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Chaotic inflation on the brane

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We consider slow-roll inflation in the context of recently proposed four-dimensional effective gravity induced on the world-volume of a three-brane in five-dimensional Einstein gravity. We find significant modifications of the simplest chaotic inflationary scenario when the five-dimensional Planck scale is below about 10^{17} GeV. We use the comoving curvature perturbation, which remains constant on super-Hubble scales, in order to calculate the spectrum of adiabatic density perturbations generated. Modifications to the Friedmann constraint equation lead to a faster Hubble expansion at high energies and a more strongly damped evolution of the scalar field. This assists slow-roll, enhances the amount of inflation obtained in any given model, and drives the perturbations towards an exactly scale-invariant Harrison-Zel'dovich spectrum. In chaotic inflation driven by a massive scalar field we show that inflation can occur at field values far below the four-dimensional Planck scale, though above the five-dimensional fundamental scale.

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I. INTRODUCTION

There is considerable interest in higher dimensional cosmological models motivated by superstring theory solutions where matter fields (related to open string modes) live on a lower dimensional brane while gravity (closed string modes) can propagate in the bulk [1]. In such a scenario the extra dimension need not be small [2], and may even be infinite if non-trivial geometry can lead gravity to be bound to the three-dimensional subspace on which we live at low energies [3–6]. One possibility of great importance arising from these ideas is the notion that the fundamental Planck scale M_{4+d} in $4+d$ dimensions can be considerably smaller than the effective Planck scale, $M_4=1.2\times10^{19}$ GeV, in our four-dimensional spacetime, which would have profound consequences for models of the very early universe.

In this Rapid Communication we investigate the impact of such a scenario when $d=1$ [4] for simple chaotic inflation models. Specific models of inflation have previously been discussed with finite compactified dimensions, scalar fields in the bulk and/or multiple branes (see, e.g., [7–9]). Our aim is to quantify the minimal modification of slow-roll inflation in the brane scenario for arbitrary inflaton potentials on the brane, independent of the dynamics of the bulk, while assuming stability of the brane. If Einstein's equations hold in the five-dimensional bulk, with a cosmological constant as source, and the matter fields are confined to the 3-brane, then Shiromizu et al. [10] have shown that the four-dimensional Einstein equations induced on the brane can be written as

$$G_{\mu\nu}=-\Lambda_4 g_{\mu\nu}+\left(\frac{8\pi}{M_4^2}\right)T_{\mu\nu}+\left(\frac{8\pi}{M_5^3}\right)^2\pi_{\mu\nu}-E_{\mu\nu}, \tag{1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter on the brane, $\pi_{\mu\nu}$ is a tensor quadratic in $T_{\mu\nu}$, and $E_{\mu\nu}$ is a projection of the five-dimensional Weyl tensor, describing the effect of bulk graviton degrees of freedom on brane dynamics. The effective cosmological constant Λ_4 on the brane is determined by the five-dimensional bulk cosmological constant Λ and the 3-brane tension λ as

$$\Lambda_4=\frac{4\pi}{M_5^3}\left(\Lambda+\frac{4\pi}{3M_5^3}\lambda^2\right), \tag{2}$$

and the four-dimensional Planck scale is given by

$$M_4=\sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\lambda}}\right)M_5. \tag{3}$$

In a cosmological scenario in which the metric projected onto the brane is a spatially flat Friedmann-Robertson-Walker model, with scale factor $a(t)$, the Friedmann equation on the brane has the generalized form [11]

$$H^2=\frac{\Lambda_4}{3}+\left(\frac{8\pi}{3M_4^2}\right)\rho+\left(\frac{4\pi}{3M_5^3}\right)^2\rho^2+\frac{\mathcal{E}}{a^4}, \tag{4}$$

where $\mathcal{E}$ is an integration constant arising from $E_{\mu\nu}$, and thus transmitting bulk graviton influence onto the brane. This term appears as a form of ''dark radiation'' [11,12] affecting primordial nucleosynthesis and the heights of the acoustic peaks in the cosmic microwave background radiation, because it is decoupled from matter on the brane and behaves like an additional collisionless (and isotropic) massless component. Thus observations can be used to place limits on $|\mathcal{E}|$. However, during inflation this term will be rapidly diluted, and we can neglect it. We will also assume that the bulk cosmological constant $\Lambda\approx-4\pi\lambda^2/3M_5^3$ so that Λ_4 is negligible, at least in the early universe. This fine-tuning is the restatement in the brane-world scenario of the cosmological constant problem and we do not attempt to solve it here.

The crucial correction in what follows is the term quadratic in the density, which modifies the expansion dynamics at densities $\rho\gtrsim\lambda$. This can be seen on rewriting Eq. (4) using Eq. (3), when $\Lambda_4=0$ and $\mathcal{E}=0$, to give

$$H^2=\frac{8\pi}{3M_4^2}\rho\left[1+\frac{\rho}{2\lambda}\right]. \tag{5}$$

Note that in the limit $\lambda\to\infty$ we recover standard four-dimensional general relativistic results (neglecting $\mathcal{E}$). The quadratic modification will dominate at high energies for moderate λ, but must be sub-dominant at nucleosynthesis. Since it decays as a^{-8} during the radiation era, it will rapidly become negligible thereafter. The nucleosynthesis limit implies that $\lambda\gtrsim(1\text{ MeV})^4$, and by Eq. (3) this gives [9]

$$M_5\gtrsim\left(\frac{1\text{ MeV}}{M_4}\right)^{2/3}M_4\sim10\ \text{ TeV}. \tag{6}$$

A more stringent constraint may be obtained if the fifth dimension is infinite, by requiring that relative corrections to the Newtonian law of gravity, which are of order $M_5^6\lambda^{-2}r^{-2}$ (see, e.g., [4]), should be small on scales $r\gtrsim1$ mm. Using Eq. (3), this gives $M_5>10^5$ TeV.

II. SLOW-ROLL INFLATION ON THE BRANE

We will consider the case where the energy-momentum tensor $T_{\mu\nu}$ on the brane is dominated by a scalar field ϕ (confined to the brane) with self-interaction potential $V(\phi)$. The field satisfies the Klein-Gordon equation

$$\ddot{\phi}+3H\dot{\phi}+V'(\phi)=0, \tag{7}$$

since $\nabla^\nu T_{\mu\nu}=0$ on the brane. In four-dimensional general relativity, the condition for inflation is $\dot{\phi}^2<V(\phi)$, i.e., $p<-\frac{1}{3}\rho$, where $\rho=\frac{1}{2}\dot{\phi}^2+V$ and $p=\frac{1}{2}\dot{\phi}^2-V$. This guarantees $\ddot{a}>0$. The modified Friedmann equation leads to a *stronger condition for inflation:* using Eqs. (5) and (7), we find that

$$\ddot{a}>0\Rightarrow p<-\left[\frac{\lambda+2\rho}{\lambda+\rho}\right]\frac{\rho}{3}. \tag{8}$$

As $\lambda\to\infty$, this reduces to the violation of the strong energy condition, but for $\rho>\lambda$, a more stringent condition on p is required for accelerating expansion. In the limit $\rho/\lambda\to\infty$, we have $p<-\frac{2}{3}\rho$. When the only matter in the universe is a self-interacting scalar field, the condition for inflation becomes

$$\dot{\phi}^2-V+\frac{\dot{\phi}^2+2V}{8\lambda}(5\dot{\phi}^2-2V)<0, \tag{9}$$

which reduces to $\dot{\phi}^2<V(\phi)$ when $(\dot{\phi}^2+2V)\ll\lambda$.

Assuming that the "brane energy condition" in Eq. (8) is satisfied, we now discuss the dynamics of the last 50 or so e-foldings of inflation. Within the slow-roll approximation, we assume that the energy density is dominated by the self-interaction energy of the scalar field and that the scalar field evolution is strongly damped, which implies

$$H^2\simeq\left(\frac{8\pi}{3M_4^2}\right)V\left[1+\frac{V}{2\lambda}\right], \tag{10}$$

$$\dot{\phi}\simeq-\frac{V'}{3H}, \tag{11}$$

where we use "$\simeq$" to denote equality within the slow-roll approximation. The term in square brackets is the brane-modification to the standard slow-roll expression for the Hubble rate. For $V\gg\lambda$, Eqs. (3) and (10) give $H\simeq(4\pi/3)V/M_5^3$ consistent with the "non-linear" regime discussed in Ref. [7].

Requiring the slow-roll approximation to remain consistent with the full evolution equations places constraints on the slope and curvature of the potential. We can define two slow-roll parameters

$$\epsilon\equiv\frac{M_4^2}{16\pi}\left(\frac{V'}{V}\right)^2\left[\frac{2\lambda(2\lambda+2V)}{(2\lambda+V)^2}\right], \tag{12}$$

$$\eta\equiv\frac{M_4^2}{8\pi}\left(\frac{V''}{V}\right)\left[\frac{2\lambda}{2\lambda+V}\right]. \tag{13}$$

Self-consistency of the slow-roll approximation then requires $\max\{\epsilon,|\eta|\}\ll1$. At low energies, $V\ll\lambda$, the slow-roll parameters reduce to the standard form (see, e.g., Refs. [13,14]). However at high energies, $V\gg\lambda$, the extra contribution to the Hubble expansion helps damp the rolling of the scalar field and the new factors in square brackets become $\approx\lambda/V$. Thus *brane effects ease the condition for slow-roll inflation for a given potential.*

The number of e-folds during inflation is given by $N=\int_{t_i}^{t_f}Hdt$, which in the slow-roll approximation becomes

$$N\simeq-\frac{8\pi}{M_4^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}\left[1+\frac{V}{2\lambda}\right]d\phi. \tag{14}$$

The effect of the modified Friedmann equation at high energies is to increase the rate of expansion by a factor $[V/2\lambda]$, yielding more inflation between any two values of ϕ for a given potential. Thus we can obtain a given number of e-folds for a *smaller* initial inflaton value ϕ_i. For $V\gg\lambda$, Eq. (14) becomes $N\simeq-(128\pi^3/3M_5^6)\int_i^f(V^2/V')d\phi$.

III. PERTURBATIONS ON THE BRANE

The key test of any inflation model, or any modified gravity theory during inflation, will be the spectrum of perturbations produced due to quantum fluctuations of the fields about their homogeneous background values. To date there has been no study of linear perturbations about a four-dimensional Friedmann-Robertson-Walker universe on the brane for the modified four-dimensional Einstein equations given in Eq. (1). The key uncertainty here comes from the tensor $E_{\mu\nu}$, which describes the effect of tidal forces and gravitational waves in the vacuum five-dimensional bulk and whose evolution is not completely determined by the four-dimensional effective theory alone. In what follows we set $E_{\mu\nu}=0$, effectively neglecting back-reaction due to metric perturbations in the fifth dimension. This is consistent with a homogeneous density of matter on the brane [10] and thus is valid even in the presence of scalar field perturbations in the slow-roll limit (where $V'\to 0$), but we note that a full investigation is required to discover when back-reaction will have a significant effect.

To quantify the amplitude of scalar (density) perturbations we evaluate the gauge-invariant quantity [15]

$$\zeta \equiv -\psi - \frac{H}{\dot{\rho}}\delta\rho, \tag{15}$$

which reduces to the curvature perturbation, ψ, on uniform density hypersurfaces where $\delta\rho=0$. The four-dimensional energy-conservation equation, $\nabla^\nu T_{\mu\nu}=0$, for linear perturbations (in an arbitrary gauge) on large scales, requires that

$$\dot{\delta\rho} + 3H(\delta\rho + \delta p) - 3(\rho + p)\dot{\psi} = 0, \tag{16}$$

where we have neglected spatial gradients. We can apply Eq. (16) on uniform density hypersurfaces, where $\delta\rho=0$ and $\psi=\zeta$, [or, equivalently, use the gauge-invariant definition of ζ given in Eq. (15)] to obtain

$$\dot{\zeta} = -H\frac{\delta p_{\rm nad}}{\rho + p}. \tag{17}$$

Hence ζ is conserved on large scales for purely adiabatic perturbations, for which the non-adiabatic pressure perturbation $\delta p_{\rm nad}\equiv \delta p/\dot{p} - \delta\rho/\dot{\rho}$ vanishes. This gauge-invariant result is a consequence of the local conservation of energy-momentum in four dimensions, and is independent of the form of the gravitational field equations [16].

The curvature perturbation on uniform density hypersurfaces is given in terms of the scalar field fluctuations on spatially flat hypersurfaces, $\delta\phi$, by

$$\zeta = \frac{H\delta\phi}{\dot{\phi}}. \tag{18}$$

The field fluctuations at Hubble crossing ($k=aH$) in the slow-roll limit are given by $\langle\delta\phi^2\rangle \simeq (H/2\pi)^2$. Note that this result for a massless field in de Sitter space is also independent of the gravity theory [16]. For a single scalar field the perturbations are adiabatic and hence the curvature perturbation ζ can be related to the density perturbations when modes re-enter the Hubble scale during the matter dominated era which is given (using the notation of Ref. [14]) by $A_S^2 = 4\langle\zeta^2\rangle/25$. Using the slow-roll equations and Eq. (18), this gives

$$A_S^2 \simeq \left(\frac{512\pi}{75M_4^6}\right)\frac{V^3}{V'^2}\left[\frac{2\lambda+V}{2\lambda}\right]^3\Bigg|_{k=aH}. \tag{19}$$

Thus the amplitude of scalar perturbations is *increased* relative to the standard result at a fixed value of ϕ for a given potential.

The scale-dependence of the perturbations is described by the spectral tilt

$$n_S - 1 \equiv \frac{d\ln A_S^2}{d\ln k} \simeq -6\epsilon + 2\eta, \tag{20}$$

where the slow-roll parameters are given in Eqs. (12) and (13). Because these slow-roll parameters are both suppressed by an extra factor λ/V at high energies, we see that the spectral index is *driven towards the Harrison-Zel'dovich spectrum*, $n_S\to 1$, as $V/\lambda\to\infty$.

The tensor (gravitational wave) perturbations are bound to the brane at long-wavelengths [4] and decoupled from the matter perturbations to first-order, so that the amplitude on large scales is simply determined by the Hubble rate when each mode leaves the Hubble scale during inflation. The amplitude of tensor perturbations at Hubble crossing is given by [14]

$$A_T^2 = \frac{4}{25\pi}\left(\frac{H}{M_4}\right)^2\Bigg|_{k=aH}. \tag{21}$$

In the slow-roll approximation this yields

$$A_T^2 \simeq \frac{32}{75M_4^4}V\left[\frac{2\lambda+V}{2\lambda}\right]\Bigg|_{k=aH}. \tag{22}$$

Again, the tensor amplitude is *increased* by brane effects, but by a smaller factor than the scalar perturbations. The tensor spectral tilt is

$$n_T \equiv \frac{d\ln A_T^2}{d\ln k} \simeq -2\epsilon, \tag{23}$$

so that the ratio between the amplitude of tensor and scalar perturbations is given by

$$\frac{A_T^2}{A_S^2} \simeq \epsilon\left[\frac{\lambda}{\lambda+V}\right]\Bigg|_{k=aH}. \tag{24}$$

Thus the standard observational test for consistency condition [14] between this ratio and the tilt of the gravitational wave spectrum is modified by the pre-factor $\lambda/(\lambda+V)$, which becomes small at high energies. Although the amplitude of both tensor and scalar perturbations is enhanced due to the increased Hubble rate, the overall effect is to suppress the contribution of tensor perturbations relative to the scalar modes for a given potential V.

IV. A SIMPLE MODEL

As an example we investigate the simplest chaotic inflation model driven by a scalar field with potential $V=\frac{1}{2}m^2\phi^2$. Equation (14) gives the integrated expansion from $\phi_{\rm i}$ to $\phi_{\rm f}$ as

$$N \simeq \frac{2\pi}{M_4^2}(\phi_{\rm i}^2 - \phi_{\rm f}^2) + \frac{\pi^2 m^2}{3M_5^6}(\phi_{\rm i}^4 - \phi_{\rm f}^4). \tag{25}$$

The new term on the right arising from the modified Friedmann equation on the brane means that we always get more inflation for a given initial inflaton value $\phi_{\rm i}$.

In the usual chaotic inflation scenario [17] based on Einstein gravity in four dimensions, the value of the inflaton mass m is required to be $\approx 10^{13}$ GeV in order to obtain the observed level of anisotropies in the cosmic microwave background (see below). This corresponds to an energy scale $\approx 10^{16}$ GeV when the relevant scales left the Hubble scale during inflation, but crucially also an inflaton field value of order $3M_4$. Chaotic inflation has been criticized for requiring super-Planckian field values to solve both the problems of the standard background cosmology and lace the microwave background with anisotropies of the observed magnitude. The problem with super-Planckian field values is that one generically expects non-renormalizable quantum corrections $\sim(\phi/M_4)^n$, $n>4$ to completely dominate the potential, depriving one of control over the potential and typically destroying the flatness of the potential required for inflation (the η-problem [18]).

If the brane tension λ is much below 10^{16} GeV, corresponding to $M_5<10^{17}$ GeV, then the terms quadratic in the energy density dominate the modified Friedmann equation. In particular the condition for the end of inflation given in Eq. (9) becomes $\dot{\phi}^2<\frac{2}{5}V$. In the slow-roll approximation [using Eqs. (10) and (11)] $\dot{\phi}\simeq -M_5^3/2\pi\phi$ and this yields

$$\phi_{\rm end}^4 \simeq \frac{5}{4\pi^2}\left(\frac{M_5}{m}\right)^2 M_5^4. \tag{26}$$

In order to estimate the value of ϕ when scales corresponding to large-angle anisotropies on the microwave background sky left the Hubble scale during inflation, we take[1] $N_{\rm cobe}\approx 55$ in Eq. (25) and $\phi_{\rm f}=\phi_{\rm end}$. The second term on the right of Eq. (25) dominates, and we obtain

$$\phi_{\rm cobe}^4 \approx \frac{165}{\pi^2}\left(\frac{M_5}{m}\right)^2 M_5^4. \tag{27}$$

Imposing the Cosmic Background Explorer (COBE) normalization [19] on the curvature perturbations given by Eq. (19) requires

$$A_S \simeq \left(\frac{8\pi^2}{45}\right)\frac{m^4\phi_{\rm cobe}^5}{M_5^6} \approx 2\times 10^{-5}. \tag{28}$$

Substituting in the value of $\phi_{\rm cobe}$ given by Eq. (27) shows that in the limit of strong brane corrections, observations require

$$m \approx 5\times 10^{-5} M_5, \quad \phi_{\rm cobe}\approx 3\times 10^2 M_5. \tag{29}$$

Thus for $M_5<10^{17}$ GeV, chaotic inflation can occur for field values below the four-dimensional Planck scale, $\phi_{\rm cobe}<M_4$, although still above the five-dimensional scale M_5. The relation determined by COBE constraints for arbitrary brane tension is shown in Fig. 1, together with the high-energy approximation used above, which provides an excellent fit at low brane tension relative to M_4.

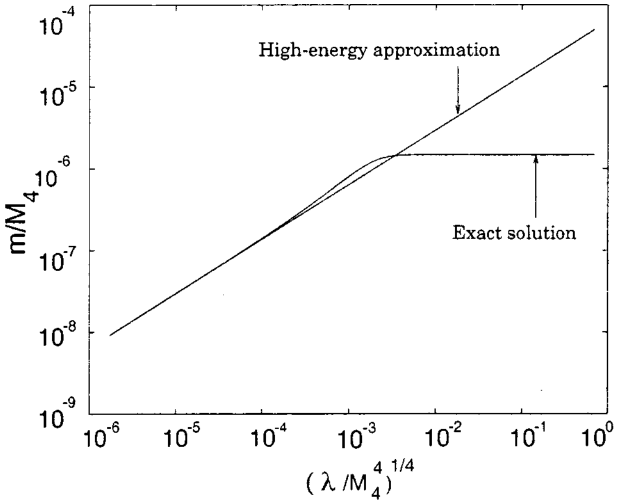

FIG. 1. The scaling of m/M_4 vs $(\lambda/M_4^4)^{1/4}$ to satisfy the COBE constraints. The straight line is the approximation used in Eq. (27), which at high energies is in excellent agreement with the exact solution, evaluated numerically in slow-roll.

V. CONCLUSION

In summary, we have found that slow-roll inflation is enhanced by the modifications to the Friedmann equation in a cosmological scenario where matter, including the inflaton field, is confined to a three-dimensional brane, in five-dimensional Einstein gravity. This enables the simplest chaotic inflation models, where the inflaton potential is a polynomial in ϕ, to inflate at field values below the four-dimensional Planck scale.

We have calculated the expected amplitude of density perturbations using the curvature perturbation ζ on uniform density hypersurfaces, which we have argued will remain constant on very large scales even in the presence of modifications to the Einstein equations at high energies, so long as the perturbations are adiabatic. Our calculations neglect the effect of gravitons in the five-dimensional bulk which is always a consistent solution for homogeneous matter fields [10]. However we note that a full calculation should include the effect of back-reaction from gravitational radiation in the bulk which might play an important role for the high momentum wavemodes, possibly modifying the amplitude of field fluctuations expected at Hubble-crossing.

Our results show that the additional friction term due to the enhanced expansion at high energies drives the expected tilt of the spectrum of density perturbations to zero, leading to the canonical scale-invariant Harrison-Zel'dovich spectrum. The modified dynamics alters the usual consistency relation between the tilt of the gravitational wave spectrum

[1] The precise value is dependent upon the actual energy scale during inflation and the reheat temperature [13]. Our results are only very weakly dependent upon the value of N chosen.

and the ratio of tensor to scalar perturbations expected in single-field slow-roll inflation. At the same time the amplitude of tensor perturbations is suppressed making an observational test of this prediction more difficult. Conversely, the detection of a tensor signal would be evidence against this scenario.

Note added in proof. The amplitude of quantum fluctuations in the five-dimensional graviton field has recently been calculated [20] and shown to be enhanced by a factor $\sqrt{3V/2\lambda}$ compared with the standard result assumed in Eqs. (21) and (22).

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