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Twistors and actions on coset manifolds

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Particle and string actions on coset spaces typically lack a quadratic kinetic term, making their quantization difficult. We define a notion of twistors on these spaces, which are hypersurfaces in a vector space that transform linearly under the isometry group of the coset. By associating the points of the coset space with these hypersurfaces, and the internal coordinates of these hypersurfaces with momenta, it is possible to construct manifestly symmetric actions with leading quadratic terms. We give a general algorithm and work out the case of a particle on AdS_p explicitly. In this case, the resulting action is a world-line gauge theory with sources (the gauge group depending on p), which is equivalent to a nonlocal world-line σ model.

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I. INTRODUCTION

The standard action for a particle or string on a coset space G/H is manifestly invariant under G but does not have a quadratic kinetic term. This obstructs the usual quantization procedure. Moreover, the isometries are nonlinearly realized on the coordinates and so even if the action were quadratic, the fields would not automatically form a G representation. This makes it difficult to directly study systems such as strings on $\mathrm{AdS}_{p+2}\times \mathrm{S}^{d-p-2}$ which are important for understanding holography [1–3].

A hint of how to work around this comes from twistors. These were originally set up by Penrose to study conformal Minkowski space [4], and have since been generalized to conformal superspace [5] and AdS_5 [6,7]. In all of these cases, twistors associated a hypersurface in some vector space which transforms linearly under the isometry group with every point of the coset space. The internal coordinates of these hypersurfaces were associated with momenta and constrained quantities.

This construction can be generalized to arbitrary coset manifolds[1] G/H. A mapping between points of the coset and hypersurfaces in a vector space can be constructed which naturally mimics the geometric structure of the coset. The isometries, for example, can easily be extracted from the linear isometry transformations of the twistors.

If the vector space is also a Hilbert space (i.e. posesses an appropriate inner product) then one can naturally construct objects out of twistors which are manifestly invariant under the coset isometries. Using the twistor mapping, these can be written as (typically fairly complicated) functions of the coset coordinates and the internal coordinates of the twistor. Since these quantities are manifestly G-invariant, one can construct actions out of them which are equivalent to ordinary coset actions if the internal coordinates are identified with momenta. Since the twistor mapping is typically very complicated, very simple twistor actions are equivalent to very complicated coset actions.

We demonstrate this construction for a particle on AdS_p. Twistors are built in a vector space which transforms in the spinor representation of $SO(p-1,2)$. A world-line gauge theory can be built out of these twistors which is equivalent to the ordinary action for a massive particle on the coset. This theory is equivalent to a nonlocal σ model whose target space is the vector space. This construction can probably be generalized to the study of particles and strings on anti–de Sitter superspaces such as those important for the AdS-CFT correspondence.

II. THE TWISTOR CONSTRUCTION

We begin by describing cosets in a language which naturally leads to twistors. A point in a coset G/H is associated with a hypersurface in the group manifold by the relation

$$\phi(\hat{x}\in G)=\{\hat{x}h:h\in H\}. \tag{1}$$

The $\hat{x}$ which generate distinct $\phi(\hat{x})$ are given by

$$\hat{x}:=v(\xi)\hat{x}_0, \tag{2}$$

where $\hat{x}_0\in G$ is the origin of the coset space (an arbitrary point), and ξ is a collective coordinate on G/H. The function $v(\xi)$ is a coset representative, which for our purposes is a function from the coordinates to the group such that $v(0)=1$ and $\phi\circ v$ is 1-1, so distinct hypersurfaces are associated with distinct coordinates. A particular form of $v(\xi)$ which we will often use is

$$v(\xi)=e^{\xi\cdot K}h(\xi), \tag{3}$$

where the K are the generators of G not in H, and $h(\xi)$ is some H-valued function chosen to simplify the resulting expression.

Using this function, we can associate coset coordinates with hypersurfaces in the group manifold in a natural way:

$$\phi(\xi)=\phi(\hat{x}(\xi))=v(\xi)\phi(\hat{x}_0). \tag{4}$$

These hypersurfaces are naturally invariant under the right action of H.

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[1]The results below apply both to cosets and supercosets, with no additional restrictions (reductivity, symmetry, etc.) except where explicitly noted.

Such a construction cannot always be globally performed. The problem is analogous to the selection of coordinates on a sphere $S^2 = SO(3)/SO(2)$. Technically, it arises because the coset representative $v(x)$ is a section of the principal bundle $G \rightarrow G/H$ and so in general cannot be globally defined. We may resolve this issue analogously to the problem of coordinates by covering G/H with patches (open sets whose intersections are contractible) and performing this construction on each open set. Transition functions on intersections are naturally induced from the transition functions of the principal G-bundle of which v is a section. It is also necessary to choose a different $\hat{x}_0$ for each patch, which is analogous to using the north and south poles as origins of the two coordinate systems on S^2. The result of such a construction will (as we will see below) be a twistor bundle rather than a global twistor space, but this will not introduce any unfamiliar complexities.

Using this construction, the geometry on the coset space may be defined by the invariances of the Cartan form $L = v^{-1}dv$. This form is canonically separated into $L = E \cdot K + \Omega \cdot H$, where E is the vielbein and Ω the H-connection. (This generalizes the spin connection of Minkowski space.) When G is semisimple, this can be contracted with a restricted Cartan-Killing metric to give a metric on the coset,

$$g^{\mu\nu} = \eta^{AB} L_A^{\mu} L_B^{\nu}, \tag{5}$$

where the indices A and B run over only the coset (K) generators[2] of G. In the more general case (which includes Minkowski space) the procedure is somewhat more subtle. For some groups at least, there is an invariant symmetric two-form which may take the place of the Cartan-Killing metric η; however, there is no general existence proof nor is there a method of computing such forms. In this case one assumes that the transformations which lead to covariant transformations of the K-components of the Cartan form will become isometries of the coset spaces.

The isometries of this space are given implicitly by the action of G on the coset representative:

$$\delta\xi : \delta v(\xi) = g v(\xi), \tag{6}$$

where $g \in G$. This implies that

$$\delta\phi(\xi) = g\,\phi(\xi). \tag{7}$$

It is straightforward to compute the actual transformations of the ξ from this relationship if we write Eq. (7) in an explicit representation. This motivates us to define twistors to be explicit group representations of these hypersurfaces $\phi(\xi)$. Specifically, if we represent the group on a vector space Λ, a twistor is a mapping of coordinates to H-invariant hypersurfaces in Λ given by

$$\mathcal{Z}(\xi) = v(\xi)\mathcal{Z}_0, \tag{8}$$

where $v(\xi)$ is the Λ-representation of the coset representative, and $\mathcal{Z}_0$ is an H-invariant hypersurface in Λ. [The Λ-representation of $\phi(\hat{x}_0)$.] As before, the mapping $\mathcal{Z}$ must be 1-1 for the set of $\mathcal{Z}(\xi)$ to be isomorphic to the coset, which means that the codimension of $\mathcal{Z}_0$ in Λ must be no less than the dimension of the coset. [$\operatorname{codim} \mathcal{Z}(\xi) = \operatorname{codim} \mathcal{Z}_0$ for any ξ since $v(x)$ is surjective.] We will also restrict ourselves to $\dim \mathcal{Z}_0 > 0$, since otherwise $\mathcal{Z}$ would be a mapping of points onto points and so would lose several of the interesting features which we will discover below.

We next write $\mathcal{Z}_0$ explicitly as a linear function of some coordinates on Λ. (These coordinates may be curvilinear, although we do not consider such possibilities in depth here.) Then using the explicit form of the coset representative $v(\xi)$, we may write each $\mathcal{Z}(\xi)$ as a function of the coordinates ξ and the internal coordinates λ of $\mathcal{Z}_0$ which is linear in λ and typically fairly complicated in ξ.

This process has two advantages. First, since the $\mathcal{Z}$ are given as explicit functions of the coset coordinates, it is straightforward to use Eq. (6) to compute the geometric properties of the space. This is especially valuable in the case of complicated cosets such as $\mathrm{AdS}_p \times S^{d-p}$ superspace, where traditional (differential-equation) methods of calculating isometries are very difficult. Second, since Λ is a Hilbert space there is a natural continuous and complete inner product of twistors which is manifestly invariant under the action of G. This allows one to easily construct quantities with a very complicated dependence on ξ and λ which are invariant under G. This invariance persists even though $H \subset G$ is typically nonlinearly realized on the coset space. If (as we will do later) we identify the λ with some internal parameters of a system such as momenta, it is possible to use these invariants to construct very simple twistor-based actions which are equivalent to very complicated coordinate-space actions.

As the preceding discussion was somewhat abstract, it is useful to consider some explicit examples. We begin with the case of conformal Minkowski space $SO(3,2)/ISO(3,1) \times D$, where D is the dilatation operator. We choose as our representation the 4-component spinor representation of $SO(3,2)$, which decomposes into a 2-component spinor and a 2-component conjugate spinor of $ISO(3,1)$ of conformal weights[3] $\pm 1/2$. In this representation, a group element has the form

$$g = \begin{pmatrix} L_{\beta}{}^{\alpha} + \frac{1}{2} D\,\delta_{\beta}{}^{\alpha} & -iK_{\alpha\dot{\alpha}} \\ -iP^{\dot{\alpha}\alpha} & -\bar{L}^{\dot{\alpha}}{}_{\dot{\beta}} - \dfrac{1}{2}\delta^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \tag{9}$$

and the initial hypersurface $\mathcal{Z}_0$ has the form

$$\mathcal{Z}_0 = \begin{pmatrix} \lambda_{\alpha} \\ \mu^{\dot{\alpha}} \end{pmatrix}, \tag{10}$$

[2]This procedure is discussed in detail in [8].

[3]Although we use spinorial representations here, this is by no means a general feature of twistors.

where the α ($\dot{\alpha}$) are (conjugate) spinor indices of $SO(3,1)$ and λ and μ are complex. The stability group H is generated by the L, K, and D. Lorentz invariance implies that if $\mathcal{Z}_0$ has any point with $\lambda \neq 0$, it must contain all such points, and likewise for μ. Thus the dimension constraint $0 < \dim \mathcal{Z}_0 \leq 4$ requires that exactly one of the two be independent. Without loss of generality, we choose λ and let μ be a linear function thereof. The remaining part of H-invariance then requires $\mu = 0$.

Now let us choose the canonical coset representative

$$v(x) = e^{-ix\cdot P} = \begin{pmatrix} 1 & 0 \\ -ix^{\dot{\alpha}\alpha} & 1 \end{pmatrix}. \tag{11}$$

The twistor mapping is now

$$\mathcal{Z}(x) = \begin{pmatrix} \lambda_\alpha \\ -ix^{\dot{\alpha}\alpha}\lambda_\alpha \end{pmatrix}. \tag{12}$$

This is the familiar Penrose twistor formula. [4] We will not discuss isometries and invariants in this case, saving that instead for the more detailed example of AdS_p below.

It is worth noting that this procedure was by no means unique. The freedoms of choice are in the selection of an appropriate coset representative [which will typically be determined by algebraic simplicity, subject to the requirement that $\mathcal{Z}(\xi)$ is 1-1] and in the choice of the initial hypersurface $\mathcal{Z}_0$.

We can also naturally ask about the invariants which may be constructed out of these twistors. The simplest world-line action which one may construct out of these twistors is clearly

$$\mathcal{L} = i\bar{\mathcal{Z}}\partial\mathcal{Z}, \tag{13}$$

where contraction has been performed with the standard spinor metric. If we substitute in Eq. (12), and write

$$P_{\dot{\alpha}\alpha} = \bar{\lambda}_{\dot{\alpha}}\lambda_\alpha, \tag{14}$$

then this action reduces to the simple form

$$S = i\int d\tau P\cdot\partial x \tag{15}$$

which is the usual world-line action for a massless particle. P is automatically null in this case because the spinor metric $\epsilon^{\alpha\beta}$ is antisymmetric. Massive actions cannot easily be written in terms of these twistors, which is unsurprising since we are here working in conformal Minkowski space.

In this case we have put the internal coordinates λ_α of the twistor to use as momenta of the particle. It is not clear how general such an interpretation is; clearly a precondition for the possibility of so doing is that the twistor bundle [the set of these hypersurfaces $\mathcal{Z}(x)$ over every point, with open sets as discussed above] contains the tangent bundle of G/H as a subbundle. Even when this is not possible, the procedure above will turn the λ_α into Lagrange multipliers for various quantities; when the quantities are derivatives of the coordinates, there is a somewhat natural momentum interpretation. We will see more of this construction later.

III. TWISTORIZATION OF AdS_p

We now turn to the case of particles on $\mathrm{AdS}_p = SO(p-1,2)/SO(p-1,1)$. The ordinary world-line action for these particles is manifestly G-invariant but does not have a quadratic kinetic term, so it is useful to try to rephrase this in terms of twistors. This is reasonable since the first-order action

$$\mathcal{L} = \frac{1}{2}P\cdot\partial x + P_\rho\partial\rho + u\left[\frac{1}{2\rho^2}P^2 - \rho^2P_\rho^2 - m^2R^2\right] \tag{16}$$

contains only terms of the form $P\cdot\partial x$, which are similar to those found in the conformal Minkowski action (13), and a constraint term which is G-invariant although not manifestly so. In a twistor construction one hopes that this can be rewritten in a manifestly symmetric (and preferably simple) way, and we will see that this is indeed the case.

Twistorization must begin with a choice of G-representation. The two simplest choices are the fundamental and the spinor. The fundamental has simpler group generators, but since its dimension is $(p+1)$ such twistors would have only one internal coordinate and so momenta could not be encoded by the twistor. Therefore we use the spinor representation, which has complex dimension $2^{[(p+1)/2]} \equiv 2d$. The group elements in this representation are[4]

$$g^A{}_B = \begin{pmatrix} L_\beta{}^\alpha + \frac{1}{2}D\delta_\beta{}^\alpha & -iK_{\alpha\dot{\alpha}} \\ -iP^{\dot{\alpha}\alpha} & -\bar{L}^{\dot{\alpha}}{}_{\dot{\beta}} - \frac{1}{2}D\delta^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix}. \tag{17}$$

The K and P generate conformal transformations and conformal momentum, which are related to AdS conformal transformations and momenta by

$$\tilde{K} = (K-P)/2$$

$$\tilde{P} = (K+P)/2. \tag{18}$$

The $L_\beta{}^\alpha$ generate the Lorentz group and the D are dilatations. The stability group H is generated by the L's and the $\tilde{K}$'s.

First we must choose an H-invariant initial hypersurface. We can write this surface in the form

$$\mathcal{Z}_0^A = \begin{pmatrix} \lambda_{0\alpha} \\ \mu_0^{\dot{\alpha}} \end{pmatrix}. \tag{19}$$

[4]Our index notation is $\mu = 0 \ldots p-2$ is an $SO(p-2,1)$ (Lorentz) vector index, α,β and $\dot{\alpha},\dot{\beta} = 1 \ldots d$ are Lorentz spinor and conjugate spinor indices, respectively. A,B and $\dot{A},\dot{B} = 1 \ldots 2d$ are spinor and conjugate spinor indices of $SO(p-1,2)$.

As in the Penrose case, L-invariance requires that if $\mathcal{Z}_0$ contains any points with $\lambda_0 \neq 0$, then it contains all such points, and similarly for μ_0. Since we want $0 < \dim \mathcal{Z}_0 \leqslant 2d-p$, only one of these two should be independent of the other. Without loss of generality we choose λ_0 to be independent, and fix μ_0 by

$$\mu_0^{\dot{\alpha}} = F^{\dot{\alpha}\beta}\lambda_{0\beta} + G^{\dot{\alpha}\dot{\beta}}\overline{\lambda}_{0\dot{\beta}} \tag{20}$$

for some $F^{\dot{\alpha}\beta}$ and $G^{\dot{\alpha}\dot{\beta}}$ which parametrize our twistorization. $\widetilde{K}$-invariance then requires that

$$F\gamma_\mu F + G\gamma_\mu \overline{G} = \gamma_\mu \tag{21}$$

$$F\gamma_\mu G + G\gamma_\mu \overline{F} = 0 \tag{22}$$

where the $\gamma_\mu^{\alpha\dot{\alpha}}$ are the Dirac matrices for $SO(p-2,1)$. For simplicity we will consider the case $F=0$, so

$$\mathcal{Z}_0^A = \begin{pmatrix} \lambda_{0\alpha} \\ G^{\dot{\alpha}\dot{\beta}}\overline{\lambda}_{0\dot{\beta}} \end{pmatrix}. \tag{23}$$

A simple choice of coset representative is

$$v(x^\mu,\rho) = e^{x\cdot P}e^{D\log\rho} = \begin{pmatrix} \rho^{1/2} & 0 \\ -i\rho^{1/2}x^{\dot{\alpha}\alpha} & \rho^{-1/2} \end{pmatrix}; \tag{24}$$

using this, and defining $\lambda = \rho^{1/2}\lambda_0$, the twistor is

$$\mathcal{Z}^A(x^\mu,\rho) = \begin{pmatrix} \lambda_\alpha \\ -ix^{\dot{\alpha}\alpha}\lambda_\alpha + \rho^{-1}G^{\dot{\alpha}\dot{\beta}}\overline{\lambda}_{\dot{\beta}} \end{pmatrix}. \tag{25}$$

As a check, the isometries of the space can be calculated from

$$\delta\mathcal{Z}^A = g^A{}_B\mathcal{Z}^B. \tag{26}$$

Varying both sides of Eq. (25), one finds

$$\delta\lambda_\alpha = \left(L_\alpha{}^\beta + \frac{1}{2}D\delta_\alpha{}^\beta + K_{\alpha\dot{\alpha}}x^{\dot{\alpha}\beta}\right)\lambda_\beta - i\rho^{-1}K_{\alpha\dot{\alpha}}G^{\dot{\alpha}\dot{\beta}}\overline{\lambda}_{\dot{\beta}} \tag{27}$$

and so

$$\delta x^{\dot{\alpha}\alpha} = P^{\dot{\alpha}\alpha} - x^{\dot{\alpha}\beta}L_\beta{}^\alpha - \overline{L}^{\dot{\alpha}}{}_{\dot{\beta}}x^{\dot{\beta}\alpha} + Dx^{\dot{\alpha}\alpha} + x^{\dot{\alpha}\beta}K_{\beta\dot{\beta}}x^{\dot{\beta}\alpha} - \rho^{-2}K^{\dot{\alpha}\alpha} \tag{28}$$

$$\delta\rho = -D\rho - 2\rho x\cdot K \tag{29}$$

which are the well-known isometries of anti–de Sitter space.

Geometric invariants may now be constructed by contracting $\mathcal{Z}$ with the $SO(p-2,1)$ metric

$$H_{\dot{A}}{}^B = \begin{pmatrix} 0 & \mathcal{C}^{\dot{\alpha}\dot{\beta}} \\ \overline{\mathcal{C}}^{\dot{\alpha}\dot{\beta}} & 0 \end{pmatrix} \tag{30}$$

where $\mathcal{C}$ is the charge conjugation matrix, so

$$\overline{\mathcal{Z}}_1\cdot\mathcal{Z}_2 = \overline{\lambda}_1\mu_2 + \overline{\mu}_1\lambda_2. \tag{31}$$

A natural first guess for a particle action is

$$\mathcal{L} = i\overline{\mathcal{Z}}\partial\mathcal{Z}. \tag{32}$$

This matches the kinetic term in Eq. (16) if we identify components of λ with the momenta as follows:

$$P_{\alpha\dot{\alpha}} = 2\lambda_\alpha\overline{\lambda}_{\dot{\alpha}}$$

$$P_\rho = \frac{i}{2\rho^2}[\overline{\lambda}G\overline{\lambda} - \lambda\overline{G}\lambda]. \tag{33}$$

In Eq. (32), however, all the components of λ are independent and so their dynamics must be specified. The first condition is the mass-shell constraint,

$$\frac{1}{2\rho^2}P^2 - \rho^2P_\rho^2 = \frac{1}{4}(\overline{\mathcal{Z}}\mathcal{Z})^2 = M^2R^2. \tag{34}$$

There exist further independent components of λ for most values of p. These may be fixed by fixing the values of a set of twistor bilinears

$$\phi_i \equiv \overline{\mathcal{Z}}T_i\mathcal{Z} \tag{35}$$

where $(T_i)_{\dot{A}}{}^B$ are some constant matrices which transform in the $(\frac{1}{2},\frac{1}{2})$ of $SO(p-1,2)$. The number of independent ϕ_i that must be set depends on p. In an action, these will be constrained to values m_i. For example, using Eq. (34) the mass-shell constraint is $\phi_{T=1} = 2MR$. So the complete twistor action takes the simple form

$$\mathcal{L} = i\overline{\mathcal{Z}}(\partial - iu^iT_i)\mathcal{Z} - u^im_i \tag{36}$$

where the u^i are Lagrange multipliers. This action is equivalent to Eq. (16). It has several important features:

(1) The action is manifestly $SO(p-1,2)$ invariant and has a quadratic kinetic term. It has the structure of a world-line gauge theory with sources. The "gauge fields" u^i are nondynamical since there is no field strength in one dimension.

This statement can be made somewhat more precise by noting that Eq. (32) implies that the Poisson brackets (which will become commutators in the quantized theory) are

$$\{\mathcal{Z}_A, \overline{\mathcal{Z}}^{\dot{B}}\}_{PB} = -2iH_A{}^{\dot{B}} \tag{37}$$

with all other brackets vanishing, and so

$$\{\phi_i,\phi_j\}_{PB} = -2i\overline{\mathcal{Z}}[T_i,T_j]\mathcal{Z}. \tag{38}$$

Since the set of constraints under Poisson brackets forms a Lie algebra, the set of T_i form one as well, and this algebra is invariant under $SO(p-1,2)$. This guarantees that the action (36) indeed has a gauge symmetry.

(2) The gauge group contains a $U(1)$ factor corresponding to the mass-shell constraint $T=1$, $m=2MR$. The rest of the group may be calculated explicitly for small p by constructing the ϕ_i; they are

p	1	2	3	4	5	6	7
dim $\mathcal{Z}_0$	2	2	4	4	8	8	16
N_ϕ	1	0	1	0	3	2	9
Group	$U(1)^2$	$U(1)$	$U(1)^2$	$U(1)$	$U(1)\times SU(2)$	$U(1)^3$?	$U(1)\times SU(2)^3$?

The final two are conjectured but have not been explicitly calculated.

This is related to the result of [6,7] for AdS_5. In that case, the 8-component spinors were decomposed into a pair of 4-component spinors of the stability group $H=SO(4,1)$ indexed by $I,J=1,2$, and

$$(T_i)_{aI}{}^{bJ}=(\sigma_i)_I{}^J\mathcal{C}_a{}^b. \tag{39}$$

[The a,b are $SO(4,1)$ spinor indices.]

(3) This twistor Lagrangian can be quantized following a procedure similar to that used in [7,9], leading to solutions which transform in representations of $SO(p-1,2)$.

(4) For $i\neq 0$, The ϕ_i may be chosen to be independent of the momenta. In these cases it is not clear what meaning one could assign to a nonzero m_i. The analogous quantities in [6,7] are all zero.

(5) The Lagrange multipliers u^i can be integrated out to give

$$\mathcal{L}'(k)=i\bar{\mathcal{Z}}(\partial+iT\cdot m)\mathcal{Z}|_k+\int\frac{dq}{2\pi}(\bar{\mathcal{Z}}T^i\mathcal{Z})|_{k+q}(\bar{\mathcal{Z}}T_i\mathcal{Z})|_{k-q} \tag{40}$$

which is therefore equivalent to Eq. (36). (This can also be seen by explicitly resumming Feynman diagrams involving the u^i.)

The actions (36) and (40) represent a considerable simplification over their classical counterpart (16). Because they have leading quadratic terms and manifest G-symmetry, their quantum solutions automatically fill out representations of the isometry group. A similar construction can be carried out for an arbitrary coset manifold, or even a supercoset, and (similarly to [9]) can be used to construct string actions on these spaces. Since the known superstring actions are manifestly invariant under the isometries, it is likely that these systems will be amenable to a twistor interpretation which would allow their quantization and analysis, including interactions with Ramond-Ramond and Neveu-Schwarz background fields.

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