PHYSICAL REVIEW D, VOLUME 62, 037301

Modification of Z boson properties in the quark-gluon plasma

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(Received 18 February 2000; published 7 July 2000)

We calculate the change in the effective mass and width of a Z boson in the environment of a quark-gluon plasma under the conditions expected in Pb-Pb collisions at the CERN LHC. The change in width is predicted to be only about 1 MeV at a temperature of 1 GeV, compared to the natural width of 2490 ± 7 MeV. The mass shift is even smaller. Hence no observable effects are to be expected.

PACS number(s): 13.38.Dg, 12.38.Mh, 25.75.−q

The Large Hadron Collider (LHC) at CERN will collide lead nuclei at a center-of-mass energy of 2.75 TeV per nucleon, more than an order of magnitude greater than the Relativistic Heavy Ion Collider (RHIC) at BNL. At this energy Z bosons will be produced in measurable numbers. The mass and total width of the Z boson are $m_Z = 91.153 \pm 0.007$ and $\Gamma_{\rm vac} = 2.490 \pm 0.007$ GeV, respectively [1]. Kinematics and phase space then dictate that they will normally be produced with small transverse velocities and, as we shall see, most will decay in a dense environment of quarks and gluons. How much will their mass and width be modified due to immersion in this quark-gluon plasma, and how can these effects be measured?

The standard picture of a central collision between high energy heavy nuclei is the boost invariant hydrodynamic model of Bjorken [2]. Following an initial pre-equilibrium phase, quarks and gluons are approximately thermalized in the local rest frame at proper time t_0 after the highly Lorentz contracted nuclei overlap with an initial temperature T_0. Based on very general considerations these numbers were estimated to be 0.07 fm/c and 1 GeV, respectively, for central Pb-Pb collisions at the LHC [3]. Thereafter the local temperature falls with proper time as

$$T(t) = \left(\frac{t_0}{t}\right)^{1/3} T_0 . \tag{1}$$

The quark-gluon plasma expands longitudinally until it hadronizes when the local temperature reaches $T_c \approx 160$ MeV. After that, transverse expansion sets in. Eventually the hadrons lose thermal contact and begin a free-streaming phase. By the uncertainty principle a Z boson is created approximately $1/m_Z = 0.002$ fm/c after nuclear contact, and it decays with a (vacuum) lifetime of 0.08 fm/c. Therefore, Z bosons will decay in an environment of quark-gluon plasma with a temperature of order 1 GeV and so have the potential to be an excellent probe of the highest temperatures attained in these collisions.

The spectral density of the Z boson evaluated at small velocity $v \ll c$ in the rest frame of the plasma is

$$\rho(s,T) = \frac{1}{\pi}\frac{m_Z \Gamma}{(s-m_Z^2)^2 + m_Z^2\Gamma^2} \tag{2}$$

where $\Gamma = \Gamma_{\rm vac} + \Gamma_{\rm mat}(T)$, the latter contribution denoting the effect due to matter, namely the quark-gluon plasma. The invariant mass distribution of dimuons coming from Z decay is directly proportional to the spectral density. Given that the Z bosons are invariably created at time $1/m_Z \equiv t_Z$ after nuclear overlap, the number of them remaining at time $t > t_Z$ is

$$N_Z(t) = N_Z(t_Z)\exp\left[-\int_{t_Z}^{t} \Gamma(t')\,dt'\right]. \tag{3}$$

Here Γ will depend on the time because it depends on temperature and that in turn depends on time. The final observed distribution of dimuons coming from Z decay will be

$$\frac{dN_{\mu^+\mu^-}}{ds} = \Gamma_{\mu^+\mu^-}\int_{t_Z}^{\infty} dt N_Z(t)\rho(s,T(t)). \tag{4}$$

At the LHC the lifetime of the Z closely matches the thermalization time t_0. Therefore it is a good approximation to evaluate the spectral density at this time, resulting in the distribution

$$\frac{dN_{\mu^+\mu^-}}{ds} = \frac{\Gamma_{\mu^+\mu^-}}{\Gamma(T_0)} N_Z(t_Z)\rho(s,T_0). \tag{5}$$

The total number of Z bosons $N_Z(t_Z)$ ought to scale as A^2 for impact parameter averaged collisions (A is the atomic number) because production of a Z is such a hard process. Simulations done for the CMS [4] show 11 000 $Z \to \mu^+\mu^-$ events with pseudorapidity between -2.4 and 2.4 in two weeks of running. Keeping only 10% of those as being associated with central collisions producing a plasma, one arrives at 1100 every two weeks. How large might one expect the many-body effects to be? The position of the pole of the Z boson propagator at finite temperature may be computed with standard techniques. The lowest order correction to the real part of the Z self-energy comes from the single quark loop diagram shown in Fig. 1. In the limit that the Z moves nonrelativistically in the plasma the longitudinal and transverse self-energies are degenerate. The real part is (we use the conventions of Cheng and Li [5])

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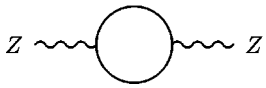

FIG. 1. One-loop contribution to the Z self-energy at finite temperature. The solid line represents a light quark (u,d,s).

$$\mathrm{Re}\,\Pi_{\text{1-loop}} = -\frac{14\pi^2}{15}\frac{T^4}{m_Z^2}\sum_f \,[g_A^2(f)+g_V^2(f)]$$
$$= -\frac{7\sqrt{2}}{5}\pi^2\left[1-\frac{4}{9}\sin^2(2\theta_W)\right]G_F T^2. \qquad (6)$$

Here it is assumed that $T \ll m_Z$. The summation is over the relevant flavors of quarks. The second line includes only u, d and s quarks. Using the latest values [1] for the Fermi constant, $G_F = 1.166\times10^{-5}$ GeV^{-2}, and weak angle, $\sin^2\theta_W = 0.2312$, the mass shift is

$$\mathrm{Re}\,\Pi_{\text{1-loop}} = \Delta m_Z^2 = -1.56\times10^{-4}\,T_{\mathrm{GeV}}^4 \qquad (7)$$

with temperature measured in GeV. The mass shift is negative but uninterestingly small. The finite temperature imaginary part of the one-loop diagram simply amounts to a Pauli blocking factor for the quarks in the final state. This is $2\exp(-m_Z/2T)\approx 3\times10^{-20}$ for $T=1$ GeV which is totally irrelevant.

The dominant finite temperature effect on the imaginary part of the self-energy comes from the two-loop diagram shown in Fig. 2. Cutting that diagram in all possible ways corresponds to scattering processes some of which are shown in Fig. 3. They represent the reactions $q+Z\to q+g$ (Compton), $g+Z\to q+\bar{q}$ (fusion), $Z\to g+q+\bar{q}$ (3-body decay), an interference between the amplitudes for $g+Z\to g+q+\bar{q}$ and $Z\to q+\bar{q}$ with a spectator gluon (interference-gluon), and an interference between the amplitudes for $q(\bar{q})+Z\to q(\bar{q})+q+\bar{q}$ and $Z\to q+\bar{q}$ with a spectator quark or antiquark (interference-quark). Averaged over initial spins and summed over final spins and color the invariant amplitudes are

$$|\mathcal{M}_C|^2 \propto -\frac{s}{u}-\frac{u}{s}-2t\left(\frac{1}{s}+\frac{1}{u}+\frac{t}{su}\right) \qquad (8)$$

$$|\mathcal{M}_F|^2 \propto \frac{t}{u}+\frac{u}{t}+2s\left(\frac{1}{t}+\frac{1}{u}+\frac{s}{tu}\right) \qquad (9)$$

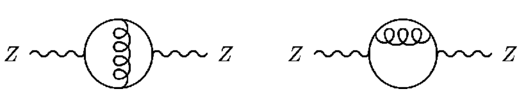

FIG. 2. Two-loop contributions to the Z self-energy at finite temperature. The solid line represents a light quark (u,d,s) and the curly line represents a gluon.

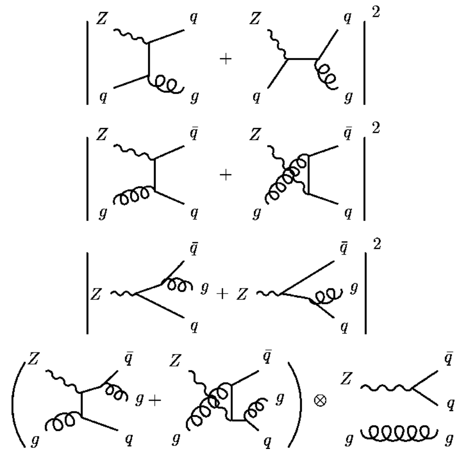

FIG. 3. Cutting the two-loop self-energy in Fig. 2 results in these contributions to the imaginary part: $q+Z\to q+g$ (Compton), $g+Z\to q+\bar{q}$ (fusion), $Z\to q+\bar{q}+g$ (3-body decay). There are also interferences between the amplitudes for $g+Z\to q+\bar{q}+g$ and $Z\to q+\bar{q}$ together with a spectator gluon, an example of which is shown here (the incoming and outgoing gluons have the same energy and momentum). There are similar interference terms involving a spectator quark or antiquark.

$$|\mathcal{M}_D|^2 \propto 2\left(\frac{p\cdot p'+k\cdot p'}{p\cdot q}+\frac{p\cdot p'+k\cdot p}{p'\cdot q}+\frac{2(p\cdot p')^2}{(p\cdot q)(p'\cdot q)}\right). \qquad (10)$$

In all cases the constant of proportionality is $16g_s^2(g_V^2+g_A^2)/3$. The four-momenta are p for quark, p' for antiquark, q for gluon and k for the Z boson. The contribution to the imaginary part of Π involves integrating these amplitudes over phase space in the usual way, including thermal distributions of quarks and gluons in the initial state and Pauli blocking or Bose enhancement factors in the final state. The interference terms are more complicated and are not displayed here. A detailed exposition of how they are derived and evaluated in this and other theories is deferred to a separate paper [6].

The contribution to the imaginary part of Π involving one gluon thermal distribution, arising from the fusion, decay and interference processes may be written as (limits $m_f \ll T \ll m_Z$ assumed throughout)

$$\mathrm{Im}\,\Pi_{F+D+I} = -\frac{2}{3}\frac{\alpha_s}{\pi^2}\sum_f\,[g_A^2(f)+g_V^2(f)]$$
$$\times\int_0^\infty d\omega\,\frac{1}{\exp(\omega/T)-1}(F+D+I) \qquad (11)$$

where

$$F = -2\omega+\left[2\omega+2m_Z+\frac{m_Z^2}{\omega}\right]\ln\left(\frac{2m_Z\omega}{k_c^2}\right), \qquad (12)$$

$$D = -2\omega + \left[2\omega - 2m_Z + \frac{m_Z^2}{\omega}\right]\ln\left(\frac{2m_Z\omega}{k_c^2}\right), \tag{13}$$

$$I = 8\omega + \left[4\omega - 2\frac{m_Z^2}{\omega}\right]\ln\left(\frac{2m_Z\omega}{k_c^2}\right). \tag{14}$$

Here ω is the energy of the gluon and k_c^2 is a cutoff placed on the four momentum transfer t and/or u (or virtuality) carried by the exchanged quarks in the plasma [7]. It arises naturally in the infrared resummation scheme of Braaten and Pisarski [8]. The individual contributions are quadratically divergent in the infrared. The sum of the three contributions is, however, finite as expected by rather general arguments [9]:

$$\mathrm{Im}\,\Pi_{F+D+I} = -\frac{1}{9}\alpha_s \sum_f \left[g_A^2(f) + g_V^2(f)\right] \times T^2\left[8\ln\left(\frac{2m_Z T}{k_c^2}\right) + 2.8224\right]. \tag{15}$$

The contribution from processes involving one quark or antiquark thermal distribution may similarly be computed to be

$$\mathrm{Im}\,\Pi_{C+D+I} = -\frac{2}{3}\frac{\alpha_s}{\pi^2}\sum_f \left[g_A^2(f) + g_V^2(f)\right] \times \int_0^\infty d\omega \frac{1}{\exp(\omega/T)+1}(C+D+I) \tag{16}$$

where

$$C = 4\omega + [-2\omega + m_Z]\ln\left(\frac{2m_Z\omega}{k_c^2}\right), \tag{17}$$

$$D = 2\omega + [-2\omega - m_Z]\ln\left(\frac{2m_Z\omega}{k_c^2}\right), \tag{18}$$

$$I = 8\omega\ln\left(\frac{2m_Z\omega}{k_c^2}\right). \tag{19}$$

After integration, the sum of these contributions is

$$\mathrm{Im}\,\Pi_{C+D+I} = -\frac{1}{9}\alpha_s \sum_f \left[g_A^2(f) + g_V^2(f)\right] \times T^2\left[2\ln\left(\frac{2m_Z T}{k_c^2}\right) + 4.0920\right]. \tag{20}$$

The sum of all processes, F+C+D+I, is

$$\mathrm{Im}\,\Pi_{F+C+D+I} = -\frac{10}{9}\alpha_s \sum_f \left[g_A^2(f) + g_V^2(f)\right] \times T^2\left[\ln\left(\frac{2m_Z T}{k_c^2}\right) + 0.6914\right]. \tag{21}$$

One can view the cutoff on the quark virtuality as the effective mass of a quark propagating through the plasma with a typical thermal momentum. This effective mass is $\sqrt{2}m_q$ where $m_q^2 = g_s^2 T^2/6$ [7]. Summing over the three flavors of lightest quarks,

$$\mathrm{Im}\,\Pi = -\sqrt{2}\left[1 - \frac{4}{9}\sin^2(2\theta_W)\right]\alpha_s m_Z^2 G_F T^2 \ln\left(\frac{m_Z}{1.049\alpha_s T}\right). \tag{22}$$

The rate Γ is determined from the self-energy via $\Gamma = -\mathrm{Im}\,\Pi/m_Z$. Numerically the thermal contribution to the rate is

$$\Gamma_T = 1.03\times 10^{-3}\alpha_s T_{\mathrm{GeV}}^2 \ln\left(\frac{86.9}{\alpha_s T_{\mathrm{GeV}}}\right). \tag{23}$$

For the strong coupling we use the one loop expression

$$\alpha_s(T) = \frac{6\pi}{27\ln(T/50\,\mathrm{MeV})}, \tag{24}$$

once again assuming 3 flavors of massless quarks. The argument of the logarithm is adjusted to fit lattice results [10]. Numerically $\alpha_s(T=200\ \mathrm{MeV}) \approx 1/2$ and $\alpha_s(T=1\ \mathrm{GeV}) \approx 1/4$. At $T=1$ GeV this gives a finite temperature contribution to the width of only 1.5 MeV and totally ignorable compared to the vacuum part.

In conclusion, we have computed the finite temperature contribution to the width of the Z boson in a quark-gluon plasma achievable in Pb-Pb collisions at the CERN LHC. This effect will not be observable in the dimuon invariant mass spectra. This is very unfortunate because the approximate equality between the Z lifetime and the formation or thermalization time of the quark-gluon plasma could have made it a fine probe of the initial state of the plasma.

We thank P. Stankus and R. Rusack for prodding us to investigate this problem. This work was supported by the US Department of Energy under grant DE-FG02-87ER40328.

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