Generality of singularity avoidance in superstring theory: Anisotropic case

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In the one-loop string effective action, we study a generality of nonsingular cosmological solutions found in the isotropic and homogeneous case. We discuss Bianchi type-I and -IX spacetimes. We find that nonsingular solutions still exist in the Bianchi type-I model around nonsingular flat Friedmann solutions. On the other hand, we cannot find any nonsingular solutions in the Bianchi type-IX model. The nonexistence of a nonsingular Bianchi type-IX universe may be consistent with the analysis of Kawai, Sakagami, and Soda; i.e., the tensor-mode perturbations against a nonsingular flat Friedmann universe are unstable, because the Bianchi type-IX model is regarded as a closed Friedmann universe with a single gravitational wave. With the stability analysis of Kawai, Sakagami, and Soda, the nonsingular universe found in the isotropic case is unstable, and a singularity avoidance may not work in generic spacetimes.

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I. INTRODUCTION

The initial singularity is one of the most serious problems in the big bang universe. Even the inflationary universe model, which resolves many difficulties in the early universe, cannot avoid are initial singularity. The quantum effect might resolve it as a scenario in quantum cosmology, but quantum gravity is not yet completed. To construct a theory of quantum gravity, a unification of fundamental interactions may be one of the most promising ways. Among the many attempts at such a unification, the superstring model would be the best candidate [1]. In fact, the origin of black hole entropy may be understood by string theory. Hence, the initial singularity problem might also be solved in the context of string theory, which may allow us to manage the physics at the Planck scale. However, a full theory has not yet been developed. We have so far been unable to discuss the early universe in string theory as it is, except for some restricted models [2]. Therefore, many attempts to find a nonsingular universe are mainly based on a low-energy effective field theory of superstring model. Such an effective field theory will be invalid beyond the Planck energy scale. However, if a superstring theory is truly the theory of everything, its effective theory may reveal some important aspects about gravity. In particular, if it has a property of singularity avoidance, it would be good evidence for a superstring.

Based on an effective theory, there is one interesting approach called the pre-big-bang universe model [3]. This model is based on T duality, which gives some relation between large and small scales. Assuming that the Universe has such a duality [4], we find a cosmological solution which consists of two distinct and disconnected branches. One of the branches ($t>0$) corresponds to the expanding Friedmann universe, and other branch ($t<0$) gives another expanding universe, ending up with a singularity at $t=0$. In the pre-big-bang scenario, however, those two disconnected branches are assumed that be connected without a singularity on account of some unknown stringy effect, which is not included in the lowest effective action.

There are several works that study whether two branches can be connected without a singularity. At the tree-level in a superstring action, however, a possibility of classical branch-changing solutions is excluded [5]. Only a quantum effect may be a method to connect two branches. In fact, taking into consideration one-loop contributions to a superstring effective action with dilaton and modulus fields [6], Antoniadis, Rizos, and Tamvakis found a nonsingular solution with a spatially flat background. Then, with the same action, Easther and one of the present authors also showed a nonsingular closed universe [7]. Although those solutions do not help the pre-big-bang scenario, they may still be interesting because of singularity avoidance.

A question about such nonsingular solutions, however, is whether or not they are generic. Although we find nonsingular solutions for some finite range of initial data, the spacetime is assumed to be isotropic and homogeneous [7]. The universe, however, may begin with an anisotropic and/or inhomogeneous geometry. Therefore, we have to study whether or not the present nonsingular solution is generic, or is stable or unstable. Recently, Kawai, Sakagami, and Soda analyzed the stability of the flat nonsingular solution against perturbations, and found that a tensor mode is unstable. However, since a closed nonsingular universe bounces in a

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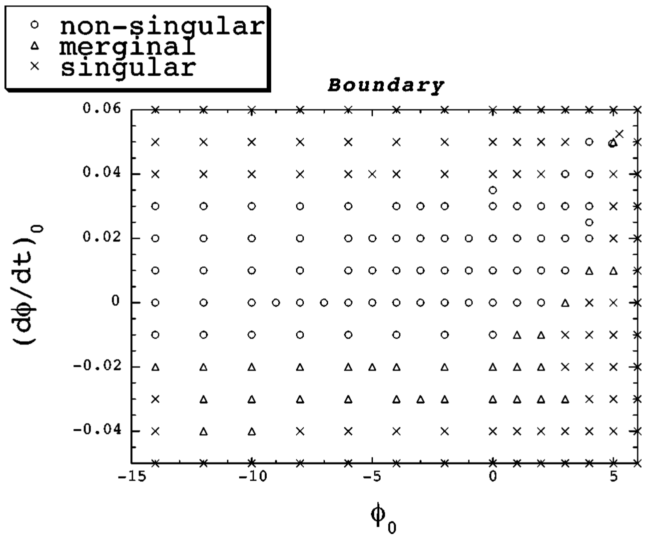

FIG. 1. We show the fate of the flat Friedmann universe in terms of initial values ϕ_0 and $\dot{\phi}_0$, in the cases of the flat Universe. ○ means regular solutions, while × means that the Universe evolves into a singularity. For △, the solution seems to be singular, although we could not confirm it because we need more CPU time.

finite time, it is not clear whether it is unstable against tensor mode perturbations. In order to clarify such a problem and to study the generality of nonsingular solutions, in this paper, we will analyze two types of Bianchi models; Bianchi type-I and type-IX models.

This paper is organized as follows. In the next section, we introduce the basic equations for Bianchi type-I and type-IX models. The numerical results are presented in Sec. III, and conclusions and discussions follow. We adopt the metric signature $(-,+,+,+)$ and units of $c=8\pi G=1$.

II. BASIC EQUATIONS

We take the following one-loop effective action [6–11]:

$$S=\int d^4x\sqrt{-g}\bigg[\frac{1}{2}R-\frac{1}{4}\nabla_\mu\phi\nabla^\mu\phi-\frac{3}{4}\nabla_\mu\sigma\nabla^\mu\sigma -\frac{1}{6}H^{\mu\nu\lambda}H_{\mu\nu\lambda}+\frac{1}{16}[\lambda e^{\phi}-\delta\xi(\sigma)]R^2_{GB} +(\text{higher curvature terms})\bigg], \tag{2.1}$$

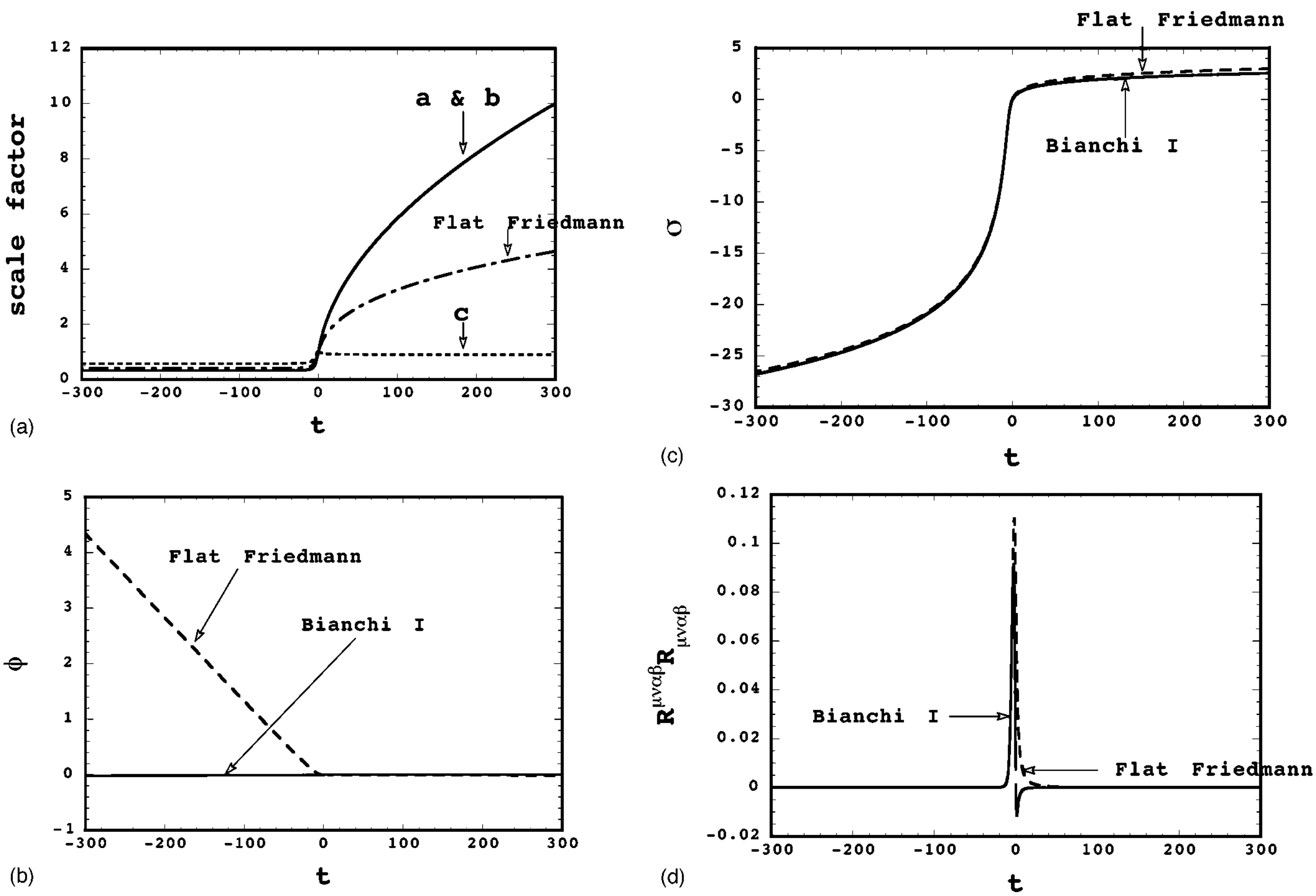

FIG. 2. We show one nonsingular solution. We choose a negative value of $\overline{\delta}$ as $\overline{\delta}=-48/\pi$. We have set $\dot{\Omega}_0=-0.1$, $\phi_0=\dot{\phi}_0=0$, $\sigma_0=0$, and $\dot{\beta}_{+0}=0.05$, $\dot{\beta}_{-0}=0.0$. $\dot{\sigma}_0=0.173\,205$ has been determined by the constraint equation ($\dot{\sigma}_0=0.200$ for the isotropic case). We show the scale factor a in (a), the dilaton field ϕ in (b), the modulus field σ in (c), and $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ in (d), respectively. The solid line, dashed line, and dotted line represent scale factors a, b, and c in the anisotropic case, respectively. The dash-dotted line represents a scale factor in the isotropic case.

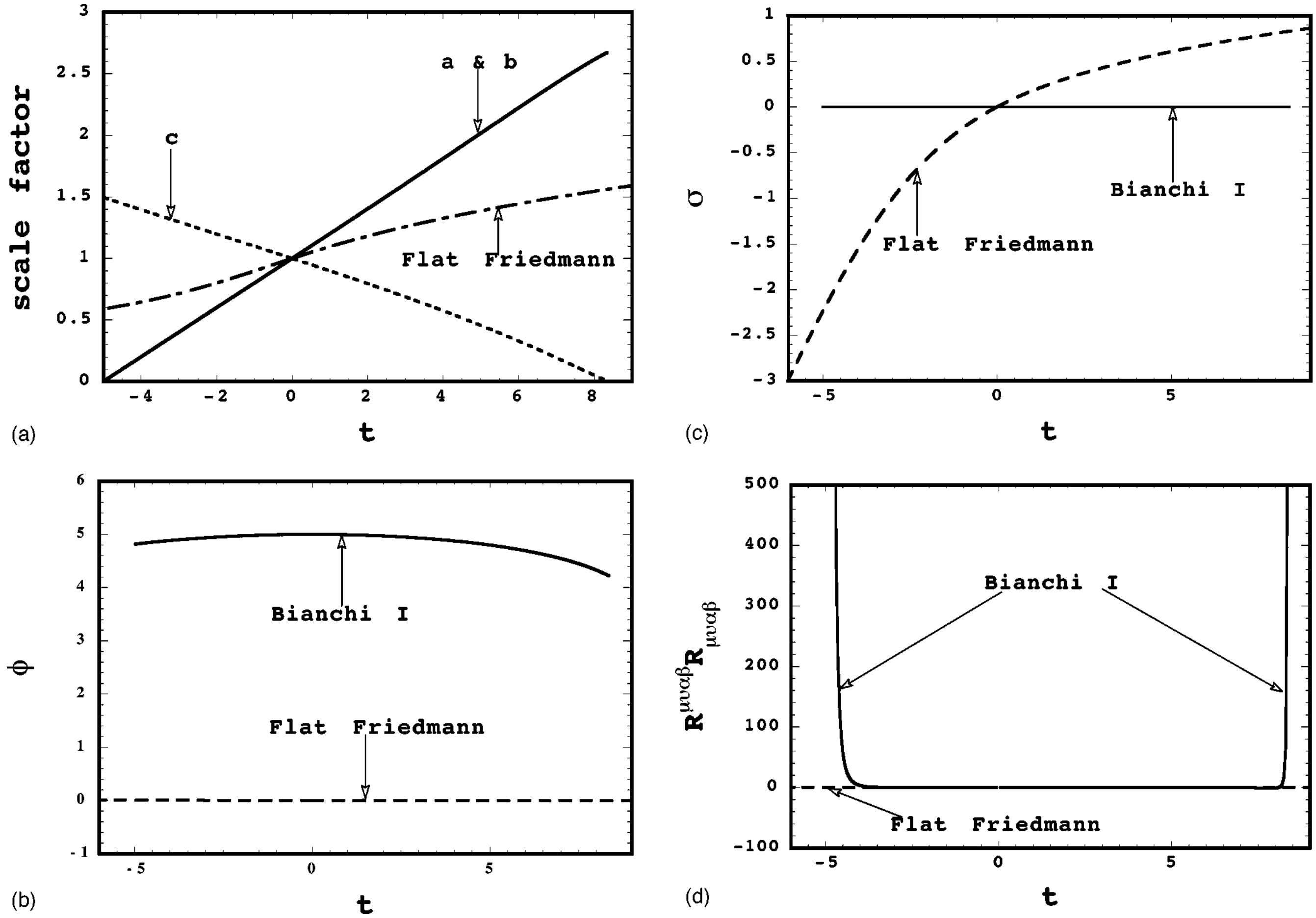

FIG. 3. We show one singular solution. We choose a negative value of $\overline{\delta}$ as $\overline{\delta}=-48/\pi$. We have set $\dot{\Omega}_0=-0.1,\ \phi_0=\dot{\phi}_0=0,\ \sigma_0=0$, and $\dot{\beta}_{+0}=0.1,\ \dot{\beta}_{-0}=0.0$. $\dot{\sigma}_0=0.00$ has been determined by the constraint equation ($\dot{\sigma}_0=0.200$ for the isotropic case). We show the scale factor a in (a), the dilaton field ϕ in (b), the modulus field σ in (c), and $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ in (d), respectively. The solid line, dashed line, and dotted line represent scale factors a, b, and c in the anisotropic case, respectively. The dash-dotted line represents a scale factor in the isotropic case.

where R, ϕ, and σ are the scalar curvature, the dilaton, and the modulus field, respectively,

$$R^2_{GB}=R^2-4R^{\mu\nu}R_{\mu\nu}+R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \tag{2.2}$$

is the Gauss-Bonnet term, and H is the antisymmetric tensor field. The coefficient λ is positive definite and determined by the inverse string tension α'. The coefficient δ, which depends on the relative numbers of chiral, vector, and spin-$\frac{3}{2}$ massless supermultiplets, is proportional to the four-dimensional trace anomaly of the $N=2$ sector. The δ would be either positive or negative. The function $\xi(\sigma)$ is given by

$$\xi(\sigma)=\ln[2e^{\sigma}\eta^4(ie^{\sigma})] \tag{2.3}$$

with the Dedekind η function [12], defined by

$$\eta(\tau)=q^{1/12}\prod_{n=1}^{\infty}(1-q^{2n}), \quad q=e^{i\pi\tau}. \tag{2.4}$$

The first derivative of ξ with respect to σ is

$$\xi_{\sigma}(\sigma)=1-\frac{\pi e^{\sigma}}{3}+8\pi e^{\sigma}\sum_{n=1}^{\infty}\frac{ne^{-2n\pi e^{\sigma}}}{1-e^{-2n\pi e^{\sigma}}}, \tag{2.5}$$

which is approximated very well by $\sinh\sigma$ as

$$\xi_{\sigma}(\sigma)\approx-\frac{2\pi}{3}\sinh\sigma, \tag{2.6}$$

as was shown in [7]. Then we will use Eq. (2.6) in our analysis just for simplicity, although we have also checked our results by use of the exact function (2.5). We also introduce the following function $f(\phi,\sigma)$ for convenience:

$$f(\phi,\sigma)=\frac{1}{16}[e^{\phi}-\overline{\delta}\xi(\sigma)], \tag{2.7}$$

where $\overline{\delta}\equiv\delta/\lambda$. We set $H\equiv0$ and ignore higher curvature terms than second order. It is convenient to rescale time and spatial coordinates by λ as $\overline{t}=t/\sqrt{\lambda}$, $\overline{x^i}=x^i/\sqrt{\lambda}$. Hereafter we drop a bar for brevity.

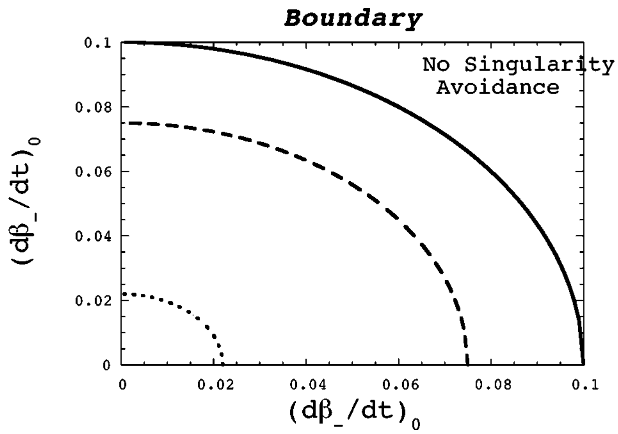

FIG. 4. Changing the anisotropic parameters $\dot{\beta}_{+0}$ and $\dot{\beta}_{-0}$, we search nonsingular solutions. For $\overline{\delta}=-48/\pi$, $\dot{\Omega}_0=-0.1$, and $\sigma_0=0$, we show the boundary on the $\dot{\beta}_{+0}$ and $\dot{\beta}_{-0}$ plane, beyond which no nonsingular solution is found. The boundary is almost a circle. The solid line, dashed line, and dotted line represent the boundary in the cases of $\phi_0=\dot{\phi}_0=0$, of $\phi_0=1.5$, $\dot{\phi}_0=0.20$, and of $\phi_0=3.0$, $\dot{\phi}_0=0.40$, respectively.

With the present model, cosmological solutions have been studied from a view point of the initial singularity problem. Antoniadis, Rizos, and Tamvakis analyzed a spatially flat Friedmann model and found a nonsingular solution [6]. Easther and Maeda extended their analysis to a closed Friedmann model and also showed a nonsingular solution [7]. Since we are interested in the generality of those nonsingular solutions, we extend their works to anisotropic spacetimes. Here we study only Bianchi type-I and type-IX models because those spacetimes include a flat and a closed Friedmann models.

Taking a variation of the action, we obtain the basic equations. With those basic equations, for Bianchi type-I and type-IX models, we can assume the following diagonal metric form:

$$ds^2=-dt^2+e^{-2\Omega}e^{2\beta_{ij}}\omega^i\omega^j, \tag{2.8}$$

where

$$\beta_{ij}=\begin{bmatrix} \beta_+ + \sqrt{3}\beta_- & 0 & 0 \\ 0 & \beta_+ - \sqrt{3}\beta_- & 0 \\ 0 & 0 & -2\beta_+ \end{bmatrix}$$

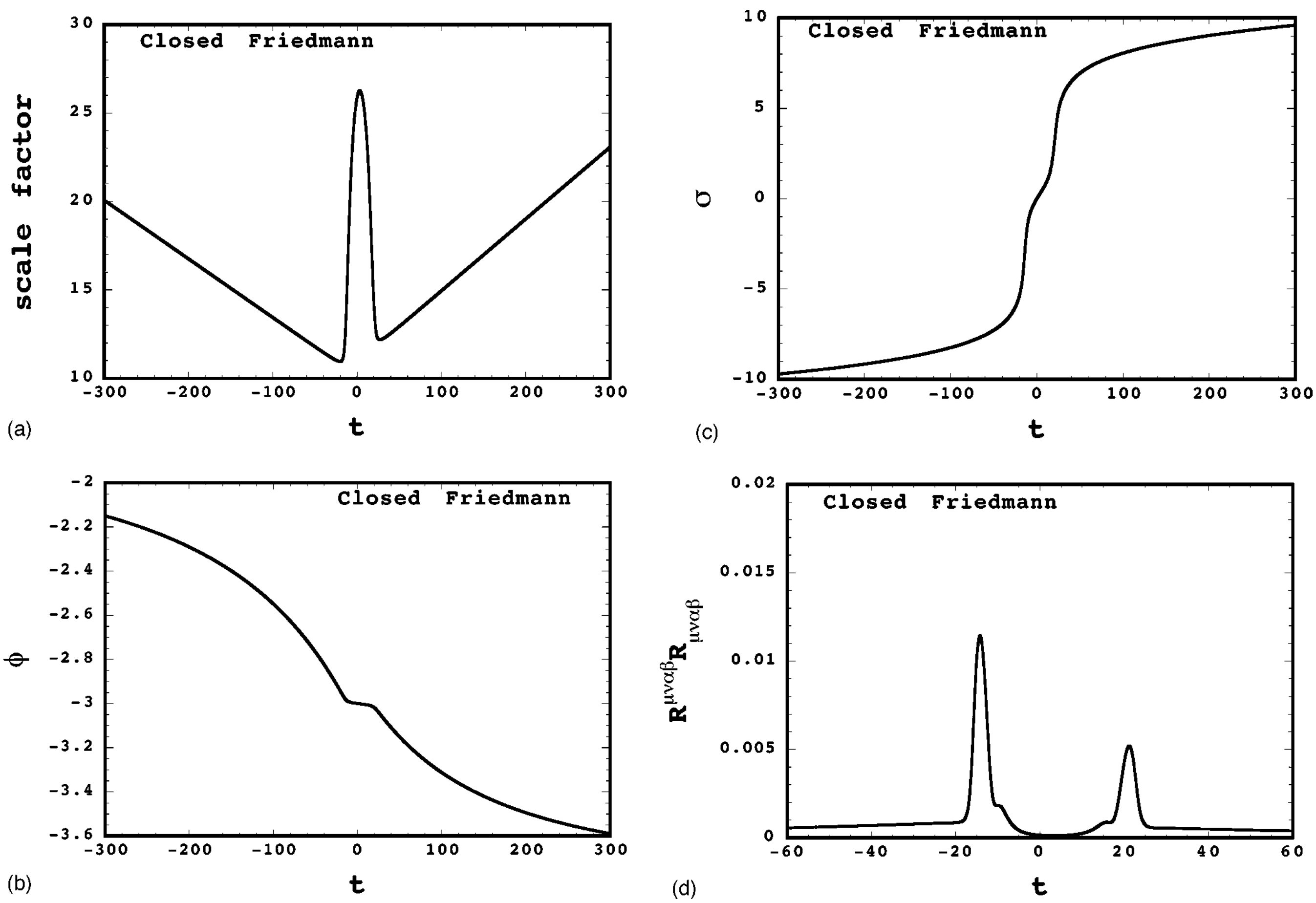

FIG. 5. We show a nonsingular solution in a closed Friedmann universe. We choose a negative value of $\overline{\delta}$ as $\overline{\delta}=-48/\pi$. We set $a_0=e^{3.2511380746580628}$, $\dot{\Omega}_0=-0.01$, $\phi_0=-3$, $\dot{\phi}_0=-5\times10^{-4}$, $\sigma_0=0$. We show the scale factor a in (a), the dilaton field ϕ in (b), the modulus field σ in (c), and $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ in (d), respectively.

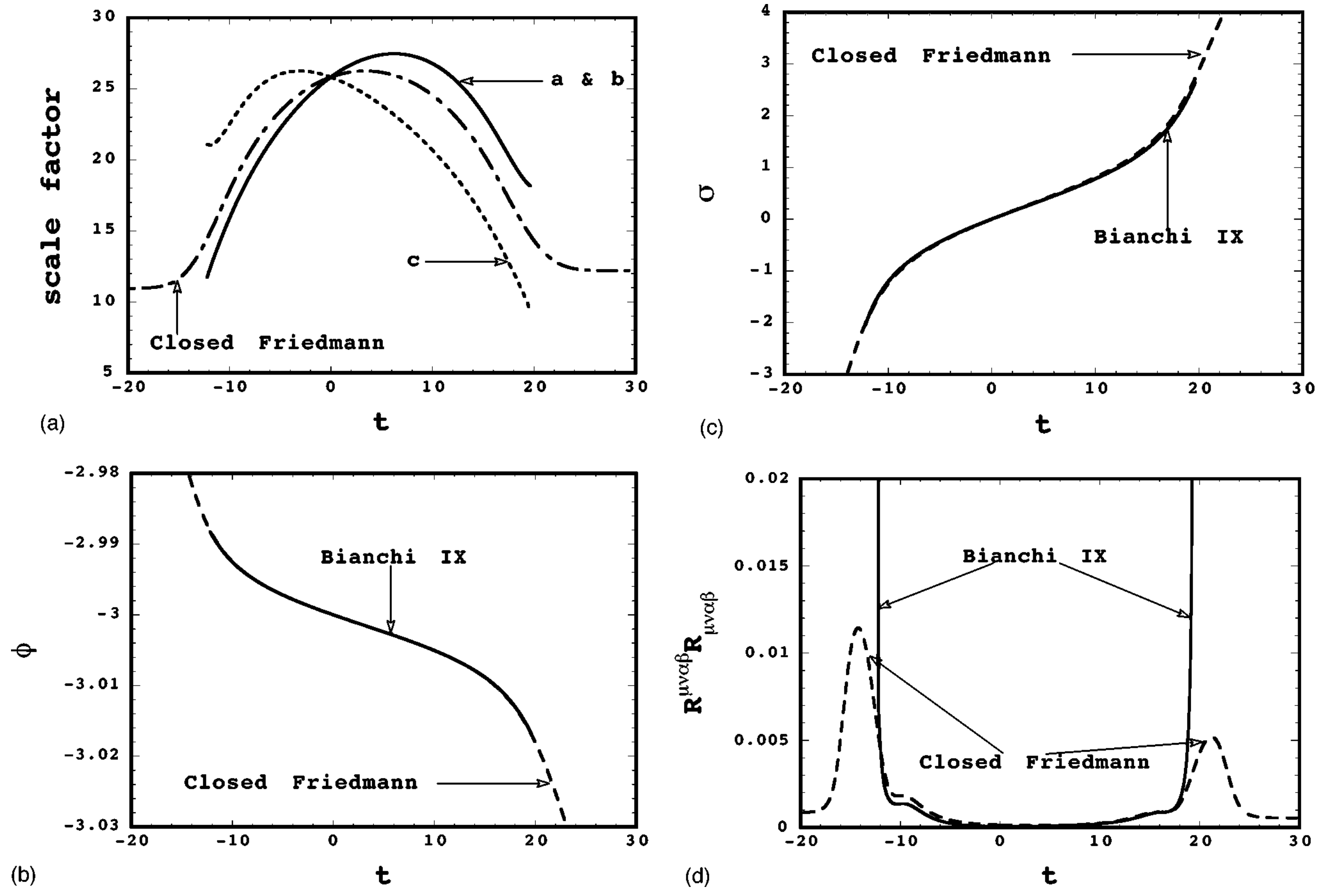

FIG. 6. We show a singular solution in the Bianchi type-IX model. We set the same initial values in Fig. 5, $\overline{\delta}=-48/\pi$, $a_0=b_0=c_0=e^{3.2511380746580628}$, $\dot{\Omega}_0=-0.01$, $\phi_0=-3$, $\dot{\phi}_0=-5\times 10^{-4}$, $\sigma_0=0$, except for anisotropy, which is chosen as $\beta_{+0}=0.01$ and $\dot{\beta}_{-0}=0.0$. We show the scale factors a, b, and c in (a), the dilaton field ϕ in (b), the modulus field σ in (c), and $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ in (d), respectively. We find that I grows almost exponentially.

and

$$\omega^i=dx^i,$$

for the Bianchi type-I model and

$$\omega^1=-\sin x^3 dx^1+\sin x^1 \cos x^3 dx^2,$$

$$\omega^2=\cos x^3 dx^1+\sin x^1 \sin x^3 dx^2,$$

$$\omega^3=\cos x^1 dx^1+dx^3,$$

for the Bianchi type-IX model. In both models, we also introduce $p(t)$, $q(t)$, and $r(t)$ as

$$e^{p(t)}=e^{-\Omega+\beta_+ +\sqrt{3}\beta_-},\quad e^{q(t)}=e^{-\Omega+\beta_+ -\sqrt{3}\beta_-},$$

$$e^{r(t)}=e^{-\Omega-2\beta_+}. \tag{2.9}$$

We can calculate volume element V as follows [13]:

$$V=e^{p(t)}e^{q(t)}e^{r(t)}=e^{-3\Omega(t)}. \tag{2.10}$$

The basic equations obtained are divided into two groups.

(1) The dynamical equations for the metric p, q, and r, the dilaton field ϕ, and the modulus field σ:

$$(1+8\dot{r}\dot{f})(\ddot{q}+\dot{q}^2)+(1+8\dot{q}\dot{f})(\ddot{r}+\dot{r}^2)+(1+8\ddot{f})\left[\dot{q}\dot{r}+\frac{1}{2}U_1\right]+\frac{1}{4}\dot{\phi}^2+\frac{3}{4}\dot{\sigma}^2=0, \tag{2.11}$$

$$(1+8\dot{r}\dot{f})(\ddot{p}+\dot{p}^2)+(1+8\dot{p}\dot{f})(\ddot{r}+\dot{r}^2)+(1+8\ddot{f})\left[\dot{r}\dot{p}+\frac{1}{2}U_2\right]+\frac{1}{4}\dot{\phi}^2+\frac{3}{4}\dot{\sigma}^2=0, \tag{2.12}$$

$$(1+8\dot{q}\dot{f})(\ddot{p}+\dot{p}^2)+(1+8\dot{p}\dot{f})(\ddot{q}+\dot{q}^2)+(1+8\ddot{f})\left[\dot{p}\dot{q}+\frac{1}{2}U_3\right]+\frac{1}{4}\dot{\phi}^2+\frac{3}{4}\dot{\sigma}^2=0, \tag{2.13}$$

$$\ddot{\phi}+(\dot{p}+\dot{q}+\dot{r})\dot{\phi}=2\frac{\partial f}{\partial \phi}R_{GB}^2, \tag{2.14}$$

$$\ddot{\sigma}+(\dot{p}+\dot{q}+\dot{r})\dot{\sigma}=\frac{2}{3}\frac{\partial f}{\partial\sigma}R_{GB}^2, \tag{2.15}$$

where an overdot denotes a differentiation with respect to t, and the Gauss-Bonnet term R_{GB}^2 is given as

$$R_{GB}^2=8\left\{[\ddot{p}+\dot{p}^2]\left[\dot{q}\dot{r}+\frac{1}{2}U_1\right]+[\ddot{q}+\dot{q}^2]\left[\dot{r}\dot{p}+\frac{1}{2}U_2\right]+[\ddot{r}+\dot{r}^2]\left[\dot{p}\dot{q}+\frac{1}{2}U_3\right]+\frac{1}{2}\dot{p}^2\frac{\partial U_1}{\partial p}+\frac{1}{2}\dot{q}^2\frac{\partial U_2}{\partial q}+\frac{1}{2}\dot{r}^2\frac{\partial U_3}{\partial r}\right.$$
$$\left.-\frac{1}{2}\dot{p}\dot{q}\left[\frac{\partial U_1}{\partial p}+\frac{\partial U_2}{\partial q}-\frac{\partial U_3}{\partial r}\right]-\frac{1}{2}\dot{q}\dot{r}\left[\frac{\partial U_2}{\partial q}+\frac{\partial U_3}{\partial r}-\frac{\partial U_1}{\partial p}\right]-\frac{1}{2}\dot{r}\dot{p}\left[\frac{\partial U_3}{\partial r}+\frac{\partial U_1}{\partial p}-\frac{\partial U_2}{\partial q}\right]\right\}. \tag{2.16}$$

(2) The constraint equation

$$\dot{p}\dot{q}+\dot{q}\dot{r}+\dot{r}\dot{p}+24\dot{p}\dot{q}\dot{r}\dot{f}+4\dot{p}\dot{f}U_1+4\dot{q}\dot{f}U_2+4\dot{r}\dot{f}U_3-\frac{1}{4}\dot{\phi}^2$$
$$-\frac{3}{4}\dot{\sigma}^2+\frac{1}{2}(U_1+U_2+U_3)=0. \tag{2.17}$$

Here the functions U_1,U_2,U_3 are defined by (A) the Bianchi type-I model:

$$U_1=U_2=U_3=0 \tag{2.18}$$

(B) the Bianchi type-IX model:

$$U_1=e^{-2q}+e^{-2r}-e^{-2p}+\frac{1}{2}(e^{2(q-r-p)}+e^{2(r-p-q)}-3e^{2(p-q-r)}),$$

$$U_2=e^{-2r}+e^{-2p}-e^{-2q}+\frac{1}{2}(e^{2(r-p-q)}+e^{2(p-q-r)}-3e^{2(q-r-p)}),$$

$$U_3=e^{-2p}+e^{-2q}-e^{-2r}+\frac{1}{2}(e^{2(p-q-r)}+e^{2(q-r-p)}-3e^{2(r-p-q)}). \tag{2.19}$$

The basic equations above are the five second-order differential equations with one constraint equation. For $p(t)=q(t)=r(t)$, i.e., the case of the isotropic and homogeneous model, the equations are reduced to the cases of a flat universe studied by Antoniadis, Rizos, and Tamvakis [6], and of a close universe by Easther and Maeda [7].

III. NUMERICAL RESULTS

We have examined the case of $\delta<0$ because in the isotropic and homogeneous case, nonsingular cosmological solutions are found only for $\delta<0$ [6,7]. We solve the basic equations numerically.

Since five second-order derivatives $\ddot{p},\ddot{q},\ddot{r},\ddot{\phi},\ddot{\sigma}$ in the basic equations (2.11)–(2.15) are coupled, we have to make an inverse transformation as follows: Defining a vector $\boldsymbol{x}=(\ddot{p},\ddot{q},\ddot{r},\ddot{\phi},\ddot{\sigma})$, the basic equations (2.11)–(2.15) are written in the matrix form as

$$Z\boldsymbol{x}=\boldsymbol{y}, \tag{3.1}$$

where 5×5 matrix $Z=Z(p,q,r,\phi,\sigma,\dot{p},\dot{q},\dot{r},\dot{\phi},\dot{\sigma})$ and vector $\boldsymbol{y}=\boldsymbol{y}(p,q,r,\phi,\sigma,\dot{p},\dot{q},\dot{r},\dot{\phi},\dot{\sigma})$ are known explicitly from the basic equations.

Unless $\Delta\equiv\det Z$ vanishes, we have

$$\boldsymbol{x}=Z^{-1}\boldsymbol{y}, \tag{3.2}$$

then we can solve the basic equations (3.2), with the initial data of $p,q,r,\phi,\sigma,\dot{p},\dot{q},\dot{r},\dot{\phi},\dot{\sigma}$. However, when Δ vanishes, then we cannot proceed further with our numerical calculations. Such an end point, which may appear in the evolution of the Universe, seems to be a spacetime singularity, however, more detailed analysis will be required as we will show later.

Although we have used scale factors p, q, and r for the basic equations, we shall describe our results by two anisotropy variables β_+ and β_-.

A. Bianchi type-I case

First we will show the results in the Bianchi type-I model. We choose $\bar{\delta}=-48/\pi$. We introduce scale factors as

$$a(t)=e^{p(t)}=e^{-\Omega+\beta_++\sqrt{3}\beta_-},$$

$$b(t)=e^{q(t)}=e^{-\Omega+\beta_+-\sqrt{3}\beta_-},$$

$$c(t)=e^{r(t)}=e^{-\Omega-2\beta_+}. \tag{3.3}$$

Without a loss of generality, we can set $\Omega_0=\beta_\pm=0$, i.e., $a_0=b_0=c_0=1$. Here the subscript 0 denotes the initial value of the variables. We set the initial time $t=t_0=0$. We have to give the initial data of $\dot{\Omega}_0$, $\dot{\beta}_{\pm0}$, ϕ_0, $\dot{\phi}_0$, σ_0, and $\dot{\sigma}_0$, which must satisfy one constraint equation (2.17). Then we have five independent initial values. Since we are interested in whether or not nonsingular solutions found in the isotropic case are generic, we shall set up the initial data in the anisotropic case around the isotropic ones.

In the isotropic and homogeneous case, we find nonsingular solutions for some finite parameter range of initial data.

In Fig. 1, setting $\dot{\Omega}_0 = -0.1$, $\sigma_0 = 0$ we show the range of initial data of ϕ_0 and $\dot{\phi}_0$, which gives for nonsingular solutions (shown by a circle). $\dot{\sigma}_0$ is fixed by the constraint equation (2.17). If $\dot{\phi}_0$ is efficiently small, then in the case of $\phi_0 < 0$ there always exist nonsingular solutions. This is because if $\phi < 0$ and $|\phi| \gg 1$, $e^{\phi} \approx 0$ and then singularity avoidance is almost independent of ϕ.

To give initial data for anisotropic spacetime giving the same value of $\dot{\Omega}_0$ and σ_0, and changing some values of ϕ_0, $\dot{\phi}_0$ in Fig. 1, we include anisotropy, i.e., $\dot{\beta}_{+0}, \dot{\beta}_{-0} (\neq 0)$, and solve the constraint for $\dot{\sigma}_0$.

We find nonsingular solutions in the Bianchi type-I model near the isotropic nonsingular solution. We show one example in Fig. 2, where we have set $\dot{\Omega}_0 = -0.1$, $\phi_0 = \dot{\phi}_0 = 0$, $\sigma_0 = 0$, and $\dot{\beta}_{+0} = 0.05$, $\dot{\beta}_{-0} = 0.0$, $\dot{\sigma}_0 = 0.173205$. Note that $\dot{\sigma}_0 = 0.200$ for the isotropic case. We also show a singular solution in Fig. 3. We have set $\dot{\Omega}_0 = -0.1$, $\phi_0 = \dot{\phi}_0 = 0$, $\sigma_0 = 0$, and $\dot{\beta}_{+0} = 0.1$, $\dot{\beta}_{-0} = 0.0$. $\dot{\sigma}_0 = 0.00$ has been determined by the constraint equation ($\dot{\sigma}_0 = 0.200$ for the isotro-

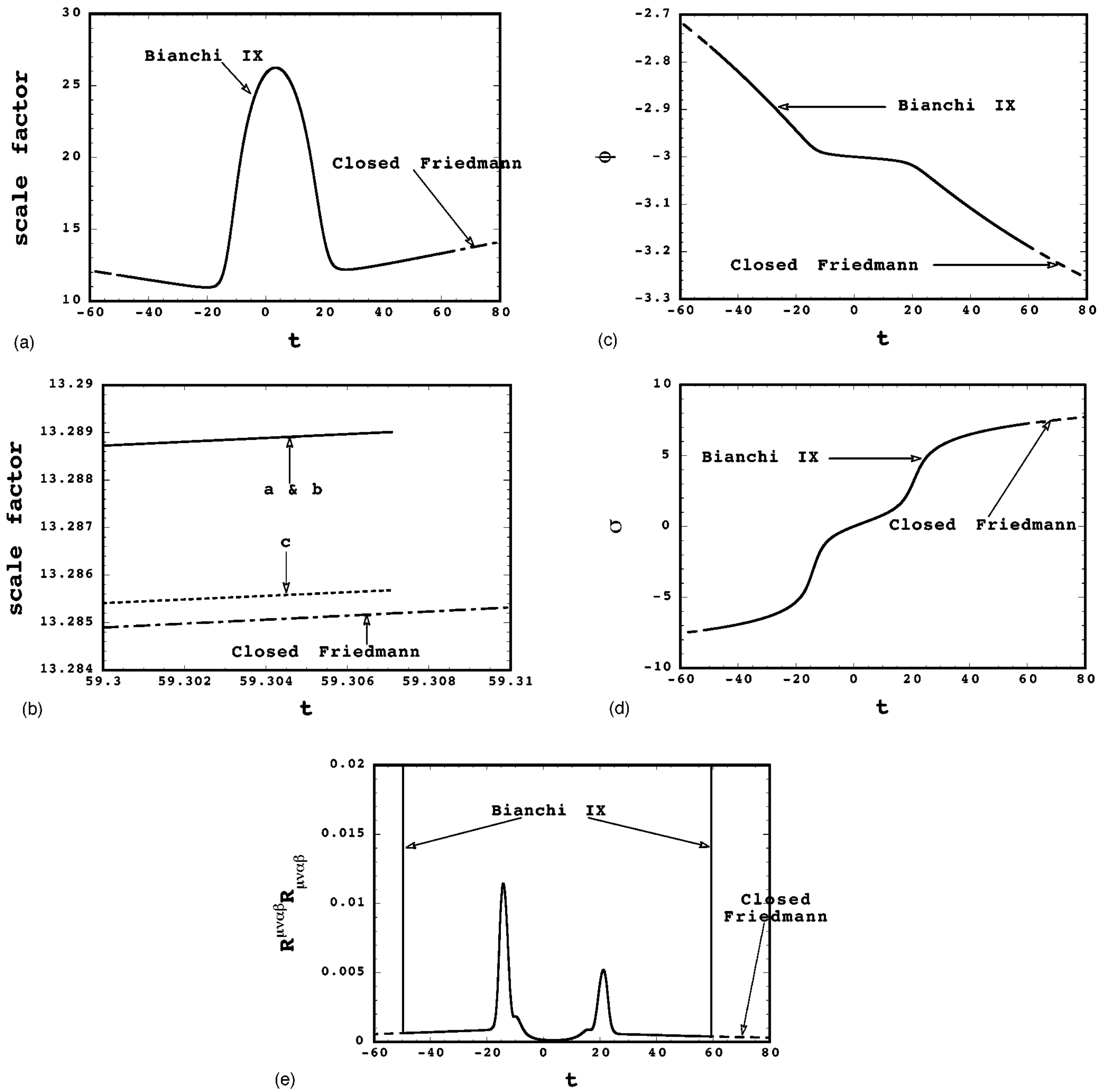

FIG. 7. We show a singular solution in Bianchi type-IX model. We set the same initial values as those in Fig. 6, except for anisotropy, i.e., $\dot{\beta}_{+0} = 10^{-14}$ and $\dot{\beta}_{-0} = 0$. We show scale factors a, b, and c in (a) and (b), dilaton field ϕ in (c), modulus field σ in (d), and $I = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ in (e), respectively. We find that I grows almost exponentially.

pic case). The volume e^{Ω} vanishes both at $t\approx-4.98$ and $t\approx8.35$, resulting in a big-bang-type singularity.

Changing the anisotropic parameters $\dot{\beta}_{+0}$, $\dot{\beta}_{-0}$, we search for nonsingular solutions in order to find how large anisotropy is possible (to allow nonsingular solutions). The result is shown in Fig. 4 for $\dot{\Omega}_0=-0.1$, $\phi_0=\dot{\phi}_0=0$, $\sigma_0=0$. We find that if anisotropy is large enough at the initial stage, then the spacetime evolves into a singularity. The boundary between nonsingular and singular solutions in the $(\dot{\beta}_{+0},\dot{\beta}_{-0})$ plane is almost a circle [the deviation of the magnitude of a shear σ, which $\propto(\dot{\beta}_{+2}+\dot{\beta}_{-2})$, is within 10^{-2}]. Whether spacetime will evolve into a singularity or avoid it, of course, strongly depends on the initial parameters $(\dot{\Omega}_0,\ \phi_0,\ \dot{\phi}_0,\ \sigma_0)$. As we shift these initial parameters to the critical values in the isotropic case beyond which no nonsingular solution is found, the radius of the boundary in the $(\dot{\beta}_{+0},\dot{\beta}_{-0})$ plane decreases, and if we set the initial parameters fairly close to the boundary values, then the range for nonsingular solutions eventually disappears. Then in this case, the singularity avoidance no longer works.

We can conclude that for Bianchi type-I anisotropy, singularity avoidance is still generic [14]. Because the range of nonsingular solutions is finite and not small.

However, we have the stability analysis done by Kawai, Sakagami, and Soda [15–17]. Their analysis is based on the same action (but only a modulus field) and the flat Friedmann background case. They found that there exists instability in the tensor mode. Then the flat Friedmann model is unstable. This result may not change even in a Bianchi type-I background universe.

B. Bianchi type-IX case

In the same way as the Bianchi type-I case, we analyze the Bianchi type-IX model . We introduce scale factors as

$$a(t)=2e^{p(t)},\quad b(t)=2e^{q(t)},\quad c=2e^{r(t)}. \tag{3.4}$$

The factor comes from definitions with invariant basis ω^i and the scale factor in a closed Friedmann model. Similar to the Bianchi type-I model, we first search for nonsingular closed Friedmann solutions, and then search for nonsingular Bianchi type-IX solutions, by setting up the initial anisotropic data around those in a closed nonsingular solution.

We show an isotropic nonsingular solution in Fig. 5. We choose a negative value of $\overline{\delta}$ as $\overline{\delta}=-48/\pi$, and set $a(0)=b(0)=c(0)=e^{3.2511380746580628}$, $\dot{\Omega}_0=-0.01$, $\phi_0=-3$, $\dot{\phi}_0=-5\times10^{-4}$ and $\sigma_0=0$, which initial values were found by Easther and Maeda. An anisotropy is added just as the same as the Bianchi type-I model. We show our numerical results in Figs. 6 and 7. In both cases, we take the same initial values as the isotropic case except for anisotropy (β_{+0}, and β_{-0}, and $\dot{\sigma}_0$, which is determined by a constraint equation). As for anisotropy we set $\dot{\beta}_{+0}=0.01$ and $\dot{\beta}_{-0}=0.0$ for Fig. 6, and $\dot{\beta}_{+0}=10^{-14}$ and $\dot{\beta}_{-0}=0$ for Fig. 7, respectively.

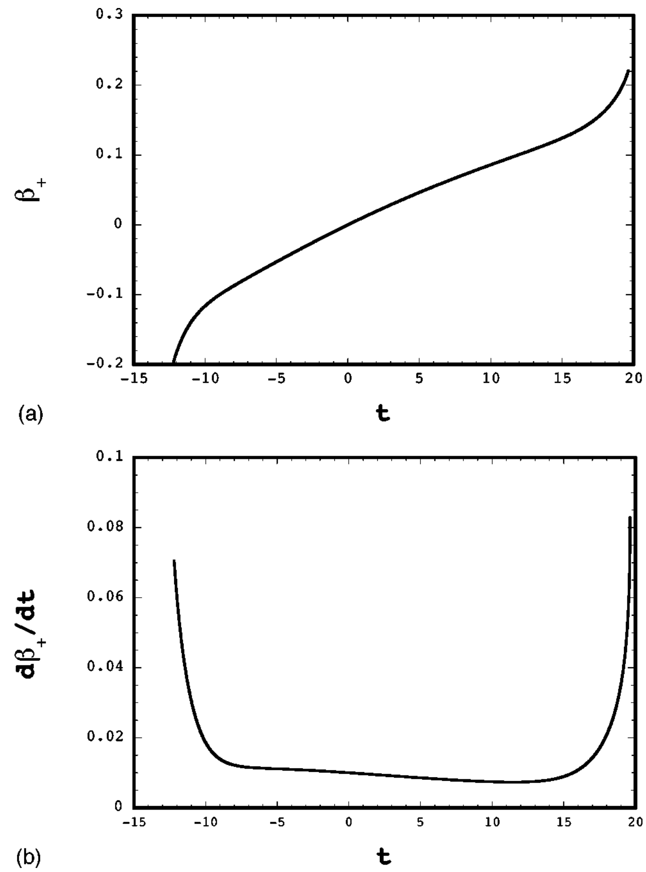

FIG. 8. We show the time evolution of β_+ and $\dot{\beta}_+$. We set the initial parameters as those in Fig. 6. The anisotropy β_+ also grows exponentially, as I diverges.

In our numerical calculation, $\Delta(=\det Z)$ eventually vanishes during the evolution of the universe. We always find a singularity at a certain finite time, so our numerical analysis has to break down. This type of breakdown appears even if an initial anisotropy is quite small, that is, both $\dot{\beta}_{+0}$ and $\dot{\beta}_{-0}$ are less than 10^{-14}. Is this singularity just a numerical divergence, or an inappropriate gauge choice? To answer for this question, we show gauge-invariant variables.

In Figs. 6 and 7, we show $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, in which suddenly diverges near the breakdown point. We show the time evolution of β_+ and $\dot{\beta}_+$ in Figs. 8 and 9, in which initial values are the same as Figs. 6 and 7, respectively. β_+, β_- or both will also increase very rapidly near the breakdown point. Because of the exponential growth of a curvature invariant $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and a shear σ $[\sigma\propto(\dot{\beta}_+^2+\dot{\beta}_-^2)]$, we believe that it is a curvature singularity. Since the volume factor e^{Ω} does not vanish there, it is not a big-bang-type singularity. The Universe finds a singularity with a finite volume [18].

We have searched the wide range of parameters where Easther and Maeda found nonsingular closed universe solutions, but we could not find any nonsingular solution even if the anisotropy was extremely small. In all cases we have examined $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and σ which always show a sharp increase near the point where Δ vanishes. Then we con-

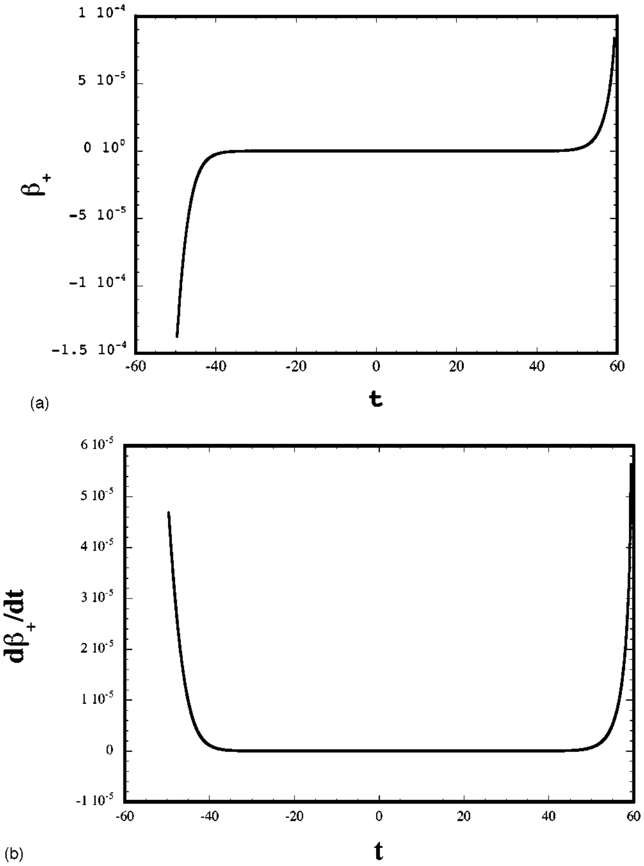

FIG. 9. We show the time evolution of β_+ and $\dot{\beta}_+$ for the solution given in Fig. 7. The anisotropy β_+ also grows exponentially, as I diverges.

cluded that there is no nonsingular solution in anisotropic Bianchi type-IX type models and all solutions except for the exact isotropic case eventually evolve into a curvature singularity.

Our results may be consistent with the stability analysis by Kawai, Sakagami, and Soda [15–17]. King showed that the Bianchi type-IX type model can be regarded as a closed Friedmann background with a single gravitational wave of a fixed wave number $k=\sqrt{6}/S$, where S is the radius of the three sphere [19]. This means that Bianchi type-IX model is the same as the closed Universe with a nonlinear tensor perturbation. Then, if the tensor mode is unstable even for a closed universe we can understand our results. From the wave number k, we can estimate the as the timescale of the tensor-mode instability from their analysis and compare it with our numerical results. We find that our breakdown time is the same order of instability as the timescale, i.e., the order of our breakdown timescale is almost $(0.1\sim 1)\sqrt{|\delta|}$, which depends on initial parameters, while the instability timescale is $\sim 1\sqrt{|\delta|}$.

IV. CONCLUSION REMARKS AND DISCUSSIONS

In this paper we have examined the generality of previously found cosmological nonsingular solutions in the heterotic superstring effective action in orbifold compactifications with one-loop correction.

In the Bianchi type-I case, many nonsingular solutions are found around nonsingular flat Friedmann solutions without fine-tuned initial conditions. On the other hand, in the Bianchi type-IX case, we cannot find any nonsingular solution even if anisotropy is quite small. In this case, anisotropy grows, and our numerical analysis breaks down at a certain point where anisotropy will diverge. At this point, both $\dot{\beta}_\pm$, which represents the anisotropy, and $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ seems to diverge, then the universe may evolve into a curvature singularity. The volume factor remains a finite value, which suggests that it is not a big-bang-type singularity.

Our result is consistent with the stability analysis by Kawai, Sakagami, and Soda, who found tensor mode instability in a flat Friedmann background. The Bianchi type-IX model can be regarded as a closed Friedmann background with a single gravitational wave of a fixed wave number.

It is also expected that nonsingular Bianchi type-I models are not generic as well, since the nonsingular flat Friedmann models are unstable against tensor perturbations. Therefore, we may conclude that the nonsingular universes found in the isotropic cases are not generic and singularity avoidance may not work even in the present model.

We have argued only a first-order expansion term of inverse string tension α' and one-loop correction and ignored the antisymmetric tensor (an axion field) which might exist in the early universe. For more rigid consideration we may need to further research a full superstring theory, though such a full theory has not yet been developed to the point where we can deal with cosmology. However, we hope to find a new effect of a superstring theory by which the nonsingular universe becomes generic. It might be given by higher curvature terms in the effective action.

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