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Electromagnetic wave tails on curved spacetime and their observational implications

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We calculate the tail term of the electromagnetic potential of a pulsed source in arbitrary bounded motion in a weak gravitational field using a higher-order Green's function approach and demonstrate that generally the received radiation tail arrives after a time delay which represents geometrical backscattering by the central gravitational source. We apply the results to a compact binary system and conclude that under certain conditions the tail energy can be a noticeable fraction of the direct pulse energy.

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It is known that the presence of gravitational fields modifies the character of electromagnetic radiation, the two principal phenomena involved being redshift and backscatter. While redshift is well understood, less attention has been paid to backscattering, the effect of which is in most situations negligible. However, in certain cases backscattering can influence observations as it weakens and disperses sharp initial pulses. An electromagnetic (or gravitational) radiation pulse from a source in the vicinity of a massive body is split, and when reaching an asymptotic observer it is generally received as two distinct pulses: one arriving along the direct route and the other, called the tail contribution, being effectively scattered off by the central body [1–3]. In our opinion, the delay effect of the wave tails may be of great importance for the observational detection of the wave tails. Indeed, in the case of a compact binary astrophysical system in which the source of electromagnetic radiation is a pulsar, the intensity and the delay of the tail significantly depend on the relative positions of the observer, the wave source and the source of gravitation. Comparing the profiles of the pulses emitted at different phases of rotation, it is possible to distinguish between the direct pulse and the tail (at least in principle if the delay effect is minute).

Recently the wave tails have come to be recognized as factors in the planned detection of the gravitational waves by forthcoming laser interferometric detectors [4–9]. It has also been shown that the tails play an important role in the generation of gravitational waves by the orbital inspiral of a compact binary system [10–12]. The close relationship between the generation of the wave tails and gravitational focusing has been demonstrated in Ref. [13].

To investigate the electromagnetic wave tails we consider first the vector wave equation [14] $\mathbf{Lu}=\mathbf{f}$, which in local coordinates can be written in a coordinate invariant form as $Lu^c := g^{ab}\nabla_a\nabla_b u^c - R^{ac}u_a = f^c$, where g^{ab} and R_{ac} are the components of the metric and Ricci tensors. In order to be able to complete the construction of the solution $\mathbf{u}$, we restrict the solutions to a causal domain $\Omega \subseteq M$ (see [15,16]). The inhomogeneous term $\mathbf{f}$ in general is a distribution, i.e. $\mathbf{f}\in\mathcal{D}'^1(\Omega)$. Let the sets $D^{\pm}(y)$ denote the respective interiors of future and past light cones $C^{\pm}(y)$ with vertex at y and let $J^{\pm}(y) := D^{\pm}(y)\cup C^{\pm}(y)$.

The process of wave propagation on a curved spacetime being quite complicated, usually the wave equation is solved in the weak-field and slow-motion limit using successive approximations. However, we have recently developed a method [17–20] for calculating the exact and approximate solutions of scalar and tensor wave equations whose source terms are arbitrary order multipoles. It is worthwhile to point out that, as distinct from most papers concerned with wave tails in which the source of the gravitational field is regarded as a point mass, within the framework of our approach [17,18] the extention of the source of gravitation is finite. The last circumstance enables us to avoid the occurrence of a non-physical singularity, the regularizing of which may bring on difficulties in the interpretation of the results.

Specifically, in the paper [18] we obtained a solution describing 2^{μ}-pole radiation of a vector field, i.e. we calculated the unique retarded exact solution $\mathbf{u}_{\mu}^{+}$ of the vector wave equation $Lu_{\mu}^{a}=\rho_{\mu}^{a}$ with a multipole source term

$$\rho_{\mu}^{a} := (-1)^{\mu} M_{\mu}^{I(\mu)j}\nabla_{A(\mu)}[g_{I(\mu)}^{A(\mu)}(x,y)g_{j}^{a}(x,y)\overline{\delta}(x,\xi)]. \tag{1}$$

Here the Greek index μ determines the multipole order of the solution, $g_{j}^{a}(x,y(t))$ and $g_{I(\mu)}^{A(\mu)}(x,y(t))$ are the parallel transport propagators for a vector and μ-order tensor, respectively. The multipole moment $\mathbf{M}_{\mu}(t)$ is a tensor field of order $\mu+1$ on the worldline ξ of the source of electromagnetic field, and is determined through the Dixon reduced multipole moments $\mathbf{Q}_{\mu}(t)$ (see [21]) by $M_{\mu}^{I(\mu)j} := 2\mu(\mu+1)^{-1}Q^{i_{\mu}i_2\cdots i_{\mu-1}i_1 j}$, $\mu\geqslant 1$. The line distribution

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$\overline{\delta}(x,\xi)$ is defined by the following relation: $(\rho^a_\mu,\phi_a) := \int_\xi M^{I(\mu)j}_\mu(t)\phi_{j;I(\mu)}(y(t))dt$ with $y(t)\in\xi$. Herein the coordinates of the points of the curve ξ are denoted by $y(t)$, where the parameter t is the proper time along the worldline ξ of the source of electromagnetic radiation. As mentioned, the exact solution $u^{+a}_\mu = {}_{\mathrm{direct}}u^{+a}_\mu + \mathcal{V}^a_\mu$ (where ${}_{\mathrm{direct}}u^{+a}_\mu$ represents the direct wave and $\mathcal{V}^a_\mu$ the wave tail) is given in Ref. [18]. Here we assume that $\mathbf{M}_\mu(t)=0$ for $t<t_0$ and write explicitly only the tail part of the solution for $\forall x \in J^+(\xi)\setminus\{\xi\}$, namely

$$\mathcal{V}^a_\mu = \frac{1}{2\pi}\int_{t_0}^{\tau(x)} V^a_{j;I(\mu)}(x,y(t))M^{I(\mu)j}_\mu(t)dt, \tag{2}$$

where the bivector $\mathbf{V}(x,y)$ is the tail term of the classical Green's function [see Eq. (21) in [18]], the retarded time $\tau(x)$ is a solution of the equation $\sigma(x,y(\tau(x)))=0$ such that the future light cone $C^+(y(t))$ is determined by the equation $\tau(x)=t$, and $\sigma(x,y)$ denotes the world function. It follows from the symmetry properties of Dixon's moments $\mathbf{Q}_\mu$ and the uniqueness of the retarded solution of the wave equation that the solution $\mathbf{u}^+_\mu$ satisfies the gauge condition $\nabla_a u^{+a}_\mu = 0$. Thus the quantity $\mathbf{u}^+_\mu$ can be interpreted as the exact retarded potential of the electromagnetic field.

We assume that the gravitational field is weak. So we expand geometrical quantities into a series around their flat spacetime values, retaining only the first-order terms. As the expansions of the Ricci tensor and the tail term begin with the first-order terms, in what follows we will use the symbols R_{ab} and $\mathbf{V}$ to mean first-order small quantities in the gravitational constant.

On the basis of Refs. [16,22] we will prove, in an extended paper, that in the first-order approximation the tail term of the fundamental solution can be written as

$$V^i_a(x,y) = \frac{1}{4\pi} g^i_a P^b(x,y)\nabla_b[\sigma^{-1}(x,y)] + F^i_a(x,y),$$
$$P^b(x,y) = \int_{\Sigma(y)} g^b_p(x,z)G^{pq}(z)d\Sigma_q(z), \tag{3}$$

where G^{pq} is the Einstein tensor and $d\Sigma_q(z)$ denotes a 3-surface element of the hypersurface $\Sigma(y) := C^+(y)\cap J^-(x)$. The quantity $F^i_a(x,y)$, which will not be explicitly used below, is nonzero only if the 2-dimensional surface $S(y) := C^+(y)\cap C^-(x)$ intersects the source of gravitation (see Fig. 1).

In order to draw Fig. 1 we have suppressed the time dimension and one space dimension. Thus all the images in Fig. 1 are obtained by projecting the Minkowski spacetime onto a hypesurface $x^0=$const, and then cutting the 3-space with x^1x^2. We have also made the following simplifying assumptions which are not used in our calculations: (i) the gravitational source is spherical and static, and (ii) the gravitational and wave sources, and the observer, do not move with respect to each other. The plane of the figure is determined by locations of the wave source y, of the observer x and of the center (not pointed out on the figure) of the gravitational source. The surface $S(y)$, which spreads with time, is the boundary of the ellipsoid of revolution $\Sigma(y)$, with foci at the locations of the observer x and of the wave source y. The ellipses S_1 and S_2 are the intersections of the plane of the figure with the surface $S(y)$ that, respectively, correspond to the instants of time at which a delta-like wave pulse emitted by the wave source reaches and passes the source of gravitation. If the surface $S(y)$ has not yet reached the gravitational source, then $\mathbf{V}=0$ and there is no wave tail. If the surface $S(y)$ has passed the gravitational source, the source will forever remain inside the region of integration $\Sigma(y)$, while $\mathbf{P}(x,y)/8\pi=$const will be the 4-momentum of the gravitational source and $F^i_a(x,y)=0$. The shadowed area K, which includes the source of gravitational field, corresponds to the region where the tail wave field is predominantly generated by gravitational focusing which deforms the direct wave fronts [13]. Thus we have the following picture. The wave source y emits an instantaneous wave pulse. The direct pulse propagates along the direct route yx to the observer x, after which there occurs a blackout before the arrival of the first tail contribution. The parts of the wave front which travel (along the routes l_1, l_2, etc.) to the points of the region K, scatter off (reflect) from there and then propagate (along l^*_1, l^*_2, etc.) to x appearing to the observer as the tail wave.

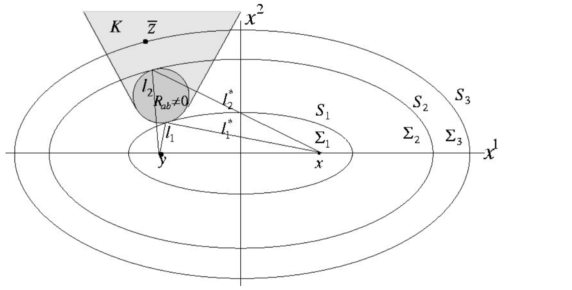

FIG. 1. A schematic representation of the geometry of wave tail generation on a spacelike plane x^1x^2. Here y is the wave source, x is the observer, the darker shadowed circular disk is the gravitational source and K is the focusing region. The remaining components in the figure have been explained in the paragraph following the figure.

Among the astrophysical applications of great interest is the case in which the source of gravitation is spatially isolated, i.e. supp $R_{ab}\subset\tilde{\Gamma}$, where $\tilde{\Gamma}$ is the world tube of the gravitational field source. In what follows we will presume that the worldline ξ of the source of electromagnetic waves remains outside the isolated gravitational source, whereas $y(t_0)$ and $y(t_1)$ are points on the worldline ξ of the wave source, t_0 and t_1 being, respectively, the proper time values when the source begins and terminates the emission. Then in the structure of the tail term $\mathcal{V}^a_\mu$ there appear features similar to the case of the tail term of the fundamental solution $\mathbf{V}$. First, we divide the spacetime domain $J^+(y(t_0))\setminus\{\xi\}$ into three subdomains: $E:=\{x|x\in J^+(y(t_0))\setminus\{\xi\},\Sigma(y(t_0))\cap\tilde{\Gamma}=\varnothing\}$, $E^*:=\{x|x\in J^+(y(t_0))\setminus\{\xi\},C^+(y(t_1))\cap\tilde{\Gamma}\subset\Sigma(y(t_1)),S(y(t_1))\cap\tilde{\Gamma}=\varnothing\}$ and $\tilde{E}:=\{x|x\in J^+(y(t_0))\setminus\{\xi\},x\notin E,x\notin E^*\}$ with $y(t)\in\xi$. If $x\in E$, then for every $t\in(t_0,\tau(x))$ neither the hypersurface

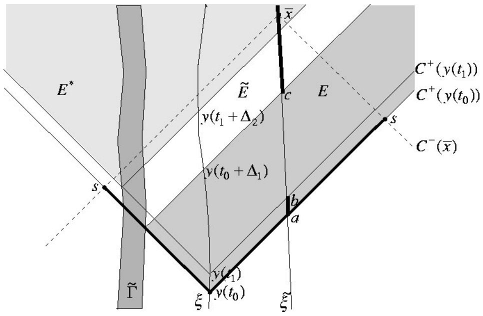

FIG. 2. A diagram in Minkowski 2-spacetime: the domain $J^+(y(t_0))$ divided into subdomains E, E^* and $\tilde{E}$. The subdomains E and $\tilde{E}$ are shadowed. The past light-cone $C^-(\bar{x})$ originating from the point $\bar{x}$ is represented by the dotted lines. The bold-faced part of the future light-cone $C^+(y(t_0))$ corresponds to the hypersurface $\Sigma(y(t_0))$ with $x=\bar{x}$, and the boundary of the hyperplane $\Sigma(y(t_0))$, i.e., $S(y(t_0))$ is seen as the two points s. $\tilde{\Gamma}$ is the worldtube of the source of gravitational field. ξ is the worldline of the wave source which radiates during a finite proper time interval $[t_0,t_1]$. $\tilde{\xi}$ is the worldline of the observer. On the observer's worldline $\tilde{\xi}$ these intervals are indicated in bold-face where the observer can in principle see the direct pulse (the interval $[a,b]$) and the wave tail (the semi-interval $[c,\infty)$). During the interval (b,c) on $\tilde{\xi}$ there occurs a blackout between the direct pulse and the wave tail.

$\Sigma(y)$ nor the 2-surface $S(y)$ intersect the world tube $\tilde{\Gamma}$. Hence it follows from Eq. (3) that $V^a_j=0$, and consequently $\mathcal{V}^a_\mu=0$, $\forall x\in E$. As $E\neq\emptyset$, *there exist points in space where the (fore)front of the wave is not simultaneously accompanied by the wave tail, the latter appearing after some time delay* (see Fig. 2).

To describe this effect in more detail, let us define the quantity $\tau_0(x,z)$ as a solution of the following set of equations: $\sigma(x,z)=0$, $z\in C^-(x)\cap\tilde{\Gamma}$, $\sigma(z,y(\tau_0(x,z)))=0$, $y(\tau_0)\in C^-(z)$. Further it is convenient to introduce the maximal and minimal values of the time interval $\tau(x)-\tau_0(x,z)$ (measured in the proper time of the wave source) for the fixed spacetime point x, namely $\Delta_1(x):=\tau(x)-\max\,\tau_0(x,z)$, $\Delta_2(x):=\tau(x)-\min\,\tau_0(x,z)$. The quantities $\Delta_1(x)$ and $\Delta_2(x)$ can be given a simple interpretation by taking into account that an instantaneous wave pulse emitted by the wave source at max $\tau_0(x,z)$ traveling at the speed of light reaches the spacetime point x exactly as if it would first travel to the source of gravitation, reflect from there and further travel to the spacetime point x. In the case of min $\tau_0(x,z)$ the reflection takes place precisely when the above-mentioned wavepulse has passed the source of gravitation. To the domain $\tilde{E}$ corresponds the interval $(t_0+\Delta_1(x),t_1+\Delta_2(x))$ of the retarded time $\tau(x)$, and in order to obtain the expression for the tail term in the domain $\tilde{E}$, one must replace the upper limit $\tau(x)$ in Eq. (2) by min $(\tau(x)-\Delta_1(x),t_1)$. Hence, in comparison with the direct pulse, the tail of the wave appears to the observer after a time delay $\Delta_1(x)$. The particular case $\Delta_1(x)>t_1-t_0$ is also of interest. In it there occurs a time lapse of duration $\Delta(x)$ between the end of the principal pulse ($\tau(x)=t_1$) and the appearance of the wave tail, with $\Delta(x):=\Delta_1(x)-(t_1-t_0)$. Thus, *instead of a single pulse, the observer would see two clearly separable pulses: the direct one and the wave tail.*

To elucidate the introduced time intervals, let us in this paragraph return to the example presented in Fig. 1. At the time $y^0=t_0$ the wave source emits a delta-like pulse. The evolution of the surface $S(y)$ is characterized by the time-varying semi-major a and semi-minor b axes which depend on the observer time x^0 as $a=(x^0-t_0)/2$ and $b=(\sigma(x,y))^{1/2}/2$. The ellipses S_1,S_2 and S_3 are the intersections of the plane of the figure with the surface $S(y)$ at times of observation $x^0_1=t_0+|\vec{x}-\vec{y}|+\Delta_1$, $x^0_2=t_0+|\vec{x}-\vec{y}|+\Delta_2$ and $x^0_3>x^0_2$, respectively. Here $|\vec{x}-\vec{y}|$ is the spatial separation between the wave source and the observer. The observer detects the direct pulse at the time $x^0_0=t_0+|\vec{x}-\vec{y}|$, the wave tail begins to appear at $x^0_1=t_0+l_1+l^*_1=x^0_0+\Delta_1$, and beginning from the time $x^0_2=t_0+l_2+l^*_2=x^0_0+\Delta_2$ the structure of the wave tail is determined by Eq. (3) with $F^i_a=0$. The time interval $\Delta_1=l_1+l^*_1-|\vec{x}-\vec{y}|$ is evidently equal to the difference of the propagation times of two pulses from the wave source to the observer: one arriving along the direct route and the other first traveling to the source of gravitation, reflecting from there and then reaching the observer.

In what follows we will examine the radiative part of the electromagnetic wave tails for the case in which the source of gravitation is spatially isolated and the distance of the source of waves from the source of gravitation is bounded. On the assumption that the local part $A^a(x,\tau)$ of a wave observable at infinity (or its zeroth-order radiative part) can be approximated with sufficient accuracy by the superposition of a finite number of multipole waves, it follows from Eq. (2) that the corresponding nonlocal radiative wave-propagation correction $E^a(x,\tau)$ has the following form:

$$E^a(x,\tau)=\int_{t_0}^{\tau(x)}\psi g^i_b A^b(x,t)V^a_i dt, \tag{4}$$

where $\psi(x,t):=-d\sigma(x,y(t))/dt$.

If $A^b(x,\tau)$ is non-zero only during a finite time interval $t_0<\tau<t_1$ and the worldline of the source of radiation lies outside the world-tube of the source of gravitation, then the above-mentioned principal properties of the tail term $\mathcal{V}^a_\mu$ of a multipole solution are also valid for the radiative part of the tail term E^a. Thus, for example, if $\tau(x)\leqslant t_0+\Delta_1(x)$, then $E^a(x,\tau)=0$; and if $\tau(x)\geqslant t_1+\Delta_2(x)$, then

$$E^a(x,\tau)=\frac{1}{4\pi}\int_{t_0}^{t_1}\frac{\sigma_{;b}(x,y(t))P^b}{\sigma(x,y(t))}\frac{d}{dt}A^a(x,t)dt. \tag{5}$$

Let us emphasize that applicability of Eqs. (4), (5) does not depend on whether the source of radiation actually emits in the pulsed mode or pulses of radiation are present at the location of an observer due to the kinematics of the radiating system (e.g. the rotation of the directed radiation cone of a pulsar).

For a source radiating in the pulsed mode the last formula enables us to predict, comparatively simply (on the basis of the parameters of a recorded direct pulse and a presumable model of an astrophysical object), the physical parameters of the radiation tail and evaluate the possibilities of observational detection of the tail. A matter of particular interest is the situation in which $\Delta_1(x)>(t_1-t_0)$, as in this case there is a blackout during the time interval $\Delta(x)=\Delta_1(x)-(t_1-t_0)$ between the end of the direct pulse and the appearance of the wave tail. This considerably simplifies the observation methods for distinguishing the profile of the direct pulse from the general relativistic radiation tail originating from compact astrophysical binary systems. The relative intensity of the direct pulse and the wave tail, and the time delay (blackout) between them can yield essential additional information, independently of other methods, about the physical characteristics of a binary (the distance between the compact objects, the mass of the source of gravitation, the orientation of the plane of the orbit with respect to the observer, etc.).

From Eq. (5) it follows that the wave tail effect of the astrophysical systems radiating in the pulsed mode is predominantly caused by the low-frequency modes ($\omega \lesssim 2\pi/(t_1-t_0)$) of the direct pulse. (Here the angular frequency ω is measured in the proper time of the wave source.) The high-frequency waves reflecting from the spacetime curvature interfere and prevalently attenuate each other in the expression of the tail. Also important, from the point of view of observational detection, is the fact ensuing from Eq. (4) that the intensity and the time delay of the tail of radiation from a pulsed source moving in a circular orbit depend substantially on the position of the observer with respect to the sources of waves and gravitation. By comparing the profiles of the pulses emitted at different points of an orbit, one can distinguish in principle the contributions of the direct pulse and the tail even if there is no considerable time delay between the tail and the principal pulse. This circumstance is significant because in the case of most astrophysically realistic models the physical conditions necessary for the occurrence of a blackout between the direct pulse and the wave tail will considerably decrease the intensity of the tail.

Let us now turn to the magnitude of electromagnetic radiative energy arriving after the direct pulse has passed, $\tau(x)>t_1+\Delta_2(x)$. The tail term being a first-order small quantity, we can regard the spacetime as flat and use Minkowskian coordinates whose origin lies inside the world tube of the source of gravitation. We denote the distance of the observer from the origin of the coordinates by r, and the position vectors of the points x and y in 3-space by $\vec{x}$ and $\vec{y}$, respectively. The power radiated into the solid angle $d\Omega$ can be calculated as follows:

$$I(\tau)\equiv\frac{dP}{d\Omega}=-\frac{r^2}{4\pi}E^a_{,0}(x,\tau)E_{a,0}(x,\tau). \tag{6}$$

In the proper reference frame of the source of gravitational field we have

$$E^a_{,0}=-2M\int_{u(t_0)}^{u(t_1)}\frac{[A^a_{,0}(x,\tau(x))]_{\tau=t(u)}}{(u(\tau)-u)^2}du, \tag{7}$$

where $u(t):=y^0(t)-\vec{n}\cdot\vec{y}(t)$, $\vec{n}:=\vec{x}/|\vec{x}|$ and M is the mass of the source of gravitational field.

To get an idea of the magnitudes involved we construct an artificial example in which

$$A^a_{,0}(x,\tau(x))|_{\tau=t(u)}=\frac{1}{r}f^a(\vartheta,\varphi)\Theta(u-u(t_0))\Theta(u(t_1)-u).$$

Here Θ is the Heaviside distribution, ϑ,φ are the polar angles of the observer's position, and f^a is an arbitrary vector function. From Eqs. (6), (7) we obtain an estimate of the ratio of the intensity of the wave tail $I(t_1+\Delta_2(x))$ to the intensity of the direct pulse I_0, namely

$$\frac{I(t_1+\Delta_2(x))}{I_0}=\left[\frac{2M}{\tilde{\Delta}_2(x)}\right]^2\left[\frac{T}{T+\tilde{\Delta}_2(x)}\right]^2, \tag{8}$$

where $T:=u(t_1)-u(t_0)$ is the duration of the direct pulse in the observer time, and $\tilde{\Delta}_2:=u(t_1+\Delta_2(x))-u(t_1)$. We see that if $\tilde{\Delta}_2(x)$ is sufficiently small, i.e. of the order of $2M$, the intensity of the tail can be comparable with that of the direct pulse. Evidently in this case we cannot confine ourselves to the first approximation but must instead go to higher approximations.

To illustrate the above estimate (8), let us consider a wave source rotating in a circular orbit (of radius r_0) around a spherical source (of radius r_s) of gravitational field. The origin of the spatial coordinates is taken to coincide with the center of the source of gravitation. Under the conditions $\vec{n}\cdot\vec{y}(t_1)<0$ and $r_0\gg d+r_s$, where $d:=r_0\sqrt{1-(\vec{n}\cdot\vec{y}(t_1)/r_0)^2}$, it follows that $\tilde{\Delta}_2(x)=(d+r_s)^2/2r_0+O(M)$. Hence, if $r_0>r_s^2/4M$, then in case the wave pulse is emitted in the region of the geometric shadow of the source of gravitation or in its vicinity, we have $\tilde{\Delta}_2(x)\sim 2M$, and according to the estimate (8) the intensity of the tail can be of the same magnitude as the intensity of the primary pulse.

For the model under discussion it is also easy to find the ratio of the energy $\mathcal{E}$ transferred by the tail term [beginning from the time $t_1+\Delta_2(x)$] to the energy $\mathcal{E}_0$ of the direct term, namely

$$\frac{\mathcal{E}}{\mathcal{E}_0}=\left[\frac{2M}{\tilde{\Delta}_2(x)}\right]^2\mathcal{F}\left(\frac{\tilde{\Delta}_2(x)}{T}\right),$$

$$\mathcal{F}(\rho):=\rho+\frac{\rho^2}{\rho+1}-2\rho^2\ln\frac{\rho+1}{\rho}.$$

It is interesting to note that the function $\mathcal{F}(\rho)$ has a maximum at $\rho\approx 0.638$. Therefore, for the given model, the amount of energy transferred by the tail after the time $t_1+\Delta_2(x)$ is maximal if the duration of the direct pulse is $T\approx 1.57\tilde{\Delta}_2(x)$, and it is equal to $\mathcal{E}\approx 0.119\mathcal{E}_0(2M)^2/\tilde{\Delta}_2^2(x)$.

As the quantity $\mathcal{F}$ does not explicitly depend on the mass of the gravitational source, then its value for any particular case (as 0.119 for the present example) is primarily determined by the profile of the direct pulse and the spatial configuration of the system consisting of the wave source, gravitational source and the observer. Somewhat unexpected is the outcome that the magnitude of the factor $(2M/\tilde{\Delta}_2)^2$, which characterizes the influence of the gravitational source, can be of the order 1, even if the potential of the gravitational source is low everywhere, i.e. $2M \ll r_s$. This conclusion can be understood on the basis of Ref. [13] where it is shown that the tail field is predominantly generated by the direct field in those regions where gravitational focusing has deformed the geometry of the direct wave fronts (see the shadowed region in Fig. 1). The deformation is characterized by the focusing function $\alpha(\overline{z},y)$. If the points $\overline{z}$ and y lie on a geodesic line which does not cross the gravitational source, then $\alpha(\overline{z},y)=0$. If the distance r_0 of the wave source from the gravitational source is much larger than the extension r_s of the latter, then for the rays originating in the wave source and passing through the gravitational source it is valid $\alpha \sim M r_0/r_s^2$. In the case of our example the condition $2M/\tilde{\Delta}_2 \approx 1$ corresponds to $\alpha \sim 1$. Let us note that the last effect is not revealed by the traditional methods based on the expansion in terms of spherical functions, as in this case it is natural to choose the effective location of the wave source within the source of gravitation. Thus, $\tilde{\Delta}_2 \approx r_s$ and $\alpha \sim M/r_s$, that is, the intensity of the tail is proportional to the square of the potential of the gravitational field on the surface of the source of gravitation.

To sum up, we emphasize that in the case of compact binary astrophysical objects the time interval (blackout) between the direct pulse and the tail caused by delay of the tail may be observable, providing, independently of other methods, in terms of the relative intensity of the tail, essential information about the characteristics of the physical system. The intensity of the electromagnetic wave pulse emanating from the wave source within a compact binary in the vicinity of the geometric shadow of the source of gravitation can be of the same magnitude as the intensity of the direct pulse, and the energy carried away by the tail can amount to 10% of the energy of the low-frequency modes of the direct pulse.

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[1] P. E. Roe, Nature (London) **227**, 154 (1970).
[2] P. C. Peters, Phys. Rev. D **7**, 368 (1973).
[3] P. C. Peters, Phys. Rev. D **1**, 1559 (1970).
[4] L. Blanchet and B. S. Sathyaprakash, Class. Quantum Grav. **11**, 2807 (1994).
[5] L. Blanchet and T. Damour, Phys. Rev. D **46**, 4304 (1992).
[6] L. Blanchet, Phys. Rev. D **55**, 714 (1997).
[7] W. B. Bonnor and M. S. Piper, Class. Quantum Grav. **15**, 955 (1998).
[8] S. W. Leonard and E. Poisson, Phys. Rev. D **56**, 4789 (1997).
[9] L. Blanchet, Class. Quantum Grav. **15**, 113 (1998).
[10] L. Blanchet and G. Schäfer, Class. Quantum Grav. **10**, 2699 (1993).
[11] E. Poisson, Phys. Rev. D **47**, 1497 (1993).
[12] C. Cutler and E. E. Flanagan, Phys. Rev. D **49**, 2658 (1994).
[13] K. S. Thorne and S. J. Kovacs, Astrophys. J. **200**, 245 (1975).
[14] Boldface letters denote vectors and tensors. We use Latin letters for tensor indices and multiindices, and assume summing over them: indices $a,b,A(\mu) := (a_1 \ldots a_\mu)$ refer to the point x, indices $i,j,I(\mu) := (i_1 \ldots i_\mu)$ to y and indices p,q to z, according to our index convention for multi-point tensors. Covariant differentiation is denoted by ∇ and semicolon. We use units in which $c=G=1$.
[15] P. Günther, *Huygens' Principle and Hyperbolic Equations* (Academic, New York, 1988).
[16] F. G. Friedlander, *The Wave Equation on a Curved Space-Time* (Cambridge University Press, Cambridge, England, 1975).
[17] R. Mankin, R. Tammelo, and T. Laas, Class. Quantum Grav. **16**, 1215 (1999).
[18] R. Mankin, R. Tammelo, and T. Laas, Gen. Relativ. Gravit. **31**, 537 (1999).
[19] R. Mankin, R. Tammelo, and T. Laas, Class. Quantum Grav. **16**, 2525 (1999).
[20] T. Laas, R. Mankin, and R. Tammelo, Class. Quantum Grav. **15**, 1595 (1998).
[21] W. G. Dixon, Proc. R. Soc. London **A319**, 509 (1970).
[22] R. Mankin and A. Sauga, Trans. Inst. Phys. Estonian Acad. Sci. **65**, 41 (1989).