

Flavor-conserving CP phases in supersymmetry and implications for exclusive B decays

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We study rare exclusive B decays based on the quark-level transition $b \rightarrow s(d)l^+l^-$, where $l = e$ or μ , in the context of supersymmetric theories with minimal flavor violation. We present analytic expressions for various mixing matrices in the presence of new CP -violating phases, and examine their impact on observables involving B and \bar{B} decays. An estimate is obtained for CP -violating asymmetries in $\bar{B} \rightarrow K^{(*)}l^+l^-$ and $\bar{B} \rightarrow \rho(\pi)l^+l^-$ decays for the dilepton invariant mass region $1.2 \text{ GeV} < M_{l^+l^-} < M_{J/\psi}$. As a typical result, we find a CP -violating partial width asymmetry of about -6% (-5%) in the case of $B \rightarrow \pi$ ($B \rightarrow \rho$) in effective supersymmetry with phases of $\mathcal{O}(1)$, taking into account the measurement of the inclusive $b \rightarrow s\gamma$ branching fraction. On the other hand, CP asymmetries of less than 1% are predicted in the case of $B \rightarrow K^{(*)}$. We argue that it is not sufficient to have additional CP phases of $\mathcal{O}(1)$ to observe large CP -violating effects in exclusive $b \rightarrow s(d)l^+l^-$ decays.

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I. INTRODUCTION

Within the standard model (SM), CP violation is caused by a non-zero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1]. While the experimentally observed indirect CP violation in the neutral kaon system, ϵ_K , can be accommodated in the SM, it is still an open question whether the SM description of CP violation is consistent with the new experimental result on direct CP violation, ϵ'/ϵ_K , since the theoretical prediction of its precise value suffers from large hadronic uncertainties [2]. On the other hand, if the baryon asymmetry of the universe has been generated via baryogenesis at the electroweak phase transition, the CKM mechanism of CP violation cannot account for the observed amount of baryon asymmetry. This feature could be a hint of the existence of CP -violating sources outside the CKM matrix [3].

Important tests of the SM are provided by flavor-changing neutral current (FCNC) reactions involving B decays [4], thus offering an opportunity to search for supersymmetric extensions of the SM [5,6]. There are at present only a few FCNC processes which have been observed experimentally, but the situation will change considerably after the completion of B factories in the near future.

In this work, we analyze the exclusive decays $\bar{B} \rightarrow K^{(*)}l^+l^-$ and $\bar{B} \rightarrow \rho(\pi)l^+l^-$ in the context of supersymmetry (SUSY) with minimal particle content and R -parity conservation [7,8]. The inclusive reaction $\bar{B} \rightarrow X_s l^+l^-$ within supersymmetric models has been extensively studied in Refs. [5,9–16] and, more recently, in Ref. [17]. New physics effects in the exclusive channels have been investigated in Refs. [18–20].

We place particular emphasis on CP -violating effects associated with the partial rate asymmetry between B and \bar{B} decays as well as the forward-backward asymmetry of the l^- . Within the SM these effects turn out to be unobservably small ($\leq 10^{-3}$) in the decays $\bar{B} \rightarrow K^{(*)}l^+l^-$ [21], and amount to only a few percent in $\bar{B} \rightarrow \rho(\pi)l^+l^-$ [21,22]. However, in models with new CP -violating phases in addition to the single phase of the CKM matrix, larger effects may occur due to the interference of amplitudes with different phases. The purpose of the present analysis is to explore CP -violating observables in the aforementioned FCNC reactions that could provide evidence of a non-standard source of CP violation, and hence may be useful in analyzing supersymmetry in future collider experiments.

The paper is organized as follows. In Sec. II, we exhibit the various mixing matrices of the minimal supersymmetric standard model (MSSM) in the presence of additional CP -violating phases. Within such a framework we discuss different scenarios for the SUSY parameters. In Sec. III, we are primarily concerned with the short-distance matrix element and Wilson coefficients governing $b \rightarrow s(d)l^+l^-$ in the MSSM. We also briefly describe an approximate procedure to incorporate quark antiquark resonant intermediate states—namely ρ, ω , and the J/ψ family—which enter through the decay chain $b \rightarrow s(d)V_{q\bar{q}} \rightarrow s(d)l^+l^-$. Section IV is devoted to the exclusive decay modes $\bar{B} \rightarrow K^{(*)}l^+l^-$ and $\bar{B} \rightarrow \rho(\pi)l^+l^-$, where formulas are given to calculate CP asymmetries which can be determined experimentally by measuring the difference of B and \bar{B} events. In Sec. V, we present our numerical results for CP -violating observables in the non-resonant domain $1.2 \text{ GeV} < M_{l^+l^-} < M_{J/\psi}$, taking into account experimental bounds on rare B decays such as $b \rightarrow s\gamma$. We summarize and conclude in Sec. VI. The analytic formulas describing the short-distance effects in the presence of SUSY as well as the explicit expressions for the form factors are relegated to the Appendixes.

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II. THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

In the MSSM, there are new sources of CP violation. In general, a large number of CP -violating phases appear in the mass matrices as well as the couplings. After an appropriate redefinition of fields one ends up with at least two new CP -violating phases, besides the phase of the CKM matrix and the QCD vacuum angle, which cannot be rotated away. For instance, in the MSSM with universal boundary conditions at some high scale only two new physical phases arise; namely φ_{μ_0} associated with the Higgsino mass parameter μ in the superpotential and φ_{A_0} connected with the soft SUSY-breaking trilinear mass terms.

In order to fulfill the severe constraints on the electric dipole moments (EDM's) of electron and neutron, one generally assumes that the new phases are less than $\mathcal{O}(10^{-2})$. Since there is no underlying symmetry which would force the phases to be small, this requires fine-tuning. Of course, one can relax the tight constraint on these phases by having masses of the superpartners in the TeV region; this heavy SUSY spectrum may, however, lead to an unacceptably large contribution to the cosmological relic density.

It has recently been pointed out by several authors that it is possible to evade the EDM constraints so that phases of $\mathcal{O}(1)$ still remain consistent with the current experimental upper limits. Methods that have been advocated to suppress

the EDM's include cancellations among different SUSY contributions [23,24], and nearly degenerate heavy sfermions for the first two generations while being consistent with naturalness bounds. The latter can be realized within the context of so-called ‘‘effective SUSY’’ models [25], thereby solving the SUSY FCNC and CP problems.

To get an idea how supersymmetry affects CP observables in rare B decays, we will consider as illustrative examples the following types of SUSY models:

(i) MSSM coupled to $N=1$ supergravity with a universal SUSY-breaking sector at the grand unification scale.

(ii) Effective SUSY with near degeneracy of the heavy first and second generation sfermions.

In the present analysis, we restrict ourselves to the discussion of flavor-diagonal sfermion mass matrices—that is, we assume the CKM matrix to be the only source of flavor mixing.¹

A. Mixing matrices and new CP -violating phases

This subsection concerns the mass and mixing matrices relevant to our analysis. In what follows, we will adopt the conventions of Ref. [26].

1. Charged Higgs-boson mass matrix

The mass-squared matrix of the charged Higgs bosons reads

$$M_{H^\pm}^2 = \begin{pmatrix} B\mu \tan \beta + M_W^2 \sin^2 \beta + t_1/v_1 & B\mu + M_W^2 \sin \beta \cos \beta \\ B\mu + M_W^2 \sin \beta \cos \beta & B\mu \cot \beta + M_W^2 \cos^2 \beta + t_2/v_2 \end{pmatrix}, \quad (2.1)$$

with

$$B\mu = \frac{1}{2} \sin 2\beta (m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2), \quad (2.2)$$

$$|\mu|^2 = -\frac{1}{2} M_Z^2 + \frac{m_{H_2}^2 \sin^2 \beta - m_{H_1}^2 \cos^2 \beta}{\cos 2\beta}. \quad (2.3)$$

Here B and μ refer to the complex soft SUSY-breaking and Higgsino mass parameters respectively, $m_{H_{1,2}}^2$ are the soft SUSY breaking Higgs-boson masses at the electroweak scale, and $t_{1,2}$ stand for the renormalized tadpoles. The mixing angle β is defined as usual by $\tan \beta \equiv v_2/v_1$, with $v_{1,2}$ denoting the tree level vacuum expectation values (VEV's) of the two neutral Higgs fields. In Eq. (2.2) we have adjusted the phase of the μ parameter in such a way that $B\mu$ is real at tree level, thereby ensuring that the VEV's of the two Higgs fields are real. Consequently, the mass matrix becomes real and can be reduced to a diagonal form through a biorthogonal transformation $(M_{H^\pm}^{\text{diag}})^2 = O M_{H^\pm}^2 O^T$. At the tree level, i.e. $t_i=0$ in Eq. (2.1), we have

$$O = \begin{pmatrix} -\cos \beta & \sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}. \quad (2.4)$$

Before proceeding, we should mention that radiative corrections to the Higgs potential induce complex VEV's. As a matter of fact, CP violation in the Higgs sector leads to an additional phase which, in the presence of chargino and neutralino contributions, cannot be rotated away by reparametrization of fields [27]. As a result, the radiatively induced phase modifies the squark, chargino, and neutralino mass matrices. In the present analysis, we set this phase equal to zero.

2. Squark mass matrices

We now turn to the 6×6 squark mass-squared matrix which can be written as

¹One should keep in mind that renormalization group effects induce flavor off-diagonal entries in the sfermion mass matrices at the weak scale (see below).

$$M_q^2 = \begin{pmatrix} M_{qLL}^2 & M_{qLR}^2 e^{-i\varphi_{\tilde{q}}} \\ M_{qLR}^2 e^{i\varphi_{\tilde{q}}} & M_{qRR}^2 \end{pmatrix}, \quad \tilde{q} = \tilde{U}, \tilde{D}, \quad (2.5)$$

in the $(\tilde{q}_L, \tilde{q}_R)$ basis, and can be diagonalized by a unitary matrix $R_{\tilde{q}}$ such that

$$(M_{\tilde{q}}^{\text{diag}})^2 = R_{\tilde{q}} M_q^2 R_{\tilde{q}}^\dagger. \quad (2.6)$$

For subsequent discussion it is useful to define the 6×3 matrices

$$(\Gamma^{qL})_{ai} = (R_{\tilde{q}}^-)_{ai}, \quad (\Gamma^{qR})_{ai} = (R_{\tilde{q}}^-)_{a,i+3}, \quad q = U, D, \quad (2.7)$$

with U and D denoting up- and down-type quarks respectively. Working in the so-called ‘‘super-CKM’’ basis [6] in which the 3×3 quark mass matrices M_U and M_D are real and diagonal, the submatrices in Eq. (2.5) take the form

$$M_{\tilde{U}LL}^2 = (M_{\tilde{U}}^2)_{LL} + M_U^2 + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 4 \sin^2 \theta_W) \mathbb{1}, \quad (2.8a)$$

$$M_{\tilde{U}LR}^2 = M_U |A_U - \mu^* \cot \beta|, \quad (2.8b)$$

$$M_{\tilde{U}RR}^2 = (M_{\tilde{U}}^2)_{RR} + M_U^2 + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W \mathbb{1}, \quad (2.8c)$$

$$\varphi_{\tilde{U}} = \arg(A_U - \mu^* \cot \beta), \quad (2.8d)$$

$$M_{\tilde{D}LL}^2 = V_{\text{CKM}}^\dagger (M_{\tilde{D}}^2)_{LL} V_{\text{CKM}} + M_D^2 - \frac{1}{6} M_Z^2 \cos 2\beta (3 - 2 \sin^2 \theta_W) \mathbb{1}, \quad (2.9a)$$

$$M_{\tilde{D}LR}^2 = M_D |A_D - \mu^* \tan \beta|, \quad (2.9b)$$

$$M_{\tilde{D}RR}^2 = (M_{\tilde{D}}^2)_{RR} + M_D^2 - \frac{1}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W \mathbb{1}, \quad (2.9c)$$

$$\varphi_{\tilde{D}} = \arg(A_D - \mu^* \tan \beta). \quad (2.9d)$$

Here θ_W denotes the Weinberg angle, $\mathbb{1}$ represents a 3×3 unit matrix, $(M_{\tilde{q}}^2)_{LL}$ and $(M_{\tilde{q}}^2)_{RR}$ are Hermitian scalar soft mass matrices, and V_{CKM} is the usual CKM matrix. In deriving Eq. (2.9a), we have used the relation $(M_{\tilde{D}}^2)_{LL} = V_{\text{CKM}}^\dagger (M_{\tilde{U}}^2)_{LL} V_{\text{CKM}}$, which is due to $SU(2)$ gauge invariance. Since we ignore flavor-mixing effects among squarks, $(M_{\tilde{q}}^2)_{LL}$ and $(M_{\tilde{q}}^2)_{RR}$ in Eqs. (2.8) and (2.9) are diagonal—and hence real—whereas the A_q 's are given by

$$A_U = \text{diag}(A_u, A_c, A_t), \quad A_D = \text{diag}(A_d, A_s, A_b), \quad (2.10)$$

$$A_i \equiv |A_i| e^{i\varphi_{A_i}}.$$

Consequently, the squark mass-squared matrix, Eq. (2.5), in the up-squark sector decomposes into a series of 2×2 matrices. As far as the scalar top quark is concerned, we have

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 4 \sin^2 \theta_W) & m_t |A_t - \mu^* \cot \beta| e^{-i\varphi_{\tilde{t}}} \\ m_t |A_t - \mu^* \cot \beta| e^{i\varphi_{\tilde{t}}} & m_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W \end{pmatrix}, \quad (2.11)$$

where $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ are diagonal elements of $(M_{\tilde{U}}^2)_{LL}$ and $(M_{\tilde{U}}^2)_{RR}$ respectively, while $\varphi_{\tilde{t}}$ can be readily inferred from Eq. (2.8d). Diagonalization of the stop mass-squared matrix then leads to the physical mass eigenstates \tilde{t}_1 and \tilde{t}_2 , namely

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{t}} & \sin \theta_{\tilde{t}} e^{-i\varphi_{\tilde{t}}} \\ -\sin \theta_{\tilde{t}} e^{i\varphi_{\tilde{t}}} & \cos \theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \equiv \begin{pmatrix} \Gamma_{33}^{U_L} & \Gamma_{33}^{U_R} \\ \Gamma_{63}^{U_L} & \Gamma_{63}^{U_R} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad (2.12)$$

where the mixing angle $\theta_{\tilde{t}}$ is given by the expression $(-\pi/2 \leq \theta_{\tilde{t}} \leq \pi/2)$

$$\tan 2\theta_{\tilde{t}} = \frac{2m_t |A_t - \mu^* \cot \beta|}{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2) + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 8 \sin^2 \theta_W)}. \quad (2.13)$$

3. Chargino mass matrix

The chargino mass matrix can be written as

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & |\mu| e^{i\varphi_\mu} \end{pmatrix}, \quad (2.14)$$

where we have adopted a phase convention in which the mass term of the W-ino field, M_2 , is real and positive. Note that without loss of generality, we can always perform a global rotation to remove one of the three phases from the gaugino mass parameters M_i ($i = 1, 2, 3$).

The mass matrix can be cast in diagonal form by means of a biunitary transformation, namely

$$M_{\tilde{\chi}^\pm}^{\text{diag}} = U^* M_{\tilde{\chi}^\pm} V^\dagger, \quad (2.15)$$

where $M_{\tilde{\chi}^\pm}^{\text{diag}}$ is diagonal with positive eigenvalues, and U, V are unitary matrices. Solving the eigenvalue problem

$$(M_{\tilde{\chi}^\pm}^{\text{diag}})^2 = U^* M_{\tilde{\chi}^\pm} M_{\tilde{\chi}^\pm}^\dagger U^T = V M_{\tilde{\chi}^\pm}^\dagger M_{\tilde{\chi}^\pm} V^\dagger, \quad (2.16)$$

we find

$$U = \begin{pmatrix} \cos \theta_U & \sin \theta_U e^{-i\varphi_U} \\ -\sin \theta_U e^{i\varphi_U} & \cos \theta_U \end{pmatrix}, \quad (2.17)$$

$$V = \begin{pmatrix} \cos \theta_V e^{-i\phi_1} & \sin \theta_V e^{-i(\varphi_V + \phi_1)} \\ -\sin \theta_V e^{i(\varphi_V - \phi_2)} & \cos \theta_V e^{-i\phi_2} \end{pmatrix}, \quad (2.18)$$

with the mixing angles

$$\tan 2\theta_U = \frac{2\sqrt{2}M_W[M_2^2 \cos^2 \beta + |\mu|^2 \sin^2 \beta + |\mu|M_2 \sin 2\beta \cos \varphi_\mu]^{1/2}}{M_2^2 - |\mu|^2 - 2M_W^2 \cos 2\beta}, \quad (2.19)$$

$$\tan 2\theta_V = \frac{2\sqrt{2}M_W[M_2^2 \sin^2 \beta + |\mu|^2 \cos^2 \beta + |\mu|M_2 \sin 2\beta \cos \varphi_\mu]^{1/2}}{M_2^2 - |\mu|^2 + 2M_W^2 \cos 2\beta}, \quad (2.20)$$

$$\tan \varphi_U = -\frac{|\mu| \sin \varphi_\mu \sin \beta}{M_2 \cos \beta + |\mu| \sin \beta \cos \varphi_\mu}, \quad (2.21)$$

$$\tan \varphi_V = -\frac{|\mu| \sin \varphi_\mu \cos \beta}{M_2 \sin \beta + |\mu| \cos \beta \cos \varphi_\mu}, \quad (2.22)$$

$$\tan \phi_1 = \frac{|\mu| \sin \varphi_\mu M_W^2 \sin 2\beta}{M_2(m_{\tilde{\chi}_1^\pm}^2 - |\mu|^2) + |\mu|M_W^2 \sin 2\beta \cos \varphi_\mu}, \quad (2.23)$$

$$\tan \phi_2 = -\frac{|\mu| \sin \varphi_\mu (m_{\tilde{\chi}_2^\pm}^2 - M_2^2)}{M_2 M_W^2 \sin 2\beta + |\mu| (m_{\tilde{\chi}_2^\pm}^2 - M_2^2) \cos \varphi_\mu}. \quad (2.24)$$

Here we have chosen $-\pi/2 \leq \theta_i \leq \pi/2$, $-\pi \leq \varphi_i, \phi_i \leq \pi$, where $i = U, V$, and the chargino mass eigenvalues read

$$m_{\tilde{\chi}_{1,2}^\pm}^2 = \frac{1}{2} [M_2^2 + |\mu|^2 + 2M_W^2 \mp \{(M_2^2 - |\mu|^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2 [M_2^2 + |\mu|^2 + 2|\mu|M_2 \sin 2\beta \cos \varphi_\mu]\}^{1/2}]. \quad (2.25)$$

B. SUSY particles and FCNC interactions

We present here the SUSY Lagrangian relevant to the FCNC processes of interest which will also serve as a means of fixing our notation. The interactions of charged Higgs bosons, charginos, neutralinos, and gluinos in the presence of new CP phases can be written as [11,26]

$$\begin{aligned} \mathcal{L}_{\text{SUSY}} = & \frac{g}{\sqrt{2}M_W} [\cot \beta (\bar{u} M_U V_{\text{CKM}} P_L d) + \tan \beta (\bar{u} V_{\text{CKM}} M_D P_R d)] H^+ + \sum_{j=1}^2 \bar{\tilde{\chi}}_j^- [\tilde{u}^\dagger (X_j^{UL} P_L + X_j^{UR} P_R) d + \tilde{v}^\dagger (X_j^{LL} P_L + X_j^{LR} P_R) l] \\ & + \sum_{k=1}^4 \bar{\tilde{\chi}}_k^0 [\tilde{d}^\dagger (Z_k^{DL} P_L + Z_k^{DR} P_R) d + \tilde{l}^\dagger (Z_k^{LL} P_L + Z_k^{LR} P_R) l] - \sqrt{2} g_s \sum_{b=1}^8 \bar{g}^b \tilde{d}^\dagger (G^{DL} P_L - G^{DR} P_R) T^b d + \text{H.c.}, \end{aligned} \quad (2.26)$$

where generation indices have been suppressed, and $P_{L,R} = (1 \mp \gamma_5)/2$. The mixing matrices in the super-CKM basis are given by

$$X_j^{UL} = g \left[-V_{j1}^* \Gamma^{UL} + V_{j2}^* \Gamma^{UR} \frac{M_U}{\sqrt{2} M_W \sin \beta} \right] V_{\text{CKM}}, \quad (2.27a)$$

$$X_j^{UR} = g U_{j2} \Gamma^{UL} V_{\text{CKM}} \frac{M_D}{\sqrt{2} M_W \cos \beta}, \quad (2.27b)$$

$$X_j^{LL} = -g V_{j1}^* R_{\bar{\nu}}, \quad X_j^{LR} = g U_{j2} R_{\bar{\nu}} \frac{M_E}{\sqrt{2} M_W \cos \beta}, \quad (2.27c)$$

$$Z_k^{DL} = -\frac{g}{\sqrt{2}} \left[\left(-N_{k2}^* + \frac{1}{3} \tan \theta_W N_{k1}^* \right) \Gamma^{DL} + N_{k3}^* \Gamma^{DR} \frac{M_D}{M_W \cos \beta} \right], \quad (2.27d)$$

$$Z_k^{DR} = -\frac{g}{\sqrt{2}} \left[\frac{2}{3} \tan \theta_W N_{k1} \Gamma^{DR} + N_{k3} \Gamma^{DL} \frac{M_D}{M_W \cos \beta} \right], \quad (2.27e)$$

$$Z_k^{LL} = \frac{g}{\sqrt{2}} \left[(N_{k2}^* + \tan \theta_W N_{k1}^*) \Gamma^{LL} - N_{k3}^* \Gamma^{LR} \frac{M_E}{M_W \cos \beta} \right], \quad (2.27f)$$

$$Z_k^{LR} = -\frac{g}{\sqrt{2}} \left[2 \tan \theta_W N_{k1} \Gamma^{LR} + N_{k3} \Gamma^{LL} \frac{M_E}{M_W \cos \beta} \right], \quad (2.27g)$$

$$G^{DL} = e^{-i\varphi_3/2} \Gamma^{DL}, \quad G^{DR} = e^{i\varphi_3/2} \Gamma^{DR}, \quad (2.27h)$$

φ_3 being the phase of the gluino mass term M_3 . (For scalar lepton as well as neutralino mass and mixing matrices, we refer the reader to Ref. [26].) In the remainder of this section, we briefly discuss two SUSY models with quite distinct scenarios for the CP -violating phases.

C. Different scenarios for the SUSY parameters

1. Constrained MSSM

In order to solve the FCNC problem in the MSSM and to further reduce the number of unknown parameters, the MSSM is generally embedded in a grand unified theory (GUT). This leads to the minimal supergravity (MSUGRA) inspired model, commonly referred to as the constrained MSSM [7]. In this model one assumes universality of the soft terms at some high scale, which we take to be the scale of gauge coupling unification, M_{GUT} , implying that (i) all gaugino mass parameters are equal to a common mass $M_{1/2}$; (ii) all the scalar mass parameters share a common value m_0 ; and (iii) all the soft trilinear couplings are equal to A_0 . As a result, the MSUGRA model has only two new independent

phases which are associated with the μ_0 and A_0 parameters. After all, we have at the GUT scale

$$\tan \beta, M_{1/2}, m_0, |A_0|, |\mu_0|, \varphi_{A_0}, \varphi_{\mu_0}, \quad (2.28)$$

with $M_{1/2}$ and m_0 being real. The parameters at the electroweak scale are then obtained by solving the renormalization group equations (RGE's).

A few remarks are in order here. First, the phases of the gaugino mass terms M_i are not affected by the renormalization group evolution, and therefore the low energy gaugino mass parameters are real. Second, the phase that appears together with the μ parameter does not run at one-loop level so that $\varphi_\mu = \varphi_{\mu_0}$. Moreover, to satisfy the constraints on the EDM's of electron and neutron, φ_μ has to be of $\mathcal{O}(10^{-2})$ unless strong cancellations between different contributions occur. Third, solving the RGE's for the evolution of the CP -violating phase of the A_t -term yields a small value for φ_{A_t} , and thus is not constrained by the EDM's [13,28]. Lastly, off-diagonal entries occur in the squark mass matrices due to renormalization group evolution of the parameters even in the absence of flavor mixing at the GUT scale. However, these effects are found to be small and therefore the squark mass matrix is essentially flavor diagonal at the electroweak scale (see also Refs. [11,29]).

2. Effective supersymmetry

As an example of SUSY models with large CP phases, we consider the effective supersymmetry picture [25] without assuming universality of sfermion masses at a high scale. Within such a framework, the first and second generation sfermions are almost degenerate and have masses above the TeV scale, while third generation sfermions can be light enough to be accessible at future hadron colliders. Consequently, FCNC reactions as well as one-loop contributions to the EDM's of electron and neutron are well below the current experimental bounds.

However, it should be noted that the EDM's also receive contributions at two-loop level involving scalar bottom and top quarks that may become important for phases of order unity in the large $\tan \beta$ regime [30]. In our numerical work, $\tan \beta$ is assumed to be in the interval $2 \leq \tan \beta \leq 30$.

III. RARE B DECAYS AND NEW PHYSICS

A. Short-distance matrix element

Let us start with the QCD-corrected matrix element describing the short-distance interactions in $b \rightarrow s(d)l^+l^-$ within the SM. It is given by

$$\begin{aligned} \mathcal{M}_{\text{SD}} = & \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{if}^* \left\{ \left[(c_9^{\text{eff}} - c_{10}) \langle H(k) | \bar{f} \gamma_\mu P_L b | \bar{B}(p) \rangle \right. \right. \\ & - \frac{2c_7^{\text{eff}}}{q^2} \langle H(k) | \bar{f} i \sigma_{\mu\nu} q^\nu (m_b P_R \\ & \left. \left. + m_f P_L) b | \bar{B}(p) \rangle \right] \bar{l} \gamma^\mu P_L l + (c_{10 \rightarrow} - c_{10}) \bar{l} \gamma^\mu P_R l \right\}, \quad (3.1) \end{aligned}$$

where q is the four-momentum of the lepton pair, and $H = K, K^*$ (π, ρ) in the case of $f = s$ ($f = d$). In the SM, the Wilson coefficients c_7^{eff} and c_{10} are real with values of -0.314 and -4.582 respectively, and the leading term in c_9^{eff} has the form [31,32]

$$c_9^{\text{eff}} = c_9 + (3c_1 + c_2)\{g(m_c, q^2) + \lambda_u[g(m_c, q^2) - g(m_u, q^2)]\} + \dots, \quad (3.2)$$

where $c_9 = 4.216$. The Wilson coefficients will be discussed in detail in Appendix B. In the above expression

$$\lambda_u \equiv \frac{V_{ub}V_{uf}^*}{V_{tb}V_{tf}^*} \approx \begin{cases} -\lambda^2(\rho - i\eta) & \text{for } f = s, \\ \frac{\rho(1-\rho) - \eta^2}{(1-\rho)^2 + \eta^2} - \frac{i\eta}{(1-\rho)^2 + \eta^2} & \text{for } f = d, \end{cases} \quad (3.3)$$

with ρ and η being the Wolfenstein parameters [33], where the latter reflects the presence of CP violation in the SM. For definiteness, we will assume $\rho = 0.19$ and $\eta = 0.35$ [34].

Finally, the one-loop function $g(m_i, q^2)$ at the scale $\mu_R = m_b$ is given by²

$$g(m_i, q^2) = -\frac{8}{9} \ln(m_i/m_b) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{1 - y_i} \\ \times \left\{ \Theta(1 - y_i) \left[\ln \left(\frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i\pi \right] + \Theta(y_i - 1) 2 \arctan \frac{1}{\sqrt{y_i - 1}} \right\}, \quad (3.4)$$

where $y_i = 4m_i^2/q^2$. Observe that $c\bar{c}$ and $u\bar{u}$ loops provide absorptive parts that are mandatory, as we show below, for a non-zero partial width asymmetry besides the presence of a CP -violating phase.

B. Wilson coefficients and new physics

Throughout this paper, we will assume that in the presence of new physics there are no new operators beyond those that correspond to the Wilson coefficients appearing in Eq. (3.1). (For a discussion of the implications of new operators for rare B decays, see, e.g., Ref. [35].) Thus, the effect of new physics is simply to modify the matching conditions of the Wilson coefficients, i.e. their absolute values and phases at the electroweak scale.

As a result, we are left with additional SUSY contributions at one-loop level to the Wilson coefficients c_7^{eff} , c_9^{eff} , and c_{10} in Eq. (3.1). In fact, they arise from penguin and box diagrams with (i) charged Higgs boson up-type quark loops; (ii) chargino up-type squark loops; (iii) neutralino down-type squark loops; and (iv) gluino down-type squark loops. Thus,

the short-distance coefficients can be conveniently written as

$$c_i(M_W) = c_i^{\text{SM}}(M_W) + c_i^{H^\pm}(M_W) + c_i^{\tilde{\chi}^\pm}(M_W) \\ + c_i^{\tilde{\chi}^0}(M_W) + c_i^{\tilde{g}}(M_W) \quad (i = 7, \dots, 10). \quad (3.5)$$

The explicit expressions for the various Wilson coefficients are given in Appendix B. Since we limit our attention to flavor-conserving effects in the squark sector, the neutralino and gluino exchange contributions in Eq. (3.5) will be omitted in our numerical calculations.

For future reference, we parametrize the new physics contributions as follows:

$$R_i = \frac{c_i(M_W)}{c_i^{\text{SM}}(M_W)} \equiv |R_i| e^{i\phi_i}, \quad \chi = \frac{R_8 - 1}{R_7 - 1}, \quad (3.6)$$

where χ is real to a good approximation within the models under study.

C. Resonant intermediate states

We have considered so far only the short-distance interactions. A more complete analysis, however, has also to take into account resonance contributions due to $u\bar{u}$, $d\bar{d}$, and $c\bar{c}$ intermediate states, i.e. $\rho, \omega, J/\psi, \psi'$, and so forth. A detailed discussion of the various theoretical suggestions of how to describe these effects is given in Ref. [36].

We employ here the approach proposed in Ref. [37] which makes use of the renormalized photon vacuum polarization, $\Pi_{\text{had}}^\gamma(s)$, related to cross-section data³

$$R_{\text{had}}(s) \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (3.7)$$

with $s \equiv q^2$. In fact, the absorptive part of the vacuum polarization is given by

$$\text{Im} \Pi_{\text{had}}^\gamma(s) = \frac{\alpha}{3} R_{\text{had}}(s), \quad (3.8)$$

whereas the dispersive part may be obtained via a once-subtracted dispersion relation [38]

$$\text{Re} \Pi_{\text{had}}^\gamma(s) = \frac{\alpha s}{3\pi} P \int_{4M_\pi^2}^\infty \frac{R_{\text{had}}(s')}{s'(s' - s)} ds', \quad (3.9)$$

with P denoting the principal value. For example, in the case of the J/ψ family (i.e. $J/\psi, \psi', \dots$) the imaginary part of the one-loop function $g(m_c, s)$, Eq. (3.4), can be expressed as

²In order to avoid confusion with the μ parameter of the superpotential, we use the notation μ_R for the renormalization scale.

³This method assumes quark-hadron duality and rests on the factorization assumption.

$$\text{Im } g(m_c, s) = \frac{\pi}{3} R_{\text{had}}^{J/\psi}(s), \quad R_{\text{had}}^{J/\psi}(s) \equiv R_{\text{cont}}^{\bar{c}c}(s) + R_{\text{res}}^{J/\psi}(s), \quad (3.10)$$

where the subscripts ‘‘cont’’ and ‘‘res’’ stand for continuum and resonance contributions respectively, while the real part is given by

$$\begin{aligned} \text{Re } g(m_c, s) = & -\frac{8}{9} \ln(m_c/m_b) - \frac{4}{9} \\ & + \frac{s}{3} P \int_{4M_\pi^2}^{\infty} \frac{R_{\text{had}}^{J/\psi}(s')}{s'(s'-s)} ds'. \end{aligned} \quad (3.11)$$

The contributions from the continuum can be determined by means of experimental data given in Ref. [39], whereas the narrow resonances are well described by a relativistic Breit-Wigner distribution.

However, in order to reproduce correctly the branching ratio for direct J/ψ production via the relation ($V_{c\bar{c}} = J/\psi, \psi', \dots$)

$$\mathcal{B}(B \rightarrow HV_{c\bar{c}} \rightarrow Hl^+l^-) = \mathcal{B}(B \rightarrow HV_{c\bar{c}}) \mathcal{B}(V_{c\bar{c}} \rightarrow l^+l^-), \quad (3.12)$$

where H stands for pseudoscalar and scalar mesons, one has to multiply $R_{\text{res}}^{J/\psi}$ in Eqs. (3.10) and (3.11) by a phenomenological factor κ , regardless of which method one uses for the description of the resonances [40,41].⁴ Using the form factors of Ref. [43] (see next section) together with experimental data on $\mathcal{B}(B \rightarrow K^{(*)}J/\psi)$, $\mathcal{B}(B \rightarrow K^{(*)}\psi')$, and $\mathcal{B}(B^- \rightarrow \pi^- J/\psi)$ [44], we find a magnitude for κ of 1.7 to 3.3. At this point two remarks are in order. First, the branching ratio for direct J/ψ and ψ' production is enhanced by a factor κ^2 , while it is essentially independent of κ outside the resonance region. Second, the numerical results for average CP asymmetries in the non-resonant continuum $1.2 \text{ GeV} < \sqrt{s} < 2.9 \text{ GeV}$ are not affected by the uncertainty in κ .

Similar considerations also hold for $u\bar{u}$ and $d\bar{d}$ systems except that the ρ resonance is described through

$$R_{\text{res}}^\rho(s) = \frac{1}{4} \left(1 - \frac{4M_\pi^2}{s} \right)^{3/2} |F_\pi(s)|^2, \quad (3.13)$$

where the pion form factor is given by a modified Gounaris-Sakurai formula [45].

IV. THE DECAYS $\bar{B} \rightarrow K^{(*)}l^+l^-$ AND $\bar{B} \rightarrow \pi(\rho)l^+l^-$

The hadronic matrix elements in exclusive B decays can be written in terms of q^2 -dependent form factors, where q^2 is the invariant mass of the lepton pair. In the work described here, we employ heavy-to-light $B \rightarrow K^{(*)}$ and $B \rightarrow \pi(\rho)$ form factors determined by Melikhov and Nikitin [43] within a relativistic quark model. To get an estimate of the theoretical uncertainty that is inherent to any model for the form factors, we also utilize the parametrization of Colangelo *et al.* [46], which makes use of QCD sum rule predictions.

For simplicity of presentation, we do not display corrections due to a non-zero lepton mass, which can be found in Refs. [22,47]. (The same remark applies to the light quark masses $m_{s,d}$.) Henceforth we shall denote pseudoscalar and vector mesons by $P = K, \pi$ and $V = K^*, \rho$ respectively.

A. $B \rightarrow P$ transitions

1. Form factors

The hadronic matrix elements for the decays $B \rightarrow P$ can be parametrized in terms of three Lorentz-invariant form factors (see Appendix D for details), namely

$$\langle P(k) | \bar{f} \gamma_\mu P_L b | \bar{B}(p) \rangle = \frac{1}{2} [(2p - q)_\mu f_+(q^2) + q_\mu f_-(q^2)], \quad (4.1a)$$

$$\begin{aligned} \langle P(k) | \bar{f} i \sigma_{\mu\nu} q^\nu P_{L,R} b | \bar{B}(p) \rangle = & -\frac{1}{2} [(2p - q)_\mu q^2 - (M_B^2 \\ & - M_P^2) q_\mu] s(q^2), \end{aligned} \quad (4.1b)$$

with $q = p - k$. Here we assume that the form factors are real, in the absence of final-state interactions. Note that the terms proportional to q_μ may be dropped in the case of massless leptons.

2. Differential decay spectrum and forward-backward asymmetry

Introducing the shorthand notation

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac), \quad (4.2)$$

$$X_i = \frac{1}{2} \lambda^{1/2}(M_B^2, M_i^2, s), \quad (4.3)$$

and recalling $s \equiv q^2$, the differential decay rate can be written as (neglecting m_l and m_f)

$$\frac{d\Gamma(\bar{B} \rightarrow Pl^+l^-)}{ds d \cos \theta_l} = \frac{G_F^2 \alpha^2}{2^8 \pi^5 M_B^3} |V_{tb} V_{t\bar{f}}^*|^2 X_P^3 [c_9^{\text{eff}} f_+(s) + 2c_7^{\text{eff}} m_b s(s) + |c_{10} f_+(s)|^2] \sin^2 \theta_l. \quad (4.4)$$

Here θ_l is the angle between l^- and the outgoing hadron in the dilepton center-of-mass system, and the Wilson coefficients are collected in Appendix B. Defining the forward-backward (FB) asymmetry as

⁴Strictly speaking, it is the term $(3c_1 + c_2)R_{\text{res}}^{J/\psi}$ —in the approximation of Eq. (3.2)—which has to be corrected to $\kappa(3c_1 + c_2)R_{\text{res}}^{J/\psi}$, taking into account non-factorizable contributions in two-body B decays (see, e.g., Ref. [42]).

$$A_{\text{FB}}(s) = \frac{\int_0^1 d \cos \theta_l \frac{d\Gamma}{ds d \cos \theta_l} - \int_{-1}^0 d \cos \theta_l \frac{d\Gamma}{ds d \cos \theta_l}}{\int_0^1 d \cos \theta_l \frac{d\Gamma}{ds d \cos \theta_l} + \int_{-1}^0 d \cos \theta_l \frac{d\Gamma}{ds d \cos \theta_l}}, \quad (4.5)$$

which is equivalent to the energy asymmetry discussed in Ref. [11], it follows directly from the distribution in Eq. (4.4) that A_{FB} vanishes in the case of $\bar{B} \rightarrow Pl^+l^-$ transitions. We note in passing that, given an extended operator basis (e.g. in models with neutral Higgs-boson exchange), new Dirac structures $\bar{l}l$ and $\bar{l}\gamma_5l$ may occur in Eq. (3.1), giving rise to a non-zero FB asymmetry in $\bar{B} \rightarrow Pl^+l^-$.

B. $B \rightarrow V$ transitions

1. Form factors

The hadronic matrix elements describing the decays $B \rightarrow V$ are characterized by seven independent form factors, which we present in Appendix D, defined through ($\epsilon_{0123} = +1$)

$$\langle V(k) | \bar{f} \gamma_\mu P_L b | \bar{B}(p) \rangle = i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta g(q^2) - \frac{1}{2} \{ \epsilon_\mu^* f(q^2) + (\epsilon^* \cdot q) [(2p-q)_\mu a_+(q^2) + q_\mu a_-(q^2)] \}, \quad (4.6a)$$

$$\begin{aligned} \langle V(k) | \bar{f} i \sigma_{\mu\nu} q^\nu P_{R,L} b | \bar{B}(p) \rangle &= i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta g_+(q^2) \mp \frac{1}{2} \epsilon_\mu^* [g_+(q^2)(M_B^2 - M_V^2) + q^2 g_-(q^2)] \\ &\quad \pm \frac{1}{2} (\epsilon^* \cdot q) \left\{ (2p-q)_\mu \left[g_+(q^2) + \frac{1}{2} q^2 h(q^2) \right] + q_\mu \left[g_-(q^2) - \frac{1}{2} (M_B^2 - M_V^2) h(q^2) \right] \right\}, \end{aligned} \quad (4.6b)$$

ϵ^μ being the polarization vector of the final-state meson, and $q = p - k$.

2. Differential decay spectrum and forward-backward asymmetry

The differential decay rate for $\bar{B} \rightarrow Vl^+l^-$ in the case of massless leptons and light quarks takes the form ($f = s$ or d)

$$\frac{d\Gamma(\bar{B} \rightarrow Vl^+l^-)}{ds d \cos \theta_l} = \frac{G_F^2 \alpha^2}{2^9 \pi^5 M_B^3} |V_{tb} V_{if}^*|^2 X_V [A(s) + B(s) \cos \theta_l + C(s) \cos^2 \theta_l], \quad (4.7)$$

with X_V as in Eq. (4.3). The quantities A , B , and C are

$$A(s) = \frac{2X_V^2}{M_V^2} \left[sM_V^2 \alpha_1(s) + \frac{1}{4} \left(1 + \frac{2sM_V^2}{X_V^2} \right) \alpha_2(s) + X_V^2 \alpha_3(s) + (k \cdot q) \alpha_4(s) \right], \quad (4.8)$$

$$B(s) = 8X_V \text{Re} \{ c_{10}^* [c_9^{\text{eff}} s A_x A_y - c_7^{\text{eff}} m_b (A_x B_y + A_y B_x)] \}, \quad (4.9)$$

$$C(s) = \frac{2X_V^2}{M_V^2} \left[sM_V^2 \alpha_1(s) - \frac{1}{4} \alpha_2(s) - X_V^2 \alpha_3(s) - (k \cdot q) \alpha_4(s) \right], \quad (4.10)$$

where $k \cdot q = (M_B^2 - M_V^2 - s)/2$, and

$$\alpha_1(s) = (|c_9^{\text{eff}}|^2 + |c_{10}|^2) A_x^2 + \frac{4|c_7^{\text{eff}}|^2 m_b^2}{s^2} B_x^2 - \frac{4 \text{Re}(c_7^{\text{eff}} c_9^{\text{eff}*}) m_b}{s} A_x B_x, \quad (4.11a)$$

$$\alpha_2(s) = \alpha_1(s)_{x \rightarrow y}, \quad \alpha_3(s) = \alpha_1(s)_{x \rightarrow z}, \quad (4.11b)$$

$$\alpha_4(s) = (|c_9^{\text{eff}}|^2 + |c_{10}|^2) A_y A_z + \frac{4|c_7^{\text{eff}}|^2 m_b^2}{s^2} B_y B_z - \frac{2 \text{Re}(c_7^{\text{eff}} c_9^{\text{eff}*}) m_b}{s} (A_y B_z + A_z B_y), \quad (4.11c)$$

in which the A_i 's and B_i 's are defined as

$$A_x = g(s), \quad A_y = f(s), \quad A_z = a_+(s), \quad (4.12a)$$

$$B_x = g_+(s), \quad B_y = g_+(s)(M_B^2 - M_V^2) + s g_-(s),$$

$$B_z = - \left[g_+(s) + \frac{1}{2} s h(s) \right]. \quad (4.12b)$$

The complex Wilson coefficients c_7^{eff} , c_9^{eff} , and c_{10} are given in Appendix B.

Finally, using Eqs. (4.5) and (4.7), we derive the forward-backward asymmetry

$$A_{\text{FB}}(s) = 12X_V \frac{\text{Re} \{ c_{10}^* [c_9^{\text{eff}} s A_x A_y - c_7^{\text{eff}} m_b (A_x B_y + A_y B_x)] \}}{[3A(s) + C(s)]}. \quad (4.13)$$

C. CP -violating observables

To discuss CP -violating asymmetries, let us first recall the necessary ingredients. Suppose the decay amplitude for $\bar{B} \rightarrow F$ has the general form

$$\mathcal{A}(\bar{B} \rightarrow F) = e^{i\phi_1} A_1 e^{i\delta_1} + e^{i\phi_2} A_2 e^{i\delta_2}, \quad (4.14)$$

where δ_i and ϕ_i denote strong phases (CP -conserving) and weak phases (CP -violating) respectively (A_1 and A_2 being real). Together with the decay amplitude for the conjugate process

$$\bar{\mathcal{A}}(B \rightarrow \bar{F}) = e^{-i\phi_1} A_1 e^{i\delta_1} + e^{-i\phi_2} A_2 e^{i\delta_2}, \quad (4.15)$$

which can be obtained from Eq. (4.14) by means of CPT invariance, we may define the CP asymmetry as

$$A_{CP} \equiv \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2} = \frac{-2r \sin \phi \sin \delta}{1 + 2r \cos \phi \cos \delta + r^2}, \quad (4.16)$$

with $r = A_2/A_1$, $\phi = \phi_1 - \phi_2$, and $\delta = \delta_1 - \delta_2$. As can be easily seen from Eq. (4.16), a non-zero partial rate asymmetry requires the simultaneous presence of a CP -violating phase ϕ as well as a CP -conserving dynamical phase δ , the latter being provided by the one-loop functions $g(m_c, s)$ and $g(m_u, s)$ present in the Wilson coefficient c_9^{eff} [Eq. (3.2)]. Notice that in the limit in which the charm quark mass equals the up quark mass there is no CP violation in the SM.

Given the differential decay distribution in the variables s and $\cos \theta_l$, we can construct the following CP -violating observables:

$$A_{CP}^{D,S}(s) = \frac{\int_{D,S} d \cos \theta_l \frac{d\Gamma_{\text{diff}}}{ds d \cos \theta_l}}{\int_S d \cos \theta_l \frac{d\Gamma_{\text{sum}}}{ds d \cos \theta_l}}, \quad \int_{D,S} \equiv \int_0^1 \int_{-1}^0, \quad (4.17a)$$

where we have introduced

$$\Gamma_{\text{diff}} = \Gamma(\bar{B} \rightarrow H l^+ l^-) - \bar{\Gamma}(B \rightarrow \bar{H} l^+ l^-), \quad (4.17b)$$

$$\Gamma_{\text{sum}} = \Gamma(\bar{B} \rightarrow H l^+ l^-) + \Gamma(B \rightarrow \bar{H} l^+ l^-), \quad (4.17c)$$

with $H = K^{(*)}, \pi, \rho$. It should be noted that the asymmetry A_{CP}^D represents a CP -violating effect in the angular distribution of l^- in B and \bar{B} decays while A_{CP}^S is the asymmetry in the partial widths of these decays. As can be seen from Eqs. (4.4), (4.7), and (4.13), the latter involves the phases of c_7^{eff} and c_9^{eff} while the former is also sensitive to the phase of the Wilson coefficient c_{10} .

V. NUMERICAL ANALYSIS

Given the SUSY contributions presented in the preceding sections, we now proceed to study the implications of supersymmetry for exclusive B decays.

A. Experimental constraints

In our numerical analysis, we scan the SUSY parameter space as given in Ref. [15] and take as input the parameters displayed in Appendix A. In addition, we take into account the following experimental constraints:

(i) From the measurement of the inclusive branching ratio $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$, which probes $|c_7^{\text{eff}}|$, one can derive upper and lower limits [48]:

$$2.0 \times 10^{-4} < \mathcal{B}(\bar{B} \rightarrow X_s \gamma) < 4.5 \times 10^{-4} \quad (95\% \text{ C.L.}). \quad (5.1)$$

This is specially useful to constrain extensions of the SM. Indeed, following the model-independent analysis performed in Ref. [49], and taking the Wilson coefficient c_7^{eff} in leading-log approximation [see Eq. (B2) of the Appendix], we obtain

$$\begin{aligned} \mathcal{B}(\bar{B} \rightarrow X_s \gamma) \approx & [0.801 + 0.444 |R_7|^2 + 0.002 |R_8|^2 \\ & + 1.192 \text{Re } R_7 + 0.083 \text{Re } R_8 \\ & + 0.061 \text{Re}(R_7 R_8^*)] \times 10^{-4}, \end{aligned} \quad (5.2)$$

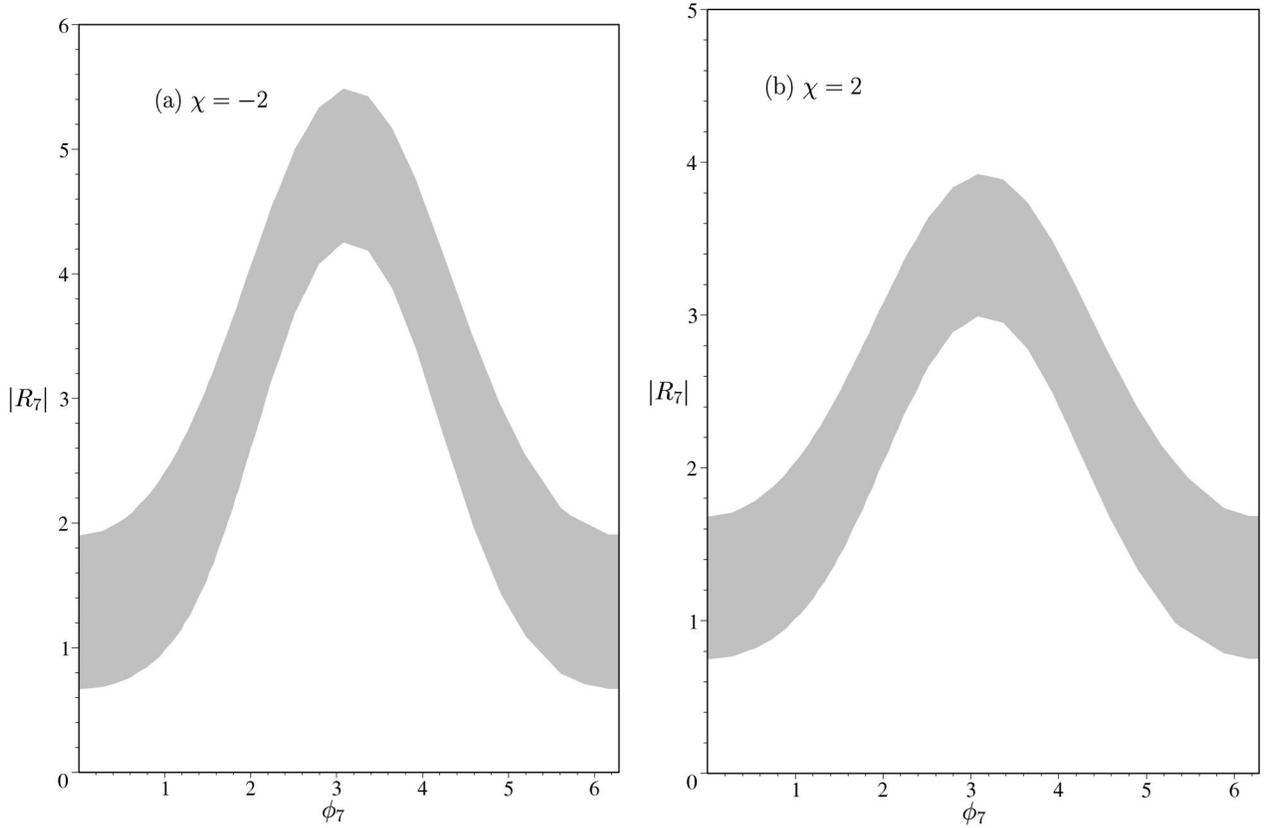


FIG. 1. Allowed region for $|R_7|$ and the corresponding CP -violating phase ϕ_7 as determined from the inclusive measurement of $b \rightarrow s\gamma$ rate, using the leading-log expression for c_7^{eff} . Diagrams (a) and (b) correspond to different values of χ [Eq. (3.6)].

where R_7 and R_8 are as in Eq. (3.6). From Fig. 1 we infer that the present $b \rightarrow s\gamma$ measurement already excludes many solutions for R_7 .

(ii) A CDF search for the exclusive decays of interest yields the upper limits $\mathcal{B}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) < 4.0 \times 10^{-6}$ and $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) < 5.2 \times 10^{-6}$ at the 90% C.L. [50]. Note that the $K^{*0} \mu^+ \mu^-$ upper limit is close to the SM prediction [19,51]. As for the modes $B \rightarrow \pi l^+ l^-$ and $B \rightarrow \rho l^+ l^-$, we are not aware of any such limits.

(iii) Non-observation of any SUSY signals at CERN e^+e^- collider LEP 2 and the Fermilab Tevatron imposes the following lower bounds [44]:

$$m_{\tilde{\chi}^\pm} > 86 \text{ GeV}, \quad m_{\tilde{\nu}} > 43 \text{ GeV}, \quad m_{\tilde{t}_1} > 86 \text{ GeV}, \\ m_{\tilde{q}} > 260 \text{ GeV}, \quad m_{H^\pm} > 90 \text{ GeV}. \quad (5.3)$$

B. CP asymmetries

As mentioned earlier, we investigate CP asymmetries in the low dilepton invariant mass region, i.e. $1.2 \text{ GeV} < \sqrt{s} < (M_{J/\psi} - 200 \text{ MeV})$, which is of particular interest because the low- s region is sensitive to the Wilson coefficient c_7^{eff} (in the case of $B \rightarrow V l^+ l^-$). In fact, it can receive large SUSY contributions and be complex, as well as change sign, while being consistent with the experimental measurement of $b \rightarrow s\gamma$. On the other hand, the new-physics effects are known to alter the remaining Wilson coefficients c_9^{eff} and c_{10} only

slightly within MSUGRA and effective SUSY with no additional flavor structure beyond the usual CKM mechanism [5,12,19]. Moreover, CP asymmetries above the J/ψ resonance are dominated by $c\bar{c}$ resonant intermediate states, whereas below 1 GeV the ρ resonance has a strong influence on the asymmetry. This can be seen from Fig. 2, where we show the CP asymmetry A_{CP}^S [Eq. (4.17a)] between $B^- \rightarrow \pi^- l^+ l^-$ and $B^+ \rightarrow \pi^+ l^+ l^-$ as a function of the dilepton invariant mass within the SM and in the presence of SUSY contributions with new CP -violating phases. It is evident that the predictions for CP asymmetries suffer from large theoretical uncertainties in the neighborhood of the ρ resonance and above the J/ψ , as discussed in Sec. III.

Using Eqs. (4.4) and (4.7) together with the definition for CP -violating asymmetries, Eqs. (4.17), we can summarize our main findings as follows.

1. CP violation in $B \rightarrow P l^+ l^-$

The CP -violating asymmetry in the l^- spectra of B and \bar{B} decays, A_{CP}^D , vanishes in the case of $B \rightarrow P$ transitions. Within the framework of the constrained MSSM with phases of $\mathcal{O}(10^{-2})$ numerical values for the average asymmetry $\langle A_{CP}^S \rangle$ in the low- s region are comparable to the SM predictions with asymmetries of 0.1% and -5% for $b \rightarrow s$ and $b \rightarrow d$ transitions respectively.

Our results for A_{CP}^S between the decays $\bar{B} \rightarrow P l^+ l^-$ and $B \rightarrow \bar{P} l^+ l^-$ in the context of effective SUSY with a light stop

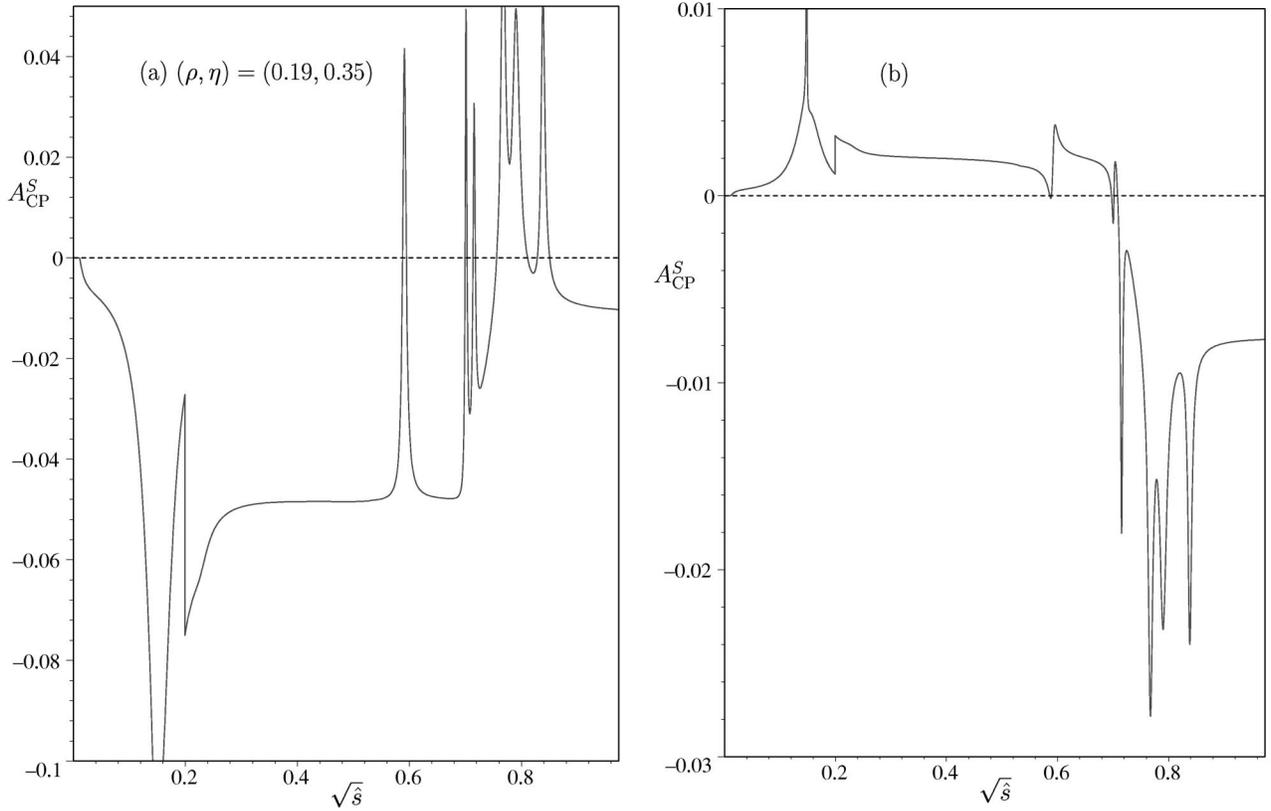


FIG. 2. CP -violating partial width asymmetry A_{CP}^S in the decays $B^- \rightarrow \pi^- l^+ l^-$ and $B^+ \rightarrow \pi^+ l^+ l^-$ vs $\sqrt{\hat{s}}$, $\hat{s} \equiv s/M_B^2$, including ρ , ω , and J/ψ , ψ' , etc. resonances, and employing the form factors of Ref. [43]. (a) Within the SM and (b) in the presence of new CP phases and a real CKM matrix. For the sake of illustration, we have chosen $|R_7|=1.6$, $|R_{9,10}|=1$, $\chi=1.5$, $\phi_7=\pi/2$, and $\phi_{9,10}=0.01$.

\tilde{t}_1 and phases of $\mathcal{O}(1)$ are shown in Figs. 3 and 4 for low and large $\tan\beta$ solutions which correspond to $\text{Re } R_7 > 0$ and $\text{Re } R_7 < 0$ respectively. Observe that the CP -violating asymmetry A_{CP}^S in $B \rightarrow P$ depends only weakly on the sign and phase of c_7^{eff} . This is due to the fact that c_7^{eff} , which is constrained by the $b \rightarrow s \gamma$ measurement and not enhanced by a factor of $1/s$ in the low- s region, is nearly one order of magnitude smaller than the leading term in c_9^{eff} [cf. Eq. (3.2)].

The CP asymmetry in the partial widths of $\bar{B} \rightarrow Kl^+l^-$ and $B \rightarrow \bar{K}l^+l^-$ changes sign for large values of ϕ_9 , while $|A_{CP}^S| \leq 1\%$ (see Fig. 3). However, non-standard contributions to ϕ_9 are found to be small and hence $A_{CP}^S \sim \mathcal{O}(10^{-3})$. On the other hand, average asymmetries of $-(5-6)\%$ are predicted for $\langle A_{CP}^S \rangle$ in the case of $B \rightarrow \pi$, even for values of ϕ_9 as small as 10^{-2} (see Fig. 4). Given a typical branching ratio of 10^{-8} and a nominal asymmetry of 6%, a measurement at 3σ level requires 3×10^{11} $b\bar{b}$ pairs. [This rather challenging task might be feasible at the CERN Large Hadron Collider (LHC) and the Tevatron.]

The small magnitude of the CP asymmetry is also due to a suppression factor multiplying the indispensable absorptive part in c_9^{eff} , which is only slightly affected by new-physics contributions. Indeed, it follows from Eqs. (3.2) and (4.16) that

$$A_{CP} \propto r \sin \phi \sin \delta \sim \frac{(3c_1 + c_2)}{|c_9|} \sin \phi \sin \delta \sim 10^{-2} \sin \phi \sin \delta, \quad (5.4)$$

where the weak and strong phases ϕ and δ can be of order unity.

Numerical estimates for average CP asymmetries are mildly affected by the parametrization of form factors (see also Refs. [22,52]).

2. CP violation in $B \rightarrow Vl^+l^-$

The contribution of the Wilson coefficient c_7^{eff} (or equivalently R_7) to the decay rate in $B \rightarrow V$ modes is enhanced by a factor of $1/s$ in the low- s region. As seen from Fig. 5, in the case of $B \rightarrow \rho$, the CP -violating asymmetry A_{CP}^S can change sign for $\tan\beta=2$ (i.e. $\text{Re } R_7 > 0$), while for large $\tan\beta$ it is always negative. For small values of ϕ_9 an average CP asymmetry of about -5% is predicted for both $\tan\beta=2$ and $\tan\beta=30$ solutions. Since the distributions of A_{CP}^S for $B \rightarrow K^*$ are very similar to the ones obtained for $B \rightarrow K$, we refrain from showing the corresponding plots.

As we have already mentioned, the CP -violating asymmetry in the angular distribution of l^- in B and \bar{B} decays can, in principle, probe the phase of the Wilson coefficient c_{10} . This is shown in Fig. 6, where we have plotted the CP asymmetry as a function of ϕ_9 and ϕ_{10} for large $\tan\beta$ (i.e.

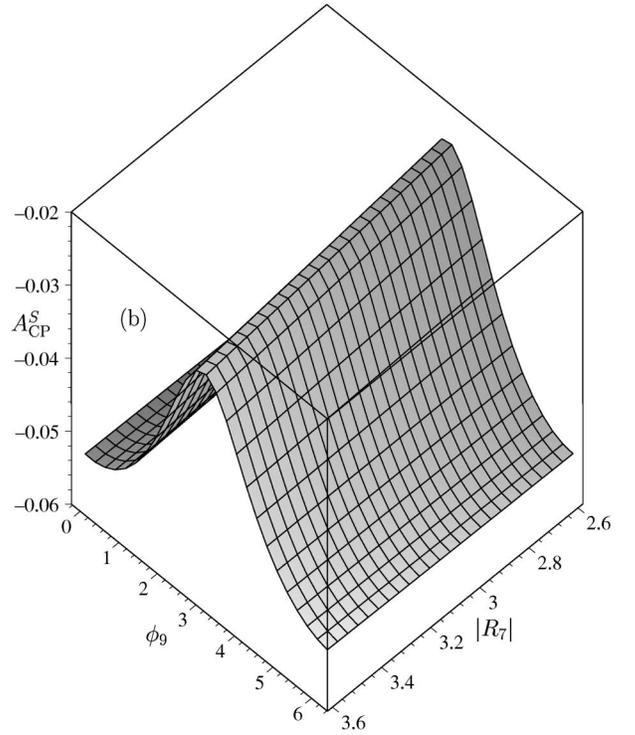
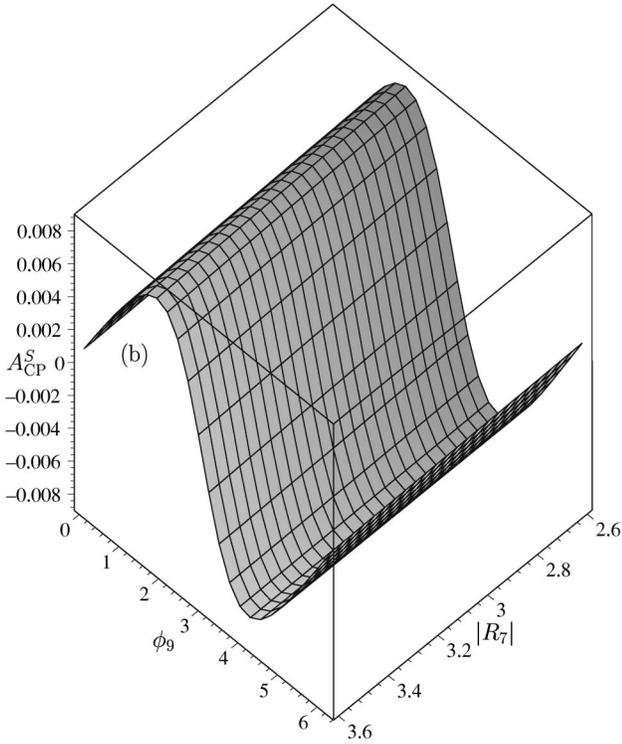
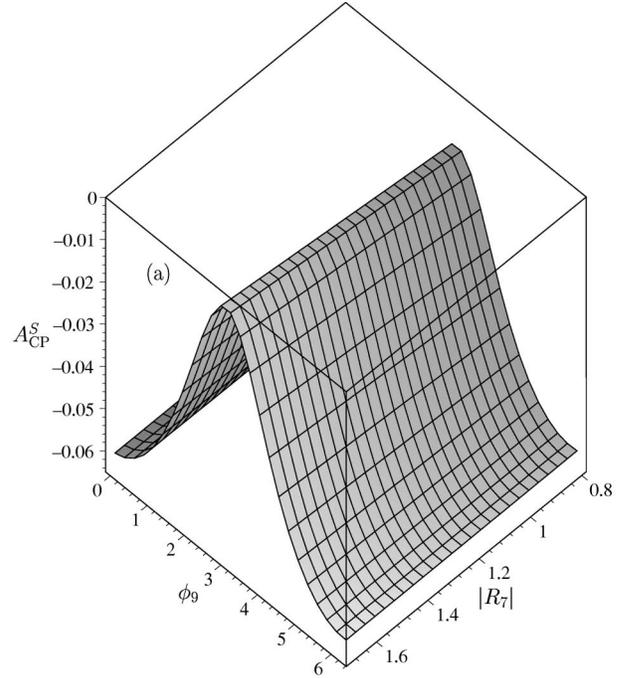
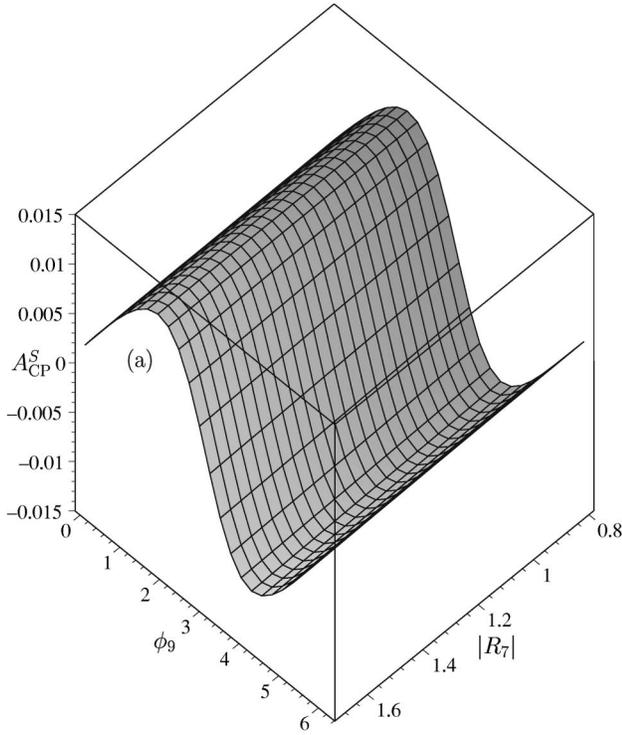


FIG. 3. CP -violating partial width asymmetry A_{CP}^S in the decays $\bar{B} \rightarrow K l^+ l^-$ and $B \rightarrow \bar{K} l^+ l^-$ as a function of ϕ_9 and $|R_7|$ for a dilepton invariant mass of $s=4 \text{ GeV}^2$ within effective SUSY. (a) $\tan \beta=2$ with $\phi_7=0.4$, $|R_9|=0.96$, $|R_{10}|=0.8$. (b) $\tan \beta=30$ with $\phi_7=2.5$, $|R_9|=0.99$, $|R_{10}|=0.9$.

$\text{Re } R_7 < 0$). Unfortunately, the MSUGRA and effective SUSY predictions for the average asymmetry $\langle A_{CP}^D \rangle$ turn out to be unobservably small.

FIG. 4. CP -violating partial width asymmetry A_{CP}^S between $B^- \rightarrow \pi^- l^+ l^-$ and $B^+ \rightarrow \pi^+ l^+ l^-$ as a function of ϕ_9 and $|R_7|$ for $s=4 \text{ GeV}^2$ within effective SUSY. (a) $\tan \beta=2$ with $\phi_7=0.4$, $|R_9|=0.96$, $|R_{10}|=0.8$. (b) $\tan \beta=30$ with $\phi_7=2.5$, $|R_9|=0.99$, $|R_{10}|=0.9$.

VI. DISCUSSION AND CONCLUSIONS

In this paper, we have studied the consequences of new CP -violating phases for exclusive B decays within the framework of supersymmetric extensions of the SM, ignoring intergenerational mixing in the squark sector. We have

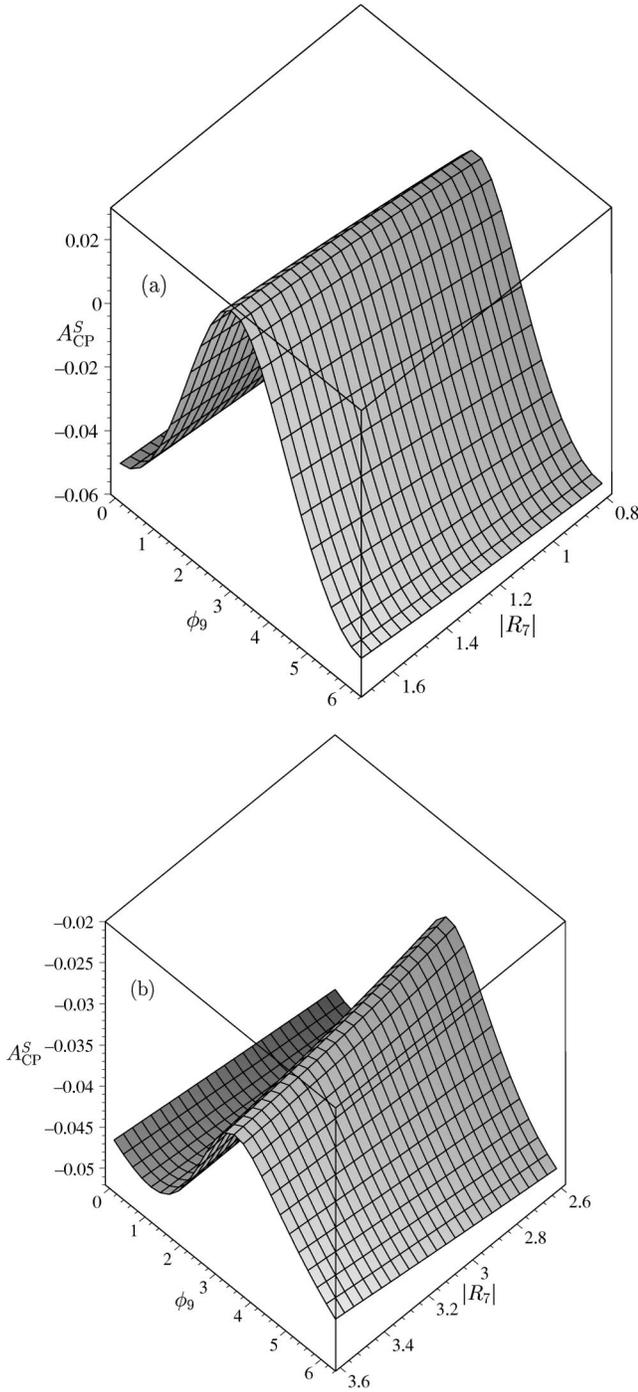


FIG. 5. CP asymmetry A_{CP}^S in the decays $B^- \rightarrow \rho^- l^+ l^-$ and $B^+ \rightarrow \rho^+ l^+ l^-$ as a function of ϕ_9 and $|R_7|$ for a dilepton invariant mass of $s=4 \text{ GeV}^2$ within effective SUSY. (a) $\tan \beta=2$ with $\phi_7=0.4$, $|R_9|=0.96$, $|R_{10}|=0.8$. (b) $\tan \beta=30$ with $\phi_7=2.5$, $|R_9|=0.99$, $|R_{10}|=0.9$. Note that (a) and (b) correspond to $\text{Re } R_7 > 0$ and $\text{Re } R_7 < 0$, respectively.

examined CP -violating asymmetries in the partial widths as well as angular distribution of l^- between the exclusive channels $b \rightarrow s(d)l^+l^-$ and $\bar{b} \rightarrow \bar{s}(\bar{d})l^+l^-$ in the invariant mass region $1.2 \text{ GeV} < M_{l^+l^-} < 2.9 \text{ GeV}$. The essential conclusion of our analysis is that it is not sufficient to have

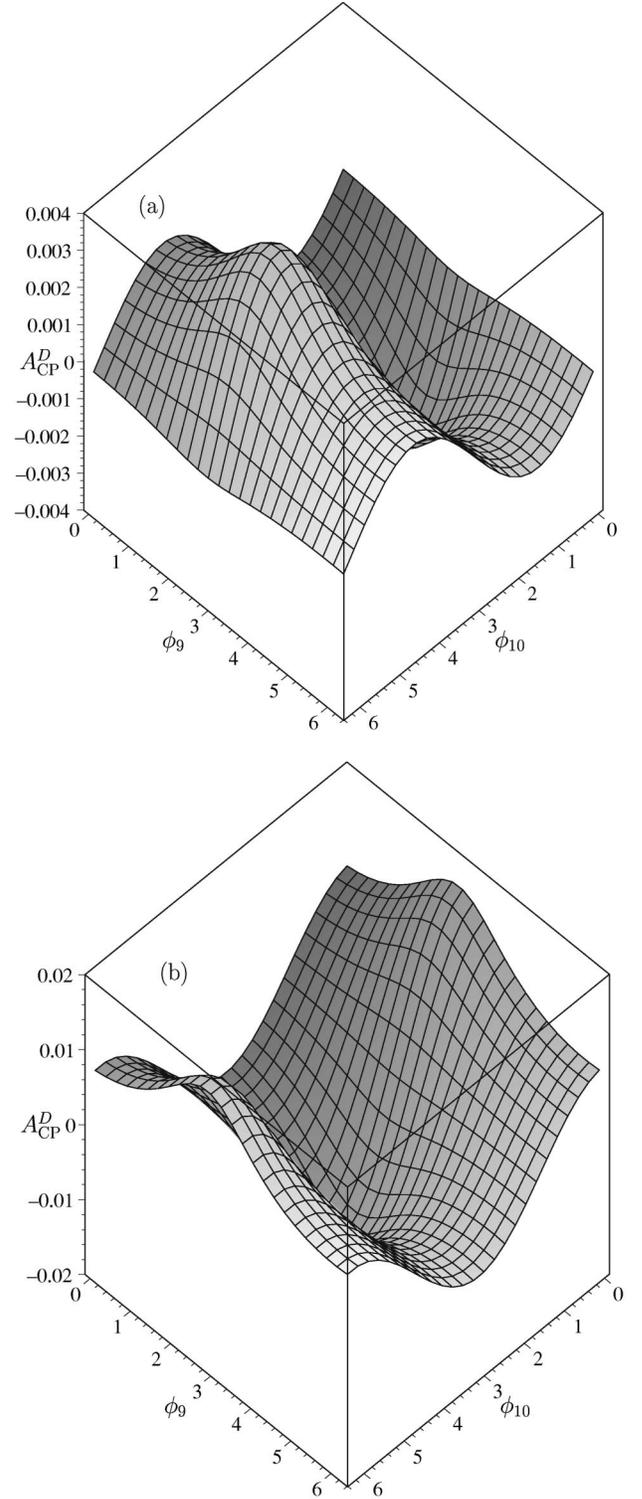


FIG. 6. CP -violating asymmetry A_{CP}^D between (a) $\bar{B} \rightarrow K^* l^+ l^-$ and $B \rightarrow \bar{K}^* l^+ l^-$, and (b) $B^- \rightarrow \rho^- l^+ l^-$ and $B^+ \rightarrow \rho^+ l^+ l^-$ for large $\tan \beta$ as in Fig. 3.

additional CP phases of $\mathcal{O}(1)$ in order to obtain large CP -violating effects.

Within the constrained MSSM and effective SUSY with a complex CKM matrix and additional CP phases, we obtain

values for the average asymmetry $\langle A_{CP}^S \rangle$ of about -6% (-5%) in the decays $\bar{B} \rightarrow \pi(\rho)l^+l^-$ and $B \rightarrow \bar{\pi}(\bar{\rho})l^+l^-$, taking into account experimental constraints on EDM's of electron and neutron, as well as rare B decays such as $b \rightarrow s\gamma$. As for the asymmetry in the angular distribution, $\langle A_{CP}^D \rangle$, it probes the phase of the Wilson coefficient c_{10} , but will be unobservable at future colliders. Numerical estimates of the CP asymmetries in the decays $\bar{B} \rightarrow K^{(*)}l^+l^-$ and $B \rightarrow \bar{K}^{(*)}l^+l^-$ turn out to be small (less than 1%) and are comparable to the SM result.

Our analysis shows that the smallness in CP asymmetries is mainly due to the coefficient $(3c_1 + c_2)/|c_9|$ which multiplies the requisite absorptive part in c_9^{eff} [Eq. (3.2)], and which is only slightly affected by the new-physics contributions discussed in Sec. III. Therefore, any sizable CP -violating effect in the low- s region requires large non-standard contributions to the short-distance coefficient c_9^{eff} and/or c_{10} , as well as additional CP phases of $\mathcal{O}(1)$. By the same token, any large CP -violating effect would provide a clue to physics beyond the SM. A detailed discussion of this point will be given elsewhere.

One could argue, however, that the inclusion of flavor off-diagonal contributions (i.e. gluino and neutralino diagrams) to the Wilson coefficients might lead to higher CP asymmetries. In fact, it has been pointed out in Refs. [29,53] that even in the presence of large supersymmetric CP phases, a non-trivial flavor structure in the soft-breaking terms is necessary in order to obtain sizable contributions to CP violation in the K system and to CP asymmetries in two-body neutral B decays (see also Ref. [24]). Using the mass insertion approximation, such effects have recently been studied in Ref. [20] which predicts a partial width asymmetry for $b \rightarrow sl^+l^-$ of a few percent in the low- s domain.

Finally, we would like to recall that, nevertheless, large CP asymmetries may occur in rare B decays like the observed $b \rightarrow s\gamma$ modes, where A_{CP} can be substantial (up to $\pm 45\%$) in some part of the parameter space [14,54].

ACKNOWLEDGMENTS

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APPENDIX A: NUMERICAL INPUTS

Unless otherwise specified, we use the experimental values as compiled by the Particle Data Group [44] and the parameters displayed in Eq. (A1).

$$\begin{aligned} m_t &= 175 \text{ GeV}, & m_b &= 4.8 \text{ GeV}, & m_c &= 1.4 \text{ GeV}, \\ m_s &= 170 \text{ MeV}, & m_d &= 10 \text{ MeV}, \\ m_u &= 5 \text{ MeV}, & \alpha &= 1/129, & \Lambda_{\text{QCD}} &= 225 \text{ MeV}. \end{aligned} \quad (\text{A1})$$

APPENDIX B: WILSON COEFFICIENTS AND SUSY

For the sake of convenience, we provide in this appendix formulas for the Wilson coefficients c_7^{eff} , c_9^{eff} , and c_{10} in the presence of SUSY, using the results derived in Refs. [9,11,31,55]. Since we study the case of massless leptons, we retain only those contributions that do not vanish in the limit $m_l \rightarrow 0$. As for τ leptons in the final state, there are further charged and neutral Higgs-boson contributions [see also Eqs. (2.27)].

Introducing the shorthand notation

$$\eta_s = \frac{\alpha_s(M_W)}{\alpha_s(m_b)}, \quad r_W = \frac{m_t^2}{M_W^2}, \quad r_{H^\pm} = \frac{m_t^2}{m_{H^\pm}^2}, \quad r_B^A = \frac{m_A^2}{m_B^2}, \quad (\text{B1})$$

the Wilson coefficient c_7^{eff} evaluated at $\mu_R = m_b$ has the form (in leading-log approximation)

$$c_7^{\text{eff}} = \eta_s^{16/23} c_7(M_W) + \frac{8}{3} (\eta_s^{14/23} - \eta_s^{16/23}) c_8(M_W) + \sum_{i=1}^8 h_i \eta_s^{a_i}, \quad (\text{B2})$$

with the coefficients a_i , h_i tabulated in Ref. [31]. Recalling Eqs. (2.27) and (3.5), and using the one-loop functions f_i listed in Appendix C, the various contributions to $c_{7,8}(M_W)$ can be written as follows:

Standard model:

$$c_7^{\text{SM}}(M_W) = \frac{1}{4} r_W f_1(r_W). \quad (\text{B3})$$

Charged Higgs boson:

$$c_7^{H^\pm}(M_W) = \frac{1}{12} [r_{H^\pm} f_1(r_{H^\pm}) \cot^2 \beta + 2f_2(r_{H^\pm})]. \quad (\text{B4})$$

Chargino:⁵

$$\begin{aligned} c_7^{\tilde{\chi}^\pm}(M_W) &= -\frac{1}{6g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{j=1}^2 \frac{M_W^2}{m_{\tilde{\chi}_j^\pm}^2} \\ &\times \left[(X_j^{UL^\dagger})_{na} (X_j^{UL})_{a3} f_1\left(\frac{\tilde{r}_{\tilde{\chi}_j^\pm}}{r_{\tilde{\chi}_j^\pm}}\right) \right. \\ &\quad \left. - 2(X_j^{UL^\dagger})_{na} (X_j^{UR})_{a3} \frac{m_{\tilde{\chi}_j^\pm}}{m_b} f_2\left(\frac{\tilde{r}_{\tilde{\chi}_j^\pm}}{r_{\tilde{\chi}_j^\pm}}\right) \right], \end{aligned} \quad (\text{B5})$$

where we have defined

$$n = \begin{cases} 1 & \text{for } f=d, \\ 2 & \text{for } f=s. \end{cases} \quad (\text{B6})$$

⁵Notice that the one-loop function appearing in the last term of Eq. (B5) is actually $f_2 + 5/2$. However, using the explicit form for the squark mixing matrices [Eqs. (2.27)], the constant term vanishes—reflecting the unitarity of the mixing matrices.

For completeness, we also give the expressions for the neutralino and gluino contributions which vanish in the limit of flavor-diagonal squark mass matrices.

Neutralino:

$$c_7^{\tilde{\chi}^0}(M_W) = -\frac{1}{6g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{k=1}^4 \frac{M_W^2}{m_{\tilde{\chi}_k^0}^2} \left[(Z_k^{DL^\dagger})_{na} (Z_k^{DL})_{a3} f_3(r_{\tilde{\chi}_k^0}^{\tilde{d}_a}) + 2(Z_k^{DL^\dagger})_{na} (Z_k^{DR})_{a3} \frac{m_{\tilde{\chi}_k^0}}{m_b} f_4(r_{\tilde{\chi}_k^0}^{\tilde{d}_a}) \right]. \quad (\text{B7})$$

Gluino:

$$c_7^{\tilde{g}}(M_W) = -\frac{4g_s^2}{9g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \frac{M_W^2}{m_{\tilde{g}}^2} \left[(G^{DL^\dagger})_{na} (G^{DL})_{a3} f_3(r_{\tilde{g}}^{\tilde{d}_a}) - 2(G^{DL^\dagger})_{na} (G^{DR})_{a3} \frac{m_{\tilde{g}}}{m_b} f_4(r_{\tilde{g}}^{\tilde{d}_a}) \right]. \quad (\text{B8})$$

The corresponding expressions $c_8^{\text{SM}}(M_W), \dots, c_8^{\tilde{\chi}^0}(M_W)$ are obtained changing $f_i \rightarrow g_i$ in Eqs. (B3)–(B7), with g_i collected in Appendix C, while the gluino contribution reads

$$c_8^{\tilde{g}}(M_W) = -\frac{4g_s^2}{9g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \frac{M_W^2}{m_{\tilde{g}}^2} \left[(G^{DL^\dagger})_{na} (G^{DL})_{a3} g_5(r_{\tilde{g}}^{\tilde{d}_a}) - 2(G^{DL^\dagger})_{na} (G^{DR})_{a3} \frac{m_{\tilde{g}}}{m_b} g_6(r_{\tilde{g}}^{\tilde{d}_a}) \right]. \quad (\text{B9})$$

The Wilson coefficient c_9^{eff} at $\mu_R = m_b$ in next-to-leading approximation is given by

$$c_9^{\text{eff}} = c_9 \left[1 + \frac{\alpha_s(m_b)}{\pi} \omega(s/m_b^2) \right] + g(m_c, s)(3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) + \lambda_u [g(m_c, s) - g(m_u, s)](3c_1 + c_2) - \frac{1}{2} g(m_f, s)(c_3 + 3c_4) - \frac{1}{2} g(m_b, s)(4c_3 + 4c_4 + 3c_5 + c_6) + \frac{2}{9} (3c_3 + c_4 + 3c_5 + c_6), \quad (\text{B10})$$

where λ_u and $g(m_i, s)$ are defined in Eqs. (3.3) and (3.4) respectively, with $s \equiv q^2$. As far as the Wilson coefficients $c_1 - c_6$ are concerned, we have numerically

$$c_1 = -0.249, \quad c_2 = 1.108, \quad c_3 = 0.011, \quad c_4 = -0.026, \quad c_5 = 0.007, \quad c_6 = -0.031, \quad (\text{B11})$$

using the values given in Appendix A. Further,

$$c_9 = c_9(M_W) - \frac{4}{9} + P_0 + P_E \sum_i E^i, \quad (\text{B12})$$

with $i = \text{SM}, H^\pm, \tilde{\chi}^\pm, \tilde{\chi}^0, \tilde{g}$, and

$$c_9(M_W) = \sum_i \left(\frac{Y^i}{\sin^2 \theta_W} - 4Z^i \right) + \frac{4}{9}, \quad (\text{B13})$$

where the analytic expressions for P_0 , P_E , and E^{SM} are given in Ref. [31]. Since $P_E \ll P_0$, we shall keep only the SM contribution in the last term of Eq. (B12). Moreover, as discussed in Ref. [41], the order α_s correction in Eq. (B10) due to one-gluon exchange may be regarded as a contribution to the form factors, and hence we set $\omega = 0$ in Eq. (B10)

Turning to the Wilson coefficient c_{10} , it has the simple form

$$c_{10}(M_W) = -\sum_i \frac{Y^i}{\sin^2 \theta_W}. \quad (\text{B14})$$

Note that the corresponding operator does not renormalize and thus $c_{10}(M_W) = c_{10}(\mu_R)$.

The expressions for the various contributions to Y and Z read as follows:⁶

Standard model:

⁶Regarding the expressions for the chargino and neutralino box-diagram contributions, and the sign discrepancy between Ref. [9] and Ref. [11], we confirm the results of the latter.

$$Y^{\text{SM}} = \frac{1}{8}f_9(r_w), \quad Z^{\text{SM}} = \frac{1}{72}f_{10}(r_w). \quad (\text{B15})$$

Charged Higgs boson:

$$Y^{H^\pm} \equiv Y_Z^{H^\pm}, \quad Z^{H^\pm} \equiv Z_\gamma^{H^\pm} + Z_Z^{H^\pm}, \quad (\text{B16a})$$

$$Z_\gamma^{H^\pm} = -\frac{1}{72}f_6(r_{H^\pm})\cot^2\beta, \quad Y_Z^{H^\pm} = Z_Z^{H^\pm} = -\frac{1}{8}r_w f_5(r_{H^\pm})\cot^2\beta. \quad (\text{B16b})$$

Chargino:

$$Y^{\tilde{\chi}^\pm} \equiv Y_Z^{\tilde{\chi}^\pm} + Y_{\text{box}}^{\tilde{\chi}^\pm}, \quad Z^{\tilde{\chi}^\pm} \equiv Z_\gamma^{\tilde{\chi}^\pm} + Z_Z^{\tilde{\chi}^\pm}, \quad (\text{B17a})$$

$$Z_\gamma^{\tilde{\chi}^\pm} = \frac{1}{36g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{j=1}^2 \frac{M_W^2}{m_{u_a}^2} [(X_j^{UL\dagger})_{na} (X_j^{UL})_{a3} f_7(r_{u_a}^{\tilde{\chi}^\pm})], \quad (\text{B17b})$$

$$Y_Z^{\tilde{\chi}^\pm} = Z_Z^{\tilde{\chi}^\pm} = \frac{1}{2g^2 V_{tb} V_{tf}^*} \sum_{a,b=1}^6 \sum_{i,j=1}^2 \left\{ (X_i^{UL\dagger})_{na} (X_j^{UL})_{b3} \left[c_2(m_{\tilde{\chi}_i^\pm}^2, m_{u_a}^2, m_{u_b}^2) (\Gamma^{UL} \Gamma^{UL\dagger})_{ab} \delta_{ij} \right. \right. \\ \left. \left. - c_2(m_{u_a}^2, m_{\tilde{\chi}_i^\pm}^2, m_{\tilde{\chi}_j^\pm}^2) \delta_{ab} V_{i1}^* V_{j1} + \frac{1}{2} m_{\tilde{\chi}_i^\pm} m_{\tilde{\chi}_j^\pm} c_0(m_{u_a}^2, m_{\tilde{\chi}_i^\pm}^2, m_{\tilde{\chi}_j^\pm}^2) \delta_{ab} U_{i1} U_{j1}^* \right] \right\}, \quad (\text{B17c})$$

$$Y_{\text{box}}^{\tilde{\chi}^\pm} = \frac{M_W^2}{g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{i,j=1}^2 [(X_i^{UL\dagger})_{na} (X_j^{UL})_{a3} d_2(m_{\tilde{\chi}_i^\pm}^2, m_{\tilde{\chi}_j^\pm}^2, m_{u_a}^2, m_{\nu_{1,2}}^2) V_{i1}^* V_{j1}], \quad (\text{B17d})$$

with $m_{\tilde{\nu}_1} (m_{\tilde{\nu}_2})$ in the case of $e^+ e^- (\mu^+ \mu^-)$ in the final state.

Neutralino:

$$Y^{\tilde{\chi}^0} \equiv Y_Z^{\tilde{\chi}^0} + Y_{\text{box}}^{\tilde{\chi}^0}, \quad Z^{\tilde{\chi}^0} \equiv Z_\gamma^{\tilde{\chi}^0} + Z_Z^{\tilde{\chi}^0} + Z_{\text{box}}^{\tilde{\chi}^0}, \quad (\text{B18a})$$

$$Z_\gamma^{\tilde{\chi}^0} = -\frac{1}{216g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{k=1}^4 \frac{M_W^2}{m_{\tilde{d}_a}^2} [(Z_k^{DL\dagger})_{na} (Z_k^{DL})_{a3} f_8(r_{\tilde{d}_a}^{\tilde{\chi}^0})], \quad (\text{B18b})$$

$$Y_Z^{\tilde{\chi}^0} = Z_Z^{\tilde{\chi}^0} = \frac{1}{2g^2 V_{tb} V_{tf}^*} \sum_{a,b=1}^6 \sum_{k,l=1}^4 \left\{ (Z_k^{DL\dagger})_{na} (Z_l^{DL})_{b3} \left[c_2(m_{\tilde{\chi}_k^0}^2, m_{\tilde{d}_a}^2, m_{\tilde{d}_b}^2) (\Gamma^{DR} \Gamma^{DR\dagger})_{ab} \delta_{kl} \right. \right. \\ \left. \left. - c_2(m_{\tilde{d}_a}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2) \delta_{ab} (N_{k3}^* N_{l3} - N_{k4}^* N_{l4}) - \frac{1}{2} m_{\tilde{\chi}_k^0} m_{\tilde{\chi}_l^0} c_0(m_{\tilde{d}_a}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2) \delta_{ab} (N_{k3}^* N_{l3}^* - N_{k4}^* N_{l4}^*) \right] \right\}, \quad (\text{B18c})$$

$$Y_{\text{box}}^{\tilde{\chi}^0} = 2 \sin^2 \theta_W Z_{\text{box}}^{\tilde{\chi}^0} + \frac{M_W^2}{2g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{k,l=1}^4 \left\{ (Z_k^{DL\dagger})_{na} (Z_l^{DL})_{a3} \left[d_2(m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2, m_{\tilde{d}_a}^2, m_{\tilde{l}_{1,2}}^2) (N_{k2}^* + \tan \theta_W N_{k1}^*) (N_{l2} \right. \right. \\ \left. \left. + \tan \theta_W N_{l1}) + \frac{1}{2} m_{\tilde{\chi}_k^0} m_{\tilde{\chi}_l^0} d_0(m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2, m_{\tilde{d}_a}^2, m_{\tilde{l}_{1,2}}^2) (N_{k2} + \tan \theta_W N_{k1}) (N_{l2}^* + \tan \theta_W N_{l1}^*) \right] \right\}, \quad (\text{B18d})$$

$$Z_{\text{box}}^{\tilde{\chi}^0} = \frac{M_W^2}{g^2 V_{tb} V_{tf}^*} \sum_{a=1}^6 \sum_{k,l=1}^4 \left\{ (Z_k^{DL\dagger})_{na} (Z_l^{DL})_{a3} \sec^2 \theta_W \left[d_2(m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2, m_{\tilde{d}_a}^2, m_{\tilde{l}_{4,5}}^2) N_{k1}^* N_{l1} \right. \right. \\ \left. \left. + \frac{1}{2} m_{\tilde{\chi}_k^0} m_{\tilde{\chi}_l^0} d_0(m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_l^0}^2, m_{\tilde{d}_a}^2, m_{\tilde{l}_{4,5}}^2) N_{k1} N_{l1}^* \right] \right\}, \quad (\text{B18e})$$

TABLE I. The $B \rightarrow K^{(*)}$ and $B \rightarrow \pi(\rho)$ form factors of Melikhov and Nikitin [43].

Form factor	$B \rightarrow K$	$B^- \rightarrow \pi^-$
$f_+(q^2)$	$0.36 \left(1 - \frac{q^2}{6.88^2}\right)^{-2.32}$	$0.29 \left(1 - \frac{q^2}{6.71^2}\right)^{-2.35}$
$f_-(q^2)$	$-0.30 \left(1 - \frac{q^2}{6.71^2}\right)^{-2.27}$	$-0.26 \left(1 - \frac{q^2}{6.55^2}\right)^{-2.30}$
$s(q^2)$	$0.06 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.85^2}\right)^{-2.28}$	$0.05 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.68^2}\right)^{-2.31}$
Form factor	$B \rightarrow K^*$	$B^- \rightarrow \rho^-$
$g(q^2)$	$0.048 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.67^2}\right)^{-2.61}$	$0.036 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.55^2}\right)^{-2.75}$
$f(q^2)$	$1.61 \text{ GeV} \left(1 - \frac{q^2}{5.86^2} + \frac{q^4}{7.66^4}\right)^{-1}$	$1.10 \text{ GeV} \left(1 - \frac{q^2}{5.59^2} + \frac{q^4}{7.10^4}\right)^{-1}$
$a_+(q^2)$	$-0.036 \text{ GeV}^{-1} \left(1 - \frac{q^2}{7.33^2}\right)^{-2.85}$	$-0.026 \text{ GeV}^{-1} \left(1 - \frac{q^2}{7.29^2}\right)^{-3.04}$
$a_-(q^2)$	$0.041 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.98^2}\right)^{-2.72}$	$0.03 \text{ GeV}^{-1} \left(1 - \frac{q^2}{6.88^2}\right)^{-2.85}$
$h(q^2)$	$0.0037 \text{ GeV}^{-2} \left(1 - \frac{q^2}{6.57^2}\right)^{-3.28}$	$0.003 \text{ GeV}^{-2} \left(1 - \frac{q^2}{6.43^2}\right)^{-3.42}$
$g_+(q^2)$	$-0.28 \left(1 - \frac{q^2}{6.67^2}\right)^{-2.62}$	$-0.20 \left(1 - \frac{q^2}{6.57^2}\right)^{-2.76}$
$g_-(q^2)$	$0.24 \left(1 - \frac{q^2}{6.59^2}\right)^{-2.58}$	$0.18 \left(1 - \frac{q^2}{6.50^2}\right)^{-2.73}$

with $m_{\tilde{l}_{1,4}} (m_{\tilde{l}_{2,5}})$ for $e^+e^- (\mu^+\mu^-)$ in the final state.

Glino:

$$Y_{\tilde{Z}}^{\tilde{g}} \equiv Y_{\tilde{Z}}^{\tilde{g}}, \quad Z_{\tilde{Z}}^{\tilde{g}} \equiv Z_{\tilde{Z}}^{\tilde{g}} + Z_{\tilde{Z}}^{\tilde{g}}, \quad (\text{B19a})$$

$$Z_{\tilde{\gamma}}^{\tilde{g}} = -\frac{g_s^2}{81g^2V_{tb}V_{tf}^*} \sum_{a=1}^6 \frac{M_W^2}{m_{\tilde{d}_a}^2} [(G^{DL\dagger})_{na}(G^{DL})_{a3}f_8(r_{\tilde{d}_a}^{\tilde{g}})], \quad (\text{B19b})$$

$$Y_{\tilde{Z}}^{\tilde{g}} = Z_{\tilde{Z}}^{\tilde{g}} = \frac{4g_s^2}{3g^2V_{tb}V_{tf}^*} \sum_{a,b=1}^6 [(G^{DL\dagger})_{na}(G^{DL})_{b3}c_2(m_g^2, m_{\tilde{d}_a}^2, m_{\tilde{d}_b}^2)(G^{DR}G^{DR\dagger})_{ab}]. \quad (\text{B19c})$$

The functions f_i , c_i , and d_i are given in Eqs. (C1)–(C4) below.

APPENDIX C: AUXILIARY FUNCTIONS

Here we list the functions f_i , g_i , c_i , and d_i introduced in the previous section:

$$f_1(x) = \frac{-7+5x+8x^2}{6(1-x)^3} - \frac{x(2-3x)}{(1-x)^4} \ln x, \quad (\text{C1a})$$

$$f_2(x) = \frac{x(3-5x)}{2(1-x)^2} + \frac{x(2-3x)}{(1-x)^3} \ln x, \quad (\text{C1b})$$

$$f_3(x) = \frac{2+5x-x^2}{6(1-x)^3} + \frac{x}{(1-x)^4} \ln x, \quad (\text{C1c})$$

$$f_4(x) = \frac{1+x}{2(1-x)^2} + \frac{x}{(1-x)^3} \ln x, \quad (\text{C1d})$$

$$f_5(x) = \frac{x}{1-x} + \frac{x}{(1-x)^2} \ln x, \quad (\text{C1e})$$

$$f_6(x) = \frac{x(38-79x+47x^2)}{6(1-x)^3} + \frac{x(4-6x+3x^3)}{(1-x)^4} \ln x, \quad (\text{C1f})$$

$$f_7(x) = \frac{52-101x+43x^2}{6(1-x)^3} + \frac{6-9x+2x^3}{(1-x)^4} \ln x, \quad (\text{C1g})$$

$$f_8(x) = \frac{2-7x+11x^2}{(1-x)^3} + \frac{6x^3}{(1-x)^4} \ln x, \quad (\text{C1h})$$

$$f_9(x) = \frac{x(4-x)}{1-x} + \frac{3x^2}{(1-x)^2} \ln x, \quad (\text{C1i})$$

$$f_{10}(x) = \frac{x(108-259x+163x^2-18x^3)}{2(1-x)^3} - \frac{8-50x+63x^2+6x^3-24x^4}{(1-x)^4} \ln x, \quad (\text{C1j})$$

$$g_1(x) = -3f_3(x), \quad g_2(x) = 3[f_4(x) - 1/2],$$

$$g_3(x) = -3f_3(x), \quad g_4(x) = -3f_4(x), \quad (\text{C2a})$$

$$g_5(x) = \frac{1}{8} \left[\frac{11-40x-19x^2}{2(1-x)^3} + \frac{3x(1-9x)}{(1-x)^4} \ln x \right], \quad (\text{C2b})$$

$$g_6(x) = \frac{3}{8} \left[\frac{5-13x}{(1-x)^2} + \frac{x(1-9x)}{(1-x)^3} \ln x \right], \quad (\text{C2c})$$

$$c_0(m_1^2, m_2^2, m_3^2) = - \left[\frac{m_1^2 \ln(m_1^2/\mu_R^2)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)} + (m_1 \leftrightarrow m_2) + (m_1 \leftrightarrow m_3) \right], \quad (\text{C3a})$$

$$c_2(m_1^2, m_2^2, m_3^2) = \frac{3}{8} - \frac{1}{4} \left[\frac{m_1^4 \ln(m_1^2/\mu_R^2)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)} + (m_1 \leftrightarrow m_2) + (m_1 \leftrightarrow m_3) \right], \quad (\text{C3b})$$

$$d_0(m_1^2, m_2^2, m_3^2, m_4^2) = - \left[\frac{m_1^2 \ln(m_1^2/\mu_R^2)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_1^2 - m_4^2)} + (m_1 \leftrightarrow m_2) + (m_1 \leftrightarrow m_3) + (m_1 \leftrightarrow m_4) \right], \quad (\text{C4a})$$

$$d_2(m_1^2, m_2^2, m_3^2, m_4^2) = - \frac{1}{4} \left[\frac{m_1^4 \ln(m_1^2/\mu_R^2)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_1^2 - m_4^2)} + (m_1 \leftrightarrow m_2) + (m_1 \leftrightarrow m_3) + (m_1 \leftrightarrow m_4) \right]. \quad (\text{C4b})$$

TABLE II. The $B \rightarrow K^{(*)}$ form factors of Colangelo *et al.* [46], with $M = 5$ GeV. As for the $B \rightarrow \pi(\rho)$ transition, we use the form factors listed below with $M = 5.3$ GeV in F_1 and F_T .

Form factor	$B \rightarrow K$
$F_1(q^2)$	$0.25 \left(1 - \frac{q^2}{M^2}\right)^{-1}$
$F_0(q^2)$	$0.25 \left(1 - \frac{q^2}{49}\right)^{-1}$
$F_T(q^2)$	$-0.14 \left(1 - \frac{q^2}{M^2}\right)^{-1} \left(1 - \frac{q^2}{49}\right)^{-1}$
Form factor	$B \rightarrow K^*$
$V(q^2)$	$0.47 \left(1 - \frac{q^2}{25}\right)^{-1}$
$A_1(q^2)$	$0.37(1 - 0.023q^2)$
$A_2(q^2)$	$0.40(1 + 0.034q^2)$
$A_0(q^2)$	$0.30 \left(1 - \frac{q^2}{4.8^2}\right)^{-1}$
$T_1(q^2)$	$0.19 \left(1 - \frac{q^2}{5.3^2}\right)^{-1}$
$T_2(q^2)$	$0.19(1 - 0.02q^2)$
$T_3(q^2)$	$0.30(1 + 0.01q^2)$

APPENDIX D: FORM FACTORS

In Tables I and II we summarize the two different sets of form factors discussed in Sec. IV, which are related via

$$F_1(q^2) = f_+(q^2), \quad (\text{D1a})$$

$$F_0(q^2) = f_+(q^2) + \frac{q^2}{M_B^2 - M_P^2} f_-(q^2), \quad (\text{D1b})$$

$$F_T(q^2) = -(M_B + M_P) s(q^2), \quad (\text{D1c})$$

$$V(q^2) = (M_B + M_V) g(q^2), \quad (\text{D1d})$$

$$A_1(q^2) = \frac{f(q^2)}{M_B + M_V}, \quad (\text{D1e})$$

$$A_2(q^2) = -(M_B + M_V) a_+(q^2), \quad (\text{D1f})$$

$$A_0(q^2) = \frac{q^2 a_-(q^2) + f(q^2) + (M_B^2 - M_V^2) a_+(q^2)}{2M_V}, \quad (\text{D1g})$$

$$T_1(q^2) = -\frac{1}{2} g_+(q^2), \quad (\text{D1h})$$

$$T_2(q^2) = -\frac{1}{2} \left[g_+(q^2) + \frac{q^2 g_-(q^2)}{M_B^2 - M_V^2} \right], \quad (\text{D1i})$$

$$T_3(q^2) = \frac{1}{2} \left[g_-(q^2) - \frac{(M_B^2 - M_V^2) h(q^2)}{2} \right]. \quad (\text{D1j})$$

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