with

$$\rho\equiv\rho(r)=-v_0[1-f(r)]^2 \tag{3.5}$$

and

$$A=1+\rho(r). \tag{3.6}$$

Using coordinates (3.3) in metric (3.2) we re-derive then metric (2.6). Thus, the two-dimensional Alcubierre space can be embedded in Minkowski space in three dimensions. Since Minkowski metric (3.2) and the identifications

$$\begin{aligned}(v,w,x)\leftrightarrow(&v\cosh(nb)+w\sinh(nb),v\sinh(nb)\\&+w\cosh(nb),x),\end{aligned} \tag{3.7}$$

where b is a dimensionless arbitrary boosting quantity and n is any integer number, make the universal covering [24] of the Misner space in three dimensions, one can covert the two-dimensional Alcubierre space into a multiply connected space if we add the corresponding identifications

$$(\tau,r)\leftrightarrow\left(\tau+\frac{nb}{v_0},r\right) \tag{3.8}$$

in the original warp drive coordinates. The boost transformation in the (v,w) plane implied by identifications (3.7) will induce therefore the boost transformation (3.8) in the two-dimensional Alcubierre space. Hence, since the boost group in Alcubierre space must be a subgroup of the invariance group of the two-dimensional Alcubierre embedding, the static metric (2.6) can also be invariant under symmetry (3.7). Thus, for coordinates defined by Eqs. (3.3) leading to the static metric with an apparent horizon as metric (2.6), the symmetry (3.7) can be satisfied in the region covered by such a metric, i.e. the region $w>|v|$, where there are CTC's, with the boundaries at $w=\pm v$, and $x^2=v_0^{-2}$ being the Cauchy horizons that limit the onset of the nonchronal region from the Alcubierre causal exterior. Such boundaries are situated at r_0, defined by $f(r_0)=1-v_0^{-1}$, and become then appropriate chronology horizons [12] for the so-obtained multiply connected two-dimensional Alcubierre space with $v_0>1$.

We have in this way succeeded in coverting a two-dimensional warp drive with a constant, faster than light apparent velocity in a multiply connected warp drive with CTC's only inside the spaceship and its event horizon at r_0 in a chronology horizon. This is a totally different way of transforming warp drives into time machines of the mechanism envisaged by Everett [13] for generating causal loops using two sources of gravitational disturbance which move past one another. In our case, even though the astronaut at the center of the warp bubble is still causally separated from the external space, he can always travel into the past to help create the warp drive on demand or set up the initial conditions for the control of one once it has been created.

IV. QUANTUM STABILITY OF MULTIPLY CONNECTED WARP DRIVE

In this section we shall show that the two-dimensional multiply connected warp drive spacetime is perfectly stable to the vacuum quantum fluctuations if either a self-consistent Rindle vacuum is introduced or for microscopic warp bubbles. We have already shown that the two-dimensional warp drive spacetime can be embedded in the Minkowskian covering of the three-dimensional Misner space when the symmetries of this space implied by identifications (3.7) [which lead to identifications (3.8) in Alcubierre coordinates] are imposed to hold also in the two-dimensional Alcubierre spacetime with $v_0>1$. Since the embedding can be taken to play an analogous role to that of a maximal Kruskal extension, it follows that showing stability against vacuum quantum fluctuations in three-dimensional Misner space would imply that the two-dimensional multiply connected warp drive space with $v_0>1$ is also stable to the same fluctuations. This conclusion is in sharp contrast with some recent result obtained by Hiscock who has shown [18] that the stress-energy tensor for a conformally invariant scalar field propagating in simply connected two-dimensional Alcubierre space diverges on the event horizon if the apparent velocity of the spaceship exceeds the speed of light. He obtained an observed energy density near the horizon proportional to $[f-(1-v_0^{-1})]^{-2}$, which in fact diverges as $r\to r_0$.

Metric (3.2) can be transformed into the metric of a three-

dimensional Misner space explicitly by using the coodinate re-definitions

$$v=\theta\cosh x^1$$
$$w=\theta\sinh x^1 \tag{4.1}$$
$$x=x^2.$$

Then,

$$ds^2=-d\theta^2+\theta^2(dx^1)^2+(dx^2)^2, \tag{4.2}$$

which is the three-dimensional Misner metric for coordinates $0<\theta<\infty$, $0\leqslant x^1\leqslant 2\pi$ and $0\leqslant x^2\leqslant\infty$. This metric is singular at $\theta=0$ and, such as happens in its four-dimensional extension, it has CTC's in the region $\theta<0$. Note that one can also obtain the line element (4.2) directly from the two-dimensional Alcubierre metric with $v_0>1$ by introducing the coordinate transformations

$$\theta=\frac{i\sqrt{A(r)}}{v_0} \tag{4.3}$$

$$x^1=-i\,\mathrm{arcsin}[\cosh(v_0\tau)] \tag{4.4}$$

and

$$x^2=F(r), \tag{4.5}$$

with $F(r)$ as defined by Eq. (3.4). Let us now consider the Euclidean continuation from which metric (4.2) becomes positive definite. This is accomplished if we rotate both coordinates θ and x^1 simultaneously, so that

$$\theta=i\eta, \quad x^1=i\chi, \tag{4.6}$$

where η and χ can be expressed in terms of the Alcubierre coordinates r and τ by means of transformations (4.3) and (4.4). The covering space of the resulting metric preserves however the Lorentzian signature. This might be an artifact coming from the singular character of metric (4.2), so it appears most appropriate to extend first this metric beyond $\theta=0$ in order to get the Euclidean sector of the three-dimensional Misner metric and, hence, investigate the stability of the superluminal, multiply connected two-dimensional warp drive spacetime against vacuum quantum fluctuations. The extension beyond $\theta=0$ of metric (4.2) is conventionally made by using the new coordinates [24] $T=\theta^2$, $V=\ln\theta+x^1$ and $x^2=x^2$, so that

$$ds^2=-dTdV+TdV^2+(dx^2)^2, \tag{4.7}$$

or by re-defining $V=Y+Z$ and $T-\int TdV=Y-Z$:

$$ds^2=-dY^2+dZ^2+(dx^2)^2 \tag{4.8}$$

and

$$Y^2-Z^2=V\left(T-\int TdV\right), \quad \frac{Y-Z}{Y+Z}=\frac{t-\int TdV}{V}. \tag{4.9}$$

Metric (4.8) becomes positive definite when continuing the new coordinate Y so that $Y=i\zeta$. By using this continuation together with rotations (4.6) in expressions (4.9), we can deduce that the section on which ζ and Z are both real corresponds to the region defined by $\theta\geqslant e^{1/2}$, $x^1\geqslant 1/2$, and that there exist two possible *Ansätze* according to which the continuation can be performed. One can first set

$$\exp(i\zeta)=i\exp\left(\frac{\zeta^2-Z^2}{Z}\right)\eta\,\exp\left[-2\chi\left(\chi+\frac{\pi}{2}\right)\right]\exp(i\chi), \tag{4.10}$$

where only the coordinate χ turns out to be periodic on the Euclidean sector, with a period $\overline{b}=2\pi$. Rotating back to the Lorentzian region, we then have $b=2\pi$. *Ansatz* (4.10) should be associated with the self-consistent Rindler vacuum considered by Li and Gott for four-dimensional Misner space [19], instead of the Minkowski vacuum with multiple images originally used by Hiscock and Konkowski [25]. Introducing then Rindler coordinates defined by $v=\xi\sinh\omega$ and $\omega=\xi\cosh\omega$ in the three-dimensional covering metric (3.2), so that it becomes

$$ds^2=-\xi^2d\omega^2+d\xi^2+dx^2,$$

we can compute the Hadamard function for a conformally invariant scalar field in Rindler vacuum to be

$$G^{(1)}(X,X')=\frac{1}{2\pi^2}\frac{\gamma}{\xi\xi'\sinh\gamma[-(\omega-\omega')^2+\gamma^2]}, \tag{4.11}$$

with $X=(\omega,\xi,x)$, $X'=(\omega',\xi',x')$, and

$$\cosh\gamma=\frac{\xi^2+\xi'^2+(x-x')^2}{2\xi\xi'}. \tag{4.12}$$

Using the method of images [25] and the usual definitions of the regularized Hadamard function and hence the renormalized stress-energy tensor [18], we finally get for the latter quantity an expression which is proportional to

$$\frac{1}{\xi^3}\left[\left(\frac{2\pi}{b}\right)^3-1\right],$$

which, if we take $b=2\pi$ according to the above *Ansatz*, obviously vanishes everywhere, even on the (Cauchy) chronology horizon at $\xi=0$.

Although in spite of the Ford-Pfenning requirement [16], this allows the existence of stable superluminal, multiply connected warp drives of any size, this choice for the vacuum has two further problems. First of all, previous work by Kay, Radzikowski and Wald [26] and by Cassidy [27] casts compelling doubts on the meaning of the Cauchy horizon and, hence, on the validity of the conclusion that the stress-energy tensor is zero also on such a horizon. On the other hand, having an Euclidean section on which time is not periodic (which implies the nonexistence of any background thermal radiation) while the spacetime has an event horizon

appears to be rather contradictory. It is for these reasons that we tend to favor the second possible *Ansatz* implementing the Euclidean continuation of the three-dimensional Misner covering, which appears to be less problematic. It reads

$$\exp\left(\frac{i\xi}{Z}\right)=\exp\left(\frac{\zeta^2-Z^2}{2Z^2}\right)\eta^{\eta^{-2}} \times\exp\left[-\frac{2\chi(\chi+\pi/2)}{\eta^2}\right]\exp\left[i\left(\frac{\chi+\pi/2}{\eta^2}\right)\right], \tag{4.13}$$

where both the Euclidean time η and the Euclidean coordinate are now periodic, with respective dimensionless periods $\Pi_\eta=1/2$ and $\Pi_\chi=2\pi\eta^2$. The physical time period in two-dimensional Alcubierre space at faster than light apparent velocity, Π_{Al}, can be related to period Π_η by means of the expression

$$v_0\Pi_{Al}=4\pi\Pi_\eta\sqrt{g^{rr}_{Al}}\,\frac{dr}{d\eta}|_{r=r_0}.$$

Using then the Euclidean continuation of Eq. (4.3), one can obtain $\Pi_{Al}=4\pi/A(r_0)'$, which corresponds to a background temperature, $T_{Al}=A(r_0)'/4\pi$, which is the same as that which is associated with the event horizon of the spaceship and was first derived by Hiscock [18]. Rotating back to the Lorentzian section we see that time θ becomes again no longer periodic, but x^1 keeps still a periodic character with period $b=2\pi\theta^2$. If we adhere to *Ansatz* (4.13), this would mean that the Misner space itself should be modified in such a way that its spatial volume would vanish as time θ approaches zero [20]. When calculating then the regularized Hadamard function, one should use a method of images which ought also to be modified accordingly with the fact that the period of the closed spatial direction is time dependent. If we use a most general automorphic field and impose constancy for the frequency of the general solution of the wave equation [20], one is led [20] to a time quantization that unavoidably implies an also strictly zero value for the renormalized stress-energy tensor, everywhere in the whole two-dimensional, superluminal Alcubierre space. The price to be paid [20] for this quantum stability is that the CTC's developing inside the warp bubble and actually the warp bubble itself should never exceed a submicroscopic size near the Planck scale, thus avoiding not just the unwanted Hiscock instability, but also any violation of negative energy-time Ford like [15,16] inequality and hence any unphysical nature of the warp drive. A possible remaining question is whether the CTC's within the bubble might produce new divergences, at least if Hawking's chronology protection conjecture [28] is correct. However, the semiclassical instabilities leading to chronology protection are actually of the kind which are precisely prevented in our above model.

V. SUMMARY AND CONCLUSIONS

Among the achronal pathologies which are present in general relativity and which can be associated with the symmetries of Misner space, we consider in this paper the two-dimensional warp drive with an apparent velocity which is faster than light and whose spaceship interior is multiply connected and therefore nonchronal. After reviewing the geometrical properties of the Alcubierre-Hiscock two-dimensional model, it has been proved that there exists a close connection between warp drives and gravitational kinky topological defects, at least in two dimensions. Generalizing to the standard Finkenstein kinked metric to allow a complete description of the one-kinks, we also generalize the Alcubierre metric in such a way that it can no longer be described in just one coordinate patch.

The geodesic incompleteness of the Alcubierre-Hiscock space at the event horizon for superluminal warp bubbles has been eliminated by first extending the metric beyond the horizon according to the Kruskal procedure and then by an embedding in a three-dimensional Minkowski space. It has been also shown that, if the latter space is provided with topological identifications that correspond to the universal covering of three-dimensional Misner space, then one can convert the interior of the warp spaceship into a multiply connected space with closed timelike curves which is able to behave like a time machine.

The problem of the quantum stability of the two-dimensional, multiply connected warp drive spacetime has been finally considered. Using the three-dimensional Misner embedding as the maximal extension of the two-dimensional warp drive space, it has also been shown that the divergence encountered by Hiscock on the event horizon for the simply connected case is smoothed out, while the apparent horizon becomes a regularized (Cauchy) chronology horizon. We argued as well that the unphysical violation of the negative energy-time inequalities can at the same time be circumvented because the most consistent quantum treatment for dealing with vacuum fluctuations in the multiply connected case leads also to the result that the size of the spaceship bubble and its closed timelike curves must necessarily be placed at scales close to the Planck length.

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