Quark-antiquark potential with retardation and radiative contributions and the heavy quarkonium mass spectra

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The charmonium and bottomonium mass spectra are calculated with the systematic account of all relativistic corrections of order v^2/c^2 and the one-loop radiative corrections. Special attention is paid to the contribution of the retardation effects to the spin-independent part of the quark-antiquark potential, and a general approach to accounting for retardation effects in the long-range (confining) part of the potential is presented. A good fit to available experimental data on the mass spectra is obtained.

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I. INTRODUCTION

The investigation of the meson properties in the framework of constituent quark models is an important problem of elementary particle physics. At present a large amount of experimental data on the masses of ground and excited states of heavy and light mesons has been accumulated [1]. By comparing theoretical predictions with experimental data, one can obtain valuable information on the form of the quark-antiquark interaction potential. Such information is of great practical interest since at present it is not possible to obtain the $q\bar{q}$ potential in the whole range of distances from the basic principles of QCD. As is well known, the growing of the strong coupling constant with distance makes perturbation theory inapplicable at large distances (in the infrared region). In this region it is necessary to account for nonperturbative effects connected with the complicated structure of the QCD vacuum. All this leads to a theoretical uncertainty in the $q\bar{q}$ potential at large and intermediate distances. It is just in this region of large and intermediate distances that most of the basic meson characteristics are formed. This makes it possible to investigate the low-energy region of strong interaction by studying the mass spectra and decays of mesons.

Some recent investigations [2–4] have shown that there could be also a linear (in radius) correction to the perturbative Coulomb potential at small distances [in contradiction with operator product expansion (OPE) predictions]. The estimates of the slope yield that it could be of the same order of magnitude as the slope of the long-range confining linear potential. It means then that the widely used Cornell potential (the sum of the Coulomb and linear confining terms) is really a correct one in the static limit both at large and at small distances.

The relativistic properties of the quark-antiquark interaction potential play an important role in analyzing different static and dynamical characteristics of heavy mesons. The Lorentz structure of the confining quark-antiquark interaction is of particular interest. In the literature there is no consent on this item. For a long time the scalar confining kernel has been considered to be the most appropriate one [5]. The main argument in favor of this choice is based on the nature of the heavy quark spin-orbit potential. The scalar potential gives a vanishing long-range magnetic contribution, which is in agreement with the flux tube picture of quark confinement of Ref. [6], and allows us to get the fine structure for heavy quarkonia in accord with experimental data. However, the calculations of electroweak decay rates of heavy mesons with a scalar confining potential alone yield results which are in worse agreement with data than for a vector potential [7,8]. The radiative $M1$ transitions in quarkonia such as, e.g., $J/\psi \to \eta_c \gamma$ are the most sensitive to the Lorentz structure of the confining potential. The relativistic corrections for these decays arising from vector and scalar potentials have different signs [7,8]. In particular, as it has been shown in Ref. [8], agreement with experiments for these decays can be achieved only for a mixture of vector and scalar potentials. In this context, it is worth remarking, that the recent study of the $q\bar{q}$ interaction in the Wilson loop approach [9] indicates that it cannot be considered as simply a scalar. Moreover, the found structure of spin-independent relativistic corrections is not compatible with a scalar potential. A similar conclusion has been obtained in Ref. [10] on the basis of a Foldy-Wouthuysen reduction of the full Coulomb gauge Hamiltonian of QCD. There, the Lorentz structure of the confinement has been found to be of vector nature. The scalar character of spin splittings in heavy quarkonia in this approach is dynamically generated through the interaction with collective gluonic degrees of freedom. Thus we see that while the spin-dependent structure of $(q\bar{q})$ interaction is well established now, the spin-independent part is still controversial in the literature. The uncertainty in the Lorentz structure of the confining interaction complicates the account for retardation corrections since the relativistic reconstruction of the static confining potential is not unique. In our previous paper [11] we gave some possible prescription of such reconstruction which, in particular, provides the satisfaction of the Barchielli-Brambilla-Prosperi (BBP) relations [12] following from the Lorentz invariance of the Wilson loop. Here we generalize this prescription and discuss its connection with the known quark potentials and the implications for the

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heavy quarkonium mass spectra.

The other important point is the inclusion of radiative corrections in the perturbative part of the quark potential. There have been considerable progress in recent years and now the perturbative QCD corrections to the static potential are known up to two loops [13,14] though for the velocity dependent and spin-dependent potentials only one-loop corrections are calculated [15–17].

The paper is organized as follows. In Sec. II we describe our relativistic quark model. The approach to accounting for retardation effects in the $q\bar{q}$ potential in the general case is presented in Sec. III. The resulting heavy quark potential containing both spin-independent and spin-dependent parts with the account of one-loop radiative corrections is given in Sec. IV. We use this potential for the calculations of the heavy quarkonium mass spectra in Sec. V. Section VI contains our conclusions and discussion of the results.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [18] of the Schrödinger type [19]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{E_aE_b}{E_a+E_b}=\frac{M^4-(m_a^2-m_b^2)^2}{4M^3}, \tag{2}$$

and E_a,E_b are given by

$$E_a=\frac{M^2-m_b^2+m_a^2}{2M},\quad E_b=\frac{M^2-m_a^2+m_b^2}{2M}. \tag{3}$$

Here $M=E_a+E_b$ is the meson mass, $m_{a,b}$ are the masses of light and heavy quarks, and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_a+m_b)^2][M^2-(m_a-m_b)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by [20]

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_a(p)\bar{u}_b(-p)\left\{\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_a^\mu\gamma_b^\nu+V_V(\mathbf{k})\Gamma_a^\mu\Gamma_{b;\mu}+V_S(\mathbf{k})\right\}u_a(q)u_b(-q), \tag{5}$$

where α_S is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k})=-\frac{4\pi}{\mathbf{k}^2},\quad D^{ij}(\mathbf{k})=-\frac{4\pi}{k^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right),$$
$$D^{0i}=D^{i0}=0, \tag{6}$$

and $\mathbf{k}=\mathbf{p}-\mathbf{q}$, γ_μ, and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{7}$$

with $\epsilon(p)=\sqrt{p^2+m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{8}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_V(r)=(1-\varepsilon)Ar+B,$$
$$V_S(r)=\varepsilon Ar, \tag{9}$$

reproducing

$$V_{\rm conf}(r)=V_S(r)+V_V(r)=Ar+B, \tag{10}$$

where ε is the mixing coefficient.

The expression for the quasipotential for the heavy quarkonia, expanded in v^2/c^2 without retardation corrections to the confining potential, can be found in Ref. [20]. The structure of the spin-dependent interaction is in agreement with the parameterization of Eichten and Feinberg [21]. All the parameters of our model, such as quark masses, parameters of the linear confining potential A and B, mixing coefficient ε, and anomalous chromomagnetic quark moment κ are fixed from the analysis of heavy quarkonium masses (see below Sec. V) and radiative decays. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.16$ GeV have usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon=-1$ has been determined from the consideration of the heavy quark expansion for the semileptonic $B\to D$ decays [22] and charmonium radiative decays [8]. Finally, the universal Pauli interaction constant $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J states [20]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa=-1$. In the present paper we will include into consideration the retardation corrections as well as one-loop radiative corrections.

III. GENERAL APPROACH TO ACCOUNTING FOR RETARDATION EFFECTS IN THE $q\bar{q}$ POTENTIAL

For the one-gluon exchange part of the $q\bar{q}$ potential it is quite easy to isolate the retardation contribution. Indeed due to the vector current conservation (gauge invariance) we have the well-known relation on the mass shell

$$\frac{1}{k^2}\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\gamma_a^\mu\gamma_{b\mu}u_a(\mathbf{q})u_b(-\mathbf{q})$$
$$=-\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\left\{\frac{\gamma_a^0\gamma_b^0}{\mathbf{k}^2}+\frac{1}{k^2}\left[\boldsymbol{\gamma}_a\cdot\boldsymbol{\gamma}_b-\frac{(\boldsymbol{\gamma}_a\cdot\mathbf{k})(\boldsymbol{\gamma}_b\cdot\mathbf{k})}{\mathbf{k}^2}\right]\right\}u_a(\mathbf{q})u_b(-\mathbf{q}), \qquad (11)$$

$$k^2=k_0^2-\mathbf{k}^2; \quad k_0=\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})=\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p});$$
$$\mathbf{k}=\mathbf{p}-\mathbf{q}.$$

The left-hand side and the right-hand side of this relation are easily recognized to be in the Feynman gauge and the Coulomb gauge, respectively. Now, if the nonrelativistic expansion in p^2/m^2 is applicable, we can immediately extract the retardation contribution. Namely, we expand the left-hand side of Eq. (11) in $k_0^2/\mathbf{k}^2$:

$$\frac{1}{k_0^2-\mathbf{k}^2}\cong-\frac{1}{\mathbf{k}^2}-\frac{k_0^2}{\mathbf{k}^4}$$

and get with needed accuracy [23]

$$-\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\left[\frac{\gamma_a^0\gamma_b^0}{\mathbf{k}^2}\left(1+\frac{k_0^2}{\mathbf{k}^2}\right)-\frac{\boldsymbol{\gamma}_a\cdot\boldsymbol{\gamma}_b}{\mathbf{k}^2}\right]u_a(\mathbf{q})u_b(-\mathbf{q}). \qquad (12)$$

In the right-hand side of Eq. (11) one should use the identity following from the Dirac equation

$$\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})(\boldsymbol{\gamma}_a\cdot\mathbf{k})(\boldsymbol{\gamma}_b\cdot\mathbf{k})u_a(\mathbf{q})u_b(-\mathbf{q})$$
$$=\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\gamma_a^0\gamma_b^0u_a(\mathbf{q})u_b(-\mathbf{q})$$
$$\times[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})][\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})].$$

After defining k_0^2 as a symmetrized product [23,24]

$$k_0^2=[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})][\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})] \qquad (13)$$

and dropping k_0^2 in the denominator we obtain the expression which is identical to Eq. (12). In this way we obtain the well-known Breit Hamiltonian (the same as in QED [23]) if we further expand Eq. (13) in p^2/m^2

$$k_0^2\cong-\frac{(\mathbf{p}^2-\mathbf{q}^2)^2}{4m_am_b}. \qquad (14)$$

This treatment allows also for the correct Dirac limit in which the retardation contribution vanishes when one of the particles becomes infinitely heavy [25].

For the confining part of the $q\bar{q}$ potential the retardation contribution is much more indefinite. This is a consequence of our poor knowledge of the confining potential especially concerning its relativistic properties: the Lorentz structure (scalar, vector, etc.) and the dependence on the covariant variables such as $k^2=k_0^2-\mathbf{k}^2$. Nevertheless we can perform some general considerations and then apply them to a particular case of the linearly rising potential. To this end we note that for any nonrelativistic potential $V(-\mathbf{k}^2)$ the simplest relativistic generalization is to replace it by $V(k_0^2-\mathbf{k}^2)$.

In the case of the Lorentz-vector confining potential we can use the same approach as before even with more general vertices containing the Pauli terms, since the mass-shell vector currents are conserved here as well. It is possible to introduce alongside with the "diagonal gauge" the so-called "instantaneous gauge" [26] which is the generalization of the Coulomb gauge. The relation analogous to Eq. (11) now looks as follows (up to the terms of order of p^2/m^2):

$$V_V(k_0^2-\mathbf{k}^2)\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\Gamma_a^\mu\Gamma_{b\mu}u_a(\mathbf{q})u_b(-\mathbf{q})$$
$$=\bar{u}_a(\mathbf{p})\bar{u}_b(-\mathbf{p})\{V_V(-\mathbf{k}^2)\Gamma_a^0\Gamma_b^0-[V_V(-\mathbf{k}^2)\boldsymbol{\Gamma}_a\cdot\boldsymbol{\Gamma}_b$$
$$+V_V'(-\mathbf{k}^2)(\boldsymbol{\Gamma}_a\cdot\mathbf{k})(\boldsymbol{\Gamma}_b\cdot\mathbf{k})]\}u_a(\mathbf{q})u_b(-\mathbf{q}), \qquad (15)$$

where

$$V_V(k_0^2-\mathbf{k}^2)\cong V_V(-\mathbf{k}^2)+k_0^2V_V'(-\mathbf{k}^2)$$

and as in the case of the one-gluon exchange above we put

$$k_0^2=[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})][\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})]\cong-\frac{(\mathbf{p}^2-\mathbf{q}^2)^2}{4m_am_b} \qquad (16)$$

again with the correct Dirac limit.

For the case of the Lorentz-scalar potential we can make the same expansion in k_0^2, which yields

$$V_S(k_0^2-\mathbf{k}^2)\cong V_S(-\mathbf{k}^2)+k_0^2V_S'(-\mathbf{k}^2). \qquad (17)$$

But in this case we have no reasons to fix k_0^2 in the only way (13). The other possibility is to take a half sum instead of a symmetrized product, namely, to set (see, e.g., Refs. [24,25])

$$k_0^2=\frac{1}{2}\{[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})]^2+[\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})]^2\}$$
$$\cong\frac{1}{8}(\mathbf{p}^2-\mathbf{q})^2\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right). \quad (18)$$

The Dirac limit is not fulfilled by this choice, but this cannot serve as a decisive argument. Thus the most general expression for the energy transfer squared, which incorporates both possibilities (16) and (18) has the form

$$k_0^2=\lambda[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})][\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})]+(1-\lambda)\frac{1}{2}$$
$$\times\{[\epsilon_a(\mathbf{p})-\epsilon_a(\mathbf{q})]^2+[\epsilon_b(\mathbf{q})-\epsilon_b(\mathbf{p})]^2\}, \quad (19)$$

where λ is the mixing parameter.

After making expansion in p^2/m^2 we obtain

$$k_0^2\cong-\lambda\frac{(\mathbf{p}^2-\mathbf{q}^2)^2}{4m_am_b}+(1-\lambda)\frac{1}{8}(\mathbf{p}^2-\mathbf{q})^2\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)$$
$$=\frac{1}{8}\left[(1-\lambda)\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)-\frac{2\lambda}{m_am_b}\right]$$
$$\times[(\mathbf{k}\cdot\mathbf{p})^2+2(\mathbf{k}\cdot\mathbf{p})(\mathbf{k}\cdot\mathbf{q})+(\mathbf{k}\cdot\mathbf{q})^2]. \quad (20)$$

Thus as expected $k_0^2\sim O(p^2/m^2)\ll 1$. Then the Fourier transform of the potential

$$V(k_0^2-\mathbf{k}^2)\cong V(-\mathbf{k}^2)+k_0^2V'(-\mathbf{k})^2$$

with k_0^2 given by Eq. (20) can be represented as follows [25]:

$$\int\frac{d^3k}{(2\pi)^3}V(k_0^2-\mathbf{k}^2)e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$=V(r)+\frac{1}{4}\left[(1-\lambda)\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)-\frac{2\lambda}{m_am_b}\right]$$
$$\times\left\{V(r)\mathbf{p}^2+V'(r)\frac{1}{r}(\mathbf{p}\cdot\mathbf{r})^2\right\}_W, \quad (21)$$

where $\{\dots\}_W$ denotes the Weyl ordering of operators and

$$V(r)=\int\frac{d^3k}{(2\pi)^3}V(-\mathbf{k}^2)e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (22)$$

In the case of the one-gluon exchange potential we had $\lambda=1$,

$$V_C(-\mathbf{k}^2)=-\frac{4}{3}\frac{4\pi\alpha_s}{\mathbf{k}^2},\quad V_C(r)=-\frac{4}{3}\frac{\alpha_s}{r}. \quad (23)$$

As for the confining potential we assume it to be a mixture of scalar and vector parts. In the nonrelativistic limit we adopt the linearly rising potential

$$V_0(r)=Ar,\quad V_0(-\mathbf{k}^2)=-\frac{8\pi A}{(\mathbf{k}^2)^2}, \quad (24)$$

which we split into scalar and vector parts by introducing the mixing parameter ε. The possible constant term in V_0 has been discussed in Ref. [11]:

$$V_0=V_S+V_V,\quad V_S=\varepsilon V_0,\quad V_V=(1-\varepsilon)V_0. \quad (25)$$

Hence the retardation contribution (21) from scalar and vector potentials has the form

$$\frac{1}{4}\left[(1-\lambda_{S,V})\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)-\frac{2\lambda_{S,V}}{m_am_b}\right]$$
$$\times\left\{V_{S,V}(r)\mathbf{p}^2+V'_{S,V}(r)\frac{1}{r}(\mathbf{p}\cdot\mathbf{r})^2\right\}_W, \quad (26)$$

where we use the general ansatz (19), (20) for both the scalar and vector potentials for the sake of completeness.

The other spin-independent corrections in our model had been calculated earlier [20,11]:

$$\frac{1}{8}(1+2\kappa)\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\Delta V_V(r)+\frac{1}{m_am_b}\{V_V(r)\mathbf{p}^2\}_W$$
$$-\frac{1}{2}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\{V_S(r)\mathbf{p}^2\}_W. \quad (27)$$

Adding to the above expression the retardation contributions (26) and the nonrelativistic parts (23) and (25) we obtain the complete spin-independent $q\bar{q}$ potential

$$V_{\rm SI}(r)=V_C(r)+V_0(r)+V_{\rm VD}(r)+\frac{1}{8}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)$$
$$\times\Delta[V_C(r)+(1+2\kappa)V_V], \quad (28)$$

where the velocity-dependent part

$$V_{\rm VD}(r)=V_{\rm VD}^C(r)+V_{\rm VD}^V(r)+V_{\rm VD}^S(r), \quad (29)$$

$$V_{\rm VD}^C(r)=\frac{1}{2m_am_b}\left\{V_C(r)\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W$$
$$=\frac{1}{2m_am_b}\left\{-\frac{4}{3}\frac{\alpha_s}{r}\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W,$$

$$V_{\rm VD}^{V}(r)=\frac{1}{m_a m_b}\{V_V(r)\mathbf{p}^2\}_W+\frac{1}{4}\left[(1-\lambda_V)\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)-\frac{2\lambda_V}{m_a m_b}\right]\left\{V_V(r)\mathbf{p}^2+V_V'(r)\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r}\right\}_W$$
$$=(1-\varepsilon)\frac{(1-\lambda_V)}{4}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\times\left\{Ar\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W+\frac{(1-\varepsilon)}{m_a m_b}\left\{Ar\left[\left(1-\frac{\lambda_V}{2}\right)\mathbf{p}^2-\frac{\lambda_V}{2}\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W,$$

$$V_{\rm VD}^{S}(r)=\frac{1}{2}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\{V_V(r)\mathbf{p}^2\}_W+\frac{1}{4}\times\left[(1-\lambda_S)\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)-\frac{2\lambda_S}{m_a m_b}\right]\times\left\{V_V(r)\mathbf{p}^2+V_V'(r)\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r}\right\}_W$$
$$=-\frac{\varepsilon}{4}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\left\{Ar\left[(1+\lambda_S)\mathbf{p}^2+(\lambda_S-1)\times\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W-\frac{\varepsilon\lambda_S}{2m_a m_b}\left\{Ar\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W.$$

Making the natural decomposition

$$V_{\rm VD}(r)=\frac{1}{m_a m_b}\left\{\mathbf{p}^2V_{bc}(r)+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}V_c(r)\right\}_W+\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\times\left\{\mathbf{p}^2V_{de}(r)-\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}V_e(r)\right\}_W \tag{30}$$

we obtain, from Eqs. (29),

$$V_{bc}(r)=-\frac{2\alpha_s}{3r}+\left[(1-\varepsilon)\left(1-\frac{\lambda_V}{2}\right)-\varepsilon\frac{\lambda_S}{2}\right]Ar,$$
$$V_c(r)=-\frac{2\alpha_s}{3r}-\left[(1-\varepsilon)\frac{\lambda_V}{2}+\varepsilon\frac{\lambda_S}{2}\right]Ar,$$
$$V_{de}(r)=\frac{1}{4}[(1-\varepsilon)(1-\lambda_V)-\varepsilon(1+\lambda_S)]Ar,$$
$$V_e(r)=-\frac{1}{4}[(1-\varepsilon)(1-\lambda_V)+\varepsilon(1-\lambda_S)]Ar. \tag{31}$$

The following simple relations hold:

$$V_{bc}-V_c=(1-\varepsilon)Ar,\quad V_{de}+V_e=-\frac{\varepsilon}{2}Ar. \tag{32}$$

The exact BBP relations [12] (see also Ref. [27]) in our notations look as follows:

$$V_{de}-\frac{1}{2}V_{bc}+\frac{1}{4}(V_C+V_0)=0,$$
$$V_e+\frac{1}{2}V_c+\frac{r}{4}\frac{{\rm d}(V_C+V_0)}{{\rm d}r}=0 \tag{33}$$

(in the original version $V_{bc}\equiv -V_b-\frac{1}{3}V_c$ and $V_{de}\equiv V_d+\frac{1}{3}V_e$). The functions (31) identically satisfy the BBP relations (33) independently of values of the parameters ε, λ_V, λ_S but only with the account of retardation corrections.

In our model [20,11] we have $\varepsilon=-1$ and $\lambda_V=1$, if we assume further that $\lambda_S=1$ [11] then we get

$$V_{bc}(r)=-\frac{2\alpha_s}{3r}+\frac{3}{2}Ar,\quad V_c(r)=-\frac{2\alpha_s}{3r}-\frac{1}{2}Ar,$$
$$V_{de}(r)=\frac{1}{2}Ar,\quad V_e(r)=0. \tag{34}$$

Our expressions (28) and (29) for purely vector ($\varepsilon=0$) and purely scalar ($\varepsilon=1$) interactions and for $\kappa=0$, $\lambda_S=\lambda_V=1$ coincide with those of Ref. [25].

In the minimal area low (MAL) and flux tube models [28]

$$V_{bc}(r)=-\frac{2\alpha_s}{3r}+\frac{1}{6}Ar,\quad V_c(r)=-\frac{2\alpha_s}{3r}-\frac{1}{6}Ar,$$
$$V_{de}(r)=-\frac{1}{6}Ar,\quad V_e=-\frac{1}{6}Ar. \tag{35}$$

To obtain these expressions one should set in relations (31), (32)

$$\varepsilon=\frac{2}{3},\quad \lambda_V+2\lambda_S=1. \tag{36}$$

Thus one gets a family of values for λ_V and λ_S. The most natural choice reads

$$\lambda_V=1,\quad \lambda_S=0, \tag{37}$$

which resembles the Gromes proposal [24]: the symmetrized product for the vector potential and the half sum for the scalar potential. But still the Dirac limit is not satisfied in this case.

Expression (28) for $V_{\rm SI}$ contains also the term with the Laplacian:

$$\frac{1}{8}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\Delta[V_C(r)+(1+2\kappa)V_V(r)]. \tag{38}$$

In the MAL and some other models these terms take the form [28]

$$\frac{1}{8}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\Delta[V_C(r)+V_0(r)+V_a(r)] \qquad (39)$$

and usually it is adopted that

$$\Delta V_a(r)=0. \qquad (40)$$

Lattice simulations [29] suggest that

$$\Delta V_a^L(r)=c-\frac{b}{r}, \quad b\cong 0.8\ \text{GeV}^2. \qquad (41)$$

In our model expression (38) can be recast as follows:

$$\frac{1}{8}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\Delta[V_C(r)+V_0(r)+\tilde{V}_a(r)],$$

$$\tilde{V}_a(r)=(1+2\kappa)(1-\varepsilon)V_0(r)-V_0(r) \qquad (42)$$

and for the adopted values $\varepsilon=-1$, $\kappa=-1$

$$\Delta\tilde{V}_a(r)=-3\Delta(Ar)=-6\frac{A}{r}, \quad 6A\cong 1.1\ \text{GeV}^2, \qquad (43)$$

which is close to the lattice result (41) but differs from the suggestion (40).

IV. HEAVY QUARK-ANTIQUARK POTENTIAL WITH THE ACCOUNT OF RETARDATION EFFECTS AND ONE LOOP RADIATIVE CORRECTIONS

At present the static quark-antiquark potential in QCD is known to two loops [13,14]. However the velocity-dependent and spin-dependent parts are known only to the one-loop order [15,16]. Thus we limit our analysis to one-loop radiative corrections. The resulting heavy quark-antiquark potential can be presented in the form of a sum of spin-independent and spin-dependent parts. For the spin-independent part using the relations (28), (29) with $\lambda_V=1$ and including one-loop radiative corrections in modified minimal subtraction ($\overline{\text{MS}}$) renormalization scheme [15,16] we get

$$V_{\text{SI}}(r)=-\frac{4}{3}\frac{\bar{\alpha}_V(\mu^2)}{r}+Ar+B-\frac{4}{3}\frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\frac{\ln(\mu r)}{r}+\frac{1}{8}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\Delta\Bigg[-\frac{4}{3}\frac{\bar{\alpha}_V(\mu^2)}{r}$$
$$-\frac{4}{3}\frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\frac{\ln(\mu r)}{r}+(1-\varepsilon)(1+2\kappa)Ar\Bigg]+\frac{1}{2m_am_b}\left(\left\{-\frac{4}{3}\frac{\bar{\alpha}_V}{r}\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W\right.$$
$$\left.-\frac{4}{3}\frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\left\{\mathbf{p}^2\frac{\ln(\mu r)}{r}+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\left(\frac{\ln(\mu r)}{r}-\frac{1}{r}\right)\right\}_W\right)+\left[\frac{1-\varepsilon}{2m_am_b}-\frac{\varepsilon}{4}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)\right]$$
$$\times\left\{Ar\left[\mathbf{p}^2-\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W-\frac{\varepsilon\lambda_S}{2}\left[\frac{1}{2}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)+\frac{1}{m_am_b}\right]\left\{Ar\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W+\left[\frac{1}{4}\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}\right)+\frac{1}{m_am_b}\right]B\mathbf{p}^2, \qquad (44)$$

where

$$\bar{\alpha}_V(\mu^2)=\alpha_s(\mu^2)\left[1+\left(\frac{a_1}{4}+\frac{\gamma_E\beta_0}{2}\right)\frac{\alpha_s(\mu^2)}{\pi}\right], \qquad (45)$$

$$a_1=\frac{31}{3}-\frac{10}{9}n_f,$$

$$\beta_0=11-\frac{2}{3}n_f.$$

Here n_f is a number of flavors and μ is a renormalization scale.

For the dependence of the QCD coupling constant $\alpha_s(\mu^2)$ on the renormalization point μ^2 we use the leading order result

$$\alpha_s(\mu^2)=\frac{4\pi}{\beta_0\ln(\mu^2/\Lambda^2)}. \qquad (46)$$

Comparing this expression for V_{SI} with the decomposition (30) we find

$$V_{bc}(r)=-\frac{2}{3}\frac{\bar{\alpha}_V(\mu^2)}{r}-\frac{2}{3}\frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\frac{\ln(\mu r)}{r}$$
$$+\left(\frac{1-\varepsilon}{2}-\frac{\varepsilon\lambda_S}{2}\right)Ar+B,$$

$$V_c(r)=-\frac{2}{3}\frac{\bar{\alpha}_V(\mu^2)}{r}-\frac{2}{3}\frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\left[\frac{\ln(\mu r)}{r}-\frac{1}{r}\right]-\left(\frac{1-\varepsilon}{2}+\frac{\varepsilon\lambda_S}{2}\right)Ar,$$

$$V_{de}(r)=-\frac{\varepsilon}{4}(1+\lambda_S)Ar+B,$$

$$V_e(r)=-\frac{\varepsilon}{4}(1-\lambda_S)Ar. \tag{47}$$

It is easy to check that the BBP relations are exactly satisfied.

The spin-dependent part of the quark-antiquark potential for equal quark masses ($m_a=m_b=m$) with the inclusion of radiative corrections [15,17] can be presented in our model [20] as follows:

$$V_{\rm SD}=a\mathbf{L}\cdot\mathbf{S}+b\left[\frac{3}{r^2}(\mathbf{S}_a\cdot\mathbf{r})(\mathbf{S}_b\cdot\mathbf{r})-(\mathbf{S}_a\cdot\mathbf{S}_b)\right]+c\mathbf{S}_a\cdot\mathbf{S}_b, \tag{48}$$

$$a=\frac{1}{2m^2}\left\{\frac{4\alpha_s(\mu^2)}{r^3}\left(1+\frac{\alpha_s(\mu^2)}{\pi}\left[\frac{1}{18}n_f-\frac{1}{36}+\gamma_E\left(\frac{\beta_0}{2}-2\right)+\frac{\beta_0}{2}\ln\frac{\mu}{m}+\left(\frac{\beta_0}{2}-2\right)\ln(mr)\right]\right)-\frac{A}{r}+4(1+\kappa)(1-\varepsilon)\frac{A}{r}\right\}, \tag{49}$$

$$b=\frac{1}{3m^2}\left\{\frac{4\alpha_s(\mu^2)}{r^3}\left(1+\frac{\alpha_s(\mu^2)}{\pi}\left[\frac{1}{6}n_f+\frac{25}{12}+\gamma_E\left(\frac{\beta_0}{2}-3\right)+\frac{\beta_0}{2}\ln\frac{\mu}{m}+\left(\frac{\beta_0}{2}-3\right)\ln(mr)\right]\right)+(1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right\}, \tag{50}$$

$$c=\frac{4}{3m^2}\left\{\frac{8\pi\alpha_s(\mu^2)}{3}\left(\left[1+\frac{\alpha_s(\mu^2)}{\pi}\left(\frac{23}{12}-\frac{5}{18}n_f-\frac{3}{4}\ln 2\right)\right]\times\delta^3(r)+\frac{\alpha_s(\mu^2)}{\pi}\left[-\frac{\beta_0}{8\pi}\nabla^2\left(\frac{\ln(\mu/m)}{r}\right)+\frac{1}{\pi}\left(\frac{1}{12}n_f-\frac{1}{16}\right)\nabla^2\left(\frac{\ln(mr)+\gamma_E}{r}\right)\right]\right)+(1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right\}, \tag{51}$$

where $\mathbf{L}$ is the orbital momentum and $\mathbf{S}_{a,b}$, $\mathbf{S}=\mathbf{S}_a+\mathbf{S}_b$ are the spin momenta.

The correct description of the fine structure of the heavy quarkonium mass spectrum requires the vanishing of the vector confinement contribution. This can be achieved by setting $1+\kappa=0$, i.e., the total long-range quark chromomagnetic moment equals zero, which is in accord with the flux tube [6] and minimal area [30,28] models. One can see from Eq. (48) that for the spin-dependent part of the potential this conjecture is equivalent to the assumption about the scalar structure of confinement interaction [5].

V. HEAVY QUARKONIUM MASS SPECTRA

Now we can calculate the mass spectra of heavy quarkonia with the account of all relativistic corrections (including retardation effects) of order v^2/c^2 and one-loop radiative corrections. For this purpose we substitute the quasipotential which is a sum of the spin-independent (44) and spin-dependent (48) parts into the quasipotential equation (1). Then we multiply the resulting expression from the left by the quasipotential wave function of a bound state and integrate with respect to the relative momentum. Taking into account the accuracy of the calculations, we can use for the resulting matrix elements the wave functions of Eq. (1) with the static potential[1]

$$V_{\rm NR}(r)=-\frac{4}{3}\frac{\bar{\alpha}_V(\mu^2)}{r}+Ar+B. \tag{52}$$

As a result we obtain the mass formula ($m_a=m_b=m$)

$$\frac{b^2(M)}{2\mu_R}=W+\langle a\rangle\langle\mathbf{L}\cdot\mathbf{S}\rangle+\langle b\rangle\left\langle\left[\frac{3}{r^2}(\mathbf{S}_a\cdot\mathbf{r})(\mathbf{S}_b\cdot\mathbf{r})-(\mathbf{S}_a\cdot\mathbf{S}_b)\right]\right\rangle+\langle c\rangle\langle\mathbf{S}_a\cdot\mathbf{S}_b\rangle, \tag{53}$$

where

$$W=\langle V_{\rm SI}\rangle+\frac{\langle\mathbf{p}^2\rangle}{2\mu_R},$$

$$\langle\mathbf{L}\cdot\mathbf{S}\rangle=\frac{1}{2}[J(J+1)-L(L+1)-S(S+1)],$$

$$\left\langle\left[\frac{3}{r^2}(\mathbf{S}_a\cdot\mathbf{r})(\mathbf{S}_b\cdot\mathbf{r})-(\mathbf{S}_a\cdot\mathbf{S}_b)\right]\right\rangle=-\frac{6(\langle\mathbf{L}\cdot\mathbf{S}\rangle)^2+3\langle\mathbf{L}\cdot\mathbf{S}\rangle-2S(S+1)L(L+1)}{2(2L-1)(2L+3)},$$

$$\langle\mathbf{S}_a\cdot\mathbf{S}_b\rangle=\frac{1}{2}\left(S(S+1)-\frac{3}{2}\right),\quad \mathbf{S}=\mathbf{S}_a+\mathbf{S}_b,$$

and $\langle a\rangle$, $\langle b\rangle$, $\langle c\rangle$ are the appropriate averages over radial wave functions of Eqs. (49)–(51). We use the usual notations

[1]This static potential includes also some radiative corrections [16]. The remaining radiative correction term with logarithm in Eq. (44), also not vanishing in the static limit, is treated perturbatively.

for heavy quarkonia classification: $n^{2S+1}L_J$, where n is a radial quantum number, L is the angular momentum, $S=0,1$ is the total spin, and $J=L-S,L,L+S$ is the total angular momentum ($\mathbf{J}=\mathbf{L}+\mathbf{S}$). The first term on the right-hand side of the mass formula (53) contains all spin-independent contributions, the second term describes the spin-orbit interaction, the third term is responsible for the tensor interaction, while the last term gives the spin-spin interaction.

To proceed further we need to discuss the parameters of our model. There is the following set of parameters: the quark masses (m_b and m_c), the QCD constant Λ and renormalization point μ [see Eqs. (46), (44), (48)] in the short-range part of the $Q\bar{Q}$ potential, the slope A and intercept B of the linear confining potential (10), the mixing coefficient ε (9), the long-range anomalous chromomagnetic moment κ of the quark (8), and the mixing parameter λ_S in the retardation correction for the scalar confining potential (26). As was already discussed in Sec. II, we can fix the values of the parameters $\varepsilon=-1$ and $\kappa=-1$ from the consideration of radiative decays [8] and comparison of the heavy quark expansion in our model [22,33] with the predictions of the heavy quark effective theory. We fix the slope of the linear confining potential $A=0.18$ GeV^2 which is a rather adopted value. In order to reduce the number of independent parameters we assume that the renormalization scale μ in the strong coupling constant $\alpha_s(\mu^2)$ is equal to the quark mass.[2] We also varied the quark masses in a reasonable range for the constituent quark masses. The numerical analysis and comparison with experimental data lead to the following values of our model parameters:

$$m_c=1.55\ \text{GeV},\quad m_b=4.88\ \text{GeV},\quad \Lambda=0.178\ \text{GeV},$$

$$A=0.18\ \text{GeV}^2,\quad B=-0.16\ \text{GeV},\quad \mu=m_Q\quad (Q=c,b),$$

$$\varepsilon=-1,\quad \kappa=-1,\quad \lambda_S=0.$$

The quark masses $m_{c,b}$ have usual values for constituent quark models and coincide with those chosen in our previous analysis [20] (see Sec. II). The above value of the retardation parameter λ_S for the scalar confining potential coincides with the minimal area low and flux tube models [28], with lattice results [29] and Gromes suggestion [24]. The found value for the QCD parameter Λ gives the following values for the strong coupling constants $\alpha_s(m_c^2)\approx 0.32$ and $\alpha_s(m_b^2)\approx 0.22$.

The results of our numerical calculations of the mass spectra of charmonium and bottomonium are presented in Tables I and II. We see that the calculated masses agree with experimental values within few MeV and this difference is compatible with the estimates of the higher order corrections in v^2/c^2 and α_s. The model reproduces correctly both the positions of the centers of gravity of the levels and their fine and hyperfine splitting. Note that the good agreement of the calculated mass spectra with experimental data is achieved by systematic accounting for all relativistic corrections (including retardation corrections) of order v^2/c^2, both spin-dependent and spin-independent ones, while in most of the potential models only the spin-dependent corrections are included.

TABLE I. Charmonium mass spectrum.

State ($n^{(2S+1)}L_J$)	Particle	Theory	Experiment [1]	Experiment [31]
1^1S_0	η_c	2.979	2.9798	2.9758
1^3S_1	J/Ψ	3.096	3.09688	
1^3P_0	χ_{c0}	3.424	3.4173	3.4141
1^3P_1	χ_{c1}	3.510	3.51053	
1^3P_2	χ_{c2}	3.556	3.55617	
2^1S_0	η_c'	3.583	3.594	
2^3S_1	Ψ'	3.686	3.686	
1^3D_1		3.798	3.7699 [a]	
1^3D_2		3.813		
1^3D_3		3.815		
2^3P_0	χ_{c0}'	3.854		
2^3P_1	χ_{c1}'	3.929		
2^3P_2	χ_{c2}'	3.972		
3^1S_0	η_c''	3.991		
3^3S_1	Ψ''	4.088	4.040 [a]	
2^3D_1		4.194	4.159 [a]	
2^3D_2		4.215		
2^3D_3		4.223		

[a]Mixture of S and D states.

The calculated mass spectra of charmonium and bottomonium are close to the results of our previous calculation [20] where retardation effects in the confining potential and radiative corrections to the one-gluon exchange potential were not taken into account. Both calculations give close values for the experimentally measured states as well as for the yet unobserved ones. The inclusion of radiative corrections allowed us to get better results for the fine splittings of quarkonium states. Thus we can conclude from this comparison that the inclusion of retardation effects and spin-independent one-loop radiative corrections resulted only in the slight shift ($\approx 10\%$) in the value of the QCD parameter Λ and an approximately twofold decrease of the constant B.[3] Such changes of parameters almost do not influence the wave

[2]Our numerical analysis showed that this is a good approximation, since the variation of μ does not increase considerably the quality of the mass spectrum fit.

[3]Note that in Ref. [20] we included this constant both in vector and scalar parts, while the present analysis indicates that the better fit can be obtained if the constant B is included only in the vector part (9).

TABLE II. Bottomonium mass spectrum.

State ($n^{(2S+1)}L_J$)	Particle	Theory	Experiment [1]	Experiment [32]
1^1S_0	η_b	9.400		
1^3S_1	Υ	9.460	9.46037	
1^3P_0	χ_{b0}	9.864	9.8598	9.8630
1^3P_1	χ_{b1}	9.892	9.8919	9.8945
1^3P_2	χ_{b2}	9.912	9.9132	9.9125
2^1S_0	η_b'	9.990		
2^3S_1	Υ'	10.020	10.023	
1^3D_1		10.151		
1^3D_2		10.157		
1^3D_3		10.160		
2^3P_0	χ_{b0}'	10.232	10.232	
2^3P_1	χ_{b1}'	10.253	10.2552	
2^3P_2	χ_{b2}'	10.267	10.2685	
3^1S_0	η_b''	10.328		
3^3S_1	Υ''	10.355	10.3553	
2^3D_1		10.441		
2^3D_2		10.446		
2^3D_3		10.450		
3^3P_0	χ_{b0}''	10.498		
3^3P_1	χ_{b1}''	10.516		
3^3P_2	χ_{b2}''	10.529		
4^1S_0	η_b'''	10.578		
4^3S_1	Υ'''	10.604	10.580	

functions. As a result the decay matrix elements involving heavy quarkonium states remain mostly unchanged.[4] We plot the reduced radial wave functions $u(r)=rR(r)$ for charmonium and bottomonium in Figs. 1 and 2.

VI. CONCLUSIONS

In this paper we have considered the heavy quarkonium spectroscopy in the framework of the relativistic quark model. Both relativistic corrections of order v^2/c^2 and one-loop radiative corrections to the short-range potential have been included into the calculation. Special attention has been devoted to the role and the structure of retardation corrections to the confining interaction. Our general analysis of the retardation effects has shown that we have a good theoretical motivation to fix the form of retardation contributions to the vector potential in the form (15) which corresponds to the parameter $\lambda_V=1$ in the generalized expression (26). On the contrary, the structure of the retardation contribution to the scalar potential is less restricted from general analysis. This means that it is not possible to fix the value of λ_S in Eq. (26) on general grounds. Our numerical analysis has shown that the value of $\lambda_S=0$ is preferable. Thus for the energy transfer squared we have the symmetrized product (16) for the vector potential and a half sum (18) for the scalar potential, in agreement with lattice calculations [29] and minimal area law and flux tube models [28]. The found structure of the spin-independent interaction (44) with the account of retardation contributions satisfies the BBP [12] relations (33), which follow from the Lorentz invariance of the Wilson loop.

In our calculations we have used the heavy quark-antiquark interaction potential with the complete account of all relativistic corrections of order v^2/c^2 and one-loop radiative corrections both for the spin-independent and spin-dependent parts. The inclusion of these corrections allowed us to fit correctly the position of the centers of gravity of the heavy quarkonium levels as well as their fine and hyperfine splittings. Moreover, the account for radiative corrections re-

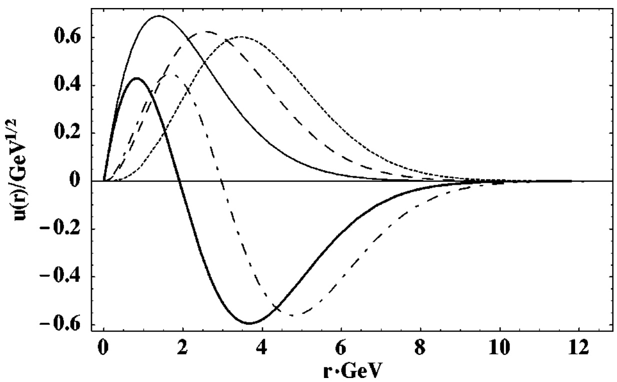

FIG. 1. The reduced radial wave functions for charmonium. The solid line is for $1S$, bold line for $2S$, long-dashed line for $1P$, dashed-dotted line for $2P$, and dotted line for $1D$ states.

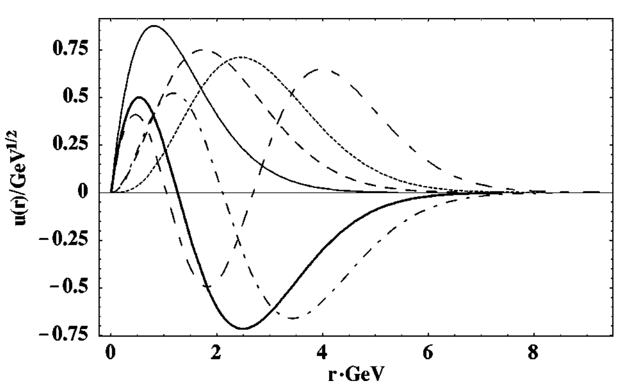

FIG. 2. The same as in Fig. 1 for bottomonium and long-short-dashed line for $3S$ state.

[4] The changes in decay matrix elements are of the same order of magnitude as the contributions of the higher order relativistic and radiative corrections.

sults in a better description of level splittings. The values of the main parameters of our quark model such as the slope of the confining linear potential $A=0.18\ \mathrm{GeV}^2$, the mixing coefficient $\varepsilon=-1$ of scalar and vector confining potentials and the long-range anomalous chromomagnetic quark moment $\kappa=-1$ used in the present analysis are kept the same as they were fixed from the previous consideration of radiative decays [8] and the heavy quark expansion [22,33]. The value of $\varepsilon=-1$ implies that the confining quark-antiquark potential in heavy mesons has predominantly a Lorentz-vector structure, while the scalar potential is anticonfining and helps to reproduce the initial nonrelativistic potential. On the other hand, the value of $\kappa=-1$ supports the conjecture that the long-range confining forces are dominated by chromoelectric interaction and that the chromomagnetic interaction vanishes, which is in accord with the dual superconductivity picture [35] and flux tube model [6].

The presented results for the charmonium and bottomonium mass spectra agree well with the available experimental data. It is of great interest to consider the predictions for the masses of the $^1S_0$ and D levels of bottomonium, which have not yet been observed experimentally. The difficulty of their experimental observation is that these states (except $^3D_1$) cannot be produced in e^+e^- collisions, since their quantum numbers are not the same as the quantum numbers of the photon. Therefore, in search for these states one must investigate decay processes of vector ($^3S_1$) levels. We discussed the possibility of observation of these states in radiative decays in Ref. [8]. Note that the small value predicted for the hyperfine splitting $M(\Upsilon)-M(\eta_b)\cong 60$ MeV leads to difficulties in observation of the η_b state.

Recently it was argued [34] that the account of relativistic kinematics substantially modifies the description of the charmonium fine structure and, in particular, leads to considerably larger values of the 2^3P_J splittings than in the nonrelativistic limit. Both our previous calculation [20] and the present one confirm this observation. Our prediction for the charmonium 2^3P_0 mass lies close to the prediction of Ref. [34] and slightly lower than the $D\bar{D}^*$ threshold. However, the fact that this state is above $D\bar{D}$ and close to $D\bar{D}^*$ thresholds makes threshold effects very important and can considerably influence the quark model prediction.

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