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Low-temperature expansion and perturbation theory in 2D models with unbroken symmetry: A new approach

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A new method for constructing weak coupling expansion of two-dimensional models with an unbroken continuous symmetry is developed. The method is based on an analogy with the Abelian XY model, respects the Mermin-Wagner theorem, and uses a link representation of the partition and correlation functions. An expansion of the free energy and of the correlation functions at small temperatures is performed and first order coefficients are calculated explicitly. They are proved to be path independent and infrared finite. We also study the free energy of the one-dimensional SU(N) model and demonstrate a nonuniformity of the low-temperature expansion in the volume for this system. Further, we investigate the contribution of holonomy operators to the low-temperature expansion in two dimensions and show that they do not survive the large volume limit. All our results agree with the conventional expansion. We discuss the applicability of our method to analysis of the uniformity of the low-temperature expansion in two dimensions.

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I. INTRODUCTION

Since two-dimensional (2D) models with a continuous global symmetry were recognized to be asymptotically free [1], they became a famous laboratory for testing many ideas and methods before applying them to more complicated gauge theories. In this paper we follow this common approach and present a method, different from conventional perturbation theory (PT), which allows one to investigate these models in the weak coupling region. Conventional PT is one of the main technical tools of modern physics. In spite of the belief of most of the physics community that this method gives the correct asymptotic expansion of such theories as 4D QCD or 2D spin systems with continuous global symmetry, recent discussion of this problem [2–6] has shown that it is rather far from an unambiguous solution. Indeed, for PT to be applicable it is necessary that the system under consideration should possess a well-ordered ground state. In two-dimensional models, such as the $O(N)$-sigma models, the Mermin-Wagner (MW) theorem guarantees the absence of such a state in the thermodynamic limit (TL), however small the coupling constant is [7]. Then, the usual argument in support of PT is that locally the system is ordered and PT is not supposed to be used for the calculation of long-distance observables. On the other hand, it should reproduce the correct behavior of fixed-distance correlations as well as all thermodynamical functions which can be expressed via short-range observables. The example of 1D models shows that even this is not always true [8], so why should one believe in the correctness of conventional PT in 2D? In fact, the only way to justify PT is to prove that it gives the correct asymptotic expansion of nonperturbatively defined models in the TL. Now, it was shown in [2] that PT results in 2D non-Abelian models depend on the boundary conditions (BC's) used to reach the TL. This result could potentially imply that the low-temperature limit and the TL do not commute in non-Abelian models. Actually, the main argumentation of [2] regarding the failure of the PT expansion is based on the fact that conventional PT is an expansion around a broken vacuum, i.e., a state which simply does not exist in the TL of 2D models. According to [2], the ground state of these systems can be described through special configurations—the so-called gas of superinstantons (SIs)—and the correct expansion should take into account these saddle points. At the present stage it is rather unclear how one could construct an expansion in the SI background. Fortunately, there exists another, more eligible way to construct the low-temperature expansion which respects the MW theorem and is *a priori* not an expansion around the broken vacuum. We develop this method in an example of the 2D SU(N)×SU(N) principal chiral model whose partition function (PF) is given by

$$Z=\int \prod_x DU_x \exp\left[\beta \sum_{x,n} \mathrm{Re}\,\mathrm{Tr}\, U_x U_{x+n}^{+}\right], \tag{1}$$

where $U_x \in \mathrm{SU}(N)$, DU_x is the invariant measure, and we impose periodic BCs. The basic idea is the following. As was rigorously proved, conventional PT gives an asymptotic expansion which is uniform in the volume for the Abelian XY model [9]. One of the basic theorems which underlies the proof states that the following inequality holds in the 3D XY model:

$$\langle \exp[\sqrt{\beta} A(\phi_x)]\rangle \leqslant C, \tag{2}$$

where C is β independent and $A(\phi+2\pi)=\phi$. Here ϕ_x is an angle parametrizing the action of the XY model, $S=\Sigma_{x,n}\cos(\phi_x-\phi_{x+n})$. It follows that at large β the Gibbs measure is concentrated around $\phi_x \approx 0$, providing the possibility of constructing an expansion around $\phi_x=0$. This in-

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equality is not true in 2D in the TL because of the MW theorem; however, the authors of [9] prove the same inequality for the link angle, i.e.,

$$\langle \exp[\sqrt{\beta} A(\phi_l)] \rangle \leqslant C, \quad \phi_l = \phi_x - \phi_{x+n}, \tag{3}$$

where the expectation value refers to the infinite volume limit. Thus, in 2D the Gibbs measure at large β is concentrated around $\phi_l \approx 0$ and the asymptotic series can be constructed by expanding the action in powers of ϕ_l. In the Abelian case such an expansion is equivalent to the expansion around $\phi_x = 0$ because (i) the action depends only on the difference $\phi_x - \phi_{x+n}$ and (ii) the integration measure is flat, $DU_x = d\phi_x$.

In 2D non-Abelian models, again because of the MW theorem, one has to expect in the TL something alike to Eq. (3), namely,

$$\langle \exp[\sqrt{\beta} \arg A(\mathrm{Tr}\, V_l)] \rangle \leqslant C, \quad V_l = U_x U^+_{x+n}. \tag{4}$$

Despite there being no rigorous proof of Eq. (4), that such an (or similar) inequality holds in 2D non-Abelian models is intuitively clear and should follow from the chessboard [10] and contour estimates [11], and from the Dobrushin-Shlosman proof of the MW theorem [7] which shows that spin configurations are distributed uniformly in the group space in the TL. Namely, the probability $p(\xi)$ that $\mathrm{Tr}(V_l - I) \leqslant -\xi$ is bounded by

$$p(\xi) \leqslant O(e^{-b\beta\xi}), \quad \beta \to \infty, \tag{5}$$

if the volume is sufficiently large, and b is a constant. Thus, until $\xi \leqslant O((\sqrt{\beta})^{-1})$ is not satisfied, all configurations are exponentially suppressed. This is equivalent to the statement that the Gibbs measure at large β is concentrated around $V_l \approx I$; therefore Eq. (4) or its analogue holds. In what follows it is assumed that Eq. (4) is correct; hence $V_l = I$ is the only saddle point for the invariant integrals.[1] Thus, the correct asymptotic expansion, if it exists, should be given via an expansion around $V_l = I$, similarly to the Abelian model. If conventional PT gives the correct asymptotics, it must reproduce the series obtained expanding around $V_l = I$. However, neither (i) nor (ii) holds in the non-Abelian models; therefore it is far from obvious that the two expansions indeed coincide. Let us parametrize $V_l = \exp(i\omega_l)$ and $U_x = \exp(i\omega_x)$. Consider the following expansion:

$$V_l = \exp[i\omega_l] \sim I + \sum_{n=1} \frac{(i\omega_l)^n}{n!}. \tag{6}$$

Standard PT states that to calculate the asymptotic expansion one has to reexpand this series at large β as

$$\omega_l = \frac{1}{\sqrt{\beta}}(\omega_x - \omega_{x+n}) + \sum_{k=2} \frac{1}{(\beta)^{k/2}} \omega_l^{(k)}, \tag{7}$$

where $\omega_l^{(k)}$ are to be calculated from the definition $U_x U^+_{x+n} = \exp(i\omega_l)$. This is presumably true in a finite volume where one can fix appropriate BCs like the Dirichlet ones. Then, making β sufficiently large one forces all the spin matrices to fluctuate around $U_x \approx I$; therefore the substitution (7) is justified. We do not see how this procedure could be justified when the volume increases and fluctuations of U_x spread up over the whole group space. It is only Eq. (6) which remains correct in the large volume limit and takes into account all the fluctuations contributing at a given order of the low-temperature expansion.

This paper is organized as follows. In the next section we introduce a link representation for the 2D SU(N) models. In this representation link matrices V_l play the role of dynamical variables, thus giving a precise mathematical meaning to the expansion (6). Section III serves as a pedagogical introduction to our method. Here we consider the XY model to construct the low-temperature expansion and to describe some features of our procedure. The next two sections are devoted to explaining the basic formalism of the expansion for non-Abelian models. In Sec. IV we introduce a certain representation for the PF and perform a general expansion for the free energy. We then calculate the generating functional and treat zero modes. In Sec. V we investigate certain ''link'' Green functions entering the generating functional and describe some of their most important properties. We also discuss briefly some of basic features of the expansion as the path independence and infrared (IR) finiteness. To show how the low-temperature expansion in the link formulation works in practice we calculate certain expectation values in Sec. VI and shortly summarize our results for various quantities. We also reanalyze the free energy of 1D non-Abelian models. As is well known, the large-β expansion in 1D non-Abelian models is nonuniform in the volume. To explain this feature, in Sec. VII we calculate the contribution of holonomy operators to the free energy of one- and two-dimensional models and show that while this contribution vanishes in the TL for 2D models it survives the large-volume limit in one dimension. In Sec. VIII we summarize our results and discuss some open problems.

II. LINK REPRESENTATION FOR THE PARTITION AND CORRELATION FUNCTIONS

To construct an expansion of the Gibbs measure and the correlation functions using Eq. (6) we use the so-called link representation for the partition and correlation functions. First, we make a change of variables $V_l = U_x U^+_{x+n}$ in (1). The PF becomes

$$Z = \int \prod_l dV_l \exp\left[\beta \sum_l \mathrm{Re}\ \mathrm{Tr}\, V_l + \ln J(V)\right], \tag{8}$$

where the Jacobian $J(V)$ is given by [12]

[1] It follows already from Eq. (5). What is important in Eq. (4) is the factor $\sqrt{\beta}$; otherwise, the very possibility of the expansion in $1/\beta$ becomes problematic.

$$J(V)=\int\prod_x dU_x\prod_l\left[\sum_r d_r\chi_r(V_l^+U_xU_{x+n}^+)\right]$$

$$=\prod_p\left[\sum_r d_r\chi_r\left(\prod_{l\in p}V_l\right)\right]. \tag{9}$$

Π_p is a product over all plaquettes of 2D lattices, the sum over r is sum over all representations of SU(N), and $d_r=\chi_r(I)$ is the dimension of the rth representation. The SU(N) character χ_r depends on the product of the link matrices V_l along a closed path (plaquette in our case):

$$\prod_{l\in p}V_l=V_n(x)V_m(x+n)V_n^+(x+m)V_m^+(x). \tag{10}$$

The expression $\Sigma_r d_r\chi_r(\Pi_{l\in p}V_l)$ is the SU(N) delta function which reflects the fact that the product of $U_xU_{x+n}^+$ around the plaquette equals I (the original model has L^2 degrees of freedom, and L^2 is the number of sites; since the number of links on the 2D periodic lattice is $2L^2$, the Jacobian must generate L^2 constraints). There are two solutions of the constraint

$$\prod_{l\in p}V_l=I, \tag{11}$$

the (1) pure gauge $V_l=U_xU_{x+n}^+$ and (2) constant solution. The first solution recovers the equivalence of the link representation and the standard representation for the partition function of the SU(N) model. To reject the constant, unphysical solution one has to constrain in addition two holonomy operators as described in Sec. VII. These holonomies, however, do not survive the TL in 2D.

Consider the two-point correlation function

$$\Gamma(x,y)=\langle\mathrm{Tr}\,U_xU_y^+\rangle. \tag{12}$$

Let C_{xy} be some path connecting points x and y. Inserting the unity $U_zU_z^+$ in every site $z\in C_{xy}$ one gets

$$\Gamma(x,y)=\left\langle\mathrm{Tr}\prod_{l\in C_{xy}}(U_xU_{x+n}^+)\right\rangle=\left\langle\mathrm{Tr}\prod_{l\in C_{xy}}W_l\right\rangle, \tag{13}$$

where $W_l=V_l$ if along the path C_{xy} the link l goes in the positive direction and $W_l=V_l^+$, otherwise. The expectation value in Eq. (13) refers to the ensemble defined in Eq. (8). Obviously, it does not depend on the path C_{xy} which can be deformed, for example, to the shortest path between sites x and y.

In this representation the series (6) acquires a well-defined meaning; therefore the expansion of the action, of the invariant measure, etc., can be done.

III. *XY* MODEL

To make a simple and clear introduction to our method we first consider the XY model where the expansion can be done in a straightforward manner. The link representation for the XY model reads

$$Z_{XY}=\int\prod_l d\omega_l\exp\left[\beta\sum_l\cos\omega_l\right]\prod_p J_p, \tag{14}$$

where the Jacobian is given by the periodic delta function

$$J_p=\sum_{r=-\infty}^{\infty}e^{ir\omega_p},$$

$$\omega_p=\omega_n(x)+\omega_m(x+n)-\omega_n(x+m)-\omega_m(x+n). \tag{15}$$

The first step is a standard one; i.e., we rescale $\omega\to\omega/\sqrt{\beta}$ and expand the Boltzmann factor in powers of $1/\beta$ as

$$\exp\left[\beta\cos\frac{\omega_l}{\sqrt{\beta}}\right]=\exp\left[\beta-\frac{1}{2}(\omega_l)^2\right]\left[1+\sum_{k=1}^{\infty}(\beta)^{-k}A_k(\omega_l^2)\right], \tag{16}$$

where A_k are known coefficients (see, e.g., [13]). In addition to this perturbation one has to extend the integration region to infinity. We do not treat this second perturbation, as usually supposing that all the corrections from this perturbation go down exponentially with β (in the Abelian case it can be proved rigorously [9]). It is more convenient now to go to a dual lattice identifying plaquettes of the original lattice with its center, i.e., $p\to x$. Let $l=(x;n)$ be a link on the dual lattice. Introducing sources h_l for the link field, one then finds

$$Z_{XY}(\beta\gg 1)=e^{\beta 2L^2-L^2\ln\beta/2}\prod_l$$

$$\times\left[1+\sum_{k=1}^{\infty}\frac{1}{\beta^k}A_k\left(\frac{\partial^2}{\partial h_l^2}\right)\right]M_{XY}(h_l), \tag{17}$$

where $M_{XY}(h_l)$ is a generating functional. Using the Poisson summation formula $M_{XY}(h_l)$ can be represented as

$$M_{XY}(h_l)=\sum_{m_x=-\infty}^{\infty}\int_{-\infty}^{\infty}\prod_x dr_x\int_{-\infty}^{\infty}\prod_l d\omega_l\exp\left[-\frac{1}{2}\sum_l\omega_l^2\right.$$

$$+i\sum_l\omega_l(r_x-r_{x+n})+2\pi i\sqrt{\beta}\sum_x r_xm_x$$

$$\left.+\sum_l\phi_lh_l\right]. \tag{18}$$

Calculating all the integrals in Eq. (18) we find, up to a constant (a sum over all repeating indices is understood),

$$M_{XY}(h_l)=e^{h_lG_{ll'}h_{l'}/4}\sum_{m_x}\delta\left(\sum_x m_x\right)$$

$$\times e^{-\pi^2\beta m_xG_{x,x'}m_{x'}+\pi\sqrt{\beta}h_lD_l(x')m_{x'}}. \tag{19}$$

The integral over the zero mode of the r field is not Gaussian and leads to a delta function in Eq. (19). Thus, the zero mode decouples from the expansion. One also sees that only the configuration $m_x=0$ for all x contributes to the asymptotic expansion, nontrivial vortex configurations are exponentially suppressed. We thus finally get

$$M_{XY}(h_l)=\exp\left[\frac{1}{4}\sum_{l,l'} h_l G_{ll'} h_{l'}\right]+O(e^{-\beta}), \tag{20}$$

where we have introduced the link Green function $G_{ll'}$ (see Sec. V for details).

The easiest way to construct the corresponding expansion for the correlation function is the following. Let C_{xy} be some path connecting points x and y and let C_{xy}^d be a path dual to the path C_{xy}, i.e., consisting of the dual links b,b' which are orthogonal to the original links $b,b'\in C_{xy}$. The correlation function is defined as

$$\Gamma_{XY}(x,y)=\left\langle \prod_{b\in C_{xy}^d} \exp\left[\frac{i}{\sqrt{\beta}}\omega_b\right]\right\rangle. \tag{21}$$

The last formula suggests that to compute the correlation function it is sufficient to make the shift $h_l \to h_l + (i/\sqrt{\beta})\Sigma_b \delta_{l,b}$ in the expression for the generating functional (20). We find

$$\Gamma_{XY}(x,y)=\exp\left[-\frac{1}{4\beta}\sum_{b,b'\in C_{xy}^d} G_{bb'}\right] Z_{XY}^{-1}\prod_l \times\left[1+\sum_{k=1}^{\infty}\frac{1}{\beta^k}A_k\left(\frac{\partial^2}{\partial h_l^2}\right)\right]\exp\left[\frac{1}{4}\sum_{l,l'} h_l G_{ll'} h_{l'} + \frac{i}{2\sqrt{\beta}}h_l\sum_b G_{lb}\right]. \tag{22}$$

It is straightforward to calculate from here all connected pieces contributing to the correlation function at a given order of $1/\beta$. For example, the second order coefficient of the correlation function expressed in terms of link Green functions reads ($b_i\in C_{xy}^d$)

$$\Gamma^{(2)}(x,y)=\frac{1}{32}\left(\sum_{b_1b_2} G_{b_1b_2}\right)^2-\frac{1}{32}\sum_{b_1b_2}\sum_l G_{b_1l}G_{lb_2}. \tag{23}$$

Using some properties of $G_{ll'}$ described in Sec. V it is easy to prove that $\Gamma^{(2)}(x,y)$ coincide with that quoted in the literature for the $O(2)$ model [14].

Let us add some comments. In the standard expansion to avoid the zero-mode problem one has to fix appropriate BC, such as Dirichlet conditions, or to fix a global gauge if one works on the periodic lattice [15]. In the present scheme the zero mode decouples automatically due to using a U(1) delta function which takes into account the periodicity of the integrand in link angles. A more important observation is that both the free energy and the correlation function are expressed only through the link Green function which is IR finite by construction. It guarantees the IR finiteness of the low-temperature expansion to all orders in $1/\beta$. This is a direct consequence of the fact that the Gibbs measure of the XY model is a function of the gradient ω_l only.

IV. WEAK COUPLING EXPANSION IN THE SU(N) MODEL

In this section we derive the low-temperature expansion for non-Abelian models. We describe the general procedure for the SU(2) group and then give a simple generalization for all SU(N) models. For some technical details we refer the reader to our earlier papers [13,16].

As usual, we want to expand the partition and correlation functions into asymptotic series whose coefficients are calculated over certain Gaussian measure, i.e., up to a constant

$$Z=1+\sum_{k=1}^{\infty}\frac{1}{\beta^k}\langle B_k\rangle_G. \tag{24}$$

We take the standard form for the SU(2) link matrix

$$V_l=\exp[i\sigma^k\omega_k(l)], \tag{25}$$

where σ^k, $k=1,2,3$ are Pauli matrices. Let us introduce

$$W_l=\left[\sum_k \omega_k^2(l)\right]^{1/2},\quad W_p=\left[\sum_k \omega_k^2(p)\right]^{1/2}, \tag{26}$$

where $\omega_k(p)$ is a plaquette angle defined as

$$V_p=\prod_{l\in p} V_l=\exp[i\sigma^k\omega_k(p)]. \tag{27}$$

It has the following expansion in powers of link angles:

$$\omega_k(p)=\omega_k^{(0)}(p)+\omega_k^{(1)}(p)+\omega_k^{(2)}(p)+\cdots. \tag{28}$$

On a dual lattice ($p\to x$) the first coefficients can be written down as

$$\omega_k^{(0)}(x)=\omega_k(l_3)+\omega_k(l_4)-\omega_k(l_1)-\omega_k(l_2), \tag{29}$$

$$\omega_k^{(1)}(x)=-\epsilon^{kpq}\sum_{i<j}^{4}\omega_p(l_i)\omega_q(l_j). \tag{30}$$

Then, the partition function (8) can be exactly rewritten in the following form (see Appendix A of [13]):

$$Z_{SU(2)}=\int\prod_l\left[\frac{\sin^2 W_l}{W_l^2}\prod_k d\omega_k(l)\right] \times\exp\left[2\beta\sum_l \cos W_l\right]\prod_x\frac{W_x}{\sin W_x} \times\prod_x\sum_{m(x)=-\infty}^{\infty}\int\prod_k d\alpha_k(x) \times\exp\left[-i\sum_k \alpha_k(x)\omega_k(x)+2\pi i m(x)\alpha(x)\right], \tag{31}$$

where $\alpha(x)=[\Sigma_k \alpha_k^2(x)]^{1/2}$. In the derivation of this representation we have used the Poisson summation formula. To perform the weak coupling expansion we make the substitution

$$\omega_k(l)\to(2\beta)^{-1/2}\omega_k(l), \quad \alpha_k(x)\to(2\beta)^{1/2}\alpha_k(x), \tag{32}$$

and then expand the integrand of Eq. (31) in powers of fluctuations of the link fields. Such a procedure is justified by the fact that at β sufficiently large link matrices fluctuate around unity as has been argued in the Introduction. Introducing now the external sources $h_k(l)$ coupled to the link field $\omega_k(l)$ and $s_k(x)$ coupled to the auxiliary field $\alpha_k(x)$ and adopting the definitions

$$\omega_k(l)\to\frac{\partial}{\partial h_k(l)}, \quad \alpha_k(x)\to\frac{\partial}{\partial s_k(x)}, \tag{33}$$

we get finally the following weak coupling expansion for the partition function (31):

$$Z=\left[1+\sum_{k=1}^{\infty}\frac{1}{\beta^k}B_k(\partial_h,\partial_s)\right]M(h,s), \tag{34}$$

where operators B_k are defined through

$$1+\sum_{k=1}^{\infty}\frac{1}{\beta^k}B_k(\partial_h,\partial_s)=\prod_l\left[\left(1+\sum_{k=1}^{\infty}\frac{1}{(2\beta)^k}\sum_{l_1,\dots,l_k}{}'\frac{a_1^{l_1}\dots a_k^{l_k}}{l_1!\dots l_k!}\right)\left(1+\sum_{k=1}^{\infty}\frac{(-1)^k}{(2\beta)^k}C_kW_l^{2k}\right)\right]$$
$$\times\prod_x\left\{\left(1+\sum_{k=1}^{\infty}\frac{J_k}{(2\beta)^k}W_x^{2k}\right)\left[1+\sum_{q=1}^{\infty}\frac{(-i)^q}{q!}\left(\sum_k\alpha_k(x)\sum_{n=1}^{\infty}\frac{\omega_k^{(n)}(x)}{(2\beta)^{n/2}}\right)^q\right]\right\}. \tag{35}$$

Here C_k and J_k are β-independent coefficients (see Ref. [13]) and

$$a_k=(-1)^{k+1}\frac{W_l^{2(k+1)}}{(2k+2)!}. \tag{36}$$

The first set of brackets on the right-hand side (RHS) of Eq. (35) comes from the expansion of the action, the second one from the invariant measure. The last two sets of brackets represent the Jacobian. As usual, one has to put $h_k=s_k=0$ after taking all the derivatives.

The generating functional $M(h,s)$ is given by

$$M(h,s)=\int_{-\infty}^{\infty}\prod_{x,k}d\alpha_k(x)\int_{-\infty}^{\infty}\prod_{l,k}d\omega_k(l)\exp\left[-\frac{1}{2}\omega_k^2(l)-i\omega_k(l)[\alpha_k(x+n)-\alpha_k(x)]\right]$$
$$\times\sum_{m(x)=-\infty}^{\infty}\exp\left[2\pi i\sqrt{2\beta}\sum_x m(x)\alpha(x)+\sum_{l,k}\omega_k(l)h_k(l)+\sum_{x,k}\alpha_k(x)s_k(x)\right]. \tag{37}$$

As in the Abelian case only the configuration with $m_x=0$ for all x contributes to the asymptotic expansion, all others being exponentially suppressed. In this case zero modes are controlled through integration over radial component of the vector $\vec{\alpha}(x)$. Integration over the constant component of this vector ensures that only neutral configurations, i.e., $\Sigma_x m_x=0$, contribute to the asymptotic expansion. Calculating all Gaussian integrals in Eq. (37) we come to

$$M(h,s)=\exp\left[\frac{1}{4}s_k(x)G_{x,x'}s_k(x')+\frac{i}{2}s_k(x)D_l(x)h_k(l)+\frac{1}{4}h_k(l)G_{ll'}h_k(l')\right], \tag{38}$$

where link functions $G_{ll'}$ and $D_l(x)$ are described in the next section. From Eq. (38) one can deduce the following simple rules:

$$\langle\omega_k(l)\omega_n(l')\rangle=\frac{\delta_{kn}}{2}G_{ll'},$$
$$\langle\alpha_k(x)\alpha_n(x')\rangle=\frac{\delta_{kn}}{2}G_{xx'},$$
$$-i\langle\omega_k(l)\alpha_n(x')\rangle=\frac{\delta_{kn}}{2}D_l(x'). \tag{39}$$

The expansion (34), representation (38) for the generating functional, and rules (39) are the main formulas of this section which allow us to calculate the weak coupling expansion of both the free energy and any short-distance observable. Extension of this expansion for the correlation function is straightforward and can be done, for example, precisely as in the XY model (i.e., through modification of the generating functional) or by making a direct expansion in powers of link angles.

Generalization for an arbitrary SU(N) model can be done as follows. As we have discussed above the low-temperature expansion arises only from the ''vacuum'' sector with $m_x=0$ for all x. One can see from Eq. (31) that in this sector the SU(N) delta function reduces to the Dirac delta function so that the partition function becomes

$$Z(\beta \gg 1)=\int \prod_l dV_l \exp\left[\beta \sum_l \text{ Re Tr } V_l\right] \prod_{p,k} \delta(\omega_p^k). \tag{40}$$

Such a naive replacement is of course plagued by the problem of (N^2-1) zero modes for auxiliary fields $\alpha_k(x)$, and therefore the delta function in Eq. (40) should be correspondingly regularized. This can be done, for instance, by introducing the heat kernel instead of the SU(N) delta function into the expression for the partition function. This procedure is equivalent to introducing a mass term for auxiliary fields. As we have shown, however, zero modes decouple from the large-β expansion in the SU(2) model [we could not generalize the consideration of Appendix A of [13] for arbitrary SU(N), though]. In what follows we work with massless Green functions, omitting zero modes from all lattice sums similarly to the SU(2) case. All general expressions remain valid if one works with the mass regulator term as mentioned above. Since all logarithmic divergences cancel, we expect that the convergence to the TL is uniform in this vacuum sector: in all cases the final result can be expressed only in terms of link Green functions and standard D functions. Then, the expansion itself is done precisely like for SU(2).

V. LINK GREEN FUNCTIONS

The main building blocks of the low-temperature expansion in the link formulation are the link Green functions $G_{ll'}$ and $D_l(x')$. In this section we describe some of their basic properties. The functions $G_{ll'}$ and $D_l(x')$ are defined as

$$G_{ll'}=2\delta_{l,l'}-G_{x,x'}-G_{x+n,x'+n'}+G_{x,x'+n'}+G_{x+n,x'}, \tag{41}$$

$$D_l(x')=G_{x,x'}-G_{x+n,x'}, \tag{42}$$

where link $l=(x,n)$ is defined by a point x and a positive direction n. Here $G_{x,x'}$ is a ''standard'' Green function on the periodic lattice:

$$G_{x,x'}=\frac{1}{L^2}\sum_{k_n=0}^{L-1}\frac{e^{(2\pi i/L)k_n(x-x_n')}}{f(k)},$$

$$f(k)=2-\sum_{n=1}^{2} \cos\frac{2\pi}{L}k_n, \quad k_n^2\neq 0. \tag{43}$$

Normalization is such that $G_{ll}=1$. In momentum space $G_{ll'}$ reads

$$G_{ll'}=\frac{2\delta_{nn'}-1}{L^2}\times\sum_{k_n=0}^{L-1}\frac{e^{(2\pi i/L)k_n(x-x')_n}}{f(k)}F(n,n')+\frac{2\delta_{nn'}}{L^2},$$

$$F(n=n')=2\left(1-\cos\frac{2\pi}{L}k_p\right), \quad n\neq p,$$

$$F(n\neq n')=(1-e^{(2\pi i/L)k_n})(1-e^{(-2\pi i/L)k_{n'}}). \tag{44}$$

Using this representation it is easy to prove the following ''orthogonality'' relations for the link functions:

$$\sum_b G_{lb}G_{bl'}=2G_{ll'},$$

$$\sum_b D_b(x)G_{bl'}=0,$$

$$\sum_b D_b(x)D_b(x')=2G_{x,x'}, \tag{45}$$

where the sum over b runs over all links of 2D lattices. Let C_{xy}^d be the path as described after Eq. (20). We then have

$$\sum_{l,l'\in C_{xy}^d} G_{ll'}=2D(x-y), \quad D(x)=\frac{1}{L^2}\sum_{k_n=0}^{L-1}\frac{1-e^{(2\pi i/L)k_n x_n}}{f(k)}. \tag{46}$$

Let $\mathcal{L}$ be any closed path. Then

$$\sum_{l,l'\in\mathcal{L}} \overline{G}_{ll'}=0, \tag{47}$$

where $\overline{G}_{ll'}=G_{ll'}$ if both links l and l' point in either the positive or negative direction and $\overline{G}_{ll'}=-G_{ll'}$ if one (and only one) of the links points in the negative direction. In particular, Eq. (47) holds for each plaquette of the lattice.

Let l_i, $i=1,\ldots,4$, be four links attached to a given site x. One sees that $G_{ll'}$ satisfies the following equation:

$$G_{l_1l'}+G_{l_2l'}-G_{l_3l'}-G_{l_4l'}=0 \tag{48}$$

for any link l'. $D_l(x')$ satisfies the lattice Laplas equation

$$D_{l_1}(x')+D_{l_2}(x')-D_{l_3}(x')-D_{l_4}(x')=2\delta_{x,x'}. \tag{49}$$

The last three equations ensure the path independence of the correlation function in the link formulation for they allow one to deform some given path to any other one.

It follows from the definition (41) that

$$|G_{ll'}|\leq 1. \tag{50}$$

Therefore, with respect to the Gaussian measure (37) the fluctuations of the link variables are bounded like

$$|\langle \omega_l \omega_{l'} \rangle_G| = \left| \frac{1}{2} G_{ll'} \right| \leq \frac{1}{2}, \tag{51}$$

due to the bound (50). One sees from the last formula that the interaction between links strongly decreases with distance. Taking an asymptotic expansion for the Green functions in Eq. (41) we find that, if $l=(0,n)$, $l=(R,m)$,

$$|\langle \omega_l \omega_{l'} \rangle_G| = O(R^{-1}). \tag{52}$$

This property justifies the low-temperature expansion in powers of fluctuations of link variables. Note that it is not the case for the original degrees of freedom which describe fluctuations of site variables. The latter fluctuations are not bounded since

$$\langle \alpha(x)\alpha(x') \rangle_G = \frac{1}{2} G_{xx'} = O(\ln L). \tag{53}$$

Probably, the most important property of the link functions is that they are IR finite by construction. And although the IR dangerous function G_{xy} also enters the generating functional for non-Abelian models, the dependence of the expectation values on G_{xy} is rather trivial since it appears in the expansion only through auxiliary fields but not through dynamical variables. In particular, we are able to exactly rewrite the partition function of non-Abelian models as

$$Z = \sum_{k=0} G_0^k \langle Z_k \rangle,$$

where Z_k are some IR finite operators and the finite-volume asymptotic expansion arises only from the term $k=0$. Details of this representation will be reported in a forthcoming publication [18].

VI. CORRELATION FUNCTION IN TWO-DIMENSIONAL MODELS

In order to demonstrate how our method works we compute the first two coefficients of the fixed-distance correlation function for the SU(N) model. Throughout this section we work on a dual lattice. Links b_i denote links belonging to the path C^d_{xy}. Expanding Eq. (13) we write down

$$\Gamma_{SU(N)}(x,y) = 1 - \frac{1}{\beta}\Gamma^{(1)}(x,y) + \frac{1}{\beta^2}\Gamma^{(2)}(x,y) + \cdots . \tag{54}$$

The first coefficient is given by

$$\Gamma^{(1)}(x,y) = \frac{N^2-1}{4N} \sum_{b_1 b_2} G_{b_1 b_2} = \frac{N^2-1}{2N} D(x-y) \tag{55}$$

and coincides with the conventional result. We split the second coefficient into three pieces:

$$\Gamma^{(2)}(x,y) = \frac{N^2-1}{16}(Q_1+Q_2+Q_3). \tag{56}$$

Q_1 describes the contribution of the second order term from the correlation function and zero order term from the Gibbs measure:

$$Q_1 = \frac{1}{N^2} \sum_{k=1}^{4} Q_1^{(k)}, \tag{57}$$

where

$$Q_1^{(1)} = \frac{2N^2-3}{6} \sum_b G_{bb}^2, \tag{58}$$

$$Q_1^{(2)} = \sum_{b_1>b_2} \left[(N^2-1) G^2_{b_1 b_1} + (N^2-2) G^2_{b_1 b_2} + \frac{4}{3}(2N^2-3) G_{b_1 b_2} \right], \tag{59}$$

$$Q_1^{(3)} = \sum_{b_1 \neq b_2 \neq b_3} [(N^2-1) G_{b_1 b_2} + (N^2-2) G_{b_1 b_2} G_{b_2 b_3}], \tag{60}$$

$$Q_1^{(4)} = 4 \sum_{b_1>b_2>b_3>b_4} [(N^2-1)(G_{b_1 b_2} G_{b_3 b_4} + G_{b_1 b_4} G_{b_2 b_3}) - G_{b_1 b_3} G_{b_2 b_4}]. \tag{61}$$

Q_2 describes the contribution of $\beta^{-3/2}$ order from the expansion of the correlation function and of $\beta^{-1/2}$ order from the expansion of the Jacobian. This ''self-connected'' piece is given by

$$Q_2 = - \sum_{b_1>b_2>b_3} \sum_x \sum_{i<j}^{4} [Q_x^{ij}(b_1,b_2,b_3) - Q_x^{ij}(b_1,b_3,b_2) + Q_x^{ij}(b_2,b_3,b_1) - Q_x^{ij}(b_2,b_1,b_3) + Q_x^{ij}(b_3,b_1,b_2) - Q_x^{ij}(b_3,b_2,b_1)], \tag{62}$$

where

$$Q_x^{ij}(b_1,b_2,b_3) = D_{b_1}(x) G_{b_2 l_i} G_{b_3 l_j}. \tag{63}$$

Finally, there are contributions of the first order terms from the expansion of the correlation function, the Gibbs measure, and the Jacobian. Q_3 describes the corresponding connected pieces:

$$Q_3 = \sum_{k=1}^{3} Q_3^{(k)}, \tag{64}$$

where

$$Q_3^{(1)} = \frac{1}{2N^2} \sum_{b_1 b_2} \sum_l G_{b_1 l} G_{l b_2}, \tag{65}$$

$$Q_3^{(2)}=-\frac{2}{3}\sum_{b_1b_2}\sum_x\Bigg[3D_{b_2}(x)(G_{b_1l_1}-G_{b_1l_4})(G_{l_1l_2}-G_{l_1l_3})+\frac{1}{2}\sum_{i=1}^{4}G_{b_1l_i}G_{b_2l_i}[D_{l_1}(x)+D_{l_2}(x)-D_{l_3}(x)-D_{l_4}(x)]$$
$$+(G_{b_1l_1}+G_{b_1l_2})[2D_{l_1}(x)G_{b_2l_2}-2D_{l_4}(x)G_{b_2l_3}-D_{l_2}(x)G_{b_2l_1}+D_{l_3}(x)G_{b_2l_4}]+[D_{l_1}(x)+D_{l_2}(x)]G_{b_1l_3}G_{b_2l_4}$$
$$-[D_{l_3}(x)+D_{l_4}(x)]G_{b_1l_1}G_{b_2l_2}\Bigg], \tag{66}$$

$$Q_3^{(3)}=\frac{1}{4}\sum_{b_1b_2}\sum_{x,x'}\sum_{i<j}^{4}\sum_{i'<j'}^{4}(G_{x,x'}I_1+I_2). \tag{67}$$

We have denoted

$$I_1=G_{b_1,l_i}G_{b_2,l'_{i'}}G_{l_j,l'_{j'}}+G_{b_1,l_j}G_{b_2,l'_{j'}}G_{l_i,l'_{i'}}-G_{b_1,l_i}G_{b_2,l'_{j'}}G_{l_j,l'_{i'}}-G_{b_1,l_j}G_{b_2,l'_{i'}}G_{l_i,l'_{j'}}, \tag{68}$$

$$I_2=D_{b_1}(x)D_{b_2}(x')(G_{l_i,l'_{j'}}G_{l_j,l'_{i'}}-G_{l_i,l'_{i'}}G_{l_j,l'_{j'}})+2D_{b_1}(x)D_{l_i}(x')(G_{b_2,l'_{i'}}G_{l_j,l'_{j'}}-G_{b_2,l'_{j'}}G_{l_j,l'_{i'}})+2D_{b_1}(x)D_{l_j}(x')$$
$$\times(G_{b_2,l'_{j'}}G_{l_i,l'_{i'}}-G_{b_2,l'_{i'}}G_{l_i,l'_{j'}})+D_{l'_{i'}}(x)G_{b_1,l'_{j'}}[D_{l_i}(x')G_{b_2,l_j}-D_{l_j}(x')G_{b_2,l_i}]$$
$$+D_{l'_{j'}}(x)G_{b_1,l'_{i'}}[D_{l_j}(x')G_{b_2,l_i}-D_{l_i}(x')G_{b_2,l_j}]. \tag{69}$$

In all formulas link l_i ($l'_{j'}$) refers to one of four links attached to a given site x (x'). As it stays, this expression for the second order coefficient is valid for any path if all links $b_i\in C^d_{xy}$ point in positive directions. If one considers a path where some links point in a negative direction, one has to change sign in the corresponding Green functions. Despite the complexity of this representation, real computations are rather straightforward if one uses the properties of link functions described in the previous section. In particular, we have proved that (i) our result for $\Gamma^{(2)}(x,y)$ coincides with the conventional answer [8], and (ii) the representation for $\Gamma^{(i)}(x,y)$ is path independent and IR finite.

The details of the proof can be found in Ref. [16].

VII. CONTRIBUTION FROM HOLONOMY OPERATORS

In this section we analyze the contribution of the holonomy operators to the low-temperature expansion of the free energy. We start with one-dimensional models. To analyze the 1D SU(N) model we note that the formula for $\Gamma^{(2)}(x,y)$ given in the previous section remains valid if we take for the link Green function the following expression:

$$G_{ll'}=2\,\delta_{l,l'}. \tag{70}$$

This equation is a trivial consequence of the fact that in the link formulation the 1D model reduces to one link integral. Then, it is straightforward to calculate, for example, the first order coefficient of the free energy. We find

$$\frac{1}{L}C^1_{f.e.}=-\frac{N^2-1}{8N}, \tag{71}$$

which agrees with the expansion of the exact result in the TL. On the other hand, it is well known that the low-temperature expansion in 1D non-Abelian models is nonuniform in volume; in particular conventional PT produces a result different from Eq. (71). To explain this nonuniformity we recall that on the periodic lattice one has to constrain a holonomy operator if one works in the link formulation. Working with 2D models we have neglected this additional constraint since it seems to us rather unlikely that such a global constraint may influence the TL in 2D (see below). This happens, however, in 1D model as we are going to show below.

The partition function is given by

$$Z=\int\prod_l dV_l\exp\Big[\beta\sum_l \mathrm{Re\,Tr}\,V_l+\ln J(V)\Big], \tag{72}$$

where $J(V)$ introduces a global constraint on link matrices:

$$J(V)=\sum_r d_r\chi_r\Bigg(\prod_{l=1}^{L}V_l\Bigg). \tag{73}$$

Again, at large β we replace the SU(N) delta function by the Dirac delta function, i.e.,

$$J(V)=\int\prod_{k=1}^{N^2-1}d\phi_k\exp[-i\phi_k\omega^k(C)], \tag{74}$$

where $\omega^k(C)$ is defined as

$$\prod_{l=1}^{L}V_l=\exp[i\lambda^k\omega^k(C)]. \tag{75}$$

We omit all technical details which are exactly the same as in 2D. For the first order coefficient of the free energy we find, in the large-volume limit,

$$\frac{1}{L}C^{1}_{SU(N)}=-\frac{N^2-1}{8N}+\frac{N(N^2-1)}{24}. \tag{76}$$

The second term on the right-hand side of the last formula comes from the expansion of $J(V)$ and modifies the correct expression (71). Our result (76) disagrees with the one given in [8]. We think it is because the result of [8] was obtained using the mass regulator term, i.e., the procedure which is known to give a wrong answer even in a finite volume [15]. To check the correctness of Eq. (76) we have compared it for $N=2$ with the $O(n=4)$ model:

$$\frac{1}{L}C^{1}_{O(n)}=\frac{n-1}{8}-\frac{(n-1)(n-2)}{24}, \tag{77}$$

where the second term comes from the Hasenfratz term which survives the TL in 1D. One sees that the results indeed coincide.[2]

On a 2D lattice one should restrict the two holonomy operators. In our previous analysis we have neglected this restriction. Since, however, this global constraint influences the TL in 1D if the low-temperature expansion is done in a finite volume, we think it is instructive to see what happens with holonomies in two-dimensional models.

Let H_i $(i=1,2)$ be any given path winding around the whole lattice. H_1 and H_2 are orthogonal to each other. One has to introduce two global constraints into the partition function (8):

$$J(H)=\int\prod_{k=1}^{N^2-1}\prod_{i=1}^{2}d\phi_k(i)\exp[-i\phi_k(i)\omega^k(H_i)], \tag{78}$$

where $\omega^k(H_i)$ is defined as

$$\prod_{l\in H_i}V_l=\exp[i\lambda^k\omega^k(H_i)]. \tag{79}$$

There are two types of contributions from $J(H)$. The first one comes from the expansion of $J(H)$ itself. It is too cumbersome to be given here in full. This contribution, however, can be expressed only through link Green functions and is proportional to the linear size of the lattice. This is the reason why it survives the TL in 1D. Correspondingly, in 2D it vanishes like $O(1/L)$. The second type is related to modification of the generating functional. Namely, one should make the following replacement of the Green functions:

$$G_{ll'}\to\overline{G}_{ll'}=G_{ll'}-\frac{1}{2}\sum_i\sum_{(bb')\in H_i}G_{lb}G_{l'b'}, \tag{80}$$

$$D_l(x)\to\overline{D}_l(x)=D_l(x)-\frac{1}{2}\sum_i\sum_{(bb')\in H_i}D_b(x)G_{lb'}, \tag{81}$$

$$G_{x,x'}\to\overline{G}_{x,x'}=G_{x,x'}+\frac{1}{2}\sum_i\sum_{(bb')\in H_i}D_b(x)D_{b'}(x'). \tag{82}$$

In particular, the corresponding replacements should be made in formulas (54)–(70). Let us take for simplicity such paths H_i which consist of links pointing only in one direction. The coordinates of the corresponding links on the dual lattice are

$$b=(x_1,0;n_2),\quad x_1\in[0,L-1],\quad b\in H_1$$

and

$$b=(0,x_2;n_1),\quad x_2\in[0,L-1],\quad b\in H_2.$$

One then proves that

$$\overline{G}_{ll'}=G_{ll'}-O(1/L^2), \tag{83}$$

$$\overline{D}_l(x)=D_l(x)-O(1/L), \tag{84}$$

$$\overline{G}_{x,x'}=G_{x,x'}+\frac{1}{2}\sum_{n=1}^{2}(2\delta_{x_n,0}-1)(2\delta_{x'_n,0}-1). \tag{85}$$

One sees that the only new term which could potentially survive the TL is the second term in the last expression. This term is to be substituted into Eq. (67) and it leads to the computation of sums of the form

$$Q_3^{(div)}=\frac{1}{8}\sum_{b_1b_2}\sum_{x,x'}\sum_{i<j}^{4}\sum_{i'<j'}^{4}(4\delta_{x_1,0}\delta_{x'_1,0}-4\delta_{x_1,0}+1)I_1. \tag{86}$$

The first term vanishes like $O(1/L)$ because of two extra deltas while the second and constant terms equal zero due to the IR finiteness of $Q_3^{(3)}$. Moreover, in general it is clear from Eqs. (83)–(85) that the holonomies may only contribute to the TL through the constant term in $\overline{G}_{x,x'}$. This is, however, equivalent to noncancellation of infrared divergences in some higher orders. In all other cases the holonomies do not survive the TL.

VIII. SUMMARY AND DISCUSSION

In this paper we proposed to use an invariant link formulation to investigate some properties of 2D models in the weak coupling region. We have argued that this approach is more suitable for the calculation of asymptotic expansions of invariant functions in cases when the Mermin-Wagner theorem forbids spontaneous symmetry breaking in the thermodynamic limit. We have found that both in the XY model and in non-Abelian SU(N) models our results for the first order coefficients of the free energy and correlation function agree with the standard PT expansion in the TL. We have demon-

[2]One needs also to replace $\beta\to2\beta$ in $O(4)$.

strated how the path independence of the correlation function manifests itself in our expansion and have proved such independence for the first two coefficients via direct calculations. We have also shown which properties of the expansion guarantee its infrared finiteness, at least in lowest orders. Moreover, since in our expansion the source of such divergences is exactly localized, it gives a good opportunity for investigation of the IR finiteness of higher order terms. It could lead to a lattice analogue of David's theorem [17] which states the IR finiteness of the weak coupling expansion of continuum models. We thus find that the low-temperature expansion performed in the link representation coincides with conventional PT in the lowest orders. In fact, it seems that these two expansions have to coincide up to arbitrary order. If conventional PT produces the correct asymptotic expansion in a finite volume, any other expansion is bound to reproduce the same coefficients when the volume is fixed. Moreover, this also refers to the TL if these coefficients are infrared finite. We may thus conclude that our calculations support conventional PT, in particular the reexpansion (7) made in the standard approach.

Now we can return to the question raised in the Introduction, namely, whether conventional PT gives a uniform asymptotic expansion for non-Abelian models. It has been well known for a long time that it is not so in one-dimensional non-Abelian models. We have reanalyzed the low-temperature expansion of 1D models in the link representation. In this representation one should impose a global constraint on link matrices (73). This global constraint vanishes in the TL if this limit is taken before the low-temperature expansion. However, if the expansion is done in a finite volume, the expansion of the holonomy operator, which imposes the global constraint, does survive the TL. It leads to the nonuniformity of the low-temperature expansion in one dimension. We also have proved that it is not the case in 2D: at least in the lowest order the holonomies do not survive the TL and there is good reason to believe that they do not in higher orders as well. Nevertheless, one cannot exclude the possibility of the nonuniformity of the low-temperature expansion arising from the remainder of the PT series [6]. This problem is extremely hard to resolve by means of standard approaches; see, for instance, [5]. On the contrary, in the link formulation we are able to calculate the exponential remainder at a given order of the low-temperature expansion. Therefore, the problem of the IR finiteness of the remainder can be addressed explicitly [19].

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