Light sterile neutrino in the top-flavor model

Ehab Malkawi*

Department of Physics, Jordan University of Science & Technology, Irbid 22110, Jordan

E. I. Lashin

Department of Physics, Ain Shams University, Cairo, Egypt

Hatem Widyan

Department of Physics, Jordan University of Science & Technology, Irbid 2210, Jordan

A scenario based on the top-flavor model is presented to explain the origin of a light sterile neutrino as indicated by all combined neutrino oscillation experiments. The model is phenomenologically well motivated and compatible with all available low-energy data. The derived neutrino mass matrix can qualitatively explain the observed hierarchy in the neutrino mass splittings as indicated by the neutrino oscillation data. Quantitative results are obtained for special cases.

I. INTRODUCTION

Recently, hints for new physics beyond the standard model (SM) have been advocated through the neutrino sector. The recent observation by the Super-Kamiokande Collaboration of the atmospheric zenith angle dependent deficit [1] has strengthened this conclusion. Also the long term puzzle of the solar neutrino deficit [2] has been a strong demonstration of the existence of new physics. Recent results on neutrino oscillation have also been reported by the Liquid Scintillator Neutrino Detector (LSND) experiment [3]. All of the observed anomalies in the neutrino data can be explained in terms of massive neutrino oscillations.

The theoretical picture for neutrino oscillation poses a real challenge in understanding the form of the lepton mass matrix as derived from the neutrino oscillation data. A definite picture is obscure due to the large number of free parameters in the neutrino and charged lepton mass matrices. The phenomenological solution is not unique and clearly the need for more data and theoretical breakthrough is essential.

Neutrino oscillation can explain both the solar and atmospheric data in terms of three-generation neutrinos [4] (ignoring the LSND results [3].) In the simplest explanation picture, the solar neutrino data can be understood in terms of $\nu_e \leftrightarrow \nu_\mu$ oscillation with a small mass splitting not influencing atmospheric data. On the other hand, atmospheric data can be explained in terms of $\nu_\mu \leftrightarrow \nu_\tau$ large mixing with a large mass splitting compared to the solar case [4]. However, if we combine the LSND result with the solar and atmospheric data then we have to include at least an extra light neutrino. The full oscillation data require the existence of three different scales of neutrino mass-squared differences. The different scales can be accommodated only if at least a light fourth neutrino exists. Such a light neutrino has to be sterile, i.e., to decouple from the low-energy observables as indicated by the measurement of the Z gauge boson width [5]. Either the atmospheric-neutrino or the solar-neutrino

mixing must involve the sterile neutrino. The solar large mixing angle solution is only valid for mixing with active neutrinos [6]. The zenith-angle dependence, for the atmospheric data, significantly favors the $\nu_\mu \leftrightarrow \nu_\tau$ channel over the $\nu_\mu \leftrightarrow \nu_s$ channel when matter effects are taken into account [7]. Some recent phenomenological studies indicate that in a minimal scheme the dominant transition of solar neutrinos is due to $\nu_e \leftrightarrow \nu_s$ mixing, while the dominant transition of atmospheric neutrinos in long-baseline experiments is due to $\nu_\mu \leftrightarrow \nu_\tau$ mixing [8].

The inclusion of a sterile neutrino still poses another theoretical challenge: namely, to understand both the origin of the sterile fermion and the very low mass it has. The small mass is probably the most difficult issue in introducing such a particle. Any successful scenario has to explain the tiny mass of the sterile neutrino in a natural way. A possible scenario would be to generate the light mass through radiative corrections [9]. Another interesting scenario would be to postulate that the extra neutrino is active at a relatively high-energy scale. At that scale the extra neutrino is massless due to an assumed gauge symmetry which forbids its mass generation. Once the high-energy symmetry is broken (probably in the TeV region), a Dirac mass can be generated while the neutrino becomes sterile. Finally, by invoking the seesaw mechanism we can understand the highly suppressed Majorana mass of the sterile neutrino.

In this work we consider the possibility of understanding the origin of a light sterile neutrino through the second scenario. A similar model has been discussed by Brahmachari [10]; however, his model suffers from theoretical anomalies. Furthermore, an explicit formulation of the model is highly complicated. The model we discuss in this work does not suffer from such theoretical drawbacks. It is based on the gauge nonuniversal symmetry $SU(3)_c \times SU(2)_l \times SU(2)_h \times U(1)_Y$ discussed extensively in Refs. [11,12]. We refer to this model as the top-flavor model. The model is anomaly free and phenomenologically well motivated as discussed by several recent phenomenological studies [12]. To account for the existence of the sterile neutrino, we modify the standard fermion content by the inclusion of a few extra fermions.

*Email address: malkawie@just.edu.jo

The extra fermion spectrum does not appreciably affect the low-energy regime because of the heavy mass of the extra active fermions, to be discussed later. Only one neutral fermion emerges with a small mass while decoupling from the low-energy observables which we then call the sterile neutrino. The inclusion of exotic fermions which span generations has been previously discussed in Ref. [13].

The rest of this paper is organized as follows. In Sec. II, we briefly review the model. In Sec. III, we enlarge the fermion spectrum by introducing extra fermions and discuss the mechanism for generating the mass of the sterile and active fermions. Finally, in Sec. IV we provide some quantitative discussion and conclusions.

II. STRUCTURE OF THE MODEL

The top-flavor model [11,12] is based on the gauge symmetry $G = SU(3)_c \times SU(2)_l \times SU(2)_h \times U(1)_Y$. In this model, the third generation of fermions (top quark t, bottom quark b, tau lepton τ, and its neutrino ν_τ) is subjected to a new gauge interaction at the high-energy scale, instead of the usual weak interaction advocated by the SM of the electroweak interaction. On the contrary, the first and second generations only feel the weak interaction supposedly equivalent to the SM case. The new gauge dynamics is attributed to the $SU(2)_h$ symmetry under which the left-handed fermions of the third generation transform in the fundamental representation (doublets), while they remain singlets under the $SU(2)_l$ symmetry. On the other hand, the left-handed fermions of the first and second generation transform as doublets under the $SU(2)_l$ group and singlets under the $SU(2)_h$ group. The $U(1)_Y$ group is the SM hypercharge group. The right-handed fermions only transform under the $U(1)_Y$ group as assigned by the SM. Finally, the QCD interactions and the color symmetry $SU(3)_c$ are the same as that in the SM.

The spontaneous symmetry breaking of the group $G = SU(3)_c \times SU(2)_l \times SU(2)_h \times U(1)_Y$ is accomplished by introducing the complex scalar fields Σ, Φ_1, and Φ_2, where $\Sigma \sim (1,2,2,0)$, $\Phi_1 \sim (1,2,1,1)$, and $\Phi_2 \sim (1,1,2,1)$. For the Σ field we explicitly write

$$\Sigma = \begin{pmatrix} \pi_1^0 & \pi_1^+ \\ \pi_2^- & \pi_2^0 \end{pmatrix}, \qquad (1)$$

where all scalar fields are taken to be complex. The group G is then broken at three different stages. The first stage of symmetry breaking is accomplished once the Σ field acquires a vacuum expectation value (VEV) u, i.e., $\langle \Sigma \rangle = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}$, where u is taken to be real. The form of $\langle \Sigma \rangle$ guarantees the breakdown of $SU(2)_l \times SU(2)_h \rightarrow SU(2)$. Therefore, the unbroken symmetry is essentially the SM gauge symmetry $SU(3)_c \times SU(2)_w \times U(1)_Y$, where $SU(2)_w$ is the usual SM weak group. At this stage, three of the gauge bosons acquire a mass of the order u, while the other gauge bosons remain massless. The phenomenology of the model imposes the constraint $u \gtrsim 1$ TeV [12]. The second and third stage of symmetry breaking (the electroweak symmetry breaking) is accomplished through the scalar fields Φ_1 and

Φ_2 by acquiring their vacuum expectation values $\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ and $\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$, respectively. The electroweak symmetry-breaking scale v is defined as $v \equiv \sqrt{v_1^2 + v_2^2} = 246$ GeV. Since the third generation of fermions is heavier than the first two generations, it is suggestive to conclude that $v_2 \gg v_1$. The surviving symmetry at low energy is $SU(3)_c \times U(1)_{em}$ gauge symmetry. Because of this pattern of symmetry breaking, the gauge couplings g_l, g_h, and g_Y of $SU(2)_l$, $SU(2)_h$, and $U(1)_Y$, respectively, are related to the $U(1)_{em}$ gauge coupling e by the relation $1/e^2 = 1/g_l^2 + 1/g_h^2 + 1/g_Y^2$ [12]. We define

$$g_l = \frac{e}{\sin\theta\cos\phi}, \quad g_h = \frac{e}{\sin\theta\sin\phi}, \quad g_Y = \frac{e}{\cos\theta}, \quad (2)$$

where θ is the usual weak mixing angle and ϕ is a new parameter of the model.

For $g_h > g_l$ (equivalently $\tan\phi < 1$), we require $g_h^2 \lesssim 4\pi$ [which implies $\sin^2\phi \gtrsim g^2/(4\pi) \sim 0.04$] so that the perturbation theory is valid. Similarly, for $g_h < g_l$, we require $\sin^2\phi \lesssim 0.96$. For simplicity, we focus on the region where $x \equiv u^2/v^2 \gg 1$, and ignore the corrections which are suppressed by higher powers of $1/x$. The light gauge boson masses are found to be $M_{W^\pm}^2 = M_Z^2 \cos^2\theta = M_0^2[1 + O(1/x)]$, where $M_0 \equiv ev/2\sin\theta$. For the heavy gauge bosons $W'^\pm$ and Z', one finds [12]

$$M_{W'^\pm}^2 = M_{Z'}^2 = M_0^2 \left(\frac{x}{\sin^2\phi\cos^2\phi} + O(1) \right). \qquad (3)$$

Up to this order, the heavy gauge bosons are degenerate in mass because they do not mix with the hypercharge gauge boson field. The SM fermions acquire their masses through their Yukawa interaction via the Φ_1 and Φ_2 scalar fields. For instance, the leptonic Yukawa sector is given by

$$\mathcal{L}_Y^l = \overline{\Psi}_L^1 \Phi_1 [g_{11}^e e_R + g_{12}^e \mu_R + g_{13}^e \tau_R]$$
$$+ \overline{\Psi}_L^2 \Phi_1 [g_{21}^e e_R + g_{22}^e \mu_R + g_{23}^e \tau_R]$$
$$+ \overline{\Psi}_L^3 \Phi_2 [g_{31}^e e_R + g_{32}^e \mu_R + g_{33}^e \tau_R] + \text{H.c.}, \qquad (4)$$

where

$$\Psi_L^1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \Psi_L^2 = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \text{and} \quad \Psi_L^3 = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}. \qquad (5)$$

The phenomenology of the model has been studied extensively in Refs. [11,12].[1] Comparisons with the CERN Large Electron Positron (LEP) and other low-energy data have been investigated and constraints on the heavy gauge boson mass are reported as $M_{W'} \gtrsim 1$ TeV. The parameter $x \equiv u^2/v^2$ is constrained by LEP data to be larger than 20. Other low-energy data such as the τ lifetime imposes a

---

[1]Although some differences in the scalar sector exist among those references.

higher constraint on x for specific scenarios of lepton mixing. Flavor changing neutral current effects in the lepton and quark sectors have been explored and contributions to different processes have been calculated [12]. Therefore, in this work we only concentrate on the leptonic mass matrix and refer the reader to Refs. [11,12] for a detailed study of the phenomenology of the model.

III. EXTRA FERMIONS

To explain the existence of the light sterile neutrino in our scenario we enlarge the particle spectrum by the inclusion of extra fermions. We consider the minimal number of particles needed to account for the existence of the light sterile neutrino without introducing anomalies in the structure of the model. Furthermore, consistency with low-energy data should be maintained and therefore all extra active fermions should decouple from the low-energy regime.

At the high-energy scale the gauge symmetry is assumed to be $G = SU(3)_c \times SU(2)_l \times SU(2)_h \times U(1)_Y$. At a lower scale, $\langle \Sigma \rangle \sim$ a few TeV, the gauge symmetry is broken into the SM symmetry group $H_1 = SU(3)_c \times SU(2)_w \times U(1)_Y$. At the electroweak scale the final stage of symmetry breaking occurs and the surviving symmetry group is $H_2 = SU(3)_c \times U(1)_{em}$. The fermion spectrum includes the standard three fermion generations with the transformation, under G, as explained in Sec. II.

We enlarge the fermion spectrum by the inclusion of three sets of extra fermions as follows:

(a) To explain the extremely light neutrino masses we invoke the seesaw mechanism. Therefore, four right-handed neutrinos are introduced, ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$, and ν_{sR}. The four right-handed neutrinos are singlets under G and are assumed to be Majorana fermions with masses of the order of the grand unified theory (GUT) scale, $M_X \sim 10^{15}$ GeV.

(b) We introduce a bi-doublet fermion field S_L with the transformation $S_L \sim (1,2,2,0)$. Explicitly, $S_L = \nu_{sL} + S_L^a \tau^a$ transforms, under G, as

$$S_L \rightarrow g_1 S_L g_2^{\dagger}, \tag{6}$$

where $g_1 \in SU(2)_l$ and $g_2 \in SU(2)_h$. Once the symmetry group G is broken down into the symmetry group H_1, the field S_L decomposes into two parts, with transformation under H_1 as $(1,1,0) + (1,3,0)$. The neutral field with the transformation $(1,1,0)$ corresponds to the sterile neutrino. The triplet field remains active and thus must acquire a heavy Dirac mass in order to be consistent with the low-energy data [5]. To prevent the uncontrolled Majorana mass we assume that S_L carries a conserved quantum number z_L at the high-energy scale. The new quantum number is due to a global $U(1)$ symmetry which causes no harmful anomaly to spoil the foundation of the model [14].

(c) We introduce another triplet fermion field $S_R = S_R^a \tau^a$ with the transformation, under G, as $(1,1,3,0)$ just for the purpose of giving a Dirac mass to the active triplet field of S_L. Similar to S_L, the field S_R is assumed to carry a quantum number z_R to prevent its unwanted Majorana mass term.

Hence, a Dirac mass for the extra active fermions is generated through the Yukawa interaction term

$$\mathcal{L}_a = \frac{g_a}{2} \text{Tr}[\overline{S_L} \Sigma S_R], \tag{7}$$

where $g_a \sim 1$ is a Yukawa coupling. Once the scalar field Σ acquires its VEV $u \gtrsim 1$ TeV, a Dirac mass for the triplet fermions is generated, $m_a = g_a u \gtrsim 1$ TeV. In order for the Yukawa term in Eq. (7) to conserve the assumed global $U(1)$ symmetry, we require Σ to carry a quantum number z_0 such that $z_L = z_R + z_0$.

The sterile neutrino ν_{sL} acquires its Dirac mass through the Yukawa term

$$\mathcal{L}_s = \frac{g_s}{2} \text{Tr}[\overline{S_L} \tau_2 \Sigma^* \tau_2] \nu_{sR}, \tag{8}$$

where $g_s \sim 1$ is a Yukawa coupling. The Yukawa terms in Eqs. (7),(8) conserve the new quantum number provided that we demand $z_L = -z_0 = z_R/2$. It is important to notice that we use the general form of Σ as given in Eq. (1). A simpler choice would be

$$\Sigma = \pi^0 + i\pi^a \tau_a = \begin{pmatrix} \pi^0 + i\pi^3 & i\sqrt{2}\pi^+ \\ i\sqrt{2}\pi^- & \pi^0 - i\pi^3 \end{pmatrix}, \tag{9}$$

where π^0 and π^a are taken to be real fields. However, for this particular choice, $\tau_2 \Sigma^* \tau_2 = \Sigma$ and therefore, the Yukawa terms in Eqs. (7),(8) are not simultaneously invariant under the assumed global $U(1)$ symmetry. It is important to emphasize that the extra active fermions S_L and S_R do not mix with the standard fermions due to the assumed global $U(1)$ symmetry. To conclude, the extra fields we introduce are the minimal number of fields required to account for the existence of a light sterile neutrino without spoiling the accuracy of the low-energy data, and also without introducing any theoretical anomalies into the model.

The full neutrino Yukawa interaction terms are given as

$$\mathcal{L}_Y^\nu = \overline{\Psi_L^1} \tilde{\Phi}_1 [g_{11}^\nu \nu_{eR} + g_{12}^\nu \nu_{\mu R} + g_{13}^\nu \nu_{\tau R} + g_{14}^\nu \nu_{sR}]$$

$$+ \overline{\Psi_L^2} \tilde{\Phi}_1 [g_{21}^\nu \nu_{eR} + g_{22}^\nu \nu_{\mu R} + g_{23}^\nu \nu_{\tau R} + g_{24}^\nu \nu_{sR}]$$

$$+ \overline{\Psi_L^3} \tilde{\Phi}_2 [g_{31}^\nu \nu_{eR} + g_{32}^\nu \nu_{\mu R} + g_{33}^\nu \nu_{\tau R} + g_{34}^\nu \nu_{sR}]$$

$$+ \frac{1}{2}\text{Tr}[\overline{S_L}\tau_2\Sigma^*\tau_2][g_{41}^\nu \nu_{eR} + g_{42}^\nu \nu_{\mu R} + g_{43}^\nu \nu_{\tau R} + g_{44}^\nu \nu_{sR}]$$

$$+ \text{H.c.}, \tag{10}$$

where $\tilde{\Phi}_{1,2} \equiv i\tau_2 \Phi_{1,2}^*$. The Dirac mass matrix, derived from Eq. (10), is written as

$$M_D = \begin{pmatrix} g_{11}^\nu v_1 & g_{12}^\nu v_1 & g_{13}^\nu v_1 & g_{14}^\nu v_1 \\ g_{21}^\nu v_1 & g_{22}^\nu v_1 & g_{23}^\nu v_1 & g_{24}^\nu v_1 \\ g_{31}^\nu v_2 & g_{32}^\nu v_2 & g_{33}^\nu v_2 & g_{34}^\nu v_2 \\ g_{41}^\nu u & g_{42}^\nu u & g_{43}^\nu u & g_{44}^\nu u \end{pmatrix}. \tag{11}$$

The right-handed neutrino Majorana mass matrix M_R is assumed to have a common mass scale of the order of the GUT scale, $M_X \sim 10^{15}$ GeV. Therefore, the full neutrino mass matrix forms an 8×8 matrix which can be written as

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \tag{12}$$

By invoking the seesaw mechanism the left-handed neutrino Majorana mass matrix is then given as

$$M_L = M_D M_R^{-1} M_D^T. \tag{13}$$

Due to the seesaw mechanism all elements of M_L are highly suppressed by the GUT scale M_X of the right-handed Majorana mass matrix M_R. Thus, we introduce a light sterile neutrino with a natural mechanism for generating its tiny mass as required by the neutrino oscillation data. In the next section we give a further discussion of the derived neutrino mass matrix.

IV. DISCUSSION AND CONCLUSIONS

The mass matrix in Eq. (11) is written in its most general form. A quantitative analysis is attainable only if the structure of the mass matrix is fully determined, which requires further theoretical assumptions to be incorporated in the structure of the model. Nevertheless, the structure of the mass matrix already suggests an interesting behavior. There are three hierarchical energy scales in the mass matrix M_D, namely, $u \gg v_2 \gg v_1$, which could be connected to the observed three hierarchical mass scales in the neutrino data, namely, $\Delta m_{\text{LSND}}^2 \gg \Delta m_{\text{atm.}}^2 \gg \Delta m_{\text{solar}}^2$. It is suggestive to conclude that

$$\Delta m_{\text{LSND}}^2 \sim \left(\frac{u^2}{M_X} \right)^2 \approx 1 \ \text{eV}^2,$$

$$\Delta m_{\text{atm.}}^2 \sim \left(\frac{v_2^2}{M_X} \right)^2 \approx 10^{-3} \ \text{eV}^2, \tag{14}$$

$$\Delta m_{\text{solar}}^2 \sim \left(\frac{v_1^2}{M_X} \right)^2 \approx 10^{-5} \ \text{eV}^2,$$

as indicated by the neutrino oscillation data [1–4]. In fact, from the LEP data, we already know that $u \gtrsim v \sqrt{20} \approx 1.2$ TeV [12]. The observed mass scales can be obtained if we simply choose $u \approx 1.2$ TeV and $v_2 \approx 230$ GeV, from which we conclude that $v_1 \approx 75$ GeV, $M_X \approx (1-10) \times 10^{15}$ GeV, and

TABLE I. Some numerical values for the mass matrix elements and the predicted mass splitting and mixing for the case of $\nu_e \leftrightarrow \nu_s$ transition. We take $u = 1200$ GeV, $M_X = 10^{18}$ GeV, and $v_1 = 75$ GeV.

g_1	g_2	g_3	$\sin^2 2\theta$	Δm^2
0.8	0.8	1.3	7×10^{-3}	3×10^{-6}
1.8	1.0	1.5	7×10^{-3}	4×10^{-6}
1.1	1.0	1.6	5×10^{-3}	5×10^{-6}

$$\frac{\Delta m_{\text{LSND}}^2}{\Delta m_{\text{solar}}^2} \sim \frac{u^4}{v_2^4} \approx 10^5,$$

$$\frac{\Delta m_{\text{LSND}}^2}{\Delta m_{\text{atm.}}^2} \sim \frac{u^4}{v_1^4} \approx 10^3, \tag{15}$$

$$\frac{\Delta m_{\text{atm.}}^2}{\Delta m_{\text{solar}}^2} \sim \frac{v_2^4}{v_1^4} \approx 10^2.$$

In the simplest scheme where oscillation data can be explained in terms of two flavor mixing, it has been argued that the dominant transition of solar neutrino is due to $\nu_e \leftrightarrow \nu_s$ mixing [8]. Such a picture can hardly be satisfied by our model with the above choice of parameters and without the need for fine-tuning. In the case of $\nu_e \leftrightarrow \nu_s$ mixing one can show that the effective 2×2 mass matrix is given as

$$M_L = \frac{1}{M_X} \begin{pmatrix} g_1 v_1^2 & g_2 v_1 u \\ g_2 v_1 u & g_3 u^2 \end{pmatrix}, \tag{16}$$

where the couplings $g_{1,2,3} \sim 1$. Therefore, one can show that the solar mass splitting is given as $\Delta m_{\text{solar}}^2 \approx u^4/M_X^2$, which is many orders of magnitude larger than the experimental fit [2,4]. If we take M_X to be of order 10^{18} GeV, which is close to the Planck scale, we get a result consistent with the experimental fit as shown in Table I, where we consider the numerical values $u = 1200$ GeV, $M_X = 10^{18}$ GeV, and $v_1 = 75$ GeV. In Table I, the numerical values of the Yukawa couplings as well as their solar neutrino solution are given. However, such a solution is not favored by the combined data as we cannot explain the apparent hierarchy among the solar, atmospheric, and LSND data.

In conclusion we have provided a scenario based on the top-flavor model to explain the existence of a light sterile neutrino. The scenario is anomaly free and phenomenologically compatible with all existing low-energy data. The scenario can also qualitatively explain the hierarchy in the observed mass scales of the neutrino oscillation data. Quantitative results are obtained for special cases.

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); **81**, 1158 (1998); Kamiokande Collaboration, S. Hatakeyama *et al.*, *ibid.* **81**, 2016 (1998).

[2] Gallex Collaboration, W. Hampel *et al.*, Phys. Lett. B **388**, 384 (1996); SAGE Collaboration, D. N. Abdurashitov *et al.*, Nucl. Phys. B (Proc. Suppl.) **48**, 299 (1996); SAGE Collaboration, V. Gavrin *et al.*, Phys. Lett. B **328**, 234 (1994); Homestake Collaboration, Nucl. Phys. **B38**, 47 (1995); Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996).

[3] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **81**, 1774 (1998); LSND Collaboration, I. Stancu, Prog. Part. Nucl. Phys. **40**, 167 (1998).

[4] R. Barbieri *et al.*, J. High Energy Phys. **12**, 017 (1998), and references therein; S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1986); G. Fogli, E. Lisi, and D. Montanino, Phys. Rev. D **54**, 2048 (1996); S. Bilenky, C. Giunti, and C. Kim, Astropart. Phys. **4**, 241 (1996); T. Teshima, T. Sakai, and O. Ingaki, Int. Mod. Phys. A **14**, 1953 (1999); M. C. Gonzalez-Garcia, P. C. de Holanda, C. Pena-Garay, and J. W. F. Valle, Nucl. Phys. **B573**, 3 (2000); E. Malkawi, Phys. Rev. D **61**, 013006 (2000).

[5] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[6] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).

[7] R. G. H. Robertson, hep-ex/0001034, and references therein.

[8] S. M. Bilenky and C. Giunti, hep-ph/9905246; S. M. Bilenky, C. Giunti, W. Grimus, and T. Schwetz, hep-ph/9904316.

[9] Yutaka Okamoto and Masaki Yasue, Prog. Theor. Phys. **101**, 1119 (1999); N. Gaur, A. Ghosal, E. Ma, and P. Roy, Phys. Rev. D **58**, 071301 (1998).

[10] Biswajoy Brahmachari, Phys. Lett. B **461**, 243 (1999).

[11] X. Li and E. Ma, Phys. Rev. Lett. **47**, 1788 (1981); E. Ma, X. Li, and S. F. Tuan, *ibid.* **60**, 495 (1988); X. Li and E. Ma, Phys. Rev. D **46**, 1905 (1992); J. Phys. G **19**, 1265 (1993).

[12] E. Malkawi, T. Tait, and C.-P. Yuan, Phys. Lett. B **385**, 304 (1996); D. J. Muller and S. Nandi, *ibid.* **383**, 345 (1996); J. C. Lee and K. Y. Lee, Phys. Rev. D **58**, 115001 (1998); J. C. Lee, K. Y. Lee, and J. K. Kim, Phys. Lett. B **424**, 133 (1998); E. Malkawi and C.-P. Yuan, Mod. Phys. Lett. A **14**, 1487 (1999); E. Malkawi and C.-P. Yuan, Phys. Rev. D **61**, 015007 (2000).

[13] E. Ma and S. F. Tuan, Phys. Rev. D **26**, 713 (1982).

[14] Several text books on quantum field theory address this issue, e.g., see George Sterman, *An Introduction to Quantum Field Theory* (Cambridge University Press, Cambridge, England, 1993).