Structure of large N_c cancellations in baryon chiral perturbation theory

Ruben Flores-Mendieta, Christoph P. Hofmann, Elizabeth Jenkins, and Aneesh V. Manohar

Department of Physics, University of California at San Diego, La Jolla, California 92093

We show how to compute loop graphs in heavy baryon chiral perturbation theory including the full functional dependence on the ratio of the Δ-nucleon mass difference to the pion mass, while at the same time automatically incorporating the $1/N_c$ cancellations that follow from the large-N_c spin-flavor symmetry of baryons in QCD. The one-loop renormalization of the baryon axial vector current is studied to demonstrate the procedure. A new cancellation is identified in the one-loop contribution to the baryon axial vector current. We show that loop corrections to the axial vector currents are exceptionally sensitive to deviations of the ratios of baryon-pion axial vector couplings from $SU(6)$ values.

I. INTRODUCTION

Baryon chiral perturbation theory can be used to systematically compute the properties of baryons as a function of the light quark masses m_q. A physical quantity's non-analytic dependence on m_q is calculable from pion-loop graphs; its analytic dependence has contributions both from pion-loop graphs and from low-energy constants that are present in the chiral Lagrangian.

It is convenient to formulate baryon chiral perturbation theory in terms of velocity-dependent baryon fields, so that the expansion of the baryon chiral Lagrangian in powers of m_q and $1/m_B$ (where m_B is the baryon mass) is manifest [1,2]. This formulation is called heavy baryon chiral perturbation theory. The earliest application of heavy baryon chiral perturbation theory was to baryon axial vector currents [2]. Two important results were obtained from this analysis. First, the baryon axial vector coupling ratios were found to be close to their $SU(6)$ values with an F/D ratio close to 2/3, the value predicted by the non-relativistic quark model. Second, there were large cancellations in the one-loop corrections to the baryon axial vector currents between loop graphs with intermediate spin-1/2 octet and spin-3/2 decuplet baryon states. It was later proved using the $1/N_c$ expansion that the baryon axial couplings ratios should have $SU(6)$ values with $F/D = 2/3$, up to corrections of order $1/N_c^2$ for pions [3,4].[1] In addition, it was shown that axial vector current loop graphs with octet and decuplet intermediate states cancel to various orders in N_c. For nucleon and Δ intermediate states, there is a cancellation of the one-loop graphs to two orders in N_c; each individual one-loop diagram is of order N_c, but the sum of all one-loop diagrams is of order $1/N_c$ [3]. Similar large-N_c cancellations also occur for other baryonic quantities [1,6].

We would like to find a calculational scheme that simultaneously exhibits both the m_q and $1/N_c$ expansions. In the chiral limit $m_q \rightarrow 0$, pions become massless Goldstone boson

states. There is an expansion about the chiral limit in powers of m_q/Λ_χ or, equivalently, in powers of m_π^2/Λ_χ^2, where $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking and m_π is the pion mass.[2] In the large-N_c limit, the nucleon and Δ become degenerate, $\Delta \equiv M_\Delta - M_N \propto 1/N_c \rightarrow 0$, and form a single irreducible representation of the contracted spin-flavor symmetry of baryons in large-N_c QCD [3,4]. There is an expansion in powers of $1/N_c$ about the large-N_c limit. We will consider a combined expansion in m_q/Λ_χ and $1/N_c$ about the double limit $m_q \rightarrow 0$ and $N_c \rightarrow \infty$.

Loop graphs in heavy baryon chiral perturbation theory have a calculable dependence on the ratio m_π/Δ. In general, this dependence is described by a function $F(m_\pi, \Delta)$. In the chiral limit $m_q \rightarrow 0$ with Δ held fixed, the function can be expanded in powers of m_π/Δ,

$$F(m_\pi, \Delta) = F_0 + \left(\frac{m_\pi}{\Delta}\right) F_1 + \left(\frac{m_\pi}{\Delta}\right)^2 F_2 + \cdots, \qquad (1)$$

whereas in the $1/N_c \rightarrow 0$ limit with m_π held fixed, the function can be expanded in powers of Δ/m_π:

$$F(m_\pi, \Delta) = \bar{F}_0 + \left(\frac{\Delta}{m_\pi}\right) \bar{F}_1 + \left(\frac{\Delta}{m_\pi}\right)^2 \bar{F}_2 + \cdots. \qquad (2)$$

The difference between the two expansions in Eqs. (1) and (2) is commonly referred to as the non-commutativity of the chiral and large-N_c limits [7].

It is important to remember, however, that the conditions for heavy baryon chiral perturbation theory (including Δ states) to be valid are that $m_\pi \ll \Lambda_\chi$ and $\Delta \ll \Lambda_\chi$. The ratio m_π/Δ is not constrained and can take any value. The entire dependence of a physical quantity on m_π/Δ is *calculable* in heavy baryon chiral perturbation theory [1], so the ratio m_π/Δ need not be small or large for calculations. In the real world, $m_\pi/\Delta \sim 0.5$, so it is useful to have a calculational scheme that retains the full functional dependence of $F(m_\pi, \Delta)$ on the ratio m_π/Δ. A straightforward approach is

---

[1] The ratio F/D is equal to 2/3 in the quark representation and to 5/8 in the Skyrme representation. The quark and Skyrme representations are equivalent in large N_c [5] up to corrections of relative order $1/N_c^2$ for pions [3]. We will use the quark representation in this paper.

[2] For $SU(3)_L \times SU(3)_R$ chiral symmetry, there are loop corrections involving the pions, kaons and η which depend on the pion, kaon and η masses, respectively. Chiral perturbation theory depends on the expansion parameters m_π^2/Λ_χ^2, m_K^2/Λ_χ^2, and m_η^2/Λ_χ^2. Large-N_c chiral perturbation theory also depends on the η' mass.
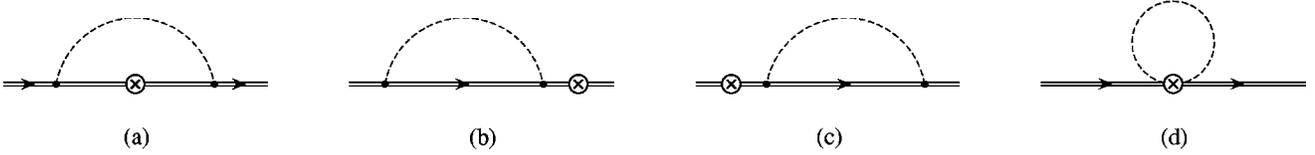
FIG. 1. One-loop corrections to the baryon axial vector current.

to simply calculate the full dependence on m_π/Δ of the loop graphs and evaluate the loop correction at the physical value $m_\pi/\Delta \sim 0.5$ [1,8]. Another common procedure advocated in the literature is to not include intermediate Δ particles explicitly in loops, but to incorporate their effects into the low-energy constants of the effective Lagrangian [9]. The disadvantage of this second approach is that one finds large numerical cancellations between loop diagrams with intermediate nucleon states and low-energy constants containing the effects of Δ states. These cancellations are guaranteed to occur as a consequence of the contracted spin-flavor symmetry which is present in the $N_c \to \infty$ limit. The large-N_c spin-flavor symmetry responsible for the cancellations is hidden in this approach because including only the spin-1/2 baryons in the chiral Lagrangian breaks the large-N_c spin-flavor symmetry explicitly, since the spin-1/2 and spin-3/2 baryons together form an irreducible representation of spin-flavor symmetry. Because the sum of the loop contributions with intermediate octet and decuplet states respects spin-flavor symmetry and is much smaller (by powers of $1/N_c$) than each individual loop contribution separately, it is important to keep the large-N_c spin-flavor symmetry of the baryon chiral Lagrangian and the large-N_c cancellations manifest.

In this paper, we will show how one can combine heavy baryon chiral perturbation theory with the $1/N_c$ expansion so that the full dependence on m_π/Δ is retained and the $1/N_c$ cancellations are explicit. This method has the advantage that the loop correction to the baryon axial isovector current, which is of the order of $1/N_c$, is automatically obtained to be of this order, instead of as the sum of two contributions (loop correction and counterterm) of order N_c which cancel to two powers in $1/N_c$. Note that at higher orders the cancellations become more severe, and it is even more important to keep the $1/N_c$ cancellations manifest. For example, at two loops, each loop diagram is naively of order N_c^2, whereas the sum of all two-loop diagrams is order $1/N_c^2$. Not including the $1/N_c$ cancellations in a systematic way gives a misleading picture of the baryon chiral expansion—one finds higher order corrections that grow with N_c, which is incorrect. Including the $1/N_c$ cancellations restores the $1/N_c$ power counting so that the loop corrections are suppressed by the factor $1/N_c^L$, where L is the number of loops.

The organization of this paper is as follows. In Sec. II, we begin with a brief overview of the $1/N_c$ cancellations occurring in the one-loop correction to the baryon axial vector currents. In Sec. III, we derive the formula for the one-loop correction to the baryon axial vector currents for arbitrary Δ/m_π, in a form that is convenient for later use. The structures of large-N_c cancellations for $\Delta/m_\pi = 0$ and $\Delta/m_\pi \neq 0$ are discussed in Secs. IV and V, respectively. The general power counting for large-N_c cancellations is derived to all

orders in the baryon hyperfine mass splitting Δ, and it is determined that the dominant large-N_c cancellations are present only in terms that are of low and finite order in Δ. A procedure for subtracting and isolating these large-N_c cancellations is given. Other contributions to axial vector current renormalization are briefly presented in Sec. VI. Our conclusions are summarized in Sec. VII.

II. OVERVIEW

A brief review of heavy baryon chiral perturbation theory and the $1/N_c$ baryon chiral Lagrangian can be found in Ref. [10], so only a few salient facts will be repeated here. The pion-baryon vertex is proportional to g_A/f, where f is the decay constant of the π meson. In the large-N_c limit, $g_A \propto N_c$ and $f \propto \sqrt{N_c}$, so that the pion-baryon vertex is of order $\sqrt{N_c}$ and grows with N_c. The baryon propagator is $i/(k \cdot v)$ and is N_c independent, as is the pion propagator. In the modified minimal subtraction scheme ($\overline{\text{MS}}$), all loop integrals are given by the pole structure of the propagators, so loop integrals do not depend on N_c.

The tree-level matrix element of the baryon axial vector current is of order N_c, since g_A is of order N_c. The one-loop diagrams that renormalize the baryon axial vector current are shown in Fig. 1. Each of the one-loop corrections in Figs. 1(a)–1(c) involves two pion-baryon vertices, and is order N_c times the tree-level graph.

The matrix elements of the space components of the baryon axial vector current between initial and final baryon states B and B' will be denoted by

$$\langle B' | \bar{\psi} \gamma^i \gamma_5 T^a \psi | B \rangle = [A^{ia}]_{B'B}, \qquad (3)$$

where B and B' are baryons in the lowest-lying irreducible representation of contracted-$SU(6)$ spin-flavor symmetry, i.e. the spin-1/2 octet and spin-3/2 decuplet baryons. The Feynman diagram amplitude for $B \to B' + \pi(k)$ is $[A^{ia}]_{B'B} k^i/f$, where $\mathbf{k}$ is the three-momentum of the emitted pion. The time component of the axial current has zero matrix element between static baryons, and is represented in the heavy baryon formulation by a higher dimension operator in the effective Lagrangian. The matrix elements $[A^{ia}]_{B'B}$ of the spatial components of the axial vector current can be written in terms of the octet and decuplet pion coupling constants F, D, $\mathcal{C}$, and $\mathcal{H}$ [2], each of which is of order N_c.

The one-loop correction to the baryon axial vector current, in the limit that the Δ-nucleon mass difference is neglected, is proportional to the double commutator

$$\delta A^{ia} \propto \frac{1}{f^2}[A^{jb},[A^{jb},A^{ia}]], \qquad (4)$$
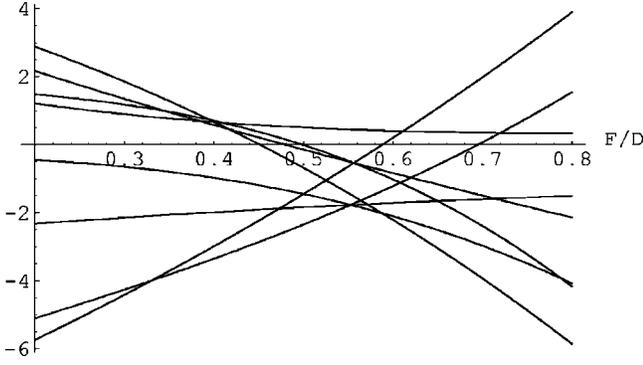
FIG. 2. One-loop pion correction to the baryon axial vector current $\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle$. The curves are $\bar{\beta}-\bar{\lambda}\alpha$ for (from top to bottom along the left hand edge of the graph) $N\to N\pi$, $\Xi\to\Lambda\bar{K}$, $\Sigma\to\Sigma\pi$, $\Xi\to\Xi\pi$, $\Xi\to\Sigma\bar{K}$, $\Sigma\to\Lambda\pi$, $\Sigma\to N\bar{K}$, $\Lambda\to N\bar{K}$.

FIG. 3. One-loop kaon correction to the baryon axial current $\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle$. The curves are $\bar{\beta}-\bar{\lambda}\alpha$ for (from top to bottom along the left hand edge of the graph) $\Xi\to\Xi\pi$, $\Sigma\to\Sigma\pi$, $\Xi\to\Lambda\bar{K}$, $\Lambda\to N\bar{K}$, $\Xi\to\Sigma\bar{K}$, $N\to N\pi$, $\Sigma\to\Lambda\pi$, $\Sigma\to N\bar{K}$.

where the sum over intermediate baryon states is given by matrix multiplication of the A^{ia} matrices. Naively, the double commutator is of order N_c^3, and $f\propto\sqrt{N_c}$ so that δA^{ia} is of order N_c^2. One of the results of the $1/N_c$ analysis for baryons is that the double commutator is of order N_c, rather than N_c^3 [3]. Each individual term in the sum Eq. (4) is of order N_c^3, but there is a cancellation in the sum over intermediate baryons, which is guaranteed by the spin-flavor symmetry of large-N_c QCD [3,4]. The cancellation only occurs when the ratios of F, D, $\mathcal{C}$, and $\mathcal{H}$ are close to their $SU(6)$ values.[3] The large-N_c cancellation implies that the one-loop correction to the axial current is $1/N_c$ times the tree-level value, instead of N_c times the tree-level value. Similarly, the two-loop correction is $1/N_c^2$ times the tree-level value, instead of N_c^2 times the tree-level value (see Ref. [11] for an explicit calculation in the degeneracy limit). The one-loop large-N_c cancellations will be discussed more fully in Secs. IV and V. The formalism for making large-N_c cancellations manifest is provided in Sec. V.

The large-N_c cancellation in the one-loop correction to the baryon axial vector current can be seen numerically from explicit computation in heavy baryon chiral perturbation theory. The baryon axial vector current matrix element at one-loop has the form

$$A=\alpha+(\bar{\beta}-\bar{\lambda}\alpha)\frac{m^2}{16\pi^2 f^2}\ln\frac{m^2}{\mu^2}+\cdots \quad (5)$$

where α is the tree-level contribution, $\bar{\beta}$ is the vertex correction, $\bar{\lambda}$ is the wave function renormalization, m is the π, K or η mass, and the Δ-nucleon mass difference has been ne-

glected for simplicity so there is only a chiral-logarithmic contribution. (The full one-loop correction will be discussed in the next section.) For the case of $\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle$, the coefficients are

$$\alpha=D+F,$$

$$\bar{\lambda}_\pi=\frac{9}{4}(F+D)^2+2\mathcal{C}^2,$$

$$\bar{\lambda}_K=\frac{1}{2}(9F^2-6FD+5D^2+\mathcal{C}^2),$$

$$\bar{\lambda}_\eta=\frac{1}{4}(3F-D)^2,$$

$$\bar{\lambda}_{\eta'}=2D^2, \quad (6)$$
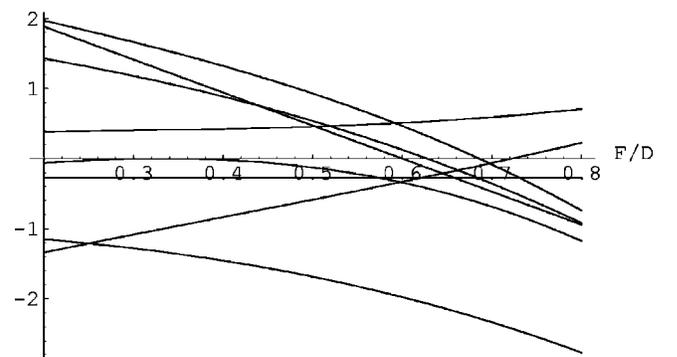
FIG. 4. One-loop eta correction to the baryon axial current $\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle$. The curves are $\bar{\beta}-\bar{\lambda}\alpha$ for (from top to bottom along the left hand edge of the graph) $\Xi\to\Xi\pi$, $\Sigma\to\Sigma\pi$, $\Xi\to\Lambda\bar{K}$, $\Lambda\to N\bar{K}$, $N\to N\pi$, $\Sigma\to\Lambda\pi$, $\Xi\to\Sigma\bar{K}$, $\Sigma\to N\bar{K}$.

---

[3]An important point to note is that large-N_c QCD predicts only the ratios of F/D, $\mathcal{C}/D$, and $\mathcal{H}/D$; the overall normalization of the coupling constants is not fixed by the symmetry. The large-N_c cancellations depend on the coupling ratios being close to their $SU(6)$ values, and do not depend on the overall normalization of the couplings.

$$\bar{\beta}_\pi = \frac{1}{4}(F+D)^3 + \frac{16}{9}(F+D)\mathcal{C}^2 - \frac{50}{81}\mathcal{H}\mathcal{C}^2 - F - D,$$

$$\bar{\beta}_K = \frac{1}{3}(-3F^3 + 3F^2D - FD^2 + D^3) + \frac{2}{9}(F+3D)\mathcal{C}^2$$

$$- \frac{10}{81}\mathcal{H}\mathcal{C}^2 - \frac{1}{2}(F+D),$$

$$\bar{\beta}_\eta = -\frac{1}{12}(F+D)(3F-D)^2,$$

$$\bar{\beta}_{\eta'} = -\frac{1}{12}(F+D)(3F-D)^2.$$

The coefficients for the other matrix elements can be found in the literature [2]. The subscripts π, K and η denote the contributions from π, K, and η loops. To illustrate the cancellation, we have plotted the one-loop coefficients ($\bar{\beta} - \bar{\lambda}\alpha$) for the axial currents (or equivalently, the couplings) $N \to N\pi$, $\Sigma \to \Lambda\pi$, $\Sigma \to \Sigma\pi$, $\Xi \to \Xi\pi$, $\Lambda \to N\bar{K}$, $\Sigma \to N\bar{K}$, $\Xi \to \Lambda\bar{K}$, and $\Xi \to \Sigma\bar{K}$ in Figs. 2, 3, and 4. For simplicity, the coefficients are plotted as a function of F/D only—the other coupling ratios have been fixed at their $SU(6)$ values $\mathcal{C}/D = -2$ and $\mathcal{H}/D = -3$. The best fit to the baryon axial currents has the axial vector coupling ratios close to their $SU(6)$ values [1,2], so this is a reasonable approximation.

The large-N_c analysis indicates that there should be some cancellation in the loop correction when F/D is close to the $SU(6)$ value of 2/3. This suppression is evident separately for the π, K and η loops for all eight processes. This is the cancellation pointed out phenomenologically in Refs. [1,2] and later proved in Refs. [3,4]. We will study this cancellation quantitatively in terms of the $1/N_c$ expansion in this work.

III. ONE-LOOP CORRECTION TO THE AXIAL VECTOR CURRENT

The one-loop diagrams that contribute to the baryon axial vector current are shown in Fig. 1. Figures 1(a)–1(c) are of order N_c times the tree-level vertex, and Fig. 1(d) is of order $1/N_c$ times the tree-level vertex. The large-N_c cancellations occur between Figs. 1(a)–1(c), so we will concentrate on these three diagrams in this section. The contribution from Fig. 1(d) is considered briefly in Sec. VI. Both contributions can be found in Ref. [2].[4]

All the loop graphs we need can be written in terms of the basic loop integral

$$\delta^{ij}F(m,\Delta,\mu) = \frac{i}{f^2}\int \frac{d^4k}{(2\pi)^4} \frac{(\mathbf{k}^i)(-\mathbf{k}^j)}{(k^2-m^2)(k\cdot v - \Delta + i\epsilon)}, \tag{7}$$

where μ is the scale parameter of dimensional regularization. Evaluating the integral gives

$$24\pi^2 f^2 \, F(m,\Delta,\mu) = \Delta\left(\Delta^2 - \frac{3}{2}m^2\right)\ln\frac{m^2}{\mu^2} - \frac{8}{3}\Delta^3 - \frac{7}{2}\Delta m^2 + \begin{cases} 2(m^2-\Delta^2)^{3/2}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\Delta}{\sqrt{m^2-\Delta^2}}\right)\right], & |\Delta| \leq m, \\ -(\Delta^2-m^2)^{3/2}\ln\left(\frac{\Delta - \sqrt{\Delta^2-m^2}}{\Delta + \sqrt{\Delta^2-m^2}}\right), & |\Delta| > m. \end{cases} \tag{8}$$

A. Wave function renormalization

The wave function renormalization graph for baryon B is shown in Fig. 5, where one sums over all possible intermediate baryons B_I. The loop graph is equal to

$$iG_{B'B} = \sum_{j,k,b,B_I} \frac{i^2}{f^2}[A^{kb}]_{B'B_I}[A^{jb}]_{B_IB}$$

$$\times \int \frac{d^4k}{(2\pi)^4} \frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{(k^2-m_b^2)[(k+p)\cdot v - (M_I-M) + i\epsilon]}, \tag{9}$$

where $b = 1, \ldots, 9$ or π, K, η, η' labels the intermediate

meson.[5] The wave function renormalization correction is $Z_{B'B} = \delta_{B'B} + z_{B'B}$,

$$z_{B'B} = \frac{\partial G_{B'B}}{\partial (p\cdot v)}\bigg|_{p\cdot v = 0}, \tag{10}$$

---

[4]Figure 1(d) is linear in the pion-baryon coupling constants, whereas Figs. 1(a)–1(c) are cubic, so it is easy to identify the two pieces in existing calculations.

[5]The η' is a ninth Goldstone boson in the large N_c limit [12,13]. Our formulas apply to the η' corrections, with flavor matrix $\lambda^9 = \sqrt{2/3}$. The formalism for including the η' is described in detail in Refs. [10,14].
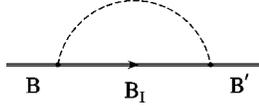
FIG. 5. One-loop wave function renormalization graph.
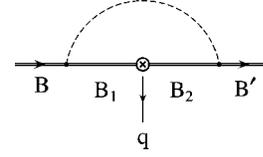
FIG. 6. One-loop vertex correction graph.

which can be written in terms of the function $F(m,\Delta,\mu)$ defined in Eq. (7) as

$$z_{B'B} = -\sum_{j,b,B_I} [A^{jb}]_{B'B_I}[A^{jb}]_{B_I B} \frac{\partial F(m_b,\Delta_{B_I B},\mu)}{\partial \Delta_{B_I B}},$$

(12)

where

$$\Delta_{B_1 B_2} \equiv M_{B_1} - M_{B_2}.$$

(13)

B. Vertex correction

The one-loop correction to the matrix element $\langle B'|A^{ia}|B\rangle$ from the vertex graph Fig. 6 can be written as

$$[\delta A^{ia}]_{B'B}^{\text{vertex}} = \sum_{j,k,b,B_1,B_2} -\frac{i}{f^2} [A^{kb}]_{B'B_2}[A^{ia}]_{B_2 B_1}[A^{jb}]_{B_1 B}$$

$$\times \int \frac{d^4k}{(2\pi)^4} \frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{(k^2-m_b^2)[k\cdot v-(M_1-M)+i\epsilon][(k-q)\cdot v-(M_2-M)+i\epsilon]},$$

(14)

where q is the outgoing momentum transfer at the axial vector vertex. For octet-octet matrix elements, $q\cdot v=0$, whereas for decuplet-octet transition matrix elements, $q\cdot v=M-M'$, the average decuplet-octet mass difference. One can rewrite the denominator of Eq. (14) using the identity

$$\frac{1}{(k^0-\Delta_1+i\epsilon)(k^0-\Delta_2+i\epsilon)} = \frac{1}{(\Delta_1-\Delta_2)}\left[\frac{1}{(k^0-\Delta_1+i\epsilon)} - \frac{1}{(k^0-\Delta_2+i\epsilon)}\right]$$

(15)

so that

$$[\delta A^{ia}]_{B'B}^{\text{vertex}} = -\sum_{j,b,B_1,B_2} [A^{jb}]_{B'B_2}[A^{ia}]_{B_2 B_1}[A^{jb}]_{B_1 B} \frac{1}{\Delta_{B_1 B}-\Delta_{B_2 B'}}[F(m_b,\Delta_{B_1 B},\mu)-F(m_b,\Delta_{B_2 B'},\mu)]$$

(16)

where $\Delta_{B_1 B_2}$ is defined in Eq. (13).

C. Total correction

The total correction to the baryon axial vector current matrix element from Figs. 1(a)–1(c) is

$$[\delta A^{ia}]_{B'B} = [\delta A^{ia}]_{B'B}^{\text{vertex}} - \frac{1}{2}\left\{\sum_{B_1} z_{B'B_1}[A^{ia}]_{B_1 B} + \sum_{B_2} [A^{ia}]_{B'B_2}z_{B_2 B}\right\}$$

(17)

$$= -\sum_{j,b,B_1,B_2} [A^{jb}]_{B'B_2}[A^{ia}]_{B_2 B_1}[A^{jb}]_{B_1 B} \frac{F(m_b,\Delta_{B_1 B},\mu)-F(m_b,\Delta_{B_2 B'},\mu)}{\Delta_{B_1 B}-\Delta_{B_2 B'}}$$

$$+ \frac{1}{2}\sum_{j,b,B_1,B_2} [A^{jb}]_{B'B_2}[A^{jb}]_{B_2 B_1}[A^{ia}]_{B_1 B} \frac{\partial F(m_b,\Delta_{B_2 B_1},\mu)}{\partial \Delta_{B_2 B_1}}$$

$$+ \frac{1}{2}\sum_{j,b,B_1,B_2} [A^{ia}]_{B'B_2}[A^{jb}]_{B_2 B_1}[A^{jb}]_{B_1 B} \frac{\partial F(m_b,\Delta_{B_1 B},\mu)}{\partial \Delta_{B_1 B}}.$$

[In addition to δA^{ia} of Eq. (17), there are also the contributions of Fig. 1(d) and the low-energy constants, which are considered in Sec. VI.] Equation (17) includes the full dependence on Δ/m of the one-loop correction. We want to rewrite this expression so that the large-N_c cancellations are manifest.

In the limit that the octet and decuplet baryons are degenerate, all the mass differences $\Delta_{AB} \to 0$, and

$$\frac{1}{\Delta_{B_1 B} - \Delta_{B_2 B'}} [F(m_b, \Delta_{B_1 B}, \mu) - F(m_b, \Delta_{B_2 B'}, \mu)]$$

$$\to F^{(1)}(m_b, 0, \mu), \tag{18}$$

where $F^{(n)}$ is defined by

$$F^{(n)}(m_b, \Delta, \mu) \equiv \frac{\partial^n F(m_b, \Delta, \mu)}{\partial \Delta^n}. \tag{19}$$

In this limit, the correction to the axial current Eq. (17) reduces to

$$[\delta A^{ia}]_{B'B} = \sum_{j,b,B_1,B_2} F^{(1)}(m_b, 0, \mu)$$

$$\times \left\{ -[A^{jb}]_{B'B_2}[A^{ia}]_{B_2 B_1}[A^{jb}]_{B_1 B} \right.$$

$$+ \frac{1}{2}[A^{jb}]_{B'B_2}[A^{jb}]_{B_2 B_1}[A^{ia}]_{B_1 B}$$

$$\left. + \frac{1}{2}[A^{ia}]_{B'B_2}[A^{jb}]_{B_2 B_1}[A^{jb}]_{B_1 B} \right\}. \tag{20}$$

Let us adopt the more compact notation that A^{ia} represents a matrix with matrix elements $[A^{ia}]_{B'B}$, and summation over intermediate baryon states is denoted by matrix multiplication. Then Eq. (20) can be written as

$$\delta A^{ia} = \sum_{j,b} F^{(1)}(m_b, 0, \mu) \left\{ -A^{jb} A^{ia} A^{jb} + \frac{1}{2} A^{jb} A^{jb} A^{ia} \right.$$

$$\left. + \frac{1}{2} A^{ia} A^{jb} A^{jb} \right\}$$

$$= \frac{1}{2} \sum_{j,b} F^{(1)}(m_b, 0, \mu) [A^{jb}, [A^{jb}, A^{ia}]], \tag{21}$$

which is the double-commutator form originally derived in Ref. [3].

The loop integral in the degeneracy limit $\Delta \to 0$ reduces to

$$F^{(1)}(m_b, 0, \mu) = -\frac{1}{16\pi^2 f^2} m_b^2 \left(\frac{11}{3} + \ln \frac{m_b^2}{\mu^2} \right). \tag{22}$$

The $\ln m_b/\mu$ term is non-analytic in the quark mass, and is called a ''chiral logarithm.'' The constant (11/3) piece is analytic in the quark masses, and has the same form as

higher dimension terms in the chiral Lagrangian. The constant term is scheme dependent, but the chiral logarithm is universal.

We discuss the structure of the large-N_c cancellations for the baryon axial vector currents in the next two sections. First, in Sec. IV, the cancellations are studied in the degeneracy limit. The generalization to non-degenerate baryons is given in Sec. V.

IV. LARGE-N_c CANCELLATIONS: $\Delta/m_\pi = 0$

The large-N_c cancellations in the degeneracy limit for the one-loop correction to the baryon axial vector current follow from the double-commutator form of Eq. (21). The pion decay constant $f \propto \sqrt{N_c}$, so the function $F^{(1)}(m_b, 0, \mu)$ is of order $1/N_c$. Each axial vector current matrix element is of order N_c (recall that g_A is of order N_c), so the correction δA^{ia} is naively of absolute order N_c^2, i.e. of order N_c relative to the tree-level value A^{ia}. The large-N_c consistency conditions derived in Ref. [3] imply that the double commutator $[A^{jb}, [A^{jb}, A^{ia}]]$ is of order N_c rather the naive order N_c^3, *provided one sums over all baryon states in a complete multiplet of the large-N_c SU(6) spin-flavor symmetry, i.e. over both the octet and decuplet, and uses axial coupling ratios given by the large-N_c spin-flavor symmetry.*

Before discussing the cancellation in the double commutator, we first review some necessary large-N_c formalism. The baryon matrix element of the axial vector current in QCD can be expanded in a $1/N_c$ expansion in terms of $SU(6)$ spin-flavor operators [3,4,16–18],[6]

$$G^{ia} = q^\dagger \frac{\sigma^i}{2} \frac{\lambda^a}{2} q, \quad T^a = q^\dagger \frac{\lambda^a}{2} q, \quad J^i = q^\dagger \frac{\sigma^i}{2} q, \tag{23}$$

where q and $q^\dagger$ are $SU(6)$ operators that create and annihilate states in the fundamental representation of $SU(6)$, and σ^i and λ^a are the Pauli spin and Gell-Mann flavor matrices. The lowest mass baryon multiplet transforms under $SU(6)$ as a completely symmetric tensor with N_c indices. For $N_c = 3$, this representation decomposes under spin and flavor into a spin-1/2 octet and a spin-3/2 decuplet. The baryon axial vector current A^{ia} in the large-N_c limit has the form [18]

$$A^{ia} = a_1 G^{ia} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{ia} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{ia}, \tag{24}$$

where the coefficients are of order 1. The operators $\mathcal{D}_n^{ia}$ are diagonal operators with nonzero matrix elements only between states with the same spin, and the operators $\mathcal{O}_n^{ia}$ are purely off-diagonal operators with nonzero matrix elements only between states of different spin. The explicit forms for

---

[6]For recent reviews of the large-N_c spin-flavor symmetry, see Ref. [15].

these operators can be found in Ref. [18]. At the physical value $N_c = 3$, Eq. (24) reduces to

$$A^{ia} = a_1 G^{ia} + b_2 \frac{1}{N_c} J^i T^a + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{ia} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{ia}, \quad (25)$$

where

$$\mathcal{D}_3^{ia} = \{J^i, \{J^j, G^{ja}\}\},$$
$$\mathcal{O}_3^{ia} = \{J^2, G^{ia}\} - \frac{1}{2}\{J^i, \{J^j, G^{ja}\}\}. \quad (26)$$

The four conventional $SU(3)$ baryon axial couplings F, D, $\mathcal{C}$ and $\mathcal{H}$ for the baryon octet and decuplet can be written as linear combinations of the coefficients a_1, b_2, b_3 and c_3 of the $1/N_c$ expansion:

$$D = \frac{1}{2} a_1 + \frac{1}{6} b_3,$$

$$F = \frac{1}{3} a_1 + \frac{1}{6} b_2 + \frac{1}{9} b_3,$$

$$\mathcal{C} = -a_1 - \frac{1}{2} c_3, \quad (27)$$

$$\mathcal{H} = -\frac{3}{2} a_1 - \frac{3}{2} b_2 - \frac{5}{2} b_3.$$

The leading order prediction of large-N_c QCD is obtained by dropping the $1/N_c$ suppressed terms in Eq. (24), i.e. the 3-body operators $\mathcal{D}_3^{ia}$ and $\mathcal{O}_3^{ia}$. The G^{ia} operator gives $F = 2D/3$, $\mathcal{C} = -2D$ and $\mathcal{H} = -3D$, so that the coupling ratios, but not necessarily their absolute normalization, are those predicted by $SU(6)$ symmetry. The 2-body operator $J^i T^a$ corrects these relations. The correction is of relative order $1/N_c^2$ for pions.

The baryon matrix elements of J^i for the low-lying baryons in the $SU(6)$ representation are of order unity. The N_c dependence of matrix elements of G^{ia} and T^a is more subtle, and depends on the particular component a chosen, as well as on the initial and final state baryon [18]. For the purposes of this paper, we will use the naive estimate that matrix elements of G^{ia} and T^a are both of order N_c, which is the largest they can be. We focus upon baryons with spins of order unity. The N_c counting rules are summarized as

$$G^{ia} \sim N_c, \quad T^a \sim N_c, \quad J^i \sim 1. \quad (28)$$

The $1/N_c$ expansion of a baryonic matrix element can be written as an expansion in powers of G^{ia}/N_c, T^a/N_c and J^i/N_c. The counting rules, Eq. (28), show that each factor J leads to a $1/N_c$ suppression factor.

We can now understand the origin of the large-N_c cancellations in Eq. (21). At leading order in N_c, the axial vector current operator A^{ia} can be replaced by $a_1 G^{ia}$, and has matrix elements of order N_c. The commutator $[A^{ia}, A^{jb}]$

$= [a_1 G^{ia}, a_1 G^{jb}]$ is naively of order N_c^2, since each G^{ia} is of order N_c. However, the commutation relation

$$[G^{ia}, G^{jb}] = \frac{i}{4} \delta^{ij} f^{abc} T^c + \frac{i}{6} \delta^{ab} \epsilon^{ijk} J^k + \frac{i}{2} d^{abc} \epsilon^{ijk} G^{kc} \quad (29)$$

shows that matrix elements of the commutator $[G^{ia}, G^{jb}]$ are at most of order N_c, since the right-hand side of Eq. (29) is at most of order N_c. Thus there is a factor of N_c cancellation between the various terms in the commutator $[G^{ia}, G^{jb}]$ from the summation over intermediate baryon states. Similarly, one finds that there is a factor of N_c cancellation in the sum over intermediate states for the commutators

$$[T^a, G^{ib}] = i f^{abc} G^{ic} \quad (30)$$

and

$$[T^a, T^b] = i f^{abc} T^c, \quad (31)$$

where the naive counting rule, Eq. (28), has been used to estimate the order in N_c of both sides of these equations. The basic reason for the cancellation is that the maximum order in N_c an r-body operator matrix element can be is N_c^r (an r-body operator is one with $r q$'s and $r q^\dagger$'s; i.e., it can be written as a polynomial of order r in J^i, G^{ia} and T^a), but the commutator of an r-body and s-body operator is at most an $(r+s-1)$-body operator. Thus, every commutator potentially leads to a cancellation by one factor of N_c. However, not every commutator gives a factor of N_c cancellation. The commutators

$$[J^i, J^j] = i \epsilon^{ijk} J^k \quad (32)$$

and

$$[J^i, G^{ja}] = i \epsilon^{ijk} G^{ka} \quad (33)$$

have no cancellations, since both sides are of order 1 and order N_c, respectively. The reason that there is no cancellation in Eqs. (32) and (33) is that J^i is a one-body operator whose matrix elements are of order unity, rather than of order N_c.

Equations (30)–(33) lead to the conclusion that each commutator produces a N_c cancellation, unless a factor of J^i is eliminated. The double commutator in Eq. (21) has a cancellation of N_c^2, because $[G^{jb}, [G^{jb}, G^{ia}]] \sim J + G + T$, so that the double commutator is order N_c, rather than N_c^3. This was the cancellation observed numerically in Ref. [2] and later proved in the $1/N_c$ expansion in Refs. [3,4].

V. LARGE-N_c CANCELLATIONS: $\Delta/m_\pi \neq 0$

In this section, we analyze the large-N_c cancellations in the renormalization of the baryon axial vector current for finite Δ/m_π.

Equation (17) can be expanded in a power series in Δ. Expanding the function $F(m, \Delta, \mu)$ in a power series and collecting terms gives

$$\delta A^{ia} = \sum_{j,b} \left\{ \frac{1}{2} F^{(1)}(m_b, 0, \mu)[A^{jb}, [A^{jb}, A^{ia}]] \right.$$

$$- \frac{1}{2} F^{(2)}(m_b, 0, \mu)\{A^{jb}, [A^{ia}, [\mathcal{M}, A^{jb}]]\}$$

$$+ \frac{1}{6} F^{(3)}(m_b, 0, \mu)\left([A^{jb}, [[\mathcal{M}, [\mathcal{M}, A^{jb}]], A^{ia}]] \right.$$

$$\left. \left. - \frac{1}{2}[[\mathcal{M}, A^{jb}], [[\mathcal{M}, A^{jb}], A^{ia}]] \right) + \cdots \right\} \qquad (34)$$

where $\mathcal{M}$ is the baryon mass matrix. In deriving this result we have converted explicit sums over intermediate baryons to implicit sums in the matrix multiplications. One can use either the baryon mass matrix $\mathcal{M}$ or the baryon mass-splitting matrix $\Delta \mathcal{M}$ in Eq. (34), since $\mathcal{M}$ differs from $\Delta \mathcal{M}$ by the average baryon mass times the unit matrix, which commutes and drops out of Eq. (34). To evaluate Eq. (34) to all orders in Δ/m_π would be extremely difficult, since one would have to sum an infinite series, with each term having a coefficient which is a complicated commutator and anti-commutator of $\mathcal{M}$'s and A^{ia}'s.

We would like to evaluate graphs in heavy baryon chiral perturbation theory so that the $1/N_c$ cancellations are manifest and do not occur as numerical cancellations at the end of the calculation. We will now show that the large N_c cancellations only occur in the first few terms of Eq. (34), so that the remaining terms can be summed using conventional heavy baryon chiral perturbation theory in the usual manner.

The expansion, Eq. (34), has a different structure depending on whether one has an even or odd number of insertions of the baryon mass operator $\mathcal{M}$. Terms with $2r$ insertions of $\mathcal{M}$ have $2r+2$ commutators, whereas terms with $2r+1$ insertions of $\mathcal{M}$ have $2r+2$ commutators and one anticommutator.

The general form of the baryon mass operator in the $1/N_c$ expansion in the $SU(3)$ limit is [3,4,16–18]

$$\mathcal{M} = N_c \left[m_0 + m_2 \frac{J^2}{N_c^2} + m_4 \frac{(J^2)^2}{N_c^4} + \cdots \right]. \qquad (35)$$

The importance of a given term in the $1/N_c$ expansion can be obtained by counting powers of J. Each factor of J/N_c leads to a $1/N_c$ suppression, since J is of order unity according to the counting rules Eq. (28).

We now have all the necessary ingredients to count the power in $1/N_c$ of a general term in Eq. (34). The operators A^{ia} and $\mathcal{M}$ are one-body operators, with naive order N_c, so the $\mathcal{M}^r$ term in the expansion in Eq. (34) is naively of order N^{3+r-1}, including the factor of $1/N_c$ from the $1/f^2$ in the loop integral F, as shown in row (C) of Table I.

The number of commutators in each term is listed in the next line in this table. Every commutator (naively) leads to a decrease in the naive N_c order by unity, since the commutator of an r-body and an s-body operator is at most an $r+s$ -1 body operator. This leads to the N_c power given in row (E). Finally, we need to count the powers of J in each term.

Each factor of $\mathcal{M}$ has at least two J's, since the $m_0 N_c$ term in Eq. (35) drops out of the expression Eq. (34). Each factor of A^{ia} can have ≥ 0 J's, as is clear from Eqs. (24)–(26). The number of J's in the original expression is listed in row (F), where $p \geq 0$ is the number of extra J's from $\mathcal{M}$ or A^{ia}, beyond the minimum values of 2 and 0, respectively. Finally, note that each commutator can be used to eliminate one power of J. Thus the net power of J left is given by subtracting the number of commutators from the number of J's. The minimum number of J's is non-negative, and is listed in row (G). Thus, the final N_c power [row (H)] is given by the net power in row (E) minus the minimum number of J's in row (G) since there is an additional $1/N_c$ factor for each J. One can compare this with the "usual" N_c counting rule listed in row (I). The usual counting rule is obtained by including a factor of N_c for each A^{ia} (i.e. for each factor of F, D, $\mathcal{C}$ or $\mathcal{H}$), a factor of $1/N_c$ for the $1/f^2$, and a factor of $1/N_c$ for each power of Δ, with $p \geq 0$ representing $1/N_c$ suppressed terms.

One interesting point can be noted from Table I. The dominant $1/N_c$ corrections from the baryon mass splittings are due to multiple insertions of the J^2 term in the baryon mass matrix. Two insertions of the J^2 term (the $p=0$ term in the $\mathcal{M}^2$ column) are N_c more important than one insertion of the J^4 term (the $p=2$ term in the $\mathcal{M}^1$ column).

There is an extra cancellation in the term linear in $\mathcal{M}$ that is not apparent in Table I. We will discuss this new large-N_c cancellation momentarily. Including this effect, one sees from Table I that all terms in the expansion of Eq. (17) with two or more powers of $\mathcal{M}$ have the same N_c behavior as one finds with the usual N_c counting; i.e., these terms have no extra cancellations. One can therefore treat all terms with two or more powers of $\mathcal{M}$ by conventional heavy baryon chiral perturbation theory—compute all the graphs, with vertices written in terms of F, D, $\mathcal{C}$ and $\mathcal{H}$. The only terms that have to be treated specially are those with zero or one power of $\mathcal{M}$. To compute graphs in the conventional way omitting the first two terms in Eq. (34) is trivial; one simply rewrites the loop integral Eq. (7) by explicitly extracting the first three terms in an expansion in Δ,

$$F(m_b, \Delta, \mu) = F(m_b, 0, \mu) + F^{(1)}(m_b, 0, \mu)\Delta$$

$$+ \frac{1}{2} F^{(2)}(m_b, 0, \mu)\Delta^2 + \tilde{F}(m_b, \Delta, \mu), \qquad (36)$$

and takes the standard expressions for the loop corrections written in terms of F, D, $\mathcal{C}$ and $\mathcal{H}$, with $F \to \tilde{F}$. This procedure sums the entire series in $(\Delta/m_\pi)^r$ starting with $r=3$. To this result is added the first two terms in Eq. (34). One needs to extract three terms from F to obtain the first two terms in Eq. (34), since $F(m_b, 0, \mu)$ cancels out of the correction.

One can now analyze the first two terms in the expansion of Eq. (17), which are given in Eq. (34). The first term is the double commutator term discussed in the previous section. We see from Table I that this term is naively of order N_c^2, but actually is at most of order N_c^0. This is consistent with the loop expansion being an expansion in $\hbar/N_c$, since the

TABLE I. Table of the order in N_c of the terms in the expansion of the one-loop correction to the axial vector currents. See the text for an explanation of the entries.

(A)	Number of $\mathcal{M}$'s	0		1	2	3	4	5	6
(B)	m_q dependence	$m_q \ln m_q$		$m_q^{1/2}$	$\ln m_q$	$m_q^{-1/2}$	m_q^{-1}	$m_q^{-3/2}$	m_q^{-2}
(C)	Naive N_c power	2		3	4	5	6	7	8
(D)	Commutators	2		2	4	4	6	6	8
(E)	Net power	0		1	0	1	0	1	0
(F)	Number of J's	p		$p+2$	$p+4$	$p+6$	$p+8$	$p+10$	$p+12$
(G)	J's left	0 ($p=0,1$); $p-2$ ($p\geq 2$)		p	p	$p+2$	$p+2$	$p+4$	$p+4$
(H)	Final N_c power	0 ($p=0,1$); $2-p$ ($p\geq 2$)		$1-p^{\text{a}}$	$-p$	$-1-p$	$-2-p$	$-3-p$	$-4-p$
(I)	Usual N_c power	$2-p$		$1-p$	$-p$	$-1-p$	$-2-p$	$-3-p$	$-4-p$

$^{\text{a}}$Actually is 0 for $p=0$. See text.

one-loop correction is of order $1/N_c$ relative to the tree-level contribution of order N_c. It is also apparent from Table I that all terms of order $\mathcal{M}^0$ with $p=0,1,2$ are equally important. Since there are no powers of the baryon mass operator, the p factors of J must all arise from $1/N_c$ corrections in the axial vertices A^{ia}. The expression for the axial vector current relevant for $N_c=3$ is given in Eq. (25), from which it follows that terms with $p=0,1,2$ in the product AAA are of the form GGG, $GG\,JT$, $G\,JT\,JT$, $GG\mathcal{D}_3$, and $GG\mathcal{O}_3$. All these terms contribute *at the same order* to the double commutator, whereas according to the usual counting one would have expected the $p=0$ product GGG to be one power of N_c more important that the $p=1$ product $GG\,JT$, which in turn would be more important by one power of N_c than the $p=2$ products. This result has an important consequence: the one-loop correction is very sensitive to the deviations of the axial vector coupling ratios from their $SU(6)$ values. While the deviations are small corrections to the couplings themselves, their importance gets enhanced in the one-loop coefficient, because the leading term (proportional to a_1^3) is $1/N_c^2$ suppressed. Thus, for example, the a_1 term is the dominant contribution to F, D, $\mathcal{C}$ and $\mathcal{H}$, and the c_3 term is a $1/N_c^2$ correction, but the a_1^3 and $a_1^2 c_3$ terms are just as important in the one-loop correction. Explicit forms for the one-loop correction in terms of a_1, b_2, b_3 and c_3 will be given elsewhere.

The second term in Eq. (34) is

$$-\frac{1}{2}F^{(2)}(m_b,0,\mu)\{A^{jb},[A^{ia},[\mathcal{M},A^{jb}]]\} \qquad (37)$$

and is at most of order N_c [using the value $(1-p)$ for $p=0$], the same order as the tree-level contribution to the axial vector current. This result is surprising, because the $1/N_c$ expansion is a semiclassical expansion in $\hbar/N_c$. One should be able to obtain the leading in $1/N_c$ contributions from classical field theory. For example, it was shown that the $N_c m_q^{3/2}$ one-loop correction to the baryon mass could be obtained from the energy of the pion cloud coupled to a classical baryon source [19]. The term in Eq. (37) involves

the baryon mass splitting, which is a quantum effect. The order N_c contribution to Eq. (37) comes from using $A^{ia}=a_1 G^{ia}$ and $\Delta\mathcal{M}=m_2 J^2/N_c$:

$$-\frac{a_1^3 m_2}{2N_c}F^{(2)}(m_b,0,\mu)\{G^{jb},[G^{ia},[J^2,G^{jb}]]\}. \qquad (38)$$

The operator factor $\{G^{jb},[G^{ia},[J^2,G^{jb}]]\}$ is naively of order N_c^3, which implies that the correction Eq. (38) is an order N_c correction to the axial currents, since $F^{(2)}(m_b,0,\mu)$ is of order $1/N_c$. However, an explicit computation of the operator product using the identities in Ref. [18] gives

$$\{G^{jb},[G^{ia},[J^2,G^{jb}]]\}=-\{J^2,G^{ia}\}+\frac{1}{2}(N_f+N_c)J^i T^a$$
$$-\frac{1}{2}(N_f-2)G^{ia}, \qquad (39)$$

which is only of order N_c^2, using the N_c-counting rules in Eq. (28). The order N_c^3 part of Eq. (39) vanishes, which is a new cancellation in the one-loop correction to the axial vector current. Consequently, Eq. (37) is of order N_c^0 rather than order N_c, and is consistent with being a quantum correction.

VI. OTHER CONTRIBUTIONS

We have computed the chiral logarithmic correction to the axial vector current from Figs. 1(a), 1(b), and 1(c). There is also the contribution from Fig. 1(d), which is

$$\delta A^{ia}=-\frac{1}{2}\sum_b [T^b,[T^b,A^{ia}]]I(m_b), \qquad (40)$$

where

FIG. 7. One-loop correction to the baryon mass.

$$I(m_b) = \frac{i}{f^2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_b^2} = \frac{m_b^2}{16\pi^2 f^2} (\ln m_b^2 / \mu^2 - 1).$$

$$(41)$$

This contribution is of order $1/N_c$ relative to the tree-level contribution, and does not involve any cancellations between Δ and nucleon states.

In addition to the loop corrections, one has the contribution from low-energy constants multiplying higher dimension operators in the heavy baryon chiral Lagrangian. These terms are analytic in the quark mass m_q. The analytic contributions from the chiral Lagrangian can be of order N_c, i.e. the same order in N_c as the tree-level contribution.

VII. CONCLUSIONS

We have shown how to rewrite loop corrections in heavy baryon chiral perturbation theory so as to include the full dependence on the Δ-nucleon mass difference, while at the same time including the cancellations that follow from the large-N_c spin-flavor symmetry of baryons. The treatment in this paper has included the decuplet-octet mass difference, but neglected the $SU(3)$ splittings of the octet and decuplet baryons. It is possible to generalize our analysis by including the $SU(3)$ mass splittings in the baryon mass operator $\mathcal{M}$.

The one-loop correction to the baryon axial vector currents is very sensitive to deviations of the axial couplings from their $SU(6)$ symmetry ratios, since the correction that depends only on the $SU(6)$ coupling ratios (the GGG term) is suppressed by $1/N_c^2$, and the first subleading correction (the $GGJT$ term) is suppressed by $1/N_c$. Thus, the normally second subleading terms with two powers of J in the axial
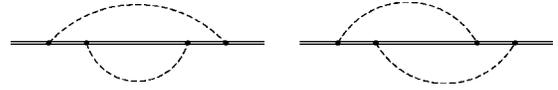
FIG. 8. Two-loop correction to the baryon mass.

vector currents are as important as these two contributions. We also have found a new cancellation in the one-loop correction to the baryon axial vector current in the term linear in the baryon mass splittings.

The large-N_c cancellations play an important role in the one-loop corrections to the axial vector current, and become more important at higher loops. They also play an important role in the one-loop corrections to other baryon properties, such as the baryon masses [3,4,14,20]. At one loop, the $m_q^{3/2}$ correction to the baryon mass from Fig. 7 is of order N_c, the same order as the tree-level baryon mass term, and there is no cancellation between nucleon and Δ states. However, at two loops, the graphs in Fig. 8 produce $m_q^{5/2}$ corrections to the baryon mass that are formally of order N_c^2, but have cancellations which make the net correction of order one.

ACKNOWLEDGMENTS

[1] E. Jenkins and A. V. Manohar, talk presented at the Workshop on Effective Field Theories of the Standard Model, Dobogoko, Hungary, 1991.

[2] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); **259**, 353 (1991).

[3] R. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993); E. Jenkins, *ibid.* **315**, 441 (1993).

[4] R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994).

[5] A. V. Manohar, Nucl. Phys. **B248**, 19 (1984).

[6] M. N. Butler, M. J. Savage, and R. P. Springer, Nucl. Phys. **B399**, 69 (1993).

[7] T. D. Cohen, Phys. Lett. B **359**, 23 (1995); hep-ph/9512275.

[8] T. D. Cohen and W. Broniowski, Phys. Lett. B **292**, 5 (1992).

[9] V. Bernard, N. Kaiser, and U. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).

[10] R. Flores-Mendieta, C. P. Hofmann, and E. Jenkins, Phys. Rev. D **61**, 116014 (2000).

[11] G. L. Keaton, Phys. Lett. B **372**, 150 (1996).

[12] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976); E. Witten, Nucl. Phys. **B156**, 269 (1979); G. Veneziano, *ibid.* **B159**, 213 (1979); P. Di Vecchia, Phys. Lett. **85B**, 357 (1979).

[13] A. Pich and E. de Rafael, Nucl. Phys. **B367**, 313 (1991).

[14] E. Jenkins, Phys. Rev. D **53**, 2625 (1996).

[15] E. Jenkins, Annu. Rev. Nucl. Part. Sci. **48**, 81 (1998); A. V. Manohar, 1997, *Large N QCD*, in *Probing the Standard Model of Particle Interactions*, edited by R. Gupta, A. Morel, E. de Rafael, and F. David, Les Houches Session LXVIII (Elsevier Science, Amsterdam, 1999), p. 1091, hep-ph/9802419.

[16] M. A. Luty and J. March-Russell, Nucl. Phys. **B426**, 71 (1994).

[17] C. Carone, H. Georgi, and S. Osofsky, Phys. Lett. B **322**, 227 (1994).

[18] R. F. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995).

[19] A. V. Manohar, Phys. Lett. B **336**, 502 (1994).

[20] Y. Oh and W. Weise, Eur. Phys. J. A **4**, 363 (1999).