$K_L \to \gamma \nu \bar{\nu}$ in the light front model

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We study the CP-conserving and -violating contributions to the decay of $K_L \to \gamma \nu \bar{\nu}$ in the standard model. In our analysis, we use form factors for $K \to \gamma$ transitions calculated directly in the entire physical range of momentum transfer within the light front model. We find that the branching ratios for the CP-conserving and -violating parts are about 1.0×10^{-13} and 1.5×10^{-15}, respectively.

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I. INTRODUCTION

With the prospect of a new generation of ongoing kaon experiments, a number of rare kaon decays have been suggested to test the Cabibbo-Kobayashi-Maskawa (CKM) [1] paradigm. However, it is sometimes a hard task to extract the short-distance contribution, which depends on the CKM matrix, because of large theoretical uncertainties in the long-distance contribution to the decays [2]. To avoid this difficulty, much recent theoretical work as well as experimental attention has been devoted to searching for the two modes $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$. It is believed that the long-distance contributions in these two modes are much smaller than the short-distance ones, and therefore they are negligible [3–6].

It has been shown that the decay branching ratio of $K^+ \to \pi^+ \nu \bar{\nu}$ is close to 10^{-10} [7,8] arising dominantly from the short-distance loop contributions containing virtual charm and top quarks. This decay is a CP-conserving process, and probably the cleanest one, in the sense of theoretical uncertainties, with which to study the absolute value of the CKM element V_{td}. Currently, the E787 group at BNL [9] has seen one event for the decay with the branching ratio of $B(K^+ \to \pi^+ \nu \bar{\nu}) = 1.5^{+3.5}_{-1.3} \times 10^{-10}$, which is consistent with the standard model prediction, and it is expected that there will be several events when the analysis is complete. The approved experiments of the E949 group at BNL and E905 at Fermilab [10] will have sensitivities of 10^{-11} and 10^{-12}, respectively.

On the other hand, the decay $K_L \to \pi^0 \nu \bar{\nu}$, depending on the imaginary part of V_{td}, is a CP-violating process [11], and offers clear information about the origin of CP violation. In the standard model, it is dominated by Z-penguin and W-box loop diagrams with virtual top quarks, and the decay branching ratio is found to be at the level of 10^{-12} [7,8], whereas the current experimental limit is less than 5.9×10^{-7} given by the experiment of the E799 group at Fermilab [12]. Several dedicated experimental searches [13] for this decay mode are underway at KEK, BNL, and Fermilab, respectively. However, from an experimental point of view, very challenging efforts are necessary to perform the experiments. This is because all the final-state particles are neutral, and the only detectable particles are 2γ's from π^0.

As an alternative search, it was proposed [14] to use the decay of $K_L \to \pi^+ \pi^- \nu \bar{\nu}$. However, the decay branching ratio is small and the background for $\pi^+ \pi^-$ is large [15]. In this paper, we study the radiative decay of $K_L \to \gamma \nu \bar{\nu}$, where there is one photon at the final states. The mode was considered previously in Refs. [16–18], and it is believed that the decay is short distance dominated [17,18]. However, the decay branching ratios predicted in Refs. [17] and [18] do not agree with one another. Furthermore, all the discussions were confined to the CP-conserving contribution due to the vector part of the structure-dependent amplitudes [17,18]. The decay branching ratio was found at levels of 10^{-11} and 10^{-13} in Refs. [17] and [18], respectively, which are different by about two orders of magnitude. To clarify this issue, we will re-examine the decay by using the form factors of the $K \to \gamma$ transition calculated directly in the entire physical range of momentum transfer within the light front framework. We will study both CP-conserving and -violating contributions to the decay branching ratio, respectively.

The paper is organized as follows. In Sec. II, we present the relevant effective Hamiltonian for the radiative decay of $K_L \to \gamma \nu \bar{\nu}$, and study the form factors in the $K^0 \to \gamma$ transition within the light front framework. In Sec. III, we calculate the decay branching ratio. We also compare our result with those in the literature [17,18]. We give our conclusions in Sec. IV.

II. EFFECTIVE HAMILTONIAN AND FORM FACTORS

The processes of $K_L \to \nu_l \bar{\nu}_l \gamma$ $(l = e, \mu, \tau)$ arise from the box and Z-penguin diagrams that contribute to $s \to d \nu_l \bar{\nu}_l$, with the photon emitted from the charged particles in the diagrams. The effective Hamiltonian for $s \to d \nu \bar{\nu}$ at the quark level in the standard model is given by

$$H_{eff}(s \to d \nu \bar{\nu}) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_{l=e,\mu,\tau} [\lambda_c X_{NL}^l + \lambda_t X(x_t)] \cdot \bar{d} \gamma_\mu (1-\gamma_5) \times s \bar{\nu}_l \gamma^\mu (1-\gamma_5) \nu_l , \tag{1}$$

where $x_t = m_t^2 / M_W^2$ and $\lambda_i = V_{is}^* V_{id}$ $(i = c, t)$ represent the products of the CKM matrix elements, and the functions of

X_{NL}^{l} and $X(x_t)$ correspond to top and charm contributions in the loops with the next-to-leading logarithmic approximation, respectively; their expressions can be found in Ref. [19]. In the Wolfenstein parametrization, we have

$$\mathrm{Re}\,\lambda_c = -\lambda\left(1-\frac{\lambda^2}{2}\right),$$

$$\mathrm{Re}\,\lambda_t = -\left(1-\frac{\lambda^2}{2}\right)A^2\lambda^5\left(1-\rho+\frac{\lambda^2}{2}\rho\right), \tag{2}$$

$$\mathrm{Im}\,\lambda_c = \mathrm{Im}\,\lambda_t = A^2\lambda^5\eta.$$

For phenomenological applications, we use

$$X(x_t) = \eta_X X_0(x_t), \tag{3}$$

where

$$\eta_X = 0.994,$$

$$X_0(x_t) = \frac{x_t}{8}\left[-\frac{2+x_t}{1-x_t}+\frac{3x_t-6}{(1-x_t)^2}\ln x_t\right], \tag{4}$$

with the $\overline{MS}$ definition of the top-quark mass, $m_t \equiv \overline{m}_t(m_t) = 166\pm 5$ GeV. For the charm sector, from Table I in Ref. [19], for example, one has

$$X_{NL}^{e,\mu} = 11.00\times 10^{-4},$$

$$X_{NL}^{\tau} = 7.47\times 10^{-4}, \tag{5}$$

with the central values of the QCD scale $\Lambda = \Lambda_{\overline{MS}}^{(4)} = 325\pm 80$ MeV, and the charm quark mass $m_c = \overline{m}_c(m_c) = 1.30\pm 0.05$ GeV.

From the effective Hamiltonian in Eq. (1), we see that, to find the decay rate, we have to evaluate the hadronic matrix element $\langle\gamma|J_\mu|K^0\rangle$, where $J_\mu = \overline{d}\gamma_\mu(1-\gamma_5)s$. The element can be parametrized as

$$\langle\gamma(q)|\overline{d}\gamma^\mu\gamma_5 s|K^0(p+q)\rangle = -e\frac{F_A}{M_K}[\epsilon^{*\mu}(p\cdot q) - (\epsilon^*\cdot p)q^\mu],$$

$$\langle\gamma(q)|\overline{d}\gamma^\mu s|K^0(p+q)\rangle = -ie\frac{F_V}{M_K}\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p_\beta q_\gamma, \tag{6}$$

where q and $p+q$ are photon and K-meson four momenta, F_A and F_V are form factors of the axial vector and vector, respectively, and ϵ is the photon polarization vector.

The form factors of F_A and F_V in Eq. (6) can be calculated in the light front quark model at timelike momentum transfers in which the physically accessible kinematic region is $0\leqslant p^2\leqslant p^2_{\max}$; they are found to be [20–22]

$$F_A(p^2) = -4M_K\int\frac{dx'\,d^2k_\perp}{2(2\pi)^3}\Phi(x,k_\perp^2)\frac{1}{1-x}\left\{\frac{1}{3}\frac{-m_s+Bk_\perp^2\Theta}{m_s^2+k_\perp^2}-\frac{2}{3}\frac{m_d-Ak_\perp^2\Theta}{m_d^2+k_\perp^2}\right\}, \tag{7}$$

$$F_V(p^2) = 4M_K\int\frac{dx'\,d^2k_\perp}{2(2\pi)^3}\Phi(x,k_\perp^2)\frac{1}{1-x}\left\{\frac{1}{3}\frac{-m_s-(1-x)(m_s-m_d)k_\perp^2\Theta}{m_s^2+k_\perp^2} -\frac{2}{3}\frac{m_d-x(m_s-m_d)k_\perp^2\Theta}{m_d^2+k_\perp^2}\right\}, \tag{8}$$

where

$$A = (1-2x')x(m_s-m_d)-2x'm_d,$$

$$B = [(1-2x')x-1]m_s+(1-2x')(1-x)m_d,$$

$$\Phi(x,k_\perp^2) = N\left(\frac{2x(1-x)}{M_0^2-(m_d-m_s)^2}\right)^{1/2}\sqrt{\frac{dk_z}{dx}}\times\exp\left(-\frac{\vec{k}^2}{2\omega_K^2}\right), \tag{9}$$

$$\Theta = \frac{1}{\Phi(x,k_\perp^2)}\frac{d\Phi(x,k_\perp^2)}{dk_\perp^2},$$

$$x = x'\left(1-\frac{p^2}{M_K^2}\right), \quad \vec{k} = (\vec{k}_\perp,\vec{k}_z),$$

with

$$N = 4\left(\frac{\pi}{\omega_K^2}\right)^{3/4},$$

$$k_z = \left(x-\frac{1}{2}\right)M_0+\frac{m_s^2-m_d^2}{2M_0}, \tag{10}$$

$$M_0^2 = \frac{k_\perp^2+m_d^2}{x}+\frac{k_\perp^2+m_s^2}{1-x}.$$

ω_K is chosen to be 0.34 GeV, fixed by the decay constant of $f_K = 160$ MeV.

To illustrate the form factors, we input the values of $m_d = 0.3$, $m_s = 0.4$, and $M_K = 0.5$ in GeV to integrate the whole range of p^2. It is interesting to note that at $p^2 = 0$, we obtain $[F_A(0), F_V(0)] = (0.0429, 0.0915)$, compared with $(0.0425, 0.0945)$ found in the chiral perturbation theory at the one-loop level [23].

III. DECAY BRANCHING RATIOS

From the effective Hamiltonian for $K^0 \to \gamma\nu\bar{\nu}$ in Eq. (1), and the form factors defined in Eq. (6), we can write the amplitude of $K^0 \to \gamma\nu\bar{\nu}$ as

$$M(K^0 \to \gamma\nu\bar{\nu}) = i\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\sum_{l=e,\mu,\tau}[\lambda_c X_{NL}^l + \lambda_t X(x_t)] \times \epsilon_\mu^* H^{\mu\nu}\overline{u}(p_{\bar{\nu}})\gamma_\nu(1-\gamma_5)v(p_\nu), \tag{11}$$

with

$$H_{\mu\nu} = \frac{F_A}{M_K}(-p'q\, g_{\mu\nu} + p'_\mu q_\nu) + i\epsilon_{\mu\nu\alpha\beta}\frac{F_V}{M_K}q^\alpha p'^\beta, \tag{12}$$

where p' is the 4 momentum of K^0, and the form factors $F_{A,V}$ are given by Eqs. (7) and (8), respectively. Since $K_L \simeq K_2 = (K^0 - \bar{K}^0)/\sqrt{2}$, we may write

$$\mathcal{M}(K_L \to \gamma\nu\bar{\nu}) = \mathcal{M}_{CPC} + \mathcal{M}_{CPV}, \tag{13}$$

where $\mathcal{M}_{CPC}$ and $\mathcal{M}_{CPV}$ are the amplitudes corresponding to CP-conserving and -violating contributions, respectively, which are given by

$$\mathcal{M}_{CPC} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\frac{2}{\sqrt{2}}\sum_{l=e,\mu,\tau}[\mathrm{Re}\,\lambda_c X_{NL}^l + \mathrm{Re}\,\lambda_t X(x_t)]\epsilon^{\mu\nu\alpha\beta}\frac{F_V}{M_K}\epsilon_\mu^* q_\alpha p'_\beta \overline{u}(p_{\bar{\nu}})\gamma_\nu \times (1-\gamma_5)v(p_\nu) \tag{14}$$

and

$$\mathcal{M}_{CPV} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\frac{6}{\sqrt{2}}\mathrm{Im}\,\lambda_t X(x_t)\frac{F_A}{M_K}\epsilon_\mu^* \times (-p'\cdot q\, g^{\mu\nu} + p'^\mu q^\nu)\overline{u}(p_{\bar{\nu}})\gamma_\nu(1-\gamma_5)v(p_\nu). \tag{15}$$

Here we have neglected the imaginary part of $\mathrm{Im}\,\lambda_c$ for $\mathcal{M}_{CPV}$.

To evaluate the branching ratio, one needs to replace p^2 into (p', q). In the physically allowed region of $K_L \to \gamma\nu\bar{\nu}$, one has

$$0 \leqslant p^2 \leqslant M_K^2. \tag{16}$$

In the K_L rest frame, the partial decay rate of $K_L \to \gamma\nu\bar{\nu}$ is given by

$$d^2\Gamma = \frac{1}{(2\pi)^3}\frac{1}{8M_K}|\mathcal{M}|^2 dE_\gamma dE_\nu, \tag{17}$$

where we have used two variables to describe the kinematic of the decay. For convention, we define $x_\gamma = 2E_\gamma/M_K$ and $x_\nu = 2E_\nu/M_K$ as the normalized energies of the photon and neutrino, respectively, and we have the form

$$p^2 = M_K^2(1 - x_\gamma). \tag{18}$$

The differential decay rate is then given by

$$\frac{d^2\Gamma}{dx_\gamma dx_\nu} = \frac{M_K}{256\pi^3}|\mathcal{M}|^2. \tag{19}$$

By integrating the variable x_ν, we obtain

$$\frac{d\Gamma}{dx_\gamma} = \frac{d\Gamma_{CPC}}{dx_\gamma} + \frac{d\Gamma_{CPV}}{dx_\gamma}, \tag{20}$$

where

$$\frac{d\Gamma_{CPC}}{dx_\gamma} = \frac{4\alpha}{3}\left(\frac{G_F\alpha}{16\pi^2\sin^2\theta_W}\right)^2\sum_{l=e,\mu,\tau}[\mathrm{Re}\,\lambda_c X_{NL}^l + \mathrm{Re}\,\lambda_t X(x_t)]^2|F_V|^2 x_\gamma^3(1-x_\gamma)M_K^5 \tag{21}$$

and

$$\frac{d\Gamma_{CPV}}{dx_\gamma} = 4\alpha\left(\frac{G_F\alpha}{16\pi^2\sin^2\theta_W}\right)^2[\mathrm{Im}\,\lambda_t X(x_t)F_A]^2 x_\gamma^3 \times (1-x_\gamma)M_K^5. \tag{22}$$

To illustrate the numerical results, we use $m_d = 0.3$ GeV, $m_s = 0.4$ GeV, $m_t = 166$ GeV, $m_c = 1.30$ GeV, $M_K = 0.5$ GeV, $\Lambda = 325$ MeV, $\alpha(M_Z) = 1/128$, $\sin^2\theta_W = 0.23$, $\omega = 0.34$, and the CKM parameters [2,24,25] $\lambda = 0.22$, $A = 0.83$, $\rho = 0.13$, and $\eta = 0.34$. The differential decay branching ratios of $dB(K_L \to \gamma\nu\bar{\nu})_{CPC}/dx_\gamma$ and $dB(K_L \to \gamma\nu\bar{\nu})_{CPV}/dx_\gamma$ as a function of $x_\gamma = 2E_\gamma/M_K$ are shown in Figs. 1 and 2, respectively. The decay branching ratios are found to be

$$B(K_L \to \gamma\nu\bar{\nu})_{CPC} = 1.0 \times 10^{-13}, \tag{23}$$

$$B(K_L \to \gamma\nu\bar{\nu})_{CPV} = 1.5 \times 10^{-15}. \tag{24}$$

From Eqs. (23) and (24), we find that the CP-conserving contribution to the decay branching ratio is about a factor of 67 larger than that of the CP-violating one. It is clear that the numerical values in Eqs. (23) and (24) depend on the values of the CKM parameters of ρ and η, respectively. Nevertheless, one could conclude that a measurement of the decay would determine the real part of V_{td}.

We now compare our numerical result for the CP-conserving contribution in Eq. (23) with those in Refs. [17,18]. Our value is about two orders of magnitude and

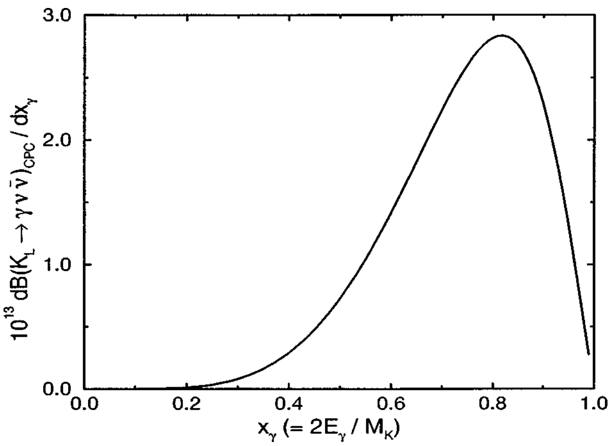

FIG. 1. The differential decay branching ratios $dB(K_L \to \gamma\nu\bar{\nu})_{CPC}/dx_\gamma$ as a function of $x_\gamma = 2E_\gamma/M_K$.

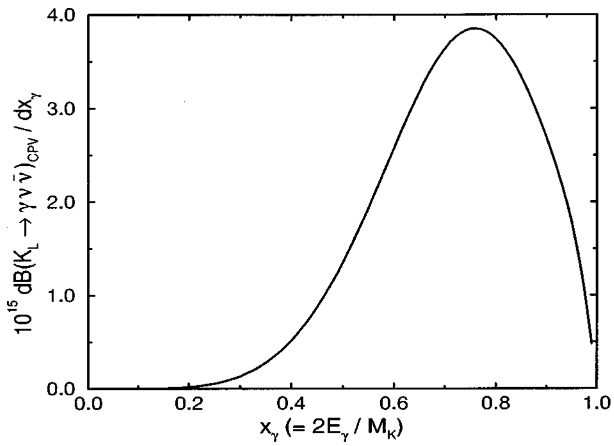

FIG. 2. The differential decay branching ratios $dB(K_L \to \gamma\nu\bar{\nu})_{CPV}/dx_\gamma$ as a function of $x_\gamma = 2E_\gamma/M_K$.

a factor 2 smaller than that in Refs. [17] and [18], respectively. The main reason for the latter difference may be due to the different form factors employed and the uncertainties inherent to such a calculation, whereas that for the former one is unclear. It seems that one needs to restudy the approach in Ref. [17]. Finally, we remark that the ratio between the CP-conserving and -violating branching rates agree with those estimated in Ref. [18].

IV. CONCLUSIONS

We have studied the CP-conserving and -violating contributions to the decay of $K_L \to \gamma\nu\bar{\nu}$ in the standard model. With the form factors for $K \to \gamma$ transitions calculated directly in the entire physical range of momentum transfer within the light front framework, we have shown that the CP-conserving part is much larger than that of the CP-violating parts. We have found that the decay branching ratio is at the level of 10^{-13}, which could be accessible at a future kaon project such as the KAMI at Fermilab [15].

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[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[2] For a review, see W. J. Marciano, summary talk presented at 1999 Chicago Conference on Kaon Physics (Kaon 99), Chicago, Illinois, 1999, hep-ph/9911382.
[3] D. Rein and L. M. Sehgal, Phys. Rev. D **39**, 3325 (1989). J. Hagelin and L. S. Littenberg, Prog. Part. Nucl. Phys. **23**, 1 (1989).
[4] M. Lu and M. B. Wise, Phys. Lett. B **324**, 461 (1994).
[5] C. Q. Geng, I. J. Hsu, and Y. C. Lin, Phys. Lett. B **355**, 569 (1995).
[6] C. Q. Geng, I. J. Hsu, and C. W. Wang, Prog. Theor. Phys. **101**, 937 (1999).
[7] G. Bélanger and C. Q. Geng, Phys. Rev. D **43**, 140 (1991).
[8] A. J. Buras and M. K. Harlander, in *Heavy Flavors*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 58; A. J. Buras, lectures given at the 14th Lake Louise Winter Institute, hep-ph/9905437, and references therein.
[9] E787 Collaboration, S. Adler *et al.*, Phys. Rev. Lett. **79**, 2204 (1997); G. Redlinger, talk presented at 1999 Chicago Conference on Kaon Physics (K 99), Chicago, Illinois, 1999.
[10] S. Kettell, talk presented at the Third International Conference on B physics and CP Violation, Taipei, Taiwan, 1999.
[11] L. Littenberg, Phys. Rev. D **39**, 3322 (1989).
[12] E799-II/KTeV Collaboration, A. Alavi-Harati *et al.*, Phys. Rev. D **61**, 072006 (2000).
[13] Y. B. Hsiung, talk presented at the Third International Conference on B physics and CP Violation, Taipei, Taiwan, 1999.
[14] C. Q. Geng, I. J. Hsu, and Y. C. Lin, Phys. Rev. D **54**, 877 (1996).
[15] Y. B. Hsiung (private communication).
[16] E. Ma and I. Okada, Phys. Rev. D **18**, 4219 (1978).
[17] S. Richardson and C. Picciotto, Phys. Rev. D **52**, 6342 (1995).
[18] W. J. Marciano and Z. Parsa, Phys. Rev. D **53**, R1 (1996).
[19] G. Buchalla and A. J. Buras, Nucl. Phys. **B548**, 309 (1999).
[20] C. Q. Geng, C. C. Lih, and W. M. Zhang, Phys. Rev. D **57**, 5697 (1998).
[21] C. C. Lih, C. Q. Geng, and Wei-Min Zhang, Phys. Rev. D **59**, 114002 (1999).
[22] C. Q. Geng, I. L. He, and I. J. Hsu (unpublished).
[23] J. Bijnens, G. Ecker, and J. Gasser, Nucl. Phys. **B396**, 81 (1993).
[24] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[25] A. Stocchi, talk presented at the Third International Conference on B physics and CP Violation, Taipei, Taiwan, 1999.