Coupled quintessence

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A new component of the cosmic medium, a light scalar field or ''quintessence,'' has been proposed recently to explain cosmic acceleration with a dynamical cosmological constant. Such a field is expected to be coupled explicitly to ordinary matter, unless some unknown symmetry prevents it. I investigate the cosmological consequences of a coupled quintessence (CQ) model, assuming an exponential potential and a linear coupling. This model is conformally equivalent to Brans-Dicke Lagrangians with any power-law potential. I evaluate the density perturbations on the cosmic microwave background and on the galaxy distribution at the present and derive bounds on the coupling constant from the comparison with observational data. A novel feature of CQ is that during the matter dominated era the scalar field has a finite and almost constant energy density. This epoch, denoted as ϕMDE, is a saddle point in the dynamical phase space. The ϕMDE is responsible of several differences with respect to uncoupled quintessence: the multipole spectrum of the microwave background is tilted at large angles, the acoustic peaks are shifted, their amplitude is changed, and the present 8 Mpc/h density variance is diminished. The present data constrain the dimensionless coupling constant to $|\beta| \lesssim 0.1$ assuming $\Omega_m = 0.3$ and a primordial fluctuation slope $n_s = 1$.

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I. INTRODUCTION

The recent evidence in favor of an accelerated cosmic expansion [1,2] has prompted theorists to hypothesize components of the cosmic medium additional to ordinary matter and radiation, whose equation of state is unable to provide the required kinematics. In a flat universe, the dark energy of such a component should provide roughly 70% of the cosmic density, and should possess an effective equation of state

$$p = (w-1)\rho, \tag{1}$$

with the present value [3,1,4]

$$w \in (0, 0.6). \tag{2}$$

The most obvious candidate, a cosmological constant, which provides $w = 0$, has unappealing features: its value would be one hundred orders of magnitude smaller than dimensionally expected; upper limits from lensing effects barely allow for a $\Omega_\Lambda = 0.7$ [5], as would be necessary to reconcile the amount of matter in clusters with the flatness suggested by inflation. The next simplest possibility is perhaps to include in the cosmic fluid a light scalar field. In fact, if the field is light enough to vary slowly during a Hubble time, its potential energy can drive an accelerated expansion, just like during inflation. The varying field equation of state can then be tuned to lie in the observed range: if this is the case, then the scalar field is sometimes denoted in the literature as ''quintessence.'' The scalar field density fraction Ω_ϕ can be made to decrease rapidly in the past, so as to pass easily the lensing constraints and to avoid discrepancies in the primordial nucleosynthesis abundances.

In addition to the acceleration argument, the study of a scalar field is interesting on its own. First, a scalar field is

predicted by many fundamental theories (string theory, pseudo-Nambu-Goldstone model, Brans-Dicke theory, etc.), so that it is natural to look at its cosmological consequences [6–9]. Second, the presence of a scalar field modifies the standard cold dark matter (CDM) spectrum [10–13], and perhaps brings it closer to real data, just as a cosmological constant does. Finally, even a small amount of scalar field density may give a detectable contribution to the standard CDM scenario, similar to what one has in the mixed dark matter (MDM) model [9].

A scalar field, however, is expected to couple explicitly (that is, beyond the gravitational coupling) to ordinary matter, with a strength comparable to gravity, as put into evidence by Carroll [14], unless some special symmetry prevents or suppresses the coupling. Such a strong coupling would render the scalar field interaction as strong as gravity, and would therefore have been already detected. However, a residual coupling still below detection cannot be excluded; moreover, if the coupling to baryons is different from the coupling to dark matter, as proposed by Damour *et al.* [15], then even a strong coupling is indeed possible. Exactly the same arguments hold if one supposes the quintessence field to be coupled to gravity, rather than to matter, as investigated by Uzan [16], Chiba [17], Chen and Kamionkowsky [18] and Perrotta *et al.* [19]. Indeed, the two models, although physically different, are related mathematically by a conformal transformation (see, e.g., Refs. [7,20]).

The nonminimal coupling of the quintessence field to ordinary matter is therefore worth investigating, especially because the wealth of high-precision data that is near to come allows the intriguing possibility of detecting the coupling on the microwave background and on the present galaxy distribution. In a previous paper (Ref. [21], hereafter paper I) I showed that a scalar field with an exponential potential [22,6] and an explicit coupling to matter may behave as a kind of hot dark matter component, as was first shown by Ferreira and Joyce [9] for zero coupling. I showed that the

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cosmic microwave background (CMB) spectrum of the model presents acoustic peaks displaced from their location without coupling, and that the galaxy power spectrum also bends in agreement to real data. In that case, the field density amounts to at most 20% of the critical density, and the expansion is not accelerated. The interesting feature was that the universe has always been in an attractor solution, independently of the initial conditions.

In this paper I focus instead on accelerated solutions. I explore first the general phase space of a homogeneous quintessence model with the same exponential potential and coupling to matter as in paper I; I will refer to this model as coupled quintessence (CQ). Once the phase-space attractors have been identified, two distinct solutions are selected that allow an accelerated epoch, and the density fluctuations on these trajectories are studied by the use of a purposely modified version of the code CMBFAST by Seljak and Zaldarriaga [23]. The linear perturbations in the uncoupled case has been already studied by Viana and Liddle [13] and Caldwell et al. [11]. As it will be shown, the coupling introduces several qualitatively new features.

II. COUPLED SCALAR FIELD MODEL

Consider two components, a scalar field ϕ and ordinary matter (e.g., baryons plus CDM) described by the energy-momentum tensors $T_{\mu\nu(\phi)}$ and $T_{\mu\nu(m)}$, respectively. General covariance requires the conservation of their sum, so that it is possible to consider a coupling such that, for instance,

$$T^{\mu}_{\nu(\phi);\mu} = CT_{(m)}\phi_{;\nu},$$

$$T^{\mu}_{\nu(m);\mu} = -CT_{(m)}\phi_{;\nu}. \tag{3}$$

Such a coupling arises for instance in string theory, or after a conformal transformation of Brans-Dicke theory. It has also been proposed to explain ''fifth-force'' experiments, since it corresponds to a new interaction that can compete with gravity and be material dependent. A coupling that violates general covariance is instead studied in Barrow and Magueijo [24].

The specific coupling (3) is only one of the possible forms. Nonlinear couplings such as $CT_{(m)}F(\phi)\phi_{;\nu}$ or more complicated functions are also possible (see, e.g., Refs. [25,26]). Also, one can think of different couplings to different matter species, for instance coupling the scalar field only to dark matter and not to baryons, as proposed by Damour et al. [15] and Casas et al. [27], and studied recently by Holden and Wands [28]. Notice that the coupling to radiation (subscript γ) vanishes, since $T_{(\gamma)}=0$. Here I restrict myself to the simplest possibility (3), which is also the same investigated earlier by Wetterich [7] and is the kind of coupling that arises from Brans-Dicke models. In fact, a field with a gravity-coupling term $\frac{1}{2}\xi\phi^2 R$ in the Lagrangian acquires, after conformal transformation, a coupling to matter of the form (3). In the limit of small positive coupling one obtains

$$C = \kappa\sqrt{\xi}, \tag{4}$$

where $\kappa^2 = 8\pi M_P^{-2}$ and M_P is the Planck mass. Moreover, if the Brans-Dicke Lagrangian contains a power-law potential $V(\phi) \sim \phi^n$, then the transformed field ϕ' acquires an exponential potential that, for small positive ξ, can be written as

$$V(\phi') \sim \exp[(n-4)\kappa\sqrt{\xi}\phi'] \tag{5}$$

(see, e.g., Refs. [29–31,20]). The CQ model with a linear coupling and an exponential potential that is studied here is therefore conformally equivalent to a large class of Brans-Dicke Lagrangians. For instance, the model studied in Perrotta et al. [19] is equivalent to $n=-1$.

There are several constraints on the coupling constant C along with constraints on the mass of the scalar field particles, reviewed by Ellis et al. [32] and Damour [15]. The constraints arise from a variety of observations, ranging from Cavendish-type experiments, to primordial nucleosynthesis bounds, to stellar structure, etc. Most of them, however, apply only if the scalar field couples universally to all matter, which is not necessarily the case. The most stringent bound for a universal coupling, quoted by Wetterich [7], amounts to

$$|C| < 0.1 M_P^{-1}. \tag{6}$$

If the coupling to dark matter is different from the baryon coupling, then the constraints on the former relaxes considerably, and becomes [15]

$$|C| < 5 M_P^{-1}. \tag{7}$$

It is also noticeable that these constraints are local both in space and time, and could be easily escaped by a time-dependent coupling constant.

The constraints from nucleosynthesis refer to the energy density in the scalar component. This has to be small enough not to perturb element production, so that at the epoch of nucleosynthesis [7,33,9]

$$\Omega_\phi(\tau_{ns}) < 0.1-0.2. \tag{8}$$

This bound is satisfied by all the models we discuss below.

III. BACKGROUND

Here I derive the background equations in the flat conformal Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = a^2(-d\tau^2 + \delta_{ij}dx^i dx^j). \tag{9}$$

The CQ scalar field equation is

$$\ddot{\phi} + 2H\dot{\phi} + a^2 U_{,\phi} = C\rho_m a^2, \tag{10}$$

where $H = \dot{a}/a$, and I adopt the exponential potential

$$U(\phi) = A e^{\sqrt{2/3}\kappa\mu\phi}. \tag{11}$$

The matter (subscript m) and the radiation (subscript γ) equations are

$$\dot{\rho}_m + 3H\rho_m = -C\rho_m\dot{\phi}, \tag{12}$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0. \tag{13}$$

Denoting with τ_0 the conformal time today, let us put

$$a(\tau_0) = 1, \quad \rho_m(\tau_0) = \frac{3H_0^2}{\kappa^2}\Omega_m = \rho_{m0},$$

$$\rho_\gamma(\tau_0) = \rho_{\gamma0}, \quad \phi(\tau_0) = \phi_0. \tag{14}$$

Without loss of generality, the scalar field can be rescaled by a constant quantity, by a suitable redefinition of the potential constant A. We put then $\phi_0 = 0$. This gives

$$\rho_m = \rho_{m0}a^{-3}e^{-C\phi},$$

$$\rho_\gamma = \rho_{\gamma0}a^{-4}. \tag{15}$$

The Friedman-Einstein equation can be written as

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\rho_{m0}}{a}e^{-C\phi} + \frac{\rho_{\gamma0}}{a^2} + \frac{1}{2}\dot{\phi}^2 + Ua^2\right). \tag{16}$$

The dynamics of the CQ model has been analyzed by Amendola [20], Holden and Wands [28] and Billyard and Coley [25] in the regime in which either matter or radiation dominates. Here I generalize the analysis to the case in which both matter and radiation are present. As we will see, this introduces some interesting new features. Generalizing Copeland *et al.* [34] the following variables are introduced:

$$x = \frac{\kappa}{H}\frac{\dot{\phi}}{\sqrt{6}}, \quad y = \frac{\kappa a}{H}\sqrt{\frac{U}{3}}, \quad z = \frac{\kappa a}{H}\sqrt{\frac{\rho_\gamma}{3}}, \tag{17}$$

along with the independent variable $\alpha = \log a$. Notice that x^2, y^2, and z^2 give the fraction of total energy density carried by the field kinetic energy, the field potential energy, and the radiation, respectively, that is $\Omega_\phi = x^2 + y^2$ and $\Omega_\gamma = z^2$. Clearly, the matter energy density fraction is the complement to unity of $x^2 + y^2 + z^2$. We can rewrite Eqs. (10) and (16) as

$$x' = x\left(\frac{z'}{z} - 1\right) - \mu y^2 + \beta(1 - x^2 - y^2 - z^2),$$

$$y' = \mu xy + y\left(2 + \frac{z'}{z}\right),$$

$$z' = -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2), \tag{18}$$

where the prime denotes here $d/d\alpha$ and where I introduced the dimensionless constant

$$\beta = \sqrt{\frac{3}{2}}\frac{C}{\kappa} \tag{19}$$

(in Ref. [20] β was defined as twice the value above). The parameters β and μ are all we need to completely specify

our model. The constraints quoted in the previous section on C become now $\beta < 0.025$ for the universal coupling and $\beta < 1$ for the dark matter coupling.

The system (18) is invariant under the change of sign of y, z and of α. Since it is also limited by the condition $\rho > 0$ to the circle $x^2 + y^2 + z^2 \leq 1$, we limit the analysis only to the quarter of unitary sphere of positive y, z. The critical points, i.e., the points that verify $x' = y' = z' = 0$, are scaling solutions, on which the scalar field equation of state is

$$w_\phi = \frac{2x^2}{x^2 + y^2} = \text{const}, \tag{20}$$

the scalar field total energy density is $\Omega_\phi = x^2 + y^2$, and the scale factor is

$$a \sim \tau^{p/1-p} = t^p, \quad p = \frac{2}{3w_{\text{eff}}} \tag{21}$$

[t being the time defined by $dt = a(\tau)d\tau$]. The effective equation of state for the total cosmic fluid $p_{\text{tot}} = (w_{\text{eff}} - 1)\rho_{\text{tot}}$ has index

$$w_{\text{eff}} = 1 + x^2 - y^2 + z^2/3 = 1 + \Omega_\gamma(w_\gamma - 1) + \Omega_\phi(w_\phi - 1) \tag{22}$$

(where $w_\gamma = 4/3$ is the radiation equation of state). As already noticed, a value $0 < w_\phi < 0.6$ is required by observations, while $w_{\text{eff}} < 2/3$ is enough for acceleration.

The system (18) with an exponential potential has up to fifteen critical points, of which only eight can be in the allowed region. They are labeled by a letter that reproduces the classification given in Ref. [20], and a subscript that denotes whether beside the field there is a component of matter (subscript M), radiation (R), both (RM) or neither of the two (in which case the energy density is taken up completely by the scalar field; no subscript in this case). The critical points are listed in Table I, where $g(\beta, \mu) \equiv 4\beta^2 + 4\beta\mu + 18$. For every value of the parameters μ, β there is one and only one stable critical point (attractor); one or more saddle point can also exist.

The regions of existence and stability of the critical points are summarized in Table II. In the table the attention is restricted to the half plane $\mu > 0$, since there is complete symmetry under simultaneous sign exchange of μ and β. I defined

$$\mu_+ = \frac{1}{2}(-\beta + \sqrt{18 + \beta^2}),$$

$$\mu_0 = -\beta - \frac{9}{2\beta}. \tag{23}$$

Figure 1 displays the regions of the parameter space (for $\mu > 0$) in which the various points are stable.

There are only two critical points that admit accelerated solutions, i.e., solutions that satisfy Eq. (2): the points a and b_M. They differ in several important aspects, so we study them separately.

TABLE I. Critical points.

Point	x	y	z	Ω_ϕ	p	w_{eff}	w_ϕ
a	$-\dfrac{\mu}{3}$	$\left(1-\dfrac{\mu^2}{9}\right)^{1/2}$	0	1	$3/\mu^2$	$2\mu^2/9$	$2\mu^2/9$
b_R	$-\dfrac{2}{\mu}$	$\dfrac{\sqrt{2}}{|\mu|}$	$\left(1-\dfrac{6}{\mu^2}\right)^{1/2}$	$\dfrac{6}{\mu^2}$	$1/2$	$4/3$	$4/3$
b_M	$-\dfrac{3}{2(\mu+\beta)}$	$\dfrac{(g-9)^{1/2}}{2|\mu+\beta|}$	0	$\dfrac{g}{4(\beta+\mu)^2}$	$\dfrac{2}{3}\left(1+\dfrac{\beta}{\mu}\right)$	$\dfrac{\mu}{\mu+\beta}$	$\dfrac{18}{g}$
c_R	0	0	1	0	$1/2$	$4/3$	$-$
c_{RM}	$\dfrac{1}{2\beta}$	0	$\left(1-\dfrac{3}{4\beta^2}\right)^{1/2}$	$\dfrac{1}{4\beta^2}$	$1/2$	$4/3$	2
c_M	$\tfrac{2}{3}\beta$	0	0	$\tfrac{4}{9}\beta^2$	$\dfrac{6}{4\beta^2+9}$	$1+\dfrac{4\beta^2}{9}$	2
d	-1	0	0	1	$1/3$	2	2
e	$+1$	0	0	1	$1/3$	2	2

Solutions of type a. The attractor a, once reached, brings to zero the matter density. To allow for the observed matter content of the universe, we have to select the initial conditions, if they exist, in such a way that the attractor is not yet reached at the present time, but the expansion is already accelerated. This is of course a kind of fine tuning: a different choice of the initial values of ϕ and $\dot\phi$ leads to different amount of Ω_ϕ at the present. On the positive side, this attractor is accelerated also for small values of the coupling constant. Let us first consider the limit $\beta=\mu=0$, that corresponds to the ordinary cosmological constant. Suppose then we put initially the field at zero kinetic energy ($x=0$). A trajectory acceptable from a cosmological point of view should start deep into the radiation era ($z\approx1$), then enter a matter dominate era ($z\approx0$), and finally fall into the attractor a, the only one still existent for $\beta=\mu=0$, which corresponds to the Λ-dominated stage. In other words, the path of a ordinary Λ universe would be $c_R\rightarrow c_M\rightarrow a$. A similar sequence of critical points characterizes all the trajectories discussed in the following.

In Fig. 2 we show the 3D phase space (x,y,z) of model a, with $\beta=0,\mu=0.1$ and $\beta=0.5,\mu=0.1$. As before, a cosmological trajectory must start in the radiation era ($z\simeq1$) and

has to provide the correct final conditions for x, y, and H. Since the scalar field has to start dominating only recently, it is clear that its initial energy density deep in the radiation era has to be very small: in the 3D phase space this means that the cosmic solutions start near $(x,y,z)=(0,0,1)$, that is, near the unstable critical point c_R. If initially the field kinetic energy dominates (unstable points d or e), there will be a fast transient towards c_R. The trajectories in Fig. 2 that begin at $(x,y,z)=(-1,0,0)$, rise to near ($0,1,0$) and then fall almost vertically are examples of such cosmic solutions. The attractor of the CQ model is the same as for the uncoupled case, at $(x,y,z)=(-0.033,0.99,0)$, but while in the ordinary quintessence case there is a saddle point c_M at $(x,y,z)=(0,0,0)$, in CQ this moves to $(x,y,z)=(2\beta/3,0,0)$, on which $\Omega_\phi=4\beta^2/9$. This appears more clearly from Fig. 3, in which the trend of Ω_m, Ω_ϕ, and Ω_γ is reported. The path of this solution (neglecting a possible initial field kinetic-dominated

TABLE II. Properties of the critical points.

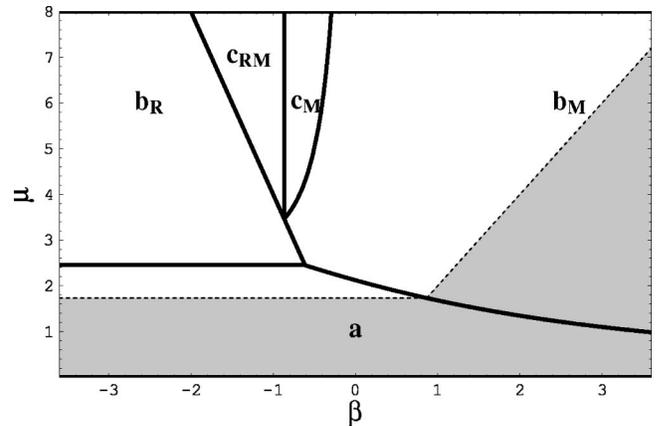| Point	Existence	Stability	Acceleration
a	$\mu<3$	$\mu<\mu_+$, $\mu<\sqrt{6}$	$\mu<\sqrt{3}$
b_R	$\mu>\sqrt{6}$	$\sqrt{6}<\mu<-4\beta$	never
b_M	$|\mu+\beta|>3/2, \mu<\mu_0$	$\mu>\mu_+, \mu>-4\beta$	$\mu<2\beta$
c_R	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never
c_{RM}	$|\beta|>\sqrt{3}/2$	$\mu>-4\beta,\beta<\sqrt{3}/2$	never
c_M	$|\beta|<3/2$	$|\beta|>\sqrt{3}/2,\mu>\mu_0$	never
d	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never
e	$\forall\mu,\beta$	unstable $\forall\mu,\beta$	never

FIG. 1. The figure shows the parameter space of the model. Each region is labeled by the critical point that is stable in that region. The shaded area indicates the values for which the attractor is accelerated.
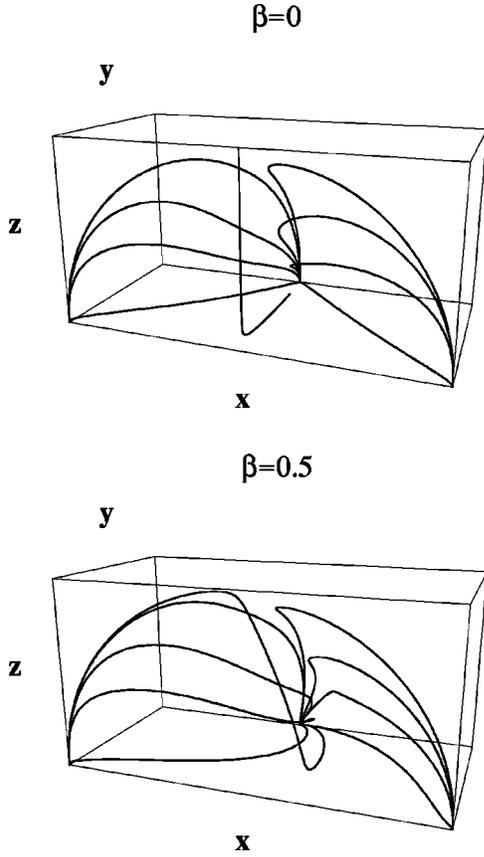
$\beta=0$

$\beta=0.5$

FIG. 2. CQ phase space for values that lie in the a region, $\mu = 0.1$ and β as indicated. The attractor a is the same as in the uncoupled model, but for $\beta \neq 0$ there is a saddle for a nonzero value of the scalar field density. The phase spaces for the values of β investigated in this paper are qualitatively similar to the $\beta=0.5$ case. The trajectory that falls almost vertically from top is similar to the background solution effectively employed.

era) is then $c_R \rightarrow c_M \rightarrow a$, just as in the pure Λ model. For $\beta > 3/2$, actually, the point c_M ceases to exist, but such high values of β are anyway unacceptable.

It is to be emphasized that the saddle-point stage of constant and finite $\Omega_\phi = 4\beta^2/9$ is typical of the CQ model, as it is absent in the limit of zero coupling. Let us call this the field-matter-dominated era, or ϕMDE. As we will see, this stage is responsible for most of the differences with respect to ordinary quintessence. We can think of the coupling as a mechanism for transferring energy from the matter to the field. Note that this is the direction of the transfer whatever the sign of β.

The equivalence time in CQ occurs earlier than in uncoupled quintessence

$$a_{\mathrm{eq}} = \left(\frac{\rho_{\gamma 0}}{\rho_{m0}} \right)^{3/(3-2\beta^2)}. \tag{24}$$

For the acceptable values of β, however, this shift has only a minor effect.

Finally, it is to be noticed that the attractor a is independent of the coupling β. Then, as the universe at $z \simeq 5$ con-
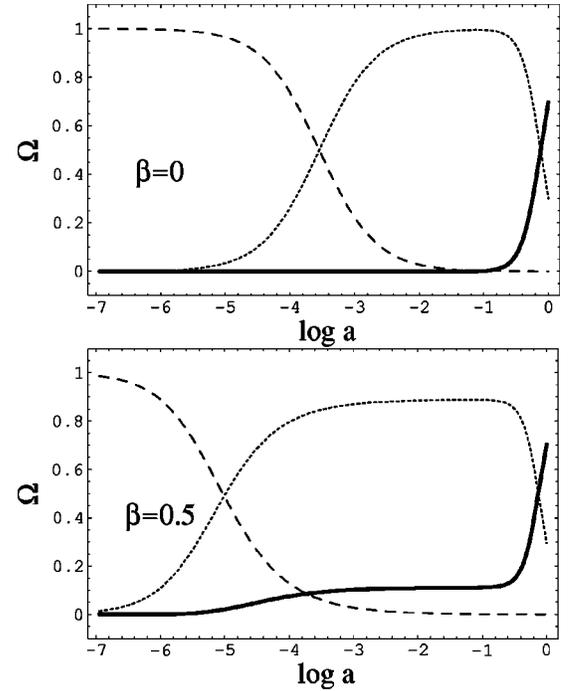

FIG. 3. Behavior of Ω_M (dotted line), Ω_R (dashed line) and Ω_ϕ (thick line) as a function of $\log(a)$ for $\mu = 0.1$ and β as indicated. Notice that for CQ there is the transient regime ϕMDE in which both the matter and the scalar field energy density are nonvanishing. Notice also that in this case, and for all values of $\beta \neq 0$, the matter-radiation equivalence occurs earlier than in the uncoupled model. For the small values of β used for the perturbation calculations, however, this is a small effect.

verges toward the point a, the dynamics becomes independent of the coupling. As a consequence, the cosmological probes at $z < 5$, similar to type Ia supernovae or the cluster abundance, are not efficient tools for discriminating between ordinary quintessence and CQ.

Solutions of type b_M. The attractor b_M is a solution for which the matter density and the scalar field share a finite and *constant* portion of the cosmic energy. For instance, if we put

$$\beta = 4.02, \quad \mu = 2.68 \tag{25}$$

we get $\Omega_M = 0.3$, $\Omega_\phi = 0.7$, and $w_\phi \simeq 0.14$, well within the requested range. These values, once reached, remain indefinitely constant. The coincidence of similar values of the energy density in the matter and field component is therefore solved, independently of the initial conditions. On the negative side, however, these solutions require a large value of the coupling constant ($\beta > \sqrt{6/5}$ to obtain $w_\phi < 0.6$) and are therefore at risk of running into conflict with constraints on the coupling derived from local experiments. The strongest objection to these solutions, however, comes from the simple fact that they lack a matter dominated era, as we will show in a moment.

In Fig. 4 we show the phase space of model b_M, assuming $\beta = 4.02$, $\mu = 2.68$. As can be seen, the phase space is now completely different. The attractor is at (x,y,z)
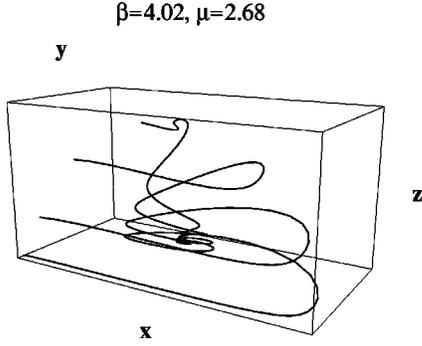
$\beta=4.02, \mu=2.68$

FIG. 4. Phase space of CQ for values that lie in the b_M region. There is a saddle c_{RM} at $(x,y,z)\simeq(0.12,0,0.95)$ that attracts the trajectory that falls from top, similar to the one adopted in the perturbation calculation.

$=(-0.22,0.81,0)$. The trajectory that falls from top is again similar to the one used in the perturbation calculations. As can be seen better in Fig. 5, there is a transient near the saddle point c_{RM}, here at $(x,y,z)\simeq(0.12,0,0.95)$. In Fig. 5 the evolution of Ω_m, Ω_ϕ, and Ω_γ in two cases is displayed, one for which the present value of w_{eff} equals 0.4, for which β and μ must be chosen as above, and the other for $w_{eff}=0.5$, which requires $\beta=\mu=2.37$. In both cases we can see the transient c_{RM}, characterized by $\Omega_\phi=1/(4\beta^2)$ and $\Omega_M=1-1/(2\beta^2)$, followed by the decay of the radiation component and the stabilization of the field and matter to their final values. The path of this solution is then $c_R \to c_{RM} \to b_M$, in contrast with the solutions of the type a. As already remarked, such a trajectory is possible only for $\beta \neq 0$.

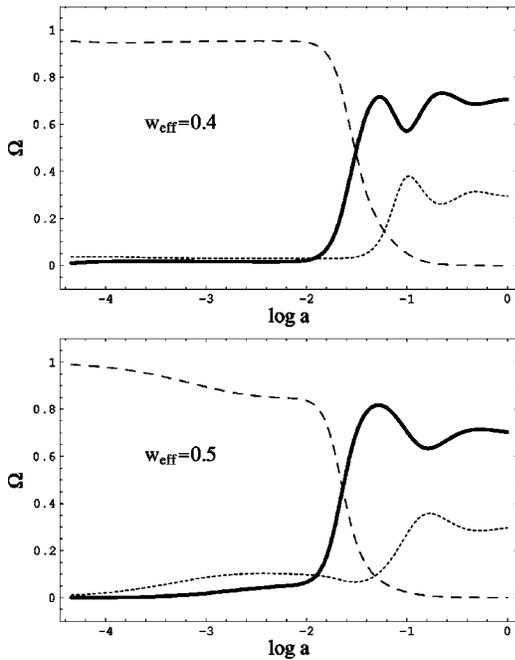FIG. 5. Behavior of Ω_M (dotted line), Ω_R (dashed line) and Ω_ϕ (thick line) as a function of $\log(a)$ for the same parameters as in Fig. 4 (label $w_{eff}=0.4$) and for $\beta=\mu=2.37$ (label $w_{eff}=0.5$). Notice the transient epoch in which radiation and field share the total energy density (saddle c_{RM}).

The most conspicuous features here are that the radiation dominates until recently ($z\simeq50$) and the matter dominated era is absent. As is intuitive, such behavior is catastrophic for the growth of the fluctuations: when a fluctuation mode re-enters the horizon, in fact, is suppressed first by the long radiation dominated era (RDE), and then by the accelerated expansion. As a consequence, the present σ_8 is unacceptably small, of the order of 10^{-4} for COBE normalized spectra. Unless some mechanism other than gravitational instability powers the fluctuations, the trajectories of type b_M are precluded. In the following we restrict therefore our attention to solutions of type a.

This concludes the analysis of the homogeneous solutions of CQ. It is to be stressed that we considered all the possible accelerated solutions. As in the next section we will span all the range of β and μ that yield cosmologically acceptable solutions of type a, we can consider exhausted the analysis of CQ for exponential potentials and linear coupling.

IV. PERTURBATIONS ALONG SOLUTIONS a

We now proceed to study the evolution of the perturbations in the coupled quintessence theory. In this section we assume a coupling to dark matter only (zero coupling to baryons), in order to explore a larger range of coupling parameters. The effect of decoupling the baryons on the homogeneous trajectories is not large, since the baryon component is much smaller than the CDM component. The effect on the perturbations is not large either, because the main effect is due to the existence of the ϕMDE, which again depends on the homogeneous solution.

The equations of the perturbations in the synchronous gauge have been derived and discussed at length in paper I and will not be repeated here. In that case it was shown that several important features of the perturbation evolution could be derived analytically, since the background evolution was always on one of the attractor, and thus particularly simple. The same occurs here, at least in some cases. As described in paper I, all the numerical results presented below have been obtained by modifying the code CMBFAST of Seljak and Zaldarriaga [23].

The key fluctuation equation in paper I was the evolution of the subhorizon perturbations in the MDE regime, the only situation in which the evolution differs from the pure CDM case. For CQ, this regime is actually the ϕMDE introduced above. Let us note first that along any attractor with $x=x_a$ one has, from Eq. (17)

$$\phi=\frac{\sqrt{6}x_a}{\kappa}\log a. \qquad (26)$$

Denoting with δ the fluctuation in the matter component, we derived in paper I an equation for subhorizon modes in MDE along an attractor that can be rewritten as follows:

$$\ddot{\delta}+H(1+2x_a\beta)\dot{\delta}-\frac{3}{2}H^2\Omega_M\delta\left(1-\frac{4\beta^2}{3}\right)=0. \qquad (27)$$

As we noticed in the previous section, the solutions a passes through the ϕMDE transient attractor c_M, before the present ϕ-dominated epoch. Therefore, from Table I, we can substitute $x_a = 2\beta/3$. It follows that Eq. (27) has a growing solution

$$\delta = Aa^m, \text{ with } m = 1 - \frac{4\beta^2}{3}. \tag{28}$$

This shows two important facts: first, the fluctuations in ϕMDE are suppressed with respect to the standard CDM behavior ($m = 1$), which also holds for the uncoupled quintessence model, for all values of β; second, the evolution does not depend on the sign of β. Before the present time, at $z \simeq 5$, the trajectory deviates from the ϕMDE solution, and ϕ begins to dominate.

With respect to paper I the initial conditions are now different, because now initially (that is, in the RDE stage) the field does not lie on an attractor. Notice that in CMBFAST one needs initial conditions at different times for each wave number. The same technique exploited in paper I to get the adiabatic initial conditions gives now a result that depends on the background solution, as worked out in detail in Ref. [12]. This gives

$$\delta\phi_i = \left(\frac{\dot{\phi}}{10a}\tau^2\delta_c\right)_i, \tag{29}$$

where $\dot{\phi}_i$ is $d\phi/d\tau$ at the initial integration time τ_i. The value of $\delta\dot{\phi}_i$ follows by differentiation. In practice, since the fluctuations for ϕ are forced by the coupling with the other fluctuation variables, we find that the results are quite independent on the choice of $\delta\phi_i$ and its time derivative. The choice $\delta\phi_i = \delta\dot{\phi}_i = 0$, for instance, adopted in Ref. [18] gives indistinguishable results.

In Fig. 6 the behavior of three density fluctuation wavelengths, calculated numerically with CMBFAST, is shown as a function of the scale factor a. I plot δ/a to enhance the differences among the various cases. It is possible to distinguish four distinct epochs. Let us first follow the intermediate wavelength of Fig. 6. First, the 100 Mpc/h fluctuation grow as a^2, while it is a superhorizon mode in RDE; then, around $z \simeq 2500$ ($a \simeq 0.0004$), it reenters the horizon and freezes until the ϕMDE begins. In uncoupled quintessence ($\beta = 0$) the fluctuation grows now as a, as usual; in CQ the growth is instead suppressed as expected from Eq. (28). Finally, around $z \simeq 5$, ϕ starts to dominate, the universe accelerates, and the fluctuation growth is definitely suppressed in all cases. The longer wavelength follows the same phases, except that it reenters directly in the ϕMDE, and therefore bypasses the freezing stage. The shorter wavelength starts in the plot already inside the horizon, and follows the same ϕMDE growth evolution of the other modes.

The same ϕMDE attractor solution can be used to derive the location of the first acoustic peak on the CMB. In fact, this depends essentially on the size of the sound horizon r_s at decoupling (subscript d). We have shown in paper I that the
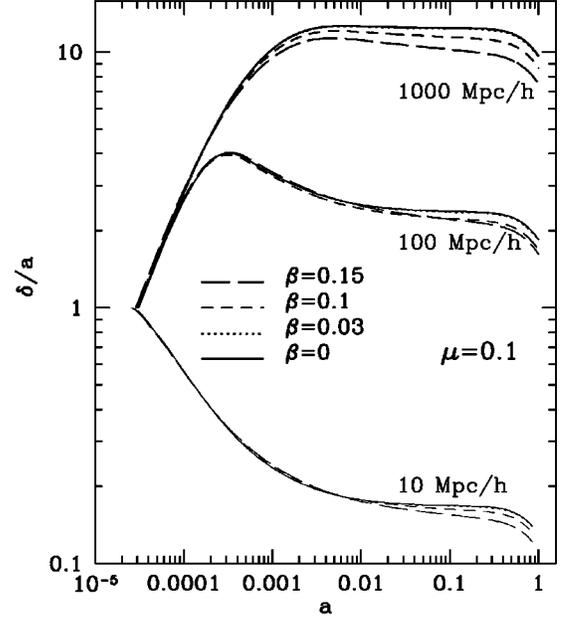

FIG. 6. Evolution of the matter density contrast δ/a for three wavelengths, $2\pi/k = \lambda = 1000$, 100, and 10 Mpc/h, for $\mu = 0.1$ and various values of β. Notice the growth suppression for $\beta \neq 0$.

following approximated law governs the size of the sound horizon along an attractor solution

$$r_s = r_0 \frac{a_d^{x_a\beta}}{1 + 2x_a\beta}, \tag{30}$$

where $r_0 = 2(a_d/3)^{1/2}H_0^{-1}$ is the standard sound horizon. Therefore, the multipole location of the first acoustic peak is

$$l_{\text{peak}} \simeq \frac{2\pi}{r_s H_0} = l_0(1 + 2x_a\beta)a_d^{-x_a\beta} \tag{31}$$

$$= l_0\left(1 + \frac{4\beta^2}{3}\right)a_d^{-2\beta^2/3}, \tag{32}$$

where $l_0 \simeq 2\pi/(r_0 H_0) \simeq 200$ is the standard peak location, and where the second line specializes to the ϕMDE attractor c_M. For instance, $\beta = 0.1$ gives a location $l_{\text{peak}} \simeq 1.06 l_0$ which, at least to a first approximation, is in agreement with the numerical results below. Notice again that the peak displacement is always toward larger ls, regardless of the sign of β, contrary to what happened in paper I along the attractor b_M. Similar behavior was found numerically by Chen and Kamionkowsky [18] and Perrotta et al. [19] in Brans-Dicke models. Here, substituting $\beta^2 = 3\xi/2$, we find that the peak shifts approximately as $(1 + 2\xi)a_d^{-\xi}$.

The solution we use as background in this section is, as anticipated, on its way to the attractor a. The initial conditions will be chosen so that at the present time (i.e., when $H^{-1} = 3000$ Mpc/h) we have $\Omega_m = 0.3$, $\Omega_\phi = 0.7$ and $w_\phi \simeq 0$. This implies that at the present time we should have

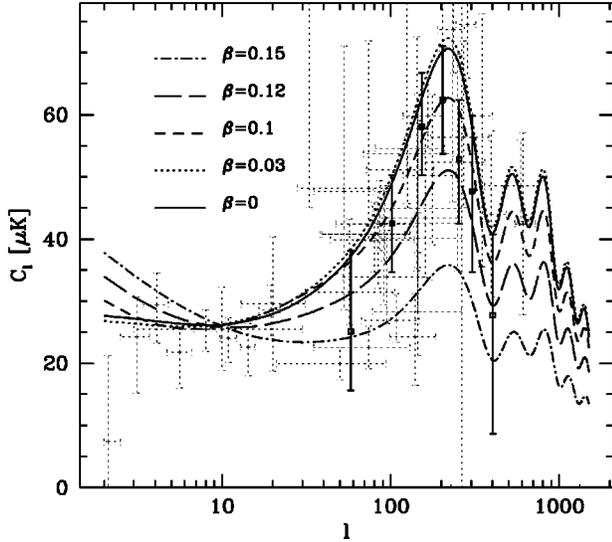$$x_0 \simeq 0, \quad y_0 = 0.837, \quad z_0 = 0.092 \tag{33}$$

FIG. 7. Multipole spectra C_l (actually we plot $[l(l+1)C_l/2\pi]^{1/2}\mu K$, as customary) for the solutions a for different couplings. The data are a selection from Tegmark's home page (http://www.sns.ias.edu/~max). The thick errorbars are from the Boomerang experiment [36].

FIG. 8. Comparison between universal or dark matter coupling, and two different values for the primordial fluctuation slope n_s. The data are from the Boomerang experiment [36].

independently of β. We begin by investigating the parameter range

$$\mu = 0.1, \quad \beta \in (0, 0.15). \qquad (34)$$

The initial conditions that produce the requested final values have been obtained by trial and error. Inserting the background solution in the modified CMBFAST code, we obtain the CMB spectra reported in Fig. 7. The other values adopted are

$$h = 0.7, \quad \Omega_b = 0.04, \quad n_s = 1. \qquad (35)$$

The gravitational wave contribution is assumed negligible. It can be noticed that the peaks move to larger multipoles, as expected. Their amplitude is generally reduced, due both to the growth suppression found above, and to the fact that now the Cosmic Background Explorer (COBE) normalization at small l includes the integrated Sachs-Wolfe (ISW) effect, no longer negligible in CQ, and as a consequence the fluctuation amplitude at decoupling is reduced (see, for instance, Ref. [35]). The ISW is also responsible for the tilt at small multipoles. Models with $\beta > 0.15$ are already ruled out by CMB observations, while models with $\beta < 0.01$ are essentially indistinguishable from uncoupled quintessence. Values smaller than $\beta < 0.06$ produce acoustic peaks which are slightly above those for the uncoupled model. The bound $\beta < 0.15$ is already stronger than Eq. (7), valid for the coupling to dark matter; the determination of the spectrum with a precision of 5%, within reach of the Boomerang or Maxima experiments, will constrain β to two decimal digits, better than current constraints to the baryon coupling (6). The effect for $\beta < 0.03$, which satisfies the constraint for the universal coupling model, will be distinguishable in the near future. The
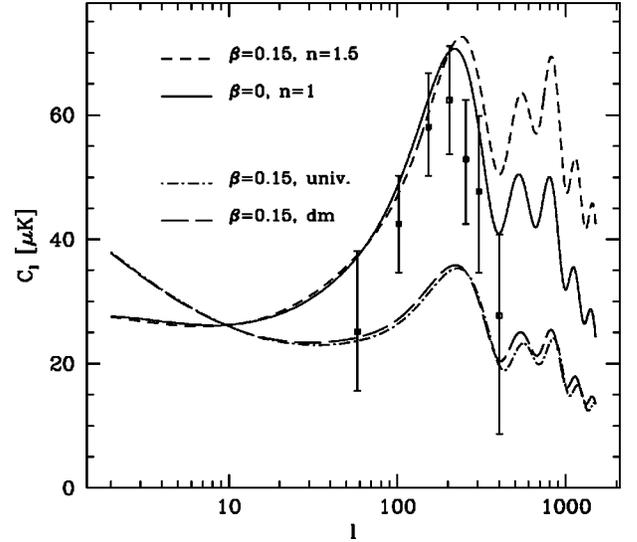
preliminary Boomerang [36] data plotted in Fig. 7, taken at face value, would allow only $\beta < 0.12$ ($\beta = 0.12$ is rejected at the 93% C.l.).

In Fig. 8 some alternative assumptions are tested. The constraint on β depends clearly on the values of the other cosmological parameters (see, for instance, Ref. [18] for a statistical analysis of the covariance among parameters in the case of the original Brans-Dicke model). The analysis of the multidimensional confidence region for $\beta, h, \Omega_\phi, \Omega_b, n_s$, etc., is very interesting but also very demanding, and will be pursued in another paper. Most of the other parameters are (or soon will be) strongly constrained by other observations. Perhaps the only parameter that depends entirely on the CMB data is the primordial fluctuation slope n_s. In Fig. 8 I compare two models, one without coupling and $n_s = 1$, and the other with $\beta = 0.15$ and $n_s = 1.5$, the highest value allowed by the COBE results. The two curves overlap pretty much in the low-l region, but are neatly distinguishable at $l > 400$. We conclude that, at least as concerns n_s, the prospects for disentangling the degeneracy are excellent. In the same Fig. 8 we consider the case of universal coupling (baryon coupling equal to dark matter coupling) and find that this decreases the peak height by less than 10%.

The reason for the small increase in the acoustic peak that is observed for $|\beta| < 0.06$ is not entirely clear. Since the numerical fluctuation growth follows very closely the analytical prediction of Eq. (28) I believe the rise is not a numerical artifact. Notice that for small β the ϕMDE transient attractor starts just past the decoupling epoch, and thus the analytical expectations based on the ϕMDE solution are not accurate.

The effect of changing μ, within the limit $\mu < \sqrt{3}$ necessary to have acceleration, is minimal, since the trajectories must anyway satisfy the same final conditions. The spectrum for $\beta = 0$ is therefore almost identical to the spectrum of a pure Λ model with $\Omega_\Lambda = 0.7$. Also, as expected from the analytical expressions, the perturbative results are almost insensitive to the sign of β. The present analysis therefore
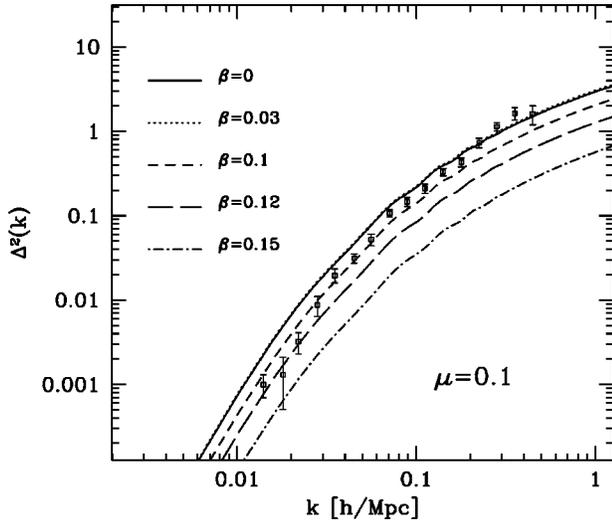
FIG. 9. Power spectrum $\Delta^2(k)=k^3 P(k)/(2\pi^2)$ for the same solutions of type a of Fig. 7. The real galaxy data are from Peacock and Dodd, 1994.

FIG. 10. The variance σ_8 versus β. The shaded area is the 95% C.l. region that matches the cluster abundance [41].

spans all the possible accelerated trajectories in CQ with $\Omega_\phi=0.7$ that are cosmologically acceptable.

In Fig. 9 I report the power spectra $\Delta^2(k)=k^3P(k)/(2\pi^2)$ normalized to COBE, compared to the data compiled and corrected for redshift and nonlinear distortions by Peacock and Dodds [37]. It can be seen that the slope of the spectrum is in rough agreement with what is observed (the largest discrepancy is for the four smallest scale data points, where nonlinearity and redshift distortions are more difficult to correct); a more precise comparison depends on the assumption that the bias between galaxies and dark matter is scale independent, and on other variables which are not of interest here, such as h.

The matter fluctuation variance in 8 Mpc/h cells σ_8 for a $\Omega_m=0.3$ universe should be around unity to fit the cluster abundance [38–40]. Wang and Steinhardt [41] find, for a constant-w model, a general expression for σ_8, that corresponds to $\sigma_8 \in (0.85,1.25)$ at the 95% C.l., adopting our cosmological parameters. It is found that this is satisfied by $|\beta| \lesssim 0.1$, so that this can be taken as the upper limit on $|\beta|$ (see Fig. 10). As found analytically, the suppression of σ_8 with respect to COBE-normalized standard CDM is due to the growth suppression in MDE. Another factor is that now the COBE normalization includes the integrated Sachs-Wolfe effect, no longer negligible. The rise in the CMB spectrum that we observed for small β induces via the COBE normalization a similar small rise in σ_8 for the same values, as can be seen in Fig. 10.

A fit to $\sigma_8(\beta)$ gives

$$\sigma_8(\beta)=\sigma_{8(0)}10^{(4.5\beta)^{1.5}-(6.4\beta)^2}, \tag{36}$$

where $\sigma_{8(0)}$ refers to uncoupled quintessence, and contains all the dependence on the other cosmological parameters, as well as on the exact COBE normalization scheme (I used here the Bunn and White [42] normalization implemented in the original CMBFAST code).
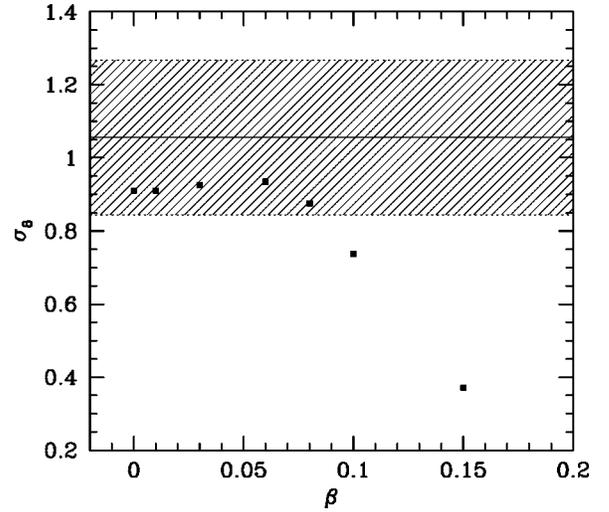
V. CONCLUSION

Soon cosmology will benefit from high-precision data that will allow unprecedented accuracy in testing fundamental theories. Quintessence models add to the battery of cosmological parameters at least two entries, one describing the potential of the field and another its coupling to the rest of the world. So far, the coupling was arbitrarily put to zero, although we know no symmetry condition that implies so. In this paper we let the coupling be nonzero, and investigated systematically all the possible trajectories in a flat space that lead to accelerated expansion at the present with $\Omega_\phi=0.7$.

The results for the homogeneous theory are applicable also to all Brans-Dicke models with any power law potential, since there is a direct correspondence between our constants β and μ and the Brans-Dicke coupling constant ξ and the potential exponent n; from Eqs. (4), (5),(19) we have

$$\beta^2=\frac{3}{2}\xi,$$

$$\mu^2=\frac{3}{2}(n-4)^2\xi. \tag{37}$$

The popular choice of an inverse power law, $n=-1$, for instance, amounts to exploring of all the parameter space, the line $\mu=5\beta$. For the fluctuations, the transformation that brings one from the fluctuation quantities in the Jordan frame (the frame in which the field is coupled to gravity) to those in the Einstein frame (in which the field is coupled to the matter) is more complicated, and a complete treatment is still to be published. However, in the limit in which the fluctuations of the scalar field are small with respect to the fluctuations in the other components the fluctuation fields are conformally invariant; since the CQ field is almost homogeneous due to its light mass, this condition is verified for most of the universe history. As a consequence, it is conjectured that also the perturbative CQ results apply to Brans-Dicke models. The verification of this conjecture is left to future work.

The main feature of the CQ model is the existence of a phase intermediate between the radiation era and the accelerated era, that we denoted ϕMDE, a saddle point in the phase space. During this era the fluctuations grow slower than in an uncoupled model. The ϕMDE has three effects on the CMB: the spectrum at low multipoles is tilted, due to the ISW effect; the acoustic peaks are shifted to higher multipoles, due to the change in the sound horizon; and their amplitude is changed in a nontrivial way. On the power spectrum at the present, the main effect is the reduction of σ_8 for large couplings and a very minor enhancement for small coupling.

We found that the potential slope is not efficiently constrained by observations, essentially because the ϕMDE is independent of μ. The coupling β is on the contrary constrained already by the present data, and is expected to be much more so in the near future, by at least an order of magnitude. From CMB and σ_8 data we can derive the bound

$$|\beta| < 0.1 \tag{38}$$

which is stronger than the dark matter constraint of Damour *et al.* [15] and not very far from the universal constraint of Wetterich [7]. Naturally this constraint holds only when $\Omega_m = 0.3$ and the other parameters have been fixed as in Eq. (35). However, all these parameters can be fixed by observations independent of the CMB, so the hope of narrowing down the bound on β is not precluded. As we have shown, the degeneracy between n_s and β can be broken at $l > 400$. A systematic study of the the constraints on β letting the other parameters vary as well is under way.

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