

Chiral shielding

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(Received 11 November 1999; published 5 July 2000)

We demonstrate how a chiral soft pion theorem (SPT) shields the scalar meson ground-state isoscalar $\sigma(600-700)$ and isospinor $\kappa(800-900)$ from detection in $a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}$, $\gamma\gamma \rightarrow 2\pi^0$, $\pi^- p \rightarrow \pi^- \pi^+ n$ and $K^- p \rightarrow K^- \pi^+ n$ processes. While pseudoscalar meson PVV transitions are known to be determined by (only) quark loop diagrams, the above SPT also constrains scalar meson SVV transitions to be governed (only) by meson loop diagrams. We apply this latter SVV theorem to $a_0 \rightarrow \gamma\gamma$ and $f_0 \rightarrow \gamma\gamma$ decays.

PACS number(s): 11.30.Rd, 12.39.-x

I. INTRODUCTION

The recent plethora of scalar meson papers appearing in the Los Alamos archives [1] stresses once again the importance but difficulty in observing the ground state $I=0$ and $I=1/2$ scalar mesons $\sigma(600-700)$ and $\kappa(800-900)$. Although these resonances were first listed in many of the 1960–1970 Particle Data Group (PDG) tables, they were later removed in the mid 1970s in favor of the higher mass $\epsilon(1300)$ and $\kappa(1400)$. Chiral symmetry shields the $\sigma(600-700)$ and $\kappa(800-900)$ for many different reasons which we shall discuss shortly.

Given the new CLEO measurement [2] of the $a_1(1230) \rightarrow \sigma\pi$ branching ratio based on $\tau \rightarrow \nu 3\pi$ decay of $\text{BR}(a_1 \rightarrow \sigma\pi) = (16 \pm 4)\%$, the average PDG value of [3] $\Gamma(a_1) \sim 425$ MeV then suggests a substantial partial width of size

$$\Gamma_{\text{CLEO}}(a_1 \rightarrow \sigma\pi) \sim (0.16)(425 \text{ MeV}) = 68 \pm 33 \text{ MeV}. \quad (1)$$

This was anticipated a decade ago by Weinberg [4], using mended chiral symmetry (MCS) to predict

$$\Gamma_{\text{MCS}}(a_1 \rightarrow \sigma\pi) = 2^{-3/2} \Gamma_\rho \approx 53 \text{ MeV}. \quad (2)$$

Moreover, assuming chiral symmetry, the needed coupling is related to $g_{a_1\sigma\pi} = g_{\rho\pi\pi} \approx 6$, the latter found from $\Gamma_\rho \approx 151$ MeV. Invoking the PDG σ mass of ~ 550 MeV [3,5] (giving $q_{\text{CM}} \approx 480$ MeV), one anticipates the width

$$\Gamma(a_1 \rightarrow \sigma\pi) = \frac{1}{3} (g_{a_1\sigma\pi}^2 / 4\pi) \frac{q_{\text{CM}}^3}{m_{a_1}^2} \approx 70 \text{ MeV}. \quad (3)$$

Considering the compatible (nonvanishing) $\Gamma_{a_1 \rightarrow \sigma\pi}$ widths in Eqs. (1)–(3) above, one might question (as Weinberg did in Ref. [4]) why the PDG listed the much smaller value $\text{BR}(a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}) < 0.7\%$ in the 1980s or the essentially vanishing width

$$\Gamma(a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}) = 1 \pm 1 \text{ MeV} \quad (4)$$

in the 1990s.

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II. VANISHING SOFT PION THEOREM

To resolve this apparent contradiction, we note that there are in fact *two* Feynman graphs to consider for $a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}$ decay, the ‘‘box’’ quark graph of Fig. 1(a) and the quark ‘‘triangle’’ graph of Fig. 1(b) (for nonstrange u and d quarks). In the soft pion limit for one soft pion in the $(\pi\pi)_{\text{swave}}$ doublet [but not the pion outside the $(\pi\pi)_{\text{swave}}$ doublet], there is a vanishing soft pion theorem (SPT) [6,7], canceling the box graph in Fig. 1(a) against the triangle graph Fig. 1(b) in the chiral soft pion limit.

Such a cancellation stems from the Dirac matrix *identity*¹

$$\frac{1}{\gamma \cdot p - m} 2m \gamma_5 \frac{1}{\gamma \cdot p - m} \equiv -\gamma_5 \frac{1}{\gamma \cdot p - m} - \frac{1}{\gamma \cdot p - m} \gamma_5. \quad (5)$$

We apply Eq. (5) together with the pseudoscalar pion quark (chiral) Goldberger–Treiman coupling $g_{\pi qq} = m/f_\pi$ for $f_\pi \approx 93$ MeV. This SPT for $p_\pi \rightarrow 0$ applied to the graphs of Figs. 1–4 results in the following.

(a) $a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}$. The box graph of Fig. 1(a) and Eq. (5) gives the amplitude as $p_\pi \rightarrow 0$,

$$M_{a_1 \rightarrow 3\pi}^{\text{box}} \rightarrow -\frac{1}{f_\pi} M(a_1 \rightarrow \sigma\pi). \quad (6)$$

But the additional σ pole quark triangle graph of Fig. 1(b) is

$$M_{a_1 \rightarrow 3\pi}^{\text{tri}} = \frac{1}{f_\pi} M(a_1 \rightarrow \sigma\pi), \quad (7)$$

because $2g_{\sigma\pi\pi} = (m_\sigma^2 - m_\pi^2)/f_\pi$ in the linear σ model ($L\sigma M$). Thus the sum of Eqs. (6) and (7) vanishes in the soft pion limit [6,7]

$$M_{a_1 \rightarrow 3\pi}|_{\text{total}} = M_{a_1 \rightarrow 3\pi}^{\text{box}} + M_{a_1 \rightarrow 3\pi}^{\text{tri}} \rightarrow 0, \quad (8)$$

compatible with data [3]: $\Gamma(a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}) = 1 \pm 1$ MeV.

(b) $\gamma\gamma \rightarrow 2\pi^0|_{s=m_\sigma^2}$. Again using pseudoscalar pion-quark couplings, it was predicted [8] five years before data ap-

¹Equation (5) reduces to $2m\gamma_5 = 2m\gamma_5$ when multiplying both sides of Eq. (5) on the left-hand side (lhs) and right-hand-side (rhs) by $(\gamma \cdot p - m)$.

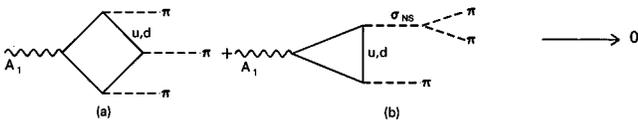


FIG. 1. Quark u, d box (a) and triangle (b) graphs contributing to $a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}$.

peared that this $\gamma\gamma \rightarrow 2\pi^0$ cross section should fall to about 10 nbarns in the 700 MeV region. Equivalently, using the SPT theorem stemming from Eq. (5), we predict the amplitude due to the quark box plus quark triangle graphs of Fig. 2

$$\langle \pi^0 \pi^0 | \gamma\gamma \rangle \rightarrow \left[-\frac{i}{f_\pi} \langle \sigma | \gamma\gamma \rangle + \frac{i}{f_\pi} \langle \sigma | \gamma\gamma \rangle \right] \rightarrow 0, \quad (9)$$

as $s \rightarrow m_\sigma^2(700)$ [7]. This picture was supported by recent Crystal Ball data [9].

(c) $\pi^- p \rightarrow \pi^- \pi^+ n$. The SPT stemming from Eq. (5) also suggests that the sum of the two π^+ peripheral-dominated $\pi^- p \rightarrow \pi^- \pi^+ n$ amplitudes of Fig. 3 vanishes:

$$M_{\pi^- p \rightarrow \pi^- \pi^+ n}|_{\text{per}} \propto [M_{\pi\pi}^{\text{box}} + M_{\pi\pi}^{\text{tri}}] \rightarrow 0. \quad (10)$$

This ‘‘chirally eaten’’ $\sigma(600-700)$ in Figs. 1(b), 2(b), 3(b) indeed did not appear in PDG tables prior to 1996, just as the SPT mandates. In fact the $\sigma(600-700)$ does not appear in recent Crystal Ball $\pi^- p \rightarrow \pi^0 \pi^0 n$ studies either [10].

(d) $K^- p \rightarrow K^- \pi^+ n$. Finally, the SPT due to Eq. (5) requires the sum of the two π^+ peripheral-dominated $K^- p \rightarrow K^- \pi^+ n$ amplitudes of Fig. 4 to vanish,

$$M_{K^- p \rightarrow K^- \pi^+ n}|_{\text{per}} \propto [M_{K\pi}^{\text{box}} + M_{K\pi}^{\text{tri}}] \rightarrow 0, \quad (11)$$

shielding this ground-state $\kappa^0(800-900)$ scalar in Fig. 4(b). Instead the $K^*(1430)$ (excited state) scalar resonance clearly appears in LASS data [11]; this $K^*(1430)$ not being eaten means it also is not a true ground-state scalar obeying the SPT. An analogous disappearance of the ground-state $\kappa(800-900)$ scalar occurs for the peripheral-dominated processes $K^- p \rightarrow \pi^- \pi^+ \Lambda, \bar{K} K \Lambda$.

None of the above four SPT processes depicted in Figs. 1–4 have been used by the experimentalists to observe such scalar mesons. Instead they study processes avoiding these four SPTs, e.g., $J/\psi \rightarrow \omega \pi \pi$ to isolate the $\sigma(500)$ resonance ‘‘bump.’’ In effect, the above s -wave SPTs (with quark boxes canceling quark triangle graphs in the soft pion limit) chirally ‘‘eat’’ the ground-state $\sigma(600-700)$ and $\kappa(800$

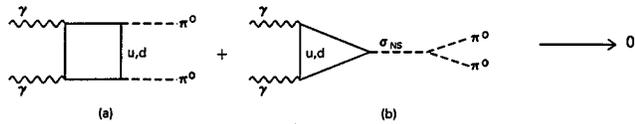


FIG. 2. Quark u, d box (a) and triangle (b) graphs contributing to $\gamma\gamma \rightarrow \pi^0 \pi^0$.

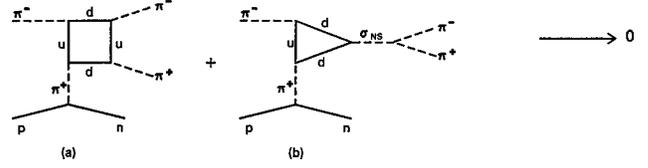


FIG. 3. Peripheral-dominated quark u, d box (a) and triangle (b) graphs contributing to $\pi^- p \rightarrow \pi^- \pi^+ n$.

–900) scalar mesons, justifying in part² why these scalar mesons have been so difficult to isolate and identify in the past.

With hindsight, the $L\sigma M$ dynamically generates ground state $\sigma(650)$ and $\kappa(850)$ scalars via (one-loop-order) tadpole graphs [12]. Even though these tadpoles can be suppressed by working in the infinite momentum frame [13], $SU(6)$ mass formulas (requiring squared masses) then kinematically favor [14] the (ground state) $\sigma(650)$ and $\kappa(820)$. This is another way (besides, e.g., $J/\psi \rightarrow \omega \pi \pi$) to circumvent the four SPTs discussed in this section.

III. QUARK LOOPS VERSUS MESON LOOPS

In most effective chiral field theories (such as the $L\sigma M$), one usually computes consistently either quark loops alone or meson loops alone for a given process. Sometimes one must add together quark and meson loops [12]. Chiral symmetry and the SPT discussed in Sec. II actually help to put order in this morass of quarks and meson loops.

Specifically for $PV V$ transitions, the anomaly [15] or simply the vanishing of, e.g., a meson $\pi\pi\pi$ vertex, etc. leads directly to a ‘‘quark loops alone’’ theory [16], such as for $\pi^0 \rightarrow 2\gamma$. However, for $SV V$ transitions, it turns out that *only* meson loop graphs contribute. This $SV V$ ‘‘meson loops alone’’ theorem also is a direct consequence of the soft pion theorem proved in Refs. [6,7] and reviewed in Sec. II above. Specifically, we study $\gamma\gamma \rightarrow \pi^0 \pi^0$ with one of the pions soft. Again the quark box plus quark triangle graphs of Fig. 2 add up to zero in the soft pion limit. Turning Fig. 2(b) around, if σ (as a 2π resonance) decays to 2γ , this SPT eats up the needed quark triangle due to the quark box. This leaves only the meson triangle $\sigma \rightarrow K^+ K^- \rightarrow 2\gamma$ dominating $SV V$ decay $\sigma \rightarrow \gamma\gamma$.

A more practical example of this theorem is for $a_0(983) \rightarrow 2\gamma$ decay. First we consider the inverse process $\gamma\gamma \rightarrow \eta\pi$, with the $\eta\pi$ final state forming an $a_0(983)$ resonance $\gamma\gamma \rightarrow a_0 \rightarrow \eta\pi$. So we should begin by first considering the quark box graph for $\gamma\gamma \rightarrow a_0$ followed by $a_0 \rightarrow \eta\pi$. Again these quark box plus triangle graphs vanish in the soft pion limit by the SPT of Sec. II. All that remains are the meson loop graphs for $a_0 \rightarrow \gamma\gamma$ decay.

Here $a_0 \rightarrow K^+ K^- \rightarrow 2\gamma$ and the charged kaon loop contributes to the $a_0 \gamma\gamma$ covariant amplitude

²Two other reasons for suppressing these scalars are: (1) they are low mass and broad, sometimes at the edge of the phase space and (2) they are usually swamped by the nearby vectors $\rho(770)$ and $\omega(783)$ and $K^*(895)$, respectively.

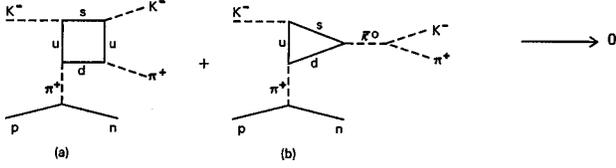


FIG. 4. Peripheral-dominated quark u, d box (a) and triangle (b) graphs contributing to $K^- p \rightarrow K^- \pi^+ n$.

$$\langle 2\gamma | a_0 \rangle = M \varepsilon_\mu(k') \varepsilon_\nu(k) (g^{\mu\nu} k' \cdot k - k'^\mu k^\nu), \quad (12)$$

where, according to Ref. [17], the effective amplitude M is given by

$$|M_{K \text{ loop}}| = \frac{2g' \alpha}{\pi m_{a_0}^2} \left[-\frac{1}{2} + \xi I(\xi) \right], \quad (13)$$

with $\xi = m_{K^+}^2 / m_{a_0}^2 = 0.2520 > 1/4$. Then the loop integral becomes

$$\begin{aligned} I(\xi) &= \int_0^1 dy y \int_0^1 dx [\xi - xy(1-y)]^{-1} \\ &= 2[\arcsin \sqrt{1/4\xi}]^2 \approx 4.39. \end{aligned} \quad (14)$$

Also the $L\sigma M a_0 K K$ coupling (g') is [17,18]

$$g' = (m_{a_0}^2 - m_K^2) / 2f_K \approx 3.18 \text{ GeV}, \quad (15)$$

so that the $a_0 \gamma \gamma$ amplitude in Eq. (13) is approximately

$$|M_{K \text{ loop}}| \approx 9.27 \times 10^{-3} \text{ GeV}^{-1}. \quad (16)$$

This results in the decay width

$$\Gamma(a_0 \rightarrow 2\gamma) = m_{a_0}^3 |M_K|^2 / 64\pi \approx 0.406 \text{ keV}. \quad (17)$$

The resonance $\kappa(900)$ contributes [17] 10% of Eq. (16), reducing Eq. (17) to

$$\Gamma(a_0 \rightarrow 2\gamma) \approx 0.406 \text{ keV} (0.90)^2 \approx 0.33 \text{ keV}. \quad (18)$$

Assuming the a_0 width is (100%) dominated by $a_0 \rightarrow \eta \pi$, the PDG tables suggest

$$\Gamma(a_0 \rightarrow 2\gamma) = (0.24_{-0.07}^{+0.08}) \text{ keV}. \quad (19)$$

Another measured SVV decay is $f_0(980) \rightarrow \gamma \gamma$ with [3]

$$\Gamma(f_0 \rightarrow 2\gamma) = 0.56 \pm 0.11 \text{ keV}. \quad (20)$$

Here σ - f_0 mixing enters the amplitude analysis with [18,19]

$$|f_0\rangle = \sin \phi_s |\text{NS}\rangle + \cos \phi_s |S\rangle, \quad (21)$$

for $f_0(980)$ being mostly strange, with $\phi_s \approx 20^\circ$. The non-strange (NS) and strange (S) quark basis states are, respectively, $|\text{NS}\rangle = (\bar{u}u + \bar{d}d) / \sqrt{2}$ and $|S\rangle = |\bar{s}s\rangle$ with singlet-octet

angle $\theta_s = \phi_s - \arctan \sqrt{2}$. The angle ϕ_s can be obtained from Eq. (21) using $\langle \sigma | f_0 \rangle = 0$ or $m_{\sigma_s}^2 = m_\sigma^2 \sin^2 \phi_s + m_{f_0}^2 \cos^2 \phi_s$, leading to [18,19]

$$\phi_s = \arcsin \left[\frac{m_{f_0}^2 - m_{\sigma_s}^2}{m_{f_0}^2 - m_\sigma^2} \right]^{1/2} \approx 20^\circ \quad (22)$$

for $m_\sigma \approx 610$ MeV and $m_{\sigma_s} \approx 2m_s \approx 940$ MeV, with constituent quark masses $m_s = (m_s / \hat{m}) \hat{m} \approx 470$ MeV, and $\hat{m} \approx 325$ MeV, $m_s / \hat{m} \approx 1.45$. Since $f_0(980)$ is mostly $\bar{s}s$ with $m_{f_0} \approx m_{a_0}$ [18], we simply scale up the width $\Gamma_{a_0 \rightarrow \gamma \gamma} \approx 0.33$ keV in Eq. (18) by $2(\cos 20^\circ)^2$ from Eq. (21) (the 2 due to [18,19] $g_{SKK} = 1/\sqrt{2}$ whereas $g_{NSKK} = 1/2$):

$$\Gamma(f_0 \rightarrow \gamma \gamma) \approx 2(\cos 20^\circ)^2 (0.33 \text{ keV}) \approx 0.58 \text{ keV}, \quad (23)$$

again for a $f_0 \rightarrow K^+ K^- \rightarrow 2\gamma$ meson loop.

We observe that the predictions (18) and (23) are in close agreement with the $a_0, f_0 \rightarrow 2\gamma$ measured decay rates in Eqs. (19) and (20), respectively.

IV. SUMMARY

In Sec. I we gave one experimental and two theoretical reasons supporting the somewhat broad width $\Gamma(a_1 \rightarrow \sigma \pi) \sim 65$ MeV. The latter appears to contradict the complementary PDG result $\Gamma(a_1 \rightarrow \pi(\pi\pi)_{\text{swave}}) = 1 \pm 1$ MeV. But in Sec. II we resolve this apparent contradiction, finding that *both* quark box and quark triangle graphs contribute to the rate $\Gamma(a_1 \rightarrow \pi(\pi\pi)_{\text{swave}})$, but the quark box-triangle sum of these amplitudes *vanishes* in the soft-pion limit. This SPT is also valid for $\sigma(\gamma\gamma \rightarrow \pi^0 \pi^0)$, and peripheral decay rates $\Gamma_{\text{per}}(\pi^- p \rightarrow \pi^- \pi^+ n)$, $\Gamma_{\text{per}}(K^- p \rightarrow K^- \pi^+ n)$. With hindsight, our quark loop chiral shielding SPTs in Sec. II parallel the $L\sigma M$ ‘‘miraculous cancellation’’ eating up the σ pole in $\pi-\pi$ scattering Ref. [20], reducing the low-energy amplitude to Weinberg’s well-known CA-PCAC result [21]. Finally, in Sec. III we turn this SPT around. Not only are pseudoscalar meson PVV decays controlled by quark loops alone (as is well known, e.g., for $\pi^0 \rightarrow 2\gamma$), but scalar meson SVV decays are governed by meson loops alone. We demonstrate how this latter SVV theorem works for $a_0 \rightarrow 2\gamma$ and $f_0 \rightarrow 2\gamma$ decays.

Without invoking this SPT, there are physicists who do appreciate the utility of a meson loop only scheme for SVV decays [22].

Note added in proof. A chiral shielding type of analysis, but in the NJL four-quark picture, was given by Bajc *et al.* [23].

ACKNOWLEDGMENTS

L.B. and M.D.S. appreciate discussions with V. Elias and partial support by the U.S. DOE. Y.A.B. and A.N.I. acknowledge discussions with V. F. Kosmach and N. I. Troitskaya.

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