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Gravitational Goldstone fields from affine gauge theory

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In order to facilitate the application of standard renormalization techniques, gravitation should be described, in the pure connection formalism, as a Yang-Mills theory of a certain spacetime group, say the Poincaré or the affine group. This embodies the translational as well as the linear connection. However, the coframe is not the standard Yang-Mills-type gauge field of the translations, since it lacks the inhomogeneous gradient term in the gauge transformations. By explicitly restoring this ''hidden'' piece within the framework of *nonlinear realizations*, the usual geometrical interpretation of the dynamical theory becomes possible, and in addition one can avoid the metric or coframe degeneracy which would otherwise interfere with the integrations within the path integral. We claim that nonlinear realizations provide the general mathematical scheme for the foundation of gauge theories of spacetime symmetries. When applied to construct the Yang-Mills theory of the affine group, tetrads become identified with nonlinear translational connections; the anholonomic metric no longer constitutes an independent gravitational potential, since its degrees of freedom reveal a correspondence to eliminateable *Goldstone* bosons. This may be an important advantage for quantization.

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I. INTRODUCTION

On a macroscopic scale, gravity is empirically rather well described by Einstein's general relativity theory (GR). However, quantum-field theoretically, Einstein's theory is perturbatively *nonrenormalizable* [1,2] and plagued by *anomalies* if coupled to fundamental matter as the Dirac field of an electron; cf. [3,4]. As a matter of fact, gravity is usually conceived as an interaction of a very different nature from the remaining forces, being supposed to be mediated by a metric potential rather than by Yang-Mills connections. Thus, it is reasonable to hope that some of the problems related to the quantization of this force might disappear if one were able to describe gravitation as an ordinary gauge theory. The search for such a formulation yielded in particular the different gauge theories of gravity proposed by Hehl and his Cologne group [5–7]. In all of them, the local treatment of translations reveals itself as a cornerstone of the Yang-Mills-type interpretation of gravitation. As Feynman [8] has put it, '' . . . gravity is that field which corresponds to a gauge invariance with respect to displacement transformations''; cf. [9].

A. Tetrads as nonlinear connections

In a first order formalism, one introduces a *local frame* field (or vielbein) $e_\alpha = e^i{}_\alpha \partial_i$ together with the *coframe* field or one-form basis $\vartheta^\beta = e_j{}^\beta dx^j$, which is dual to the frame e_α with respect to the *interior product*: $e_\alpha \rfloor \vartheta^\beta = \delta^\beta_\alpha$. Quite often, ϑ^α is advocated as the translational gauge potential, although it does *not* transform inhomogeneously under local frame rotations, as is characteristic for a connection. A rigorous explanation of this apparent paradox requires one to invoke a *nonlinear realization* (NLR) of the local spacetime group. Indeed, as we will see below, NLRs provide the necessary general foundation of gauge theories of gravitation constructed from spacetime groups in which translations are present. In this paper, we will demonstrate that ϑ^α actually is the dimensionless nonlinear translational gauge potential in metric-affine gravity (MAG), that is, in the Yang-Mills approach to the affine group. (This quite general group includes the Poincaré group of elementary particle physics as a subgroup.)

As a consequence of the nonlinear treatment, the metric tensor will reveal itself as dynamically irrelevant, no longer playing the role of a gravitational potential, since the degrees of freedom of the MAG metric become rearrangeable into redefined connections. Although this fact is derived here independently, the idea of regarding the metric as the gravitational analogue of the Higgs or Goldstone field is not new. It was first proposed by Isham, Salam, and Strathdee [10] and later on was occasionally discussed, e.g., by Nambu [11], Ne'eman and Regge [12], and Trautman [13]. Various existing models of macroscopic gravity can therefore be reinterpreted as nonlinear-affine (or Poincaré) gauge theories; cf. [14–17]. Later on, Flato and Rączka [18] as well as van der Bij [19] speculated about a *gravitational origin* of the Higgs field and its decoupling.

B. Spacetime metric as a Goldstone boson

The recent paper by Gronwald *et al.* [20] adopts the ''quartet'' of scalar fields introduced by Guendelman and Kaganovich [21–27], originally used in the study of the cosmological constant problem. Gronwald *et al.* apply these fields in a different context, in order to remedy, as far as

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possible, the unsatisfactory fact of the occurrence of the (anholonomic) metric tensor in MAG as a field different from the Yang-Mills ones. As declared in the Introduction by the authors themselves, they look for an alternative way to define a volume element without reference to the metric. We do not enter into the details of such a construction. Nevertheless, we would like to remark that the metric is not much appreciated as a fundamental gravitational potential by the principal supporters of the interpretation of gravity as a gauge theory. And the reason for it is that the gauge-theoretical *status* of the metric tensor is not clear. In other words, the search for a different volume element is motivated by a difficulty inherent to the present understanding of the foundations of MAG itself.

In fact the introduction, in addition to the linear connection and the coframe, of the metric tensor as an independent gravitational potential [7] seems to be contrary to the spirit of a pure gauge approach, where the role of gauge potentials is played exclusively by connections. In standard Yang-Mills theories, no other quantity is required to carry interactions. Contrarily, in the naive *gauge approach* to Einstein's theory as well as MAG, the metric tensor appears as a strange quantity with ten additional degrees of freedom, foreign to the otherwise standard Yang-Mills treatment. Its existence is simply assumed without deriving it from a more fundamental principle.

In our opinion, nonlinear realizations [10] provide not only a different foundation of MAG, but the necessary interpretation of the actually formulated theory in its present form [7]. In fact, most of the features of MAG—such as the vector character of the coframe despite its nature of (translational) connection, as already mentioned; the occurrence of the metric tensor, the freedom to fix it to be Minkowskian without loss of generality, the tensoriallity of the nonmetricity, etc.—are consequences of the nonlinearity, although it has not been recognized by all of the founders.

As we will see, the existence of the MAG metric derives from a particular nonlinear realization of the affine group. According to this interpretation, the metric results in being a set of ten Goldstone fields, rearrangeable in the connections. When rearranged in this way, the metric reduces to a constant Minkowski one, without any dynamical degrees of freedom. Thus, the anholonomic MAG metric tensor is not a genuine gravitational potential. Accordingly, the nonmetricity is not to be interpreted as the corresponding field strength but simply as the connection component associated to the symmetric generators of the general linear group. Under gauge transformations, nonmetricity behaves as a tensor due to the (not immediatly recognizable) underlying nonlinear realization of the affine group.

II. GAUGING SPACETIME GROUPS

As far as internal symmetries are concerned, the definition of gauge transformations as fiber-preserving bundle automorphisms [28] constitutes a satisfactory characterization of them. Accordingly, given a principal fiber bundle $P(M,H)$ with the base space M representing spacetime, a gauge transformation is identical with the action of the structure group H along local fibers, being the spacetime base manifold not being affected by such transformations.

Obviously, this scheme is not applicable to gauge theories of gravitation. In fact, they are theories of the Yang-Mills type based on the gauging of spacetime groups, and precisely these groups are symmetries which affect spacetime itself. Thus, one has to generalize the definition of gauge transformations in order to take account of such external symmetries as well. We follow Lord [29–31], who suppresses the restriction of no action on the base space. According to him, a gauge transformation is a general bundle automorphism, that is, a diffeomorphism that maps fibers to fibers.

The natural framework to define such an action is the following, based on the manifold character of Lie groups and on the simple properties of left and right multiplications of group elements. Let us choose a spacetime group G and a subgroup $H\subset G$. We will construct the gauge theory of G on the principal fiber bundle $G(G/H,H)$, where the group manifold G itself is the bundle manifold, and the subgroup H is taken to be the structure group—different H's may be chosen to play this role; the quotient space G/H will play the role of spacetime. Gauge transformations are defined on this bundle as follows. As mentioned above, the usual definition of (active) gauge transformations as vertical automorphisms along fibers, not affecting spacetime, must be modified to a more general automorphism affecting both vertical fibers and the points of the quotient space G/H the latter are attached to. Since the left and right multiplications of elements of G commute, we have in particular $L_g\circ R_h = R_h\circ L_g$, with $g\in G$, $h\in H$. Thus L_g, acting on fibers defined as orbits of the right action R_h (that is, as left cosets gH), constitutes an automorphism of the kind we are looking for, transforming in general fibers into fibers. To be explicit, we define the left action L_g of G on zero sections $\sigma: G/H\rightarrow G$ as follows:

$$L_g\circ\sigma(\xi) = R_h\circ\sigma(\xi'). \tag{2.1}$$

As observed by Lord [30], this equation coincides with the prescription for *nonlinear transformations* due to Coleman et al. [32]. In accordance with what one expects for spacetime symmetries, a transformation is induced on the quotient space G/H, reflecting the mapping from a fiber to another. Indeed, taking into account that $\pi\circ R_h\circ\sigma = \pi\circ\sigma = id$, from Eq. (2.1) then follows

$$\xi' = \pi\circ L_g\circ\sigma(\xi), \tag{2.2}$$

the fields ξ being *coset fields*, characterized as continuous labels of the elements of the quotient space G/H. In particular, for $G/H\approx R^4$, they provide the translational fields destinated to replace the ''quartet'' of scalar fields of Refs. [20,33]. Actually, in the context of gauge theories of spacetime groups, the ''Poincaré coordinates'' [34] or components [35] of ''Cartan's radius vector'' ξ^α are in fact translational coset fields. In the case of MAG, they turn out to transform as affine covectors resembling coordinates; see Eqs. (3.5) and (4.3) below.

Nonlinear connections

For practical calculational purposes, the fundamental equation (2.1) defining the nonlinear group action may be rewritten in a more explicit form in terms of $g \in G$ and $h \in H$ as

$$g\sigma(\xi) = \sigma(\xi')h(\xi,g), \tag{2.3}$$

or shortly as $\sigma' = g\sigma h^{-1}$. Because of this particular transformation law of σ, from the linear connection $\tilde{\tilde{\Gamma}}$ of G it becomes possible to define the nonlinear connection Γ with suitable transformation properties as follows:

$$\Gamma := \sigma^{-1}(d + \tilde{\tilde{\Gamma}})\sigma. \tag{2.4}$$

Indeed, given the ordinary linear transformation law of the linear connection, $\tilde{\tilde{\Gamma}}$, namely,

$$\tilde{\tilde{\Gamma}}' = g\tilde{\tilde{\Gamma}}g^{-1} + g\, dg^{-1}, \tag{2.5}$$

and the transformation (2.3) of σ in its shortened form $\sigma' = g\sigma h^{-1}$, it follows that the nonlinear connection (2.4) transforms as

$$\Gamma' = h\Gamma h^{-1} + h\, dh^{-1} \tag{2.6}$$

under local transformations. Observe that, according to Eq. (2.6), only the components of Γ defined on the Lie algebra of H transform inhomogeneously as true connections; the remaining components of Γ transform as tensors with respect to H.

The nonlinear connection allows us to construct covariant derivatives (of nonlinear fields) as follows. Consider a field φ transforming linearly under G as $\varphi' = g\varphi$, and let us schematically define a correlated nonlinear field as $\psi := \sigma^{-1}\varphi$. It is trivial to test that ψ transforms under the action of G as $\psi' = h\psi$, that is, as a representation field of the subgroup H. Accordingly, we define the covariant differential

$$D\psi := (d + \Gamma)\psi = \sigma^{-1}(d + \tilde{\tilde{\Gamma}})\varphi, \tag{2.7}$$

behaving as an H-covariant object, namely,

$$(D\psi)' = hD\psi, \tag{2.8}$$

under the left action of the whole group G.

III. GAUGE THEORETICAL ORIGIN OF THE TETRADS

By applying the previous results, taking G to be the *affine group* $A(4,R) := R^4 \rtimes GL(4,R)$, i.e., the semidirect product of translations and general linear transformations, we will show that the nonlinear approach provides the ultimate foundation of MAG. The commutation relations of the affine group are

$$[P_\alpha, P_\beta] = 0,$$

$$[L^\alpha{}_\beta, P_\gamma] = \delta^\alpha_\gamma P_\beta,$$

$$[L^\alpha{}_\beta, L^\gamma{}_\delta] = \delta^\alpha_\delta L^\gamma{}_\beta - \delta^\gamma_\beta L^\alpha{}_\delta. \tag{3.1}$$

Observe that the physical dimensions of the generators of the linear group are $[L^\alpha{}_\beta] = \hbar$, whereas those of translations are $[P_\alpha] = \hbar/\text{length}$.

Let us construct in two steps the fiber bundle descriptions of the spacetime dynamics of $G = A(4,R)$, which we denote $G(G/H_1, H_1)$ and $G(G/H_2, H_2)$, corresponding to two consecutive smaller subgroups as the structure group, namely, $H_1 = GL(4,R)$ and $\mathrm{SO}(1,3) \in H_1 > \mathrm{SO}(1,3) \subset H_1$, respectively. Other choices of H are possible, for instance, $H_3 = SO(3)$ (see Ref. [36]), but this will not be considered here. The occurrence of a certain subgroup H (or H_1, H_2, etc.), on which the action of the total group G becomes projected, is a constitutive feature of NLRs; it should not be confused with *symmetry breaking*. Indeed, in the nonlinear approach the symmetry is not broken, so that alternative choices of the subgroup H are mathematically equivalent. True symmetry breaking requires an additional mechanism involving the ground state of a dynamical theory of fundamental physics. We will not study such mechanisms here.

First we consider the gauge theory of the affine group with the general linear group $H_1 = GL(4,R)$ as a structure group. We will show that the coframe appears in a natural way as a nonlinear translative connection.

Now in the formula (2.3) for the nonlinear group action, we substitute the following quantities: The group elements g of the whole affine group $A(4,R)$ are parametrized as

$$g = e^{i\epsilon^\alpha P_\alpha} e^{i\omega_\alpha{}^\beta L^\alpha{}_\beta} \approx I + i\epsilon^\alpha P_\alpha + i\omega_\alpha{}^\beta L^\alpha{}_\beta, \tag{3.2}$$

where we also indicate the infinitesimal expansion. They act on the zero sections

$$\tilde{\sigma}(\xi) := e^{-i\xi^\alpha P_\alpha}, \tag{3.3}$$

where ξ^α are (finite) coset parameters. We introduce the tilde in order to distinguish Eq. (3.3) from the σ introduced in Eq. (4.1) below. The elements h of the structure group $GL(4,R)$ are taken to be

$$h := e^{iv_\alpha{}^\beta L^\alpha{}_\beta} \approx I + iv_\alpha{}^\beta L^\alpha{}_\beta. \tag{3.4}$$

Using the Campell-Hausdorff formula in Eq. (2.3) with Eqs. (3.2)–(3.4), the variation of the coset parameters ξ^α of Eq. (3.3) and the value of $v_\alpha{}^\beta$ [see Eq. (3.4)] are calculable, resulting in

$$\delta\xi^\alpha = -\omega^\alpha_\beta \xi^\beta - \epsilon^\alpha, \quad v_\alpha{}^\beta = \omega_\alpha{}^\beta. \tag{3.5}$$

Thus we see from Eq. (3.5) that the coset parameters ξ^α transform as affine covectors, as postulated [35] for *Cartan's generalized radius vector*. The nonlinear connection (2.4) will be constructed in terms of the linear one, namely,

$$\tilde{\tilde{\Gamma}} := -i \overset{(T)}{\Gamma}{}^\alpha P_\alpha - i \overset{(GL)}{\Gamma}{}_\alpha{}^\beta L^\alpha{}_\beta, \tag{3.6}$$

which includes the linear translational potential $\overset{(T)}{\Gamma}{}^{\alpha}$ and the $GL(4,R)$ connection $\overset{(GL)}{\Gamma}{}_{\alpha}{}^{\beta}$, whose infinitesimal transformations read

$$\delta \overset{(GL)}{\Gamma}{}_{\alpha}{}^{\beta}=\overset{(GL)}{D}\,\omega_{\alpha}{}^{\beta}, \quad \delta \overset{(T)}{\Gamma}{}^{\alpha}=\overset{(GL)}{D}\,\epsilon^{\alpha}-\omega_{\beta}{}^{\alpha}\overset{(T)}{\Gamma}{}^{\beta}. \quad (3.7)$$

Here $\overset{(GL)}{D}$ denotes the covariant differential constructed from the $GL(4,R)$ connection. Making use of the definition (2.4), we get

$$\tilde{\Gamma}:=\tilde{\sigma}^{-1}(d+\tilde{\tilde{\Gamma}})\tilde{\sigma}=-i\tilde{\vartheta}^{\alpha}P_{\alpha}-i\tilde{\Gamma}_{\alpha}{}^{\beta}L^{\alpha}{}_{\beta}, \quad (3.8)$$

with

$$\tilde{\Gamma}_{\alpha}{}^{\beta}=\overset{(GL)}{\Gamma}{}_{\alpha}{}^{\beta}, \quad \tilde{\vartheta}^{\alpha}:=\overset{(T)}{\Gamma}{}^{\alpha}+\overset{(GL)}{D}\,\xi^{\alpha}. \quad (3.9)$$

As in the case of Eq. (3.3), we denote these objects with a tilde for later convenience. Making use of Eq. (2.6), it is straightforward to prove that, whereas $\tilde{\Gamma}_{\alpha}{}^{\beta}$ transforms as a $GL(4,R)$ connection, the coframe $\tilde{\vartheta}^{\alpha}$ defined as in Eq. (3.9) transforms as a $GL(4,R)$ covector. Explicitly

$$\delta\tilde{\Gamma}_{\alpha}{}^{\beta}=\tilde{D}\omega_{\alpha}{}^{\beta}, \quad \delta\tilde{\vartheta}^{\alpha}=-\omega_{\beta}{}^{\alpha}\tilde{\vartheta}^{\beta}; \quad (3.10)$$

compare with Eq. (3.7). The nonlinear treatment of the affine group thus clarifies how the coframe can be constructed from gauge fields of the Yang-Mills type, in particular those of (3.6). The coset parameters ξ^{α} play the role of Cartan's generalized radius vector of Ref. [37], being not introduced *ad hoc*, since they are constitutive elements of the theory. They mainly contribute to the construction of the translational invariant $\tilde{\vartheta}^{\alpha}=\overset{(T)}{\Gamma}{}^{\alpha}+\tilde{D}\xi^{\alpha}$; the variation of ξ under translations [see Eq. (3.5)] is compensated for by the variation of the translative connection; see Eq. (3.7), and cf. [37]. Since $\xi=\xi^{\alpha}P_{\alpha}$ aquires its values in the ''orbit'' (coset space) $A(n,R)/GL(n,R)\approx R^{n}$, it can be regarded as an affine vector field (or ''generalized Higgs field'' according to Trautman [13]) which ''hides'' [38] the action of the local translational ''symmetry'' $\mathcal{T}(n,R)$. Accordingly, conditions such as $\overset{(T)}{\Gamma}{}^{\alpha}=0$ or $D\xi^{\alpha}=0$ break the translational symmetry. Only in the absence of gravitational interaction can we recover the specially relativistic relation $\vartheta^{\alpha}=d\xi^{\alpha}$ for the coframes (i.e., for the translational nonlinear connections), employed in Ref. [20] in order to derive a ''metric-free'' volume four-form. It is interesting to notice that, in this limit, the fields ξ^{α} play the role of ordinary coordinates; see also Eq. (3.5). In other words, the spacetime manifold of special relativity is a *residual* structure of the nonlinear approach when gravitational forces are switched off.

IV. ORIGIN OF THE METRIC IN MAG

In order to complete the MAG scheme, it only remains to explain the emergence of the metric. Indeed, until now the gauge theoretical origin of the metric tensor which is characteristic for MAG's has not been apparent. Seemingly, a metric with ten additional degrees of freedom will only appear in the gauge theory of the affine group if it is introduced by hand. But this is not exactly true. The next considerations are devoted to show how the metric tensor can be introduced in a deductive way.

Let us consider the second choice of structure subgroup in our bundle approach mentioned above, namely, $G(G/H_{2},H_{2})$ with $G=A(4,R)$, as before, and $H_{2}=SO(1,3)$. We split up the generators $L^{\alpha}{}_{\beta}$ of the general linear transformations as $L^{\alpha}{}_{\beta}=\overset{\circ}{L}{}^{\alpha}{}_{\beta}+S^{\alpha}{}_{\beta}$, $\overset{\circ}{L}{}^{\alpha}{}_{\beta}$ being the Lorentz generators and $S^{\alpha}{}_{\beta}$ those of the symmetric linear transformations. Now we apply the general formula (2.3) with the particular factorization

$$g=e^{i\epsilon^{\alpha}P_{\alpha}}e^{i\alpha^{\mu\nu}S_{\mu\nu}}e^{i\beta^{\mu\nu}\overset{\circ}{L}_{\mu\nu}}, \quad \sigma:=e^{-i\xi^{\alpha}P_{\alpha}}e^{ih^{\mu\nu}S_{\mu\nu}},$$

$$h:=e^{iu^{\mu\nu}\overset{\circ}{L}_{\mu\nu}}. \quad (4.1)$$

Here ϵ^{α}, $\alpha^{\mu\nu}$, and $\beta^{\mu\nu}$ being infinitesimal parameters of the affine group, the transformed coset parameters of σ reduce to $\xi'^{\alpha}=\xi^{\alpha}+\delta\xi^{\alpha}$ and $h'^{\mu\nu}=h^{\mu\nu}+\delta h^{\mu\nu}$; the Lorentz parameters $u^{\mu\nu}$ (the structure group H_{2} being Lorentzian) are also infinitesimal. Let us define

$$r_{\alpha}{}^{\beta}:=(e^{h})_{\alpha}{}^{\beta}:=\delta_{\alpha}{}^{\beta}+h_{\alpha}{}^{\beta}+\frac{1}{2!}h_{\alpha}{}^{\gamma}h_{\gamma}{}^{\beta}+\cdots \quad (4.2)$$

from the coset parameters $h^{\alpha\beta}$ associated with the generators of the symmetric part of $GL(4,R)$; see Eq. (4.1). (In the following, $r^{\alpha\beta}$, rather than the coset parameters $h^{\alpha\beta}$ themselves, will play the fundamental role [39,40].) We find the variations

$$\delta\xi^{\alpha}=-(\alpha_{\beta}{}^{\alpha}+\beta_{\beta}{}^{\alpha})\xi^{\beta}-\epsilon^{\alpha},$$

$$\delta r^{\alpha\beta}=(\alpha^{\alpha}{}_{\gamma}+\beta^{\alpha}{}_{\gamma})r^{\gamma\beta}+u^{\beta}{}_{\gamma}r^{\gamma\alpha}, \quad (4.3)$$

where $\alpha_{\beta}{}^{\alpha}+\beta_{\beta}{}^{\alpha}=\omega_{\beta}{}^{\alpha}$; compare with Eq. (3.5). Since $r^{\alpha\beta}$ is symmetric, the antisymmetric part of the second equation in Eqs. (4.3) vanishes. From this condition we find the explicit form of the nonlinear Lorentz parameter

$$u^{\alpha\beta}=\beta^{\alpha\beta}-\alpha^{\mu\nu}\tanh\left\{\frac{1}{2}\log[r^{\alpha}{}_{\mu}(r^{-1})^{\beta}{}_{\nu}]\right\}, \quad (4.4)$$

which obviously differs from the linear Lorentz parameter $\beta^{\alpha\beta}$. It is precisely the nonlinear $u^{\alpha\beta}$, and not the linear $\beta^{\alpha\beta}$, which is relevant for nonlinear transformations, as becomes evident in Eqs. (4.9), (4.8), and (4.10) below.

In order to define the nonlinear connection, let us first rewrite the linear-affine connection (3.6) as

$$\tilde{\tilde{\Gamma}}:=-i\overset{(T)}{\Gamma}{}^{\alpha}P_{\alpha}-i\overset{(GL)}{\Gamma}{}_{\alpha}{}^{\beta}(S^{\alpha}{}_{\beta}+\overset{\circ}{L}{}^{\alpha}{}_{\beta}). \quad (4.5)$$

Then making use of the definition (2.4), we get

$$\Gamma:=\sigma^{-1}(d+\tilde{\tilde{\Gamma}})\sigma=-i\vartheta^{\alpha}P_{\alpha}-i\Gamma_{\alpha}{}^{\beta}(S^{\alpha}{}_{\beta}+\overset{\circ}{L}{}^{\alpha}{}_{\beta}), \quad (4.6)$$

with the nonlinear $GL(4,R)$ connection $\Gamma_\alpha{}^\beta$ and the nonlinear translational connection ϑ^α, respectively, defined as

$$\Gamma_\alpha{}^\beta := (r^{-1})_\alpha{}^\gamma \overset{(GL)}{\Gamma}{}_\gamma{}^\lambda r_\lambda{}^\beta - (r^{-1})_\alpha{}^\gamma d r_\gamma{}^\beta,$$

$$\vartheta^\alpha := r_\beta{}^\alpha \left(\overset{(T)}{\Gamma}{}^\beta + \overset{(GL)}{D} \xi^\beta \right). \tag{4.7}$$

We identify the components of $\Gamma_\alpha{}^\beta$ and ϑ^α of the (nonlinear) Yang-Mills connections of the affine group with the *geometrical* linear connection and with the coframe, respectively. Thus Eqs. (4.7) establish the correspondence between the *geometrical* objects on the left-hand side (LHS) and the dynamical objects on the RHS. We find that the connection behaves as a Lorentz connection

$$\delta \Gamma_\alpha{}^\beta = D u_\alpha{}^\beta, \tag{4.8}$$

with the nonlinear Lorentz parameter (4.4). The covariant differential D in Eq. (4.8) is constructed in terms of the Lorentz connection itself. On the other hand, the coframe transforms as a Lorentz covector

$$\delta \vartheta^\alpha = - u_\beta{}^\alpha \vartheta^\beta. \tag{4.9}$$

As repeatedly mentioned, this constitutes a main result of the nonlinear approach.

Notice that, in view of the splitting of the general linear generators into a Lorentz plus a symmetric part as $L^\alpha{}_\beta = \overset{\circ}{L}{}^\alpha{}_\beta + S^\alpha{}_\beta$, the connection is actually composed of two parts, defined on different elements of the Lie algebra. In fact, only the antisymmetric part, defined on the Lorentz generators, behaves as a true connection of the Lorentz group playing the role of the structure group H_2. The symmetric part $\Gamma_{(\alpha\beta)} =: \frac{1}{2} Q_{\alpha\beta}$, i.e., the nonmetricity, is tensorial. Actually,

$$\delta Q_{\alpha\beta} = 2 u_{(\alpha}{}^\gamma Q_{\beta)\gamma}. \tag{4.10}$$

On the other hand, the structure group H_2 being the Lorentz group, the Minkowski metric $o_{\alpha\beta}$ emerges automatically in the theory as a natural invariant: $\delta o_{\alpha\beta} = 0$. Thus, a metrization of the affine theory occurs as a consequence of the nonlinear treatment—due to the particular choice of a (pseudo-)orthogonal group as the structure group, so that the corresponding Cartan-Killing metric becomes apparent. However, no degrees of freedom are related to the Minkowski metric. This seemingly makes a difference between the dynamical content of our theory and that of ordinary MAG, since in the latter the metric tensor involves ten degrees of freedom. Nevertheless, we will see immediately how these degrees of freedom, being of Goldstone nature, can be taken from the nonlinear connections where they are hidden. Actually, the Goldstone fields which will manifest themselves as the degrees of freedom of the MAG metric are those of the matrix $r^{\alpha\beta}$ defined in Eq. (4.2). They can be factorized into nonlinear connections and coframes, as shown in Eq. (4.7), in the presence of the Minkowskian metric we are discussing, or alternatively they can be explicitly displayed in the metric tensor, as we will show in Eq. (4.13) below. In this case, the metric becomes identical to the ordinary MAG metric.

In order to show how the transition between these alternative formulations takes place, we establish a correspondence between the objects of both choices H_1 and H_2 studied above. Formally, we find that this correspondence is isomorphic to a finite gauge transformation, with the matrix $r^{\alpha\beta}$ of Eq. (4.2) standing for the symmetric affine transformations. But $r^{\alpha\beta}$ is not a transformation matrix; it is constructed in terms of coset fields. The relation between Eqs. (3.9) and (4.7) reads

$$\tilde{\Gamma}_\alpha{}^\beta := \overset{(GL)}{\Gamma}{}_\alpha{}^\beta = r_\alpha{}^\gamma \Gamma_\gamma{}^\lambda (r^{-1})_\lambda{}^\beta - r_\alpha{}^\gamma d (r^{-1})_\gamma{}^\beta \tag{4.11}$$

and

$$\tilde{\vartheta}^\alpha := \overset{(T)}{\Gamma}{}^\alpha + \overset{(GL)}{D} \xi^\alpha = (r^{-1})_\beta{}^\alpha \vartheta^\beta. \tag{4.12}$$

The standard metric-affine objects of ordinary MAG, such as connections and coframes (up to the metric), are identical to those with a tilde on the LHS of Eqs. (4.11) and (4.12), studied in Sec. III, corresponding to a nonlinear realization of the affine group with $H_1 = GL(4,R)$ as the structure group. In the approach studied in Sec. III, the metric tensor was absent. However, in analogy to Eqs. (4.11) and (4.12), it can be introduced as an object with a tilde related to the Minkowski metric $o_{\alpha\beta}$ which appears in the case of $H_2 = SO(1,3)$ studied in Sec. IV. Actually, we define $\tilde{g}_{\alpha\beta}$ from $o_{\alpha\beta}$ as

$$\tilde{g}_{\alpha\beta} := r_\alpha{}^\mu r_\beta{}^\nu o_{\mu\nu}. \tag{4.13}$$

The resulting MAG-metric tensor plays the role of a *Goldstone field* (cf. [32]), which drops out after applying the inverse of the ‘‘gauge transformation’’ (4.13). By also inverting Eqs. (4.11) and (4.12), one reaches the nonlinear realization studied in Sec. IV, with the Lorentz group as the structure subgroup. This completes the correspondence between the nonlinear objects and those of the framework of metric-affine theory. As a consequence, observe that invariants such as the line element may be alternatively expressed in terms of the Lorentz-nonlinear or metric-affine objects, respectively: namely, as

$$ds^2 = o_{\alpha\beta} \vartheta^\alpha \otimes \vartheta^\beta = \tilde{g}_{\alpha\beta} \tilde{\vartheta}^\alpha \otimes \tilde{\vartheta}^\beta, \tag{4.14}$$

where the transition from $o_{\alpha\beta}$ to $\tilde{g}_{\alpha\beta}$ or vice versa takes place by means of the suitable factorization of the coset parameters associated with the symmetric affine transformations. The gauge-theoretical origin of the metric tensor in the MAGs is thus explained. Moreover, given a standard MAG, if one fixes the metric to be globally Minkowskian, the degrees of freedom of the theory automatically rearrange themselves into the nonlinear theory developed in Sec. IV, with the Lorentz group as the structure group.

Because of the transformation law (4.3) of $r^{\alpha\beta}$, which involves both general linear and Lorentz parameters, the indices of objects with a tilde behave as general linear indices; those of objects without a tilde are Lorentz indices. In the second case, the ten degrees of freedom corresponding to $r^{\alpha\beta}$ are rearranged into the coframe and connections, so that none of them remains in the metric tensor, which becomes Minkowskian. An action which is invariant under affine transformations can be alternatively expressed in terms of $GL(4,R)$ or $SO(1,3)$ tensors, respectively. This corresponds to the choice of variables with or without a tilde, as discussed above.

V. OUTLOOK: DYNAMICAL ORIGIN OF THE SIGNATURE?

Concerning the *signature* of the metric parametrized via $o_{\alpha\beta}:=\mathrm{diag}(e^{i\theta},1,1,1)$ (cf. [41]), the nonlinear approach is particularly adapted for dealing with *spontaneous symmetry breaking*. In fact, the Higgs mechanism can be understood as a way to select a particular structure group H by fixing the Goldstone fields in terms of suitable fields of the theory; see Ref. [42]. Thus, symmetry breaking could give a fundamental physical meaning to a particular structure subgroup H, fixing it dynamically.

Previously to the symmetry breaking, the choices of different structure groups H are physically equivalent in the sense that they simply provide alternative ways to rearrange the degrees of freedom of the total gauge group G. In particular, in the gauge theory of the affine group, in the absence of symmetry breaking one can freely choose the structure subgroup H to be Lorentz group or $SO(4)$, etc., so that the corresponding metric signatures become the Minkowskian or the Euclidean one, respectively. They constitute alternative realizations of the same theory, since the symmetry under the total group G is the only relevant one.

In quantum field theory, the Minkowskian or Euclidean signature is, however, quite different. Usually in the path integral approach, the Euclidean signature is chosen in order to have a well-defined measure. Moreover, tunneling between different topologies of instanton configurations may occur. (After applying a Wick rotation with $e^{i\pi}=-1$, the physical measurable quantities are regained.) In the path integral approach to quantum gravity, a summation over all *inequivalent* coframes and connections, and even topology [43], is understood. This summation will also involve *degenerate* ($\det e_j{}^{\beta}=0$) or even vanishing coframes; cf. [44]. Macroscopically, this would imply the breakdown of any length measurement performed by means of the metric (4.14). Microscopically, then also signature changes of the metric are to be admitted; cf. Refs. [45,46]. These conceptual difficulties [1] are not encountered in the quantization of internal Yang-Mills theories on a *fixed* spacetime background.

Degenerate coframes, however, tend to jeopardize the coupling of gravity to matter fields, as exemplified by Dirac or Rarita-Schwinger fields; cf. [47]. The basic reason is that the local frame e_α, even if it still exists, is not invertible any more; i.e., the relation $e_\alpha \rfloor \vartheta^\beta=\delta^\beta_\alpha$, which is needed in the formulation of matter Lagrangians, would then be lost.

These arguments seems to require the introduction of a symmetry-fixing mechanism which dynamically differentiates a particular structure group H and, thus, the signature. In other words, it remains to be seen if also the signature of the physical spacetime has a dynamical origin in such a framework, as suggested by Sakharov [48] and Greensite [49], or arises naturally in string or M theory [50,51,52].

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