Chiral symmetry in the linear sigma model in a magnetic environment

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We study the chiral symmetry structure in a linear sigma model with fermions in the presence of an external, uniform magnetic field in the ''effective potential'' approach at the one loop level. We also study the chiral phase transition as a function of density in the core of magnetized neutron stars.

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In the absence of a quark mass matrix, QCD is invariant under chiral transformation at the Lagrangian level. However, the dynamics of QCD are expected to be such that chiral symmetry is spontaneously broken with the vacuum state having a nonzero quark-antiquark condensate $\langle 0|\overline{q}q|0\rangle$ and the Goldstone theorem then requires the existence of approximately massless pseudoscalar mesons in the hadron spectrum. At high temperatures and/or at high densities, the quark condensates are expected to melt at some critical point and chiral symmetry is restored [1]. The chiral phase transition and phenomenological consequences in the form of experimentally observable signatures have been extensively discussed in the literature [2]. It has also been suggested [3,4] that systems with spontaneously broken symmetries may make a transition from a broken symmetric to restored symmetric phase in the presence of external fields. It has been shown [5,6] that there exist some realistic models where the symmetry restoration takes place. In QED uniform, external static magnetic field is known to break chiral symmetry dynamically at weak gauge couplings [7].

Large magnetic fields with strengths up to 10^{18} G have been conceived to exist [8,9] at the time of supernova collapse inside neutron stars and in other astrophysical compact objects and in the early Universe. Effect of such a strong magnetic field on chiral phase transition is thus of great interest for baryon free quark matter in the early universe and for high density baryon matter in the core of neutron stars. To study chiral phase transition in QCD we need a nonperturbative treatment. Lattice techniques and the Schwinger-Dyson equations provide specially powerful methods to study the chiral structure of QCD [10]. Klevansky and Lemmer [11] studied the chiral symmetry behavior of hadronic system described by Nambu–Jona-Lasinio (NJL) model minimally coupled to a constant electromagnetic field. Solving the gap equation they found that whereas a constant electric field restores chiral symmetry, a constant magnetic field inhibits the phase transition by stabilizing the chirally broken vacuum state. This conclusion was confirmed by later studies of the NJL model [12]. Shushpanov and Smilga [13] considered QCD with the massless flavors in the leading order of chiral perturbation theory and studied the dependence of quark condensate on an external magnetic field by studying the Schwinger-Dyson equation and showed that an external magnetic field increases the condensate.

A particularly attractive framework to study such systems is the linear sigma model originally proposed as a model for strong nuclear interactions [14]. We will consider this as an effective model for low energy phase of QCD and will examine the chiral symmetry properties at finite density and in the presence of external magnetic field. To fix ideas we consider a two flavor $SU(2)\times SU(2)$ chiral quark model given by the Lagrangian

$$\mathcal{L}=i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-g\overline{\psi}(\sigma+i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi+\frac{1}{2}(\partial_{\mu}\sigma)^2+\frac{1}{2}(\partial_{\mu}\vec{\pi})^2 - U(\sigma,\vec{\pi}), \tag{1}$$

where ψ is the quark field σ and π are the set of four scalar fields and g is the quark meson coupling constant. The potential $U(\sigma,\vec{\pi})$ is given by

$$U(\sigma,\vec{\pi})=-\frac{1}{2}\mu^2(\sigma^2+\vec{\pi}^2)+\frac{1}{4}\lambda(\sigma^2+\vec{\pi}^2)^2. \tag{2}$$

For $\mu^2>0$ chiral symmetry is spontaneously broken. The σ field can be used to represent the quark condensate, the order parameter for chiral phase transition and the pions are the Goldstone bosons. At the tree level the sigma, pion and the quark masses are given by

$$m_{\sigma}^2=3\lambda\sigma_{cl}^2-\mu^2,\quad m_{\pi}^2=\lambda\sigma_{cl}^2-\mu^2,\quad m_{\psi}^2=g\sigma_{cl}, \tag{3}$$

where $\sigma_{cl}^2=\mu^2/\lambda=f_{\pi}^2$. An elegant and efficient way to study symmetry properties of the vacuum at finite temperature, density and in the presence of external fields is through the ''effective potential'' approach discussed extensively in the literature [15]. We will compute here, in the one loop approximation, the effective potential in the presence of external magnetic field which is defined through an effective action $\Gamma(\sigma,B)$ which is the generating functional of the one particle irreducible graphs. The effective potential is then given by

$$V_{eff}(\sigma,B)=V_0(\sigma)+V_1(\sigma,B), \tag{4}$$

where $V_1(\sigma,B)$ is obtained from the propagator function $G(\sigma,B)$ by the usual relation $V_1(\sigma,B) =-(1/2i)\,\mathrm{Tr}\log G(\sigma,B)$.

Alternatively one can compute the shift in the vacuum energy density due to zero-point oscillations of the fields

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considered as an ensemble of harmonic oscillators [15]. We thus require energy eigenvalues (excitations) of particles in the magnetic field, which can be easily obtained, and in the absence of anomalous magnetic moment for uniform static magnetic field in the z-direction for a particle of mass M, charge q, and spin J, are given by [16]

$$E(k_z,n,J_z)=(k_z^2+M^2+[2n+1-\mathrm{sgn}(q)j_z]|q|B)^2, \quad (5)$$

where n represents the Landau level. Contribution of scalar particles of mass M to $V_1(M^2)$ after Wick rotation is thus given by

$$V_1(M^2)=\frac{1}{2}\int\frac{d^4k_e}{(2\pi)^4}\ln(k_e^2+M^2-i\epsilon). \quad (6)$$

In the presence of magnetic field, all we need to do is to replace the phase space integral $\int[d^4k_e/(2\pi)^4]$ by $(eB/2\pi)\Sigma_{n=0}^{\infty}[d^2k_e/(2\pi)^2]$ and the energy by expression (5) for charged particles. For a scalar field of charge $\pm e$, we thus have

$$\begin{aligned}V_1(M^2,B)&=\frac{eB}{4\pi}\sum_{n=0}^{\infty}\int\frac{d^2k_e}{(2\pi)^2}\ln(k_e^2+(2n+1)eB+M^2)\\&=-\frac{eB}{4\pi}\frac{\partial}{\partial\alpha}\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)(4\pi)^{d/2}}\\&\quad\times\sum_{n=0}^{\infty}\frac{1}{(M^2+eB+2neB)^{\alpha-d/2}}\Bigg|_{\alpha=0,d=2}\\&=-\frac{eB}{4\pi}\lim_{\alpha\to0}\frac{\partial}{\partial\alpha}\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)(4\pi)^{\alpha-d/2}}\\&\quad\times\frac{1}{(2eB)^{\alpha-d/2}}\zeta\left(\alpha-\frac{d}{2},\frac{M^2+eB}{2eB}\right)\Bigg|_{\alpha=0,d=2},\end{aligned} \quad (7)$$

where $\zeta(z,q)$ is the generalized Riemann zeta function

$$\zeta(z,q)=\sum_{n=0}^{\infty}\frac{1}{(q+n)^z}=\frac{1}{\Gamma(z)}\int_0^{\infty}dt\frac{t^{z-1}e^{-qt}}{1-e^{-t}}. \quad (8)$$

The potential (7) has poles at $\alpha=0$, 1 and 2 for d=2 which can be absorbed in the counter terms. The finite part depends on the exact renormalization conditions that are imposed. In what follows we would use the modified minimal subtraction ($\overline{\mathrm{MS}}$) renormalization scheme. From Eqs. (7) and (8) we can write

$$\begin{aligned}V_1(M^2,B)=&-\frac{eB}{32\pi^2}\lim_{\alpha\to0}\frac{\partial}{\partial\alpha}\frac{(2eB)^{1-\alpha}}{\Gamma(\alpha)}\\&\times\int dt\,t^{\alpha-2}\frac{e^{-(M^2/2eB)t}}{\sinh\frac{t}{2}},\end{aligned} \quad (9)$$

which converges for Re $\alpha>2$. We analytically continue the result in the complex α-plane and use dimensional regularization technique to extract the finite contribution. To proceed further we first consider the case $M^2/2eB<1$, expand $e^{-(M^2/2eB)t}$ and formally integrate (9) to obtain [17]

$$\begin{aligned}V_1(M^2,B)=&-\frac{|q|B}{32\pi^2}\lim_{\alpha\to0}\frac{\partial}{\partial\alpha}\sum_{n=0}^{\infty}\left(\frac{M^2}{2eB}\right)^{\alpha+\nu-1}\\&\times\frac{(-1)^{\nu}}{\nu!(M^2)^{\alpha-1}}\frac{2}{\Gamma(\alpha)}(2^{\alpha+\nu-1}-1)\\&\times\Gamma(\alpha+\nu-1)\zeta(\alpha+\nu-1).\end{aligned} \quad (10)$$

Keeping leading terms in $M^2/2eB$ we obtain

$$\begin{aligned}V_1(M^2,B)=&-\frac{1}{16\pi^2}\left[\frac{e^2B^2}{2\pi}\zeta(2)\log2eB+\frac{eBM^2}{2}\log2\right.\\&\left.-M^4\frac{\pi}{2}\log2eB+\cdots\right].\end{aligned} \quad (11)$$

The leading term for the contribution of charged Goldstone bosons relevant for symmetry considerations is

$$V_1(M^2,B)\sim-\frac{eBM^2}{32\pi^2}\log2, \quad (12)$$

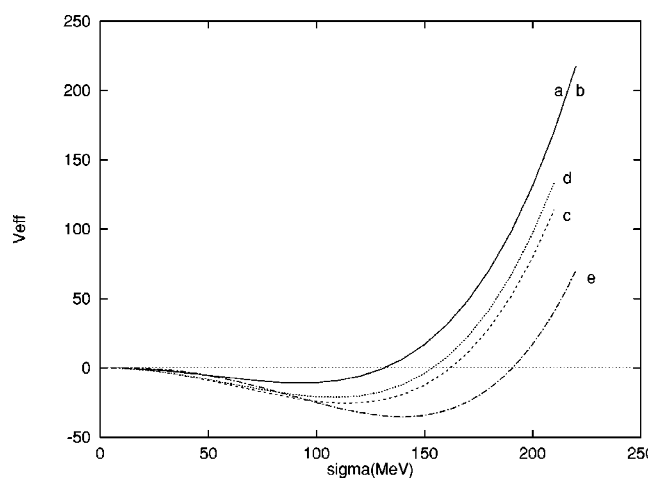

FIG. 1. Effective potential in units of $(100\ \mathrm{MeV})^4$ as a function of σ(MeV) for different values of the magnetic field. The curves a, b, c, d and e are for B=0, 10^{16},10^{18},10^{19} and 3×10^{19} G, respectively.

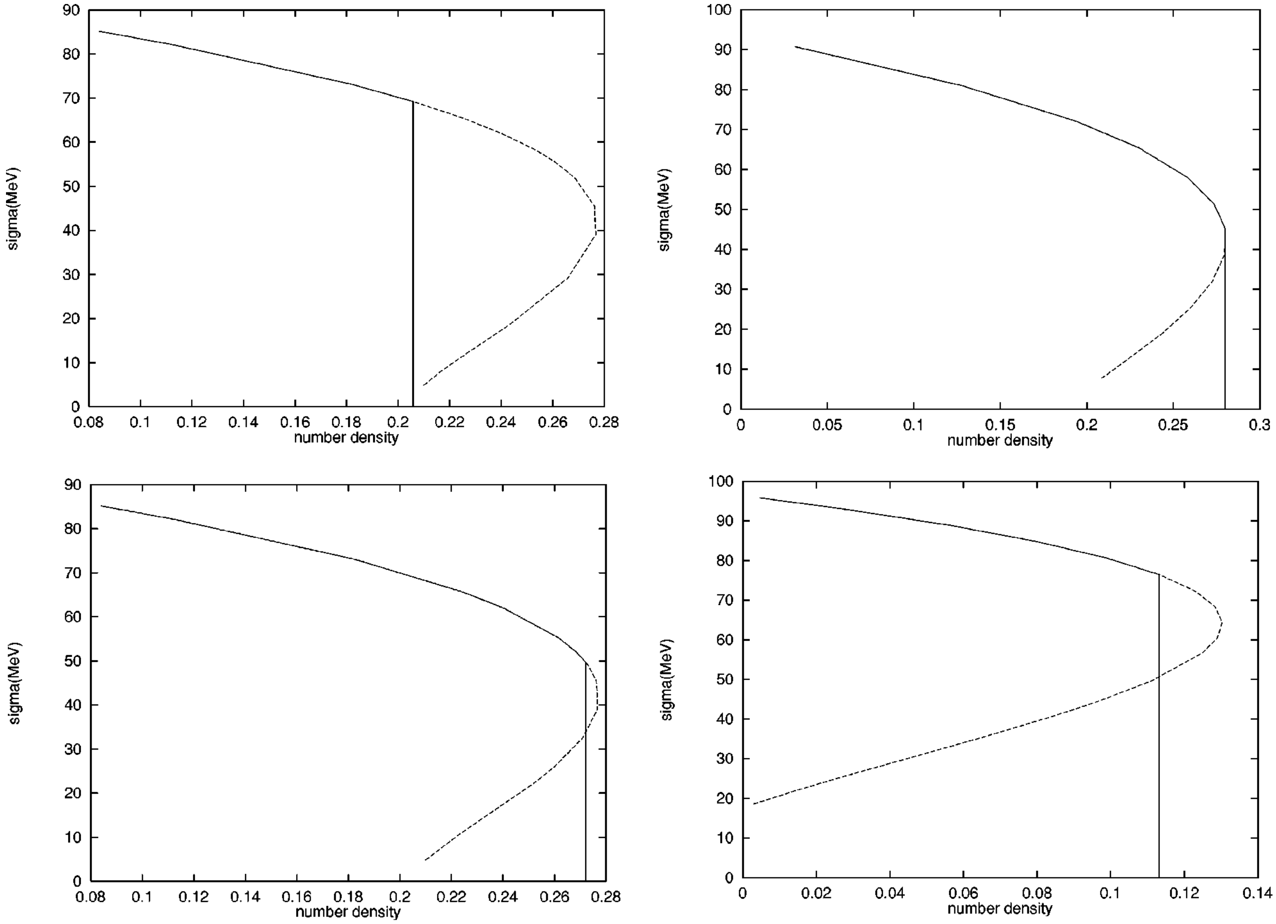

FIG. 2. (a) Chiral condensate σ(MeV) as a function of baryon density in f_m^{-3} for magnetic field B=0. (b) Chiral condensate σ(MeV) as a function of baryon density in f_m^{-3} for magnetic field B=10^{16} G. (c) Chiral condensate σ(MeV) as a function of baryon density in f_m^{-3} for magnetic field B=10^{18} G. (d) Chiral condensate σ(MeV) as a function of baryon density in f_m^{-3} for magnetic field B=10^{19} G.

which agrees with the earlier results [6,13] up to a factor of order one. For the case of $M^2/2eB>1$ we write Eq. (9) as

$$V_1(M^2,B)=-\frac{1}{32\pi^2}\lim_{\alpha\to 0}\frac{\partial}{\partial\alpha}\frac{1}{\Gamma(\alpha)}\times\int_0^\infty dx x^{\alpha-3}e^{-M^2x}\frac{eBx}{\sinh eBx} \tag{13}$$

and keeping leading terms obtain

$$V_1(M^2,B)\simeq\frac{1}{64\pi^2}\left[M^4\left(\log M^2-\frac{3}{2}\right)-\frac{2}{3}(eB)^2\log M^2\right], \tag{14}$$

which agrees with the result obtained by Salam and Strathdee [5]. Likewise for the charged fermion fields using Eq. (5) we obtain

$$V_1(M^2,B)=\frac{4|q|B}{32\pi^2}\lim_{\alpha\to 0}\frac{\partial}{\partial\alpha}\frac{(2|q|B)^{1-\alpha}}{\Gamma(\alpha)}\times\int_0^\infty dt t^{\alpha-2}e^{-(M^2/2|q|B)t}\coth\frac{t}{2}. \tag{15}$$

The factor of 4 and positive sign account for the spinor nature of the Fermi field. In the limits mentioned above, we obtain

$$V_1(M^2,B)=\frac{|q|BM^2}{8\pi^2}(1-\log M^2) \tag{16}$$

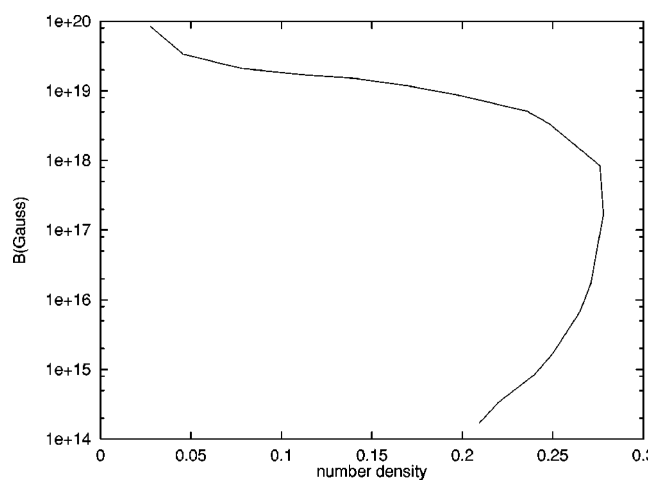

FIG. 3. Phase diagram as a plot of magnetic field versus baryon density in units of fm^{-3}.

and

$$V_1(M^2,B)\simeq -\frac{1}{16\pi^2}\left[M^4\left(\log M^2-\frac{3}{2}\right)+\frac{2}{3}(|q|B)^2\log M^2\right] \quad (17)$$

for $2|q|BM^2>1$ and <1, respectively.

The total $V_{eff}(\sigma,B)$ for the sigma model at the one loop level is thus given by

$$\begin{aligned}V_{eff}(\sigma,B)=&-\frac{1}{2}\mu^2\sigma^2+\frac{\lambda}{4}\sigma^4+\frac{1}{64\pi^2}(3\lambda\sigma^2-\mu^2)^2\\ &\times\log\left(\frac{3\lambda\sigma^2-\mu^2}{m_\sigma^2}-\frac{3}{2}\right)+\frac{1}{64\pi^2}(\lambda\sigma^2-\mu^2)^2\\ &\times\log\left(\frac{\lambda\sigma^2-\mu^2}{m^2}-\frac{3}{2}\right)-\frac{eB}{16\pi^2}(\lambda\sigma^2-\mu^2)\log 2\\ &-\frac{N_c}{16\pi^2}\sum_{flav}\left[g^4\sigma^4\left(\log\frac{g^2\sigma^2}{m_f^2}-\frac{3}{2}\right)\right.\\ &\left.+\frac{2}{3}(|q|B)^2\log\frac{g^2\sigma^2}{m_f^2}\right].\end{aligned} \quad (18)$$

For $|q|B/M^2>1$, the last term in Eq. (18) is replaced by Eq. (16) summed over flavors. In Fig. 1, we plot $V_{eff}(\sigma,B)$ as a function of σ for different values of magnetic field and compare it with the case of zero magnetic field by ignoring the B independent one loop terms. As input parameters we choose the constituent quark mass $m_f=500$ MeV, sigma mass $m_\sigma=1.2$ GeV and $f_\pi=93$ MeV. We find that in the presence of intense magnetic fields the chiral symmetry breaking is enhanced. For magnetic field large compared to m_f^2, from Eq. (16) we observe that though the fermionic contribution is towards symmetry restoration, it is not enough to offset the contribution of charged goldstone pions.

In order to study chiral symmetry restoration in the case of neutron stars as a function of chemical potential μ associated with finite baryon number density we employ the imaginary time formalism by summing over Matsubara frequencies. This amounts to adding the fermionic free energy to the one loop effective potential and is given by

$$V_1^\beta(\sigma)=-\frac{\gamma}{\beta}\int\frac{d^3k}{(2\pi)^3}\ln(1+e^{-\beta(E-\mu)}), \quad (19)$$

which in the presence of static uniform magnetic field becomes

$$V_1^\beta(\sigma)=-\frac{\gamma}{\beta}\frac{eB}{2\pi}\sum_{n=0}^{\infty}\int_0^\infty\frac{dk_z}{2\pi}\ln(1+e^{-\beta(E-\mu)}) \quad (20)$$

where γ is the degeneracy factor and is equal to $2N_c$ for each quark flavor. We consider cold dense isospin symmetric quark matter for which the integrals can be performed analytically. The baryon number density corresponding to the chemical potential μ is given by the usual thermodynamical relations:

$$N_B(\mu,0)=\frac{1}{3}\sum_{flav}\frac{\gamma}{6\pi^2}(\mu^2-g^2\sigma^2)^{3/2} \quad (21)$$

and

$$N_B(\mu,B)=\frac{1}{3}\sum_{n=0}^{n_{max}}\frac{\gamma|q|B}{4\pi^2}(2-\delta_{\mu,0})\sqrt{\mu^2-g^2\sigma^2-2n|q|B} \quad (22)$$

for zero and finite magnetic field respectively. Here $n_{max}=\mathrm{Int}|(\mu^2-g^2\sigma^2)/2|q|B|$. To study chiral symmetry behavior at finite density in the presence of uniform magnetic field, we minimize effective potential with respect to the order parameter σ for fixed values of chemical potential and magnetic field (which then fixes the baryon density). The results are shown in Fig. 2 where we have plotted the order parameter σ as a function of density at T=0 for different values of magnetic field. The solution indicates a first order phase transition. The actual transition takes place at the point where the two minima of the effective potential at $\sigma=0$ and $\sigma=\sigma(\mu,\mathrm{B})$ nonzero become degenerate. The lower values of σ (shown by dotted curves) are unphysical in the sense that they do not correspond to the lowest state of energy. We find that magnetic field continues to enhance chiral symmetry breaking at low densities as expected but as the magnetic field is raised the chiral symmetry is restored at a much lower density compared to the free field finite density case. This can be clearly seen from Fig. 3 where we have plotted the phase diagram in terms of baryon density and magnetic field.

In conclusion we have examined the chiral symmetry behavior of the linear sigma model in the presence of static, uniform magnetic field at the one loop level at zero density and at densities relevant in the core of neutron stars. We find that the contribution of scalar and fermion loops leads to an increase in chiral symmetry breaking. At high densities too, this effect persists and for magnetic fields of strength upto 10^{18} G, there is enhancement in chiral symmetry breaking resulting in the restoration of symmetry at densities higher than if no magnetic field were present. However, in the case of high magnetic field $B\geqslant 10^{19}$ G the chiral symmetry is restored at lower densities. Thus in the core of neutron stars, if the nuclear matter undergoes a transition to deconfined quark matter, the presence of magnetic field would imply the existence of massive quark matter due to enhancement in chiral symmetry breaking. This would affect the equation of state and will have astrophysical implications.

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