

Entropy of gravitons produced in the early universe

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Gravitons produced from quantum vacuum fluctuations during an inflationary stage in the early Universe have zero entropy as far as they reflect the time evolution (squeezing) of a pure state, their large occupation number notwithstanding. A nonzero entropy of the gravitons [classical gravitational waves (GW) after decoherence] can be obtained through coarse graining. The latter has to be physically justified *and* should not contradict observational constraints. We propose two ways of coarse graining for which the fixed temporal phase of each Fourier mode of the GW background still remains observable: one based on quantum entanglement, and another one following from the presence of a secondary GW background. The proposals are shown to be mutually consistent. They lead to the result that the entropy of the primordial GW background is significantly smaller than it was thought earlier. The difference can be ascribed to the information about the regular (inflationary) initial state of the Universe which is stored in this background and which reveals itself, in particular, in the appearance of primordial peaks (acoustic peaks in the case of scalar perturbations) in the multipole spectra of the CMB temperature anisotropy and polarization.

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I. INTRODUCTION

What is the entropy of primordial fluctuations from which all inhomogeneities in our Universe are assumed to originate? This fundamental problem becomes especially intriguing in the framework of the inflationary paradigm. Indeed, in that case primordial fluctuations are generated from vacuum quantum fluctuations. In this work, we shall concentrate on a primordial gravitational-wave (GW) background in inflationary cosmological models, although a similar discussion can be applied to scalar perturbations in such models as well. It is well known that in this case the unitary evolution of the quantum state of the fluctuations outside the Hubble radius leads to an effective quantum-to-classical transition of a very specific kind: the fluctuations become indistinguishable from classical fluctuations with stochastic amplitude and fixed temporal phase once the so-called decaying mode of the perturbations is neglected [1].

The approximation of simply omitting the decaying mode and then introducing the equivalent classical stochastic fluctuation is sufficient for calculating correctly the amplitudes of observable inhomogeneities in the Universe. It does not require any explicit account of interaction with an environment. In particular, inflationary predictions for the amplitude, statistics and quasiclassical temporal behavior of perturbations are independent of this interaction (at least, for sufficiently large scales). Note that the possibility to get a decoherence-independent prediction for observable quantities (inhomogeneities) is a specific property of the very mechanism of generation of inhomogeneities in the inflation-

ary scenario of the early Universe, and does not occur for general quantum systems, as has been shown in Ref. [1] with the help of the Wigner function and in Refs. [2,3] by a discussion of thought experiments involving slits. Also, it may be attributed to the fact [4] that amplitudes of perturbations constitute an (almost ideal) pointer basis in this case. Finally, this approximation fits well into the consistent-“histories” framework: as the decaying mode becomes negligible, probabilities can be consistently assigned to classical trajectories (“histories”) of perturbation modes in phase space [5].

However, this does not mean that this approximation is sufficient for the calculation of all quantities. In the present paper we consider one important (though not directly measurable) quantity for which it is definitely insufficient and for which one has to consider actual mechanisms of environment-induced decoherence—the entropy of cosmological inhomogeneities. The entropies of an initial quantum perturbation in a pure strongly squeezed state and of its (approximate) equivalent (a classical perturbation with a stochastic amplitude, but a fixed phase) are in fact both zero. It is, however, clear that, as a result of an effective coarse graining produced by environment-induced decoherence, a pure quantum state becomes mixed, while the fixed classical phase of the classical system acquires some small stochastic part, so a nonzero entropy should arise in both quantum and effective classical descriptions of perturbations.

The main question, then, is as follows: what is the correct coarse graining? Any coarse graining introduced by hand must be justified by some concrete physical mechanism. Also, it must be in agreement with present and future obser-

vational constraints. Indeed, as was already emphasized in Ref. [6], the coarse graining that one chooses for scalar (adiabatic) perturbations can be crucial with respect to the appearance, or not, of periodic oscillations—acoustic peaks—in the cosmic microwave background (CMB) anisotropy multipoles C_l . The same refers to the primordial GW background where the corresponding periodic oscillations (with a different period) are more easily seen in the CMB polarization multipoles C_{pl} [1,7] (in this case, it is more appropriate to call them primordial peaks).

In earlier papers [8] where the entropy of cosmological perturbations was considered, an averaging procedure was performed with respect to the squeezing angle φ_k of each Fourier mode, leading to the entropy $S_k = 2r_k$ per mode, where r_k is the squeezing parameter. Such a “high” entropy would, however, spoil the generic prediction of the inflationary scenario about the dominance of the quasi-isotropic (or growing, in the case of scalar perturbations) mode at the moment of the last Hubble-radius crossing, which represents the equivalent classical expression of the remaining coherence between the field-mode amplitude and its momentum. As was shown in Ref. [3], the latter coherence persists long after the loss of quantum correlation between quasi-isotropic and decaying modes of perturbations (and may survive even up to the present moment for sufficiently large perturbation wavelengths). So, in the case of cosmological perturbations generated during inflation, the quantum decoherence sufficient for the classical description of the perturbations occurs long before the time of complete relaxation (if this relaxation occurs at all).

It is just this property that leads to the appearance of the above-mentioned peaks in the CMB fluctuation spectra. The advent of high-precision cosmological observations in general, and accurate measurements of the CMB fluctuations in particular, has dramatic consequences on this problem, which seemed remote only a few years ago: the existence of peaks is incompatible with *complete* randomization of the temporal phase of the cosmological fluctuations and severely restricts the choice of possible coarse grainings.

Since, as is well known, the equations for small scalar (adiabatic) perturbations and GW (tensor) perturbations superimposed on a Friedmann-Robertson-Walker (FRW) background can be reduced to the equation of a real minimally coupled scalar field $\phi(\mathbf{r}, t)$, massless in the case of GW, while there is a mass term and a sound velocity in the case of adiabatic perturbations, we shall further consider this auxiliary field. The FRW background is assumed to be spatially flat:

$$ds^2 = dt^2 - a^2(t)dl^2, \quad (1)$$

where dl^2 is the three-dimensional Euclidean interval ($c = \hbar = 1$ is assumed throughout the paper). The quantum field $\hat{\phi}$ can then be decomposed into Fourier harmonics

$$\hat{\phi}(\mathbf{k}) = (\phi(\mathbf{k}, t)\hat{a}(\mathbf{k}) + \phi^*(-\mathbf{k}, t)\hat{a}^\dagger(-\mathbf{k}))e^{i\mathbf{k}\mathbf{r}} = \hat{\phi}^\dagger(-\mathbf{k}), \quad (2)$$

with $\hat{a}(\mathbf{k})$ and $\hat{a}^\dagger(\mathbf{k})$ being the usual annihilation and creation operators in the Fock space, respectively.

Let us take a two-mode system $\hat{\phi}(\mathbf{k}), \hat{\phi}(-\mathbf{k})$. This system exhibits a semiclassical behavior by itself soon after the first Hubble-radius crossing during an inflationary stage due to the disappearance of the decaying mode (in the Heisenberg representation), or equivalently due to large squeezing (in the Schrödinger representation). More precisely, once the wavelength of the mode $\lambda = a(t)/k$, $k \equiv |\mathbf{k}|$ is much bigger than the Hubble radius $R_H = H(t)^{-1}$, $H \equiv \dot{a}/a$, we can make $\phi(\mathbf{k}, t) = \phi(k, t)$ real by a time-independent phase rotation. Then, after the omission of the decaying mode, the quantum mode operator (2) becomes equivalent to the Gaussian stochastic quantity [1]

$$\phi(\mathbf{k}) = \phi^*(-\mathbf{k}) = c(\mathbf{k})\phi(k, t)e^{i\mathbf{k}\mathbf{r}}, \quad (3)$$

where $c(\mathbf{k}) = c^*(-\mathbf{k})$ is a complex Gaussian stochastic field with the following nonzero correlations:

$$\langle c(\mathbf{k})c^*(\mathbf{k}') \rangle = \delta(\mathbf{k} - \mathbf{k}'), \quad \langle c(\mathbf{k})c(\mathbf{k}') \rangle = \delta(\mathbf{k} + \mathbf{k}'). \quad (4)$$

The stochastic field (3) has still zero entropy since its volume in phase space is equal to zero. However, interactions with other fields, even tiny ones, are unavoidable. This is a first physical process which can lead to coarse graining. It occurs on very short decoherence time scales, rendering the field-amplitude basis a classical “pointer basis” [4,2]—the robust basis with respect to the environment [9,10]. Once $\lambda \gg R_H$, the dynamical influence of such environmental fields should be negligible since causal processes are prohibited. In particular, the quasi-isotropic mode is not affected. However, for decoherence it is just sufficient to destroy the quantum correlation between the decaying and quasi-isotropic parts of perturbations. Since the decaying mode is “local” (in the sense that it may be affected by local processes), decoherence may be easily achieved by coarse graining of quantum entanglement between (the decaying mode of) perturbations and the environment. As was pointed out above, the relaxation time scale exceeds by far the decoherence time scale (the latter being of the order of H^{-1} at the first Hubble radius crossing [2]). Since the maximum entropy $2r_k$ is associated with the relaxation time scale, the actual entropy arising from entanglement, although nonvanishing, should be much smaller.

In the following we shall consider two different coarse grainings which are physically relevant. First, a realistic situation is considered where interaction with environmental fields is efficient in the suppression of off-diagonal terms in the density matrix, but does not produce any significant back reaction (Sec. II). Since it appears that details of the interaction are not relevant, we model its effect through the introduction of a phenomenological parameter suppressing the off-diagonal terms. Second, the entropy growth due to the loss of information about the primordial GW background which occurs as a result of the appearance of a secondary GW background after the second Hubble radius crossing is calculated in Sec. III. This effect arises because we cannot distinguish one kind of gravitons from another at the present

epoch. So, in the first case a nonzero entropy arises because the primordial GW background is directly affected by an environment, while in the second case it is simply “polluted” by a secondary GW background emitted by some environment. It will be shown that both coarse grainings yield in general S_k significantly smaller than $2r_k$. Section IV contains conclusions and discussion, including the relation of this problem to the problems of entropy growth in open systems and in the Universe.

II. ENTROPY DUE TO COARSE GRAINING OF QUANTUM ENTANGLEMENT

Let us consider a squeezed vacuum state which describes the behavior of primordial fluctuations in the absence of interactions with the environment. It has the form

$$\psi_{k0} = \left(\frac{2\Omega_R}{\pi} \right)^{1/2} \exp(-[\Omega_R + i\Omega_I]|y_k|^2), \quad (5)$$

where y_k is a Fourier component of the rescaled quantum field $y(\mathbf{r}, t) = a(t)\phi(r, t)$. Since modes with different wave vectors \mathbf{k} decouple, we shall sometimes skip the index \mathbf{k} (or k) in the following. The wave function (5) can be written in terms of the squeezing parameters r and φ in the form

$$\psi_0 = \left(\frac{2k}{\pi[\cosh 2r + \cos 2\varphi \sinh 2r]} \right)^{1/2} \times \exp\left(-k \frac{1 - e^{2i\varphi} \tanh r}{1 + e^{2i\varphi} \tanh r} |y|^2 \right). \quad (6)$$

Squeezing can equivalently be expressed in terms of “particle creation” with average particle number $N(\mathbf{k}) \approx e^{2r}/4$. We are interested in the regime after the first Hubble-radius crossing (occurring during an inflationary stage) when $N(\mathbf{k}) \gg 1$.

To calculate the entropy it is sufficient to consider one of the two modes contained in the complex amplitude $y(\mathbf{k}, t) = y^*(-\mathbf{k}, t)$ (in the following, y is a real function). The entropy is additive for the various modes. The density matrix corresponding to the pure state (5) is given in the field-amplitude representation by

$$\rho_0(y, y') = \sqrt{\frac{2\Omega_R}{\pi}} \exp\left(-\frac{\Omega_R}{2}(y - y')^2 - i\Omega_I(y - y')(y + y') - \frac{\Omega_R}{2}(y + y')^2 \right). \quad (7)$$

The coefficient in front of $(y - y')^2$ is a measure of quantum coherence (size of nondiagonal elements), while the coefficient in front of $(y + y')^2$ is related to $(\Delta y)^2$ and measures the extension in configuration space of a fictitious ensemble described by the density matrix (details can be found in Appendix A2.3 of [10]). Since for large squeezing Ω_R becomes very small, both coherence and extension in phase space become large. This coherence cannot be distinguished from a classical random process, see for instance the thought experiment involving slits in Refs. [2,3,5]. Coupling

to environmental degrees of freedom [4,2] will decrease the coherence length in y , i.e., increase the coefficient in front of $(y - y')^2$. Although details may be very complicated, a very good approximation is to multiply ρ_0 by a Gaussian factor which suppresses off-diagonal elements but leaves diagonal elements (probabilities) untouched. This corresponds to the situation where dynamical back reaction is small (“ideal measurement”). Then, instead of Eq. (7), the following density matrix is obtained:

$$\rho_\xi(y, y') = \rho_0(y, y') \exp\left(\frac{-\xi}{2}(y - y')^2 \right), \quad (8)$$

where $\xi \gg \Omega_R$. For large squeezing, this condition reads

$$\frac{\xi e^{2r}}{k} \gg 1. \quad (9)$$

In the following, it is referred to as the *decoherence condition*.

To study correlations that might be present between y and (the Fourier transform of) the momentum p , it is preferable to calculate the Wigner function. This is found to be

$$W_\xi(y, p) = \frac{1}{\pi} \exp\left(-2 \frac{(p + \Omega_{Iy})^2}{(\Omega_R + \xi)} - 2\Omega_{Ry^2} \right). \quad (10)$$

The variances Δy and $\Delta(p - p_{cl})$ can be found from the extension of the corresponding contour ellipse which describes the extension of the (apparent) ensemble. This ellipse becomes a circle in the vacuum case ($r=0, \xi=0$), if we use the axes $p/k, y$. It will be convenient to introduce the variables $\tilde{y} \equiv 2\sqrt{k}y, \tilde{p} \equiv (2/\sqrt{k})p$, and we denote the major half axis of the Wigner ellipse in the \tilde{y}, \tilde{p} plane by α and the minor half axis by β . In the absence of environment ($\xi=0$), one has $\alpha_0 = e^r, \beta_0 = e^{-r}$ [1]. For $\xi \neq 0$ one generally finds rather complicated expressions, from which simple expressions are recovered in the limit of large squeezing. One finds, for $e^{2r} \rightarrow \infty$,

$$\alpha \approx e^r, \quad \beta \approx \sqrt{\xi/k} \gg \beta_0. \quad (11)$$

While the size of the major half axis remains the same, the size of the minor half axis becomes much bigger than before due to decoherence.

To preserve a correlation between y and p one has to demand that α remains much bigger than β , or

$$\frac{\xi}{k e^{2r}} \ll 1. \quad (12)$$

We shall refer to (12) as the *correlation condition*. The surface of the ellipse (divided by π) is $A = \alpha\beta = e^r \sqrt{\xi/k}$. Thus, one would expect the following relation between entropy and volume in phase space:

$$S \approx \ln A \approx (1/2) \ln(e^{2r} \xi/k). \quad (13)$$

This is what we shall demonstrate in the following.

The von Neumann entropy connected with the density matrix (8) is given by

$$S = -\text{Tr}(\rho \xi \ln \rho \xi). \quad (14)$$

It was calculated in Ref. [11] for an arbitrary Gaussian density matrix, see also Appendix A2.3 in Ref. [10]. The result is

$$S = -\ln p_0 - \frac{q}{p_0} \ln q, \quad (15)$$

where

$$p_0 = \frac{2\sqrt{\Omega_R}}{\sqrt{\Omega_R + \xi} + \sqrt{\Omega_R}}, \quad q = \frac{\sqrt{\Omega_R + \xi} - \sqrt{\Omega_R}}{\sqrt{\Omega_R + \xi} + \sqrt{\Omega_R}}. \quad (16)$$

Using $e^r \rightarrow \infty$ and $\xi \gg \Omega_R$, one finds

$$S \approx 1 - \ln 2 + \frac{1}{2} \ln \frac{e^{2r} \xi}{k} = 1 + \frac{1}{2} \ln \frac{N \xi}{k}, \quad (17)$$

in accordance with the expectations $S \approx \ln A$. Applying the decoherence condition (9), one finds

$$S \gg 1 - \ln 2 \approx 0.31, \quad (18)$$

where \gg holds here in a logarithmic sense (it directly holds for the number of states e^S). Note that this lower bound on the entropy corresponds to the loss of less than one bit of information. This is consistent with previously known results on decoherence in quantum mechanics. For example, in recent quantum-optical experiments [12], decoherence starts if, on average, one photon is lost. Thus, it may be expected that a minimal entropy $S_{\min} \approx \ln 2$ per mode would be sufficient to guarantee decoherence in the present case, too.

Applying the correlation condition (12), we get

$$S \ll 2r, \quad (19)$$

where again \ll holds in a logarithmic sense. It is evident that the entropy must be much smaller than the maximum value $2r$ which is found by integrating over the squeezing angle [8] (and which would just mean $\alpha = \beta \approx e^r$ for the Wigner ellipse).

Note that, though our initial formula $S = \ln A$ (also proposed earlier in Ref. [6]) is the same as the one introduced by Rothman and Anninos [13], our final result (19) is drastically different from the conclusion of the recent paper by Rothman [14] that this formula for the entropy leads to results identical to those obtained in Ref. [8]. The reason for this difference is evidently the fact that in Refs. [13,14], the total phase-space volume of a state with a given energy was calculated. This does not properly account for the squeezed nature of the Wigner ellipse in our case.

Although details about ξ may be complicated, it is natural to expect that the coherence length $\xi^{-1/2}$ is not smaller than the width of the ground state for $r=0$, at least during the inflationary stage, so that the quantum state really remains squeezed in some direction as compared to the ground

vacuum state. This means that $\xi < k$ and consequently $S < r$. Therefore, pure decoherence (without dynamical influence) can never totally smear out the Wigner ellipse to achieve $S \approx S_{\max} = 2r$. For complete randomization, one would thus have to invoke, for example, a thermal bath at a sufficiently high temperature. Such a model was discussed in Ref. [15], and their results seem to be consistent with our general treatment. Conditions similar to Eqs. (9) and (12) have been frequently discussed in quantum mechanics, see, e.g., Eqs. (6.32)–(6.36) in Ref. [16].

III. ENTROPY DUE TO A SECONDARY GW BACKGROUND

Let us consider now a different coarse graining due to the presence of a secondary GW background which is generated by different matter sources in a causal way after a given mode of the fluctuations has re-entered the Hubble radius [17,18]. For primordial GW, as shown in Ref. [3], the coherence with respect to the squeezing angle is maintained for a considerable time even after the second Hubble-radius crossing. On the other hand, the secondary GW background is expected to have a uniformly distributed phase. It is natural, and rather general, to assume that a \mathbf{k} -mode of this secondary background is described by a density matrix $\rho^s(\mathbf{k})$ which is diagonal in the occupation number basis, $\rho^s = \sum_n w_n |n\rangle\langle n|$ (here the argument \mathbf{k} is omitted, and $n = 0, 1, \dots$). This means in particular complete randomization of the temporal phase, or the absence of a preferred direction in phase space. As gravitons of the primary (squeezed) background are *indistinguishable* from those belonging to the secondary background, we expect some information loss when both backgrounds are mixed. The mean occupation number

$$n(\mathbf{k}) = \sum_{n=0}^{\infty} n w_n, \quad (20)$$

though expected to be significantly lower than that of the primary background $N(\mathbf{k})$, may nevertheless be large, too. We shall show that, loosely speaking, ρ^s corresponds to Eq. (8) with

$$\xi \approx \frac{16}{e^2} k n(\mathbf{k}). \quad (21)$$

Let us estimate the corresponding entropy. We consider the typical volume Γ in (half of) phase space occupied by the system. For the isolated primary background, this volume Γ_0 is the minimal one which corresponds to zero entropy. The secondary background occupies a much larger volume which corresponds to circles in phase space with the approximate radius $\sqrt{\langle y^s y^{s\dagger} \rangle}$, where

$$k \langle y^s y^{s\dagger} \rangle = n(\mathbf{k}) + \frac{1}{2}. \quad (22)$$

Here the averaging process involves also time averaging in addition to quantum average. Therefore at any time, the width of Γ in the squeezed direction of the primary background will be given by the radius of the secondary background, while the elongated direction of the primary back-

ground remains dominated by the primary background. However, since the vacuum part $1/2$ should not be counted twice (it has been already counted and “squeezed” in the primordial part of the mode), only newly created gravitons $n(\mathbf{k})$ produce an additional noise in the initially squeezed direction in phase space. If $N(\mathbf{k})n(\mathbf{k}) \gg 1$, then Γ , the volume in phase space occupied by the whole system, can be estimated as follows:

$$\Gamma^2 \simeq \frac{1}{4} \alpha_0^2 n(\mathbf{k}) = N(\mathbf{k})n(\mathbf{k}) \gg \Gamma_0^2 = \frac{1}{16}, \quad (23)$$

where Γ_0 is the phase-space volume of the initial vacuum state. The corresponding entropy $S(\mathbf{k})$ can be estimated as

$$\begin{aligned} S(\mathbf{k}) &= \ln \frac{\Gamma}{\Gamma_0} = r_k + \ln 2 + \frac{1}{2} \ln n(\mathbf{k}) \\ &= r_k + \ln 2\pi - \frac{1}{2} \ln \frac{\omega^4}{\epsilon_c} + \frac{1}{2} \ln \Omega^s, \end{aligned} \quad (24)$$

where we have used the physical quantity $\Omega^s(\omega) \equiv \omega/\epsilon_c d\epsilon^s/d\omega = \omega^4 n_\omega/\pi^2 \epsilon_c$ ($\omega = k/a_0$) which describes the secondary stochastic background, while ϵ_c denotes the critical density. Comparing Eq. (24) with Eq. (17), the factor $16/e^2$ is found in Eq. (21). It follows that the condition $N(\mathbf{k})n(\mathbf{k}) \gg 1$ used above corresponds to the decoherence condition (9), while the condition that the secondary background is much smaller than the primordial one, $N(\mathbf{k}) \gg n(\mathbf{k})$, corresponds to the correlation condition (12).

We see from Eq. (24) that $S(\mathbf{k}) < 2r_k$ as long as $n(\mathbf{k}) \ll N(\mathbf{k})$, and also $S(\mathbf{k}) \gg S^s \approx \ln n(\mathbf{k})$ [for $n(\mathbf{k}) \gg 1$]. On large cosmological scales, we have $N(\mathbf{k}) \sim 10^{100}$, $r_k \sim 115$, and we have approximately for most models $N(\mathbf{k}) \propto k^{-4}$ for 10^{-16} Hz $\ll \nu = \omega/2\pi \ll 10^{10}$ Hz. We expect that very little entropy is produced by this mechanism on large scales, hence $S(\mathbf{k}) \ll 2r_k$ for these scales.

Several possibilities can be distinguished now.

(1) $n(\mathbf{k}) \ll 1$.

This case, certainly plausible on very large cosmological scales, yields $S(\mathbf{k}) \ll r_k$.

(2) $n(\mathbf{k}) \approx 1$.

Now we have

$$S(\mathbf{k}) \approx r_k = \frac{1}{2} S_{\max}. \quad (25)$$

Note, however, that $n(\mathbf{k}) \ll N(\mathbf{k})$ and we still have a significant squeezing in phase space. Therefore, the standing-wave behavior of the primordial GW still has definite observational consequences.

(3) $1 \ll n(\mathbf{k}) \ll N(\mathbf{k})$.

In that case we have $r_k < S(\mathbf{k}) < 2r_k$. However, like for the preceding case, the system remains highly squeezed. Therefore, with respect to observations, this case is very similar to the preceding ones.

(4) $n(\mathbf{k}) \gg N(\mathbf{k})$.

Now, the squeezing of the primordial background is not apparent anymore in observations. When $n(\mathbf{k}) \gg N(\mathbf{k})$, the squeezing is completely “washed out.”

So, we have here another concrete coarse graining of the primordial GW stochastic background. For all cases for which $1 \ll n(\mathbf{k}) \ll N(\mathbf{k})$, which is a reasonable assumption on large cosmological scales, the resulting entropy is thus significantly smaller than $2r_k$. The crucial point is that a value $r_k < S(\mathbf{k}) < 2r_k$ is still fundamentally different from $S(\mathbf{k}) = S_{\max} = 2r_k$. In the first case, the composite system of both backgrounds can still reflect the squeezing of the primordial GW background, while it does not in the second case. The peaks in the B-mode polarization of the CMB produced by the total GW background would be absent in the second case.

IV. CONCLUSIONS AND DISCUSSION

We have shown that both methods of coarse graining proposed yield for the entropy per mode the result $S(\mathbf{k}) \ll 2r_k$ (in the logarithmic sense), which differs strongly from the results obtained earlier [8]. The reason is that our coarse grainings do not destroy the classical correlation (the almost deterministic temporal phase of the GW and of the adiabatic perturbations) remaining after quantum decoherence, which, as physical estimates show [3], persists for a rather long time (very long in the case of GW) after the perturbations have reentered the Hubble radius. Thus, though the maximal entropy per mode $S(\mathbf{k}) = 2r_k$ might eventually be reached after complete relaxation, it is not reached at the recombination time for sufficiently long-wave scalar perturbations and even at the present time for sufficiently long-wave GW. As was mentioned above, this leads to observable effects: periodic peaks in the multipole power spectra of the $\Delta T/T$ anisotropy and polarization of the CMB. Since the difference between entropies arising after two coarse grainings of the same system may be interpreted as the difference of amount of information loss due to these coarse grainings, the difference between previous results for the entropy of primordial GW background and our result may be interpreted as the amount of information contained in primordial peaks (acoustic peaks in the case of scalar perturbations).

Note that the energy density of the primordial GW background calculated either with the quantum operators (2) or by the use of equivalent classical stochastic fields (3) is one half of the energy density of the GW background described by a density matrix which is isotropic in phase space, with the same average number of gravitons given by $n(\mathbf{k})$ (this, of course, was taken into account in all correct calculations of the energy density of GW produced during inflation, beginning with [19]). Now for the entropy, $S = S_{\max}/2$ is also a distinguished, “average” case. However, we see that large deviations from $S_{\max}/2$ on both sides are possible; the entropy per mode can be both much larger or much smaller than r_k (in the logarithmic sense).

Let us finally discuss the temporal growth of the entropy of cosmological perturbations, which is at least partly responsible for the arrow of time in the Universe [20]. After the perturbations re-enter the Hubble radius, $N(\mathbf{k})$ remains constant (neglecting very small graviton–graviton scattering events and graviton absorption by matter), so the growth of entropy is mainly due to the growth of $n(\mathbf{k}, t)$ due to irre-

versible emission of gravitons by matter. Due to the expansion of the Universe, $n(\mathbf{k}, t)$ typically grows only as a power of time, so $\dot{S}(\mathbf{k}, t) \propto t^{-1}$. The same refers to the earlier period for $\lambda \gg R_H$, but after the end of inflation. (In inflation, one has for long wavelengths $\dot{S} \approx H \approx \text{constant}$ and therefore $S \propto t$). Then, there is no secondary GW background and we may use Eq. (17) only. Under any reasonable assumptions about the time dependence of ξ , $S(\mathbf{k}, t)$ grows logarithmically, so $\dot{S}(\mathbf{k}, t) \propto t^{-1}$, too. Comparing this behavior with those considered in the interesting discussion [21] of the paper [22], we see that the case of a two-mode subsystem of cosmological perturbations is in some sense intermediate between the two different cases discussed in Ref. [21] (decoherence due to a chaotic behavior of the subsystem itself or decoherence caused by the environment). Though our sub-

system is not chaotic but only classically unstable, the law for $\dot{S}(\mathbf{k}, t)$ is the same as for chaotic dynamical systems.

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